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Iterative Methods for Looped Network Pipeline Calculation

Dejan Brkić

Received: 14 December 2009 / Accepted: 3 February 2011 /
Published online: 17 March 2011
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Abstract Since the value of the hydraulic resistance depends on flow rate, problem of flow distribution per pipes in a gas or water distributive looped pipelines has to be solved using iterative procedure. A number of iterative methods for determining of hydraulic solution of pipeline networks, such as, Hardy Cross, Modified Hardy Cross, Node-Loop method, Modified Node method and M.M. Andrijašev method are shown in this paper. Convergence properties are compared and discussed using a simple network with three loops. In a municipal gas pipeline, natural gas can be treated as incompressible fluid. Even under this circumstance, calculation of water pipelines cannot be literary copied and applied for calculation of gas pipelines. Some differences in calculations of networks for distribution of these two fluids, i.e. water apropos natural gas are also noted.

Keywords Pipeline networks · Water distribution system · Natural gas distribution system · Calculation methods · Flow · Hydraulic pipeline systems

1 Introduction

A number of iterative methods for determining the hydraulic solution of water and natural gas pipeline networks which take ring-like form, such as, Hardy Cross, Modified Hardy Cross (including Andrijašev method), Node-Loop method and Modified Node method are compared in this paper. All presented methods assume equilibrium among pressure and friction forces in steady and incompressible flow. As a result, they cannot be successfully used in unsteady and compressible flow calculations with large pressure drops where inertia force is important. Problem of flow in pipes and open conduits was one which had been of considerable interest to engineers for nearly 250 years. Even today, this problem is not solved definitively.

D. Brkić (✉)
Ministry of Science and Technological Development, Strumička 88, 11050 Beograd, Serbia
e-mail: dejanrgf@tesla.rcub.bg.ac.rs

The difficulty to solve the turbulent flow problems lies in the fact that the friction factor is a complex function of relative surface roughness and Reynolds number. Precisely, hydraulic resistance depends on flow rate and hence flow problem in hydraulic networks has to be solved iteratively. Similar situation is with electric current when diode is in circuit. With common resistors in electrical circuits where the electric resistances do not depend on the value of electric current in the conduit, problem is linear and no iterative procedure has to be used. So problem of flow through a single tube is already complex. Despite of it, very efficient procedures are available for solution of flow problem in a complex pipeline such as looped pipeline like waterworks or natural gas distribution network is. Most of the shown methods are based on solution of the loop equations while the node equations are used only as control of accuracy. Node-Loop method is also based on solution of the loop equations but the node equations are also used in calculation and not only for control purposes. Node method and here shown Modified Node method is based on solution of the node equations while the loop equations is used for control purposes.

In this paper, speed of convergence of presented methods are compared and discussed. This is done for one simple network with three loops, both, for water and gas distribution networks. Similar example can be done for the air ventilation systems in the buildings or mines. For ventilation problem readers can consult paper of Aynsley (1997), Mathews and Köhler (1995), Wang and Hartman (1967), etc. While the disposal of water or gas distribution system is in single plane with certain elevation of pipes, ventilation network is almost always spatial (Brkić 2009).

2 Hydraulics Frictions and Flow Rates in Pipes

Each pipe is connected to two nodes at its ends. In a pipe network system, pipes are the channels used to convey fluid from one location to another. Pipes are sometimes referred to as tubes or conduits, and as a part of network to as lines, edges, arcs or branches (in the graph theory, special kinds of branches are referred to as trees). The physical characteristics of a pipe include the length, inside diameter, roughness, and minor loss coefficient. When fluid is conveyed through the pipe, hydraulic energy is lost due to the friction between the moving fluid and the stationary pipe surface. This friction loss is a major energy loss in pipe flow (Farshad et al. 2001; Kumar et al. 2010).

Various equations were proposed to determinate the head losses due to friction, including the Darcy-Weisbach, Fanning, Chezy, Manning, Hazen-Williams and Scobey formulas. These equations relate the friction losses to physical characteristics of the pipe and various flow parameters. The Darcy-Weisbach formula for calculating of friction loss is more accurate than the Hazen-Williams. Beside this, the Hazen-Williams relation is only for water flow.

2.1 Gas Flow Rates and Pressure Drops in Pipes

When a gas is forced to flow through pipes it expands to a lower pressure and changes its density. Flow-rate, i.e. pressure drop equations for condition in gas distribution networks assumes a constant density of a fluid within the pipes. This assumption applies only to incompressible, i.e. for liquids flows such as in water

distribution systems for municipalities (or any other liquid, like crude oil, etc.). For the small pressure drops in typical gas distribution networks, gas density can be treated as constant, which means that gas can be treated as incompressible fluid (Pretorius et al. 2008), but not as liquid flow. Assumption of gas incompressibility means that it is compressed and forced to convey through pipes, but inside the pipeline system pressure drop of already compressed gas is small and hence further changes in gas density can be neglected. Fact is that gas is actually compressed and hence that volume of gas is decreased and then such compressed volume of gas is conveying with constant density through gas distribution pipeline. So, mass of gas is constant, but volume is decreased while gas density is according to this, increased. Operate pressure for distribution gas network is $4 \cdot 10^5$ Pa abs i.e. $3 \cdot 10^5$ Pa gauge and accordingly volume of gas is decreased four times compared to volume of gas at normal conditions.

Inner surface of polyethylene pipes which are almost always used in gas distribution networks are practically smooth and hence flow regime in the typical network is hydraulically ‘smooth’ (Sukharev et al. 2005). For this regime is suitable Renouard’s equation adjusted for natural gas flow (1):

$$F_g = C = p_1^2 - p_2^2 = \frac{4810 \cdot Q_n^{1.82} \cdot L \cdot \rho_r}{D_{in}^{4.82}} \quad (1)$$

Regarding to Renouard’s formula has to be careful since it does not relate pressure drop but actually difference of the quadratic pressure at the input and the output of pipe. This means that \sqrt{C} is not actually pressure drop in spite of the same unit of measurement, i.e. same unit is used for the pressure (Pa). For gas pipeline calculation is very useful fact that when $\sqrt{C} \rightarrow 0$ this consecutive means that also $C \rightarrow 0$. Parameter \sqrt{C} can be noted as pseudo-pressure drop.

2.2 Liquid Flow in Pipes

In Renoard’s equation adjusted for gas pipelines (1) friction factor is rearranged in the way to be expressed using other flow parameters and also using some thermodynamic properties of natural gas. Using formulation for Darcy friction factor in hydraulically smooth region Renouard suggests his equation for liquid flow (2):

$$\lambda = \frac{0.172}{Re^{0.18}} \quad (2)$$

Then pressure drop (Ekinici and Konak 2009) can be found very easily using Darcy-Weisbach Eq. 3:

$$F_w = \Delta p = p_1 - p_2 = \lambda \cdot \frac{L}{D_{in}^5} \cdot \frac{8 \cdot Q^2}{\pi^2} \cdot \rho \quad (3)$$

Renouard’s Eq. 2 is based on power law. Liquid flow is better fit using some kind of logarithmic formula like Colebrook’s (4):

$$\frac{1}{\sqrt{\lambda}} = -2 \cdot \log \left(\frac{2.51}{Re \cdot \sqrt{\lambda}} + \frac{\varepsilon}{3.71 \cdot D_{in}} \right) \quad (4)$$

Colebrook’s equation is suitable especially for flow through steel pipes (Colebrook 1939). Some researchers adopt a modification of the Colebrook equation, using the

2.825 constant instead of 2.51 especially for gas flow calculation (Haaland 1983; Coelho and Pinho 2007).

In the case of waterworks, pressures will also be expressed in Pa, not in meter (m) equivalents. In that way, clear comparison with gas distribution network can be done. Pressures expressed in Pa can be very easily recalculated in heads expressed in meters (m) for water networks knowing water density. Example network is located in a single-flat area with no variation in elevation (otherwise the correction term must be used which is significant for water network).

3 Looped Pipeline Networks for Distribution of Fluids

A pipeline network is a collection of elements such as pipes, compressors, pumps, valves, regulators, tanks, and reservoirs interconnected in a specific way. In this article focus is on pipes. The behavior of the network is governed by the specific characteristics of the elements and how the elements are connected together. Our assumption is that pipes are connected in a smooth way, i.e. so called minor hydraulic losses are neglected. Including other elements different than pipes is a subject of sufficient diversity and complexity to merit a separate review.

The analysis of looped pipeline systems by formal algebraic procedures is very difficult if the systems are complex. Electrical models had been used in study of this problem in the time before advanced computer became available as background to support demandable numerical procedures (Mah and Shacham 1978). Here presented methods use successive corrections or better to say these methods require iterative procedure. The convergence properties of presented methods are the main subject of this article. All of such methods can be divided into two groups (1) Methods based on solution of the loop equations, and (2) Methods based on solution of the node equations.

Most of the methods used commonly in engineering practice belong to the group based on solution of the loop equations. Such method presented in this paper is Hardy Cross method (Cross 1936). Method of Andrijašev is variation of Hardy Cross method (Andrijašev 1964).¹ Contemporary with Hardy Cross, soviet author V.G. Lobačev (Latišnikov and Lobačev 1956) developed very similar method compared to original Hardy Cross method. Modified Hardy Cross method proposed Epp and Fowler (1970) which considers entire system simultaneously is also sort of loop method. Node-Loop method proposed by Wood and Charles (1972) and later improved by Wood and Rayes (1981) is combination of the loop and node oriented methods, but despite of its name is essentially belong to the group of loop methods. Node-Loop method is based on solution of the loop equations while the node equations are also involved in calculation. Only Node method proposed by Shamir and Howard (1968) is real representative of node oriented methods. Node method uses idea of Hardy Cross but to solve node equations instead of loops ones. Improved Node method uses similar idea as Epp and Fowler (1970) suggested for

¹There are some difficulties with citation of Russian methods. Both methods, Lobačev and Andrijašev, are actually from 1930s but original papers from Soviet time are problem to be found. Many books in Russian language explain methods of Lobačev and Andrijašev but in these books reference list is not available.

the improvement of original Hardy Cross method, but of course here applied to the Node method.

Example network with three loops is shown in the Fig. 1. Before calculation, maximal consumption for each node including one of more inlet nodes has to be determined. Pipe diameters and node inputs and outputs cannot be changed during the iterative procedure. Goal is to find final flow distribution for this pipeline system (simulation problem).

Pipe lengths and pipes diameters are listed in Fig. 1. Final flows do not depend on first assumed fluid flows per pipes in the case of loop oriented methods or on first assumed pressure drops or pseudo-pressure drops in the case of node oriented methods. To have a better appreciation of the utility of these representations, first will be considered the laws that govern flow rates and pressure drops in a pipeline network. These are the counterparts to Kirchhoff's laws for electrical circuits and accordingly for hydraulic networks, namely, (1) the algebraic sum of flows at each node must be zero; and (2) the algebraic sum of pressure drops for water network (i.e. pseudo-pressure drop for gas network) around any cyclic path (loop) must be approximately zero. For a hydraulic network, fact that both laws are satisfied simultaneously means that calculation for steady state for simulation problem is finished.

3.1 Methods Based on Solution of the Loop Equations

Loops are sometimes referred also to as contours or paths. Note that contours and loops are not synonyms in the M.M. Andrijašev method.

For the loop oriented methods first Kirchhoff's law must be satisfied for all nodes in all iterations. Second Kirchhoff's law for each loop must be satisfied with

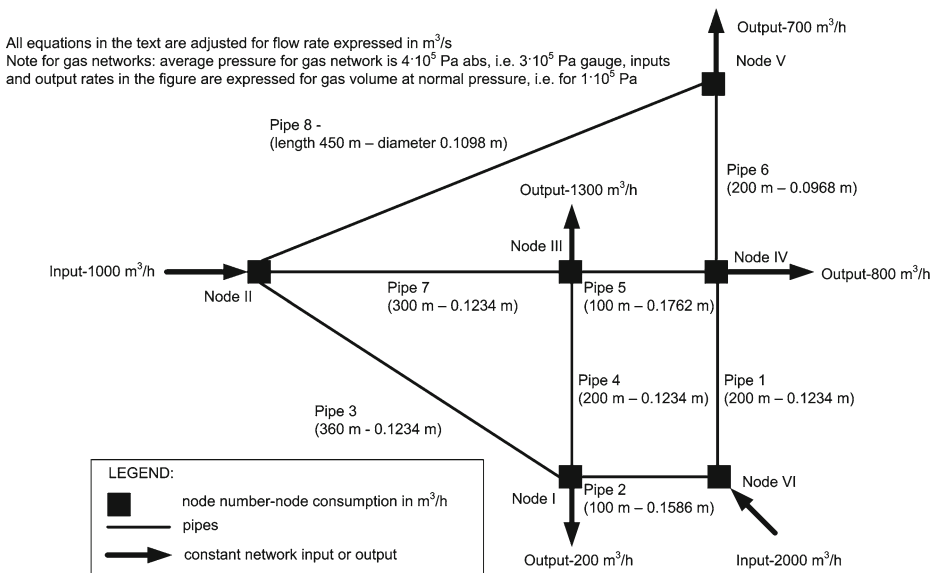


Fig. 1 Example of pipeline network

acceptable tolerance at the end of the calculation. Or, in other words, final flows are these ones which values are not changed between two successive iterations (must be satisfied for flow in each pipe).

In a loop oriented methods, first initial flow pattern must be chosen to satisfied first Kirchhoff's law. Endless number of flow combinations can satisfy this condition. Someone can conclude that it is crucial to start with good initial guesses. But, how does one obtain good initial guesses? And how sensitive are the methods to the initial guesses? Answer is that initial flow pattern does not have significant influence on convergence properties of observed method. These indicate that the choice of initial flows is not critical and need to be chosen only to satisfied first Kirchhoff's law for all nodes (Gay and Middleton 1971). It would be equally satisfactory to generate the initial distribution within the computer program. Initial flow pattern for our example network is shown in the Fig. 2. Loops are also defined in the Fig. 2.

Results of calculation using original Hardy Cross, Modified Hardy Cross, M.M. Andrijašev method are not actually flows Q but correction of flows ΔQ calculated for each loop. These corrections have to be added algebraically to flows from previous iteration for each pipe according to specific rules. A pipe common to two loops receives two corrections. The upper plus or minus sign (shown in Tables 1, 2, 3, 4, 5, 6, 7 and 8) indicates direction of flow in that conduit in these two loops and is obtained from Q for previous iteration. The upper sign is the same as the sign in front of Q if the flow direction in each loop coincides with the assumed flow direction in the particular loop under consideration, and opposite if it does not. The lower sign is copied from the primary loop for this correction (sign from the loop where this

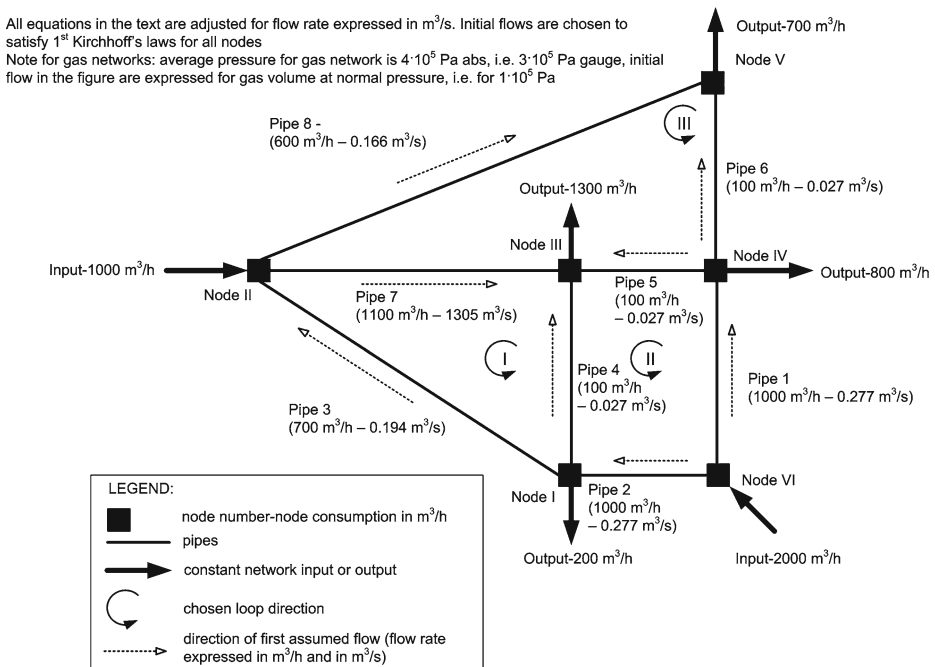


Fig. 2 Example network initial parameters prepared for loop oriented calculation

Table 1 The Hardy Cross calculation for example gas network

Loop	Pipe	Iteration 1				
		Q^a	$C = p_1^2 - p_2^{2b}$	$F' = \left \frac{\partial C(Q)}{\partial Q} \right $	$\Delta Q_1 = \frac{F^c}{F'}$	ΔQ_2^d
I	3	$-19,444 \cdot 10^{-5}$	-1,264,933,339	11,839,776,055	$+13,268 \cdot 10^{-5}$	-
	4	$+2,778 \cdot 10^{-5}$	+20,357,137	1,333,799,622	$+13,268 \cdot 10^{-5}$	$+9,722 \cdot 10^{-5} \mp$
	7	$-30,556 \cdot 10^{-5}$	-2,399,620,963	14,293,015,047	$+13,268 \cdot 10^{-5}$	$+881 \cdot 10^{-5} \ddagger$
II	Σ		$F_I = -3,644,197,165$	27,466,590,725		
	1	$+27,778 \cdot 10^{-5}$	+1,344,982,709	8,812,326,713	$-9,722 \cdot 10^{-5}$	-
	2	$-27,778 \cdot 10^{-5}$	-200,615,476	1,314,432,601	$-9,722 \cdot 10^{-5}$	-
	4	$-2,778 \cdot 10^{-5}$	-20,357,137	1,333,799,622	$-9,722 \cdot 10^{-5}$	$-13,268 \cdot 10^{-5} \pm$
	5	$+2,778 \cdot 10^{-5}$	+1,828,425	119,798,452	$-9,722 \cdot 10^{-5}$	$+881 \cdot 10^{-5} \mp$
III	Σ		$F_{II} = +1,125,838,521$	11,580,357,388		
	5	$-2,778 \cdot 10^{-5}$	-1,828,425	119,798,452	$-881 \cdot 10^{-5}$	$+9,722 \cdot 10^{-5} \ddagger$
	6	$+2,778 \cdot 10^{-5}$	+65,604,940	4,298,435,730	$-881 \cdot 10^{-5}$	-
	7	$+30,556 \cdot 10^{-5}$	+2,399,620,963	14,293,015,047	$-881 \cdot 10^{-5}$	$-13,268 \cdot 10^{-5} =$
	8	$-16,667 \cdot 10^{-5}$	-2,096,864,105	22,897,756,035	$-881 \cdot 10^{-5}$	-
	Σ		$F_{III} = +366,533,372$	41,609,005,264		
					$\Delta Q_1 = Q^e$	
						$-6,177 \cdot 10^{-5}$
						$+25,767 \cdot 10^{-5}$
						$-16,407 \cdot 10^{-5}$
						$+18,056 \cdot 10^{-5}$
						$-37,500 \cdot 10^{-5}$
						$-25,767 \cdot 10^{-5}$
						$-6,063 \cdot 10^{-5f}$
						$+6,063 \cdot 10^{-5f}$
						$+1,897 \cdot 10^{-5}$
						$+16,407 \cdot 10^{-5}$
						$-17,548 \cdot 10^{-5}$

Table 1 (continued)

Loop	Pipe	Iteration 2	$C = p_1^2 - p_2^b$	$F' = \left \frac{\partial C(Q)}{\partial Q} \right $	$\Delta Q_1 = \frac{F^c}{F'}$	ΔQ_2^d	$Q_2 = Q$
I	3	$-6,177 \cdot 10^{-5}$	$-156,904,917$	4,623,293,467	$-1,128 \cdot 10^{-5}$	–	$-7,305 \cdot 10^{-5}$
	4	$+25,767 \cdot 10^{-5}$	$+1,173,109,335$	8,285,863,414	$-1,128 \cdot 10^{-5}$	$-5,572 \cdot 10^{-5} =$	$+19,067 \cdot 10^{-5}$
	7	$-16,407 \cdot 10^{-5}$	$-773,797,561$	8,583,648,143	$-1,128 \cdot 10^{-5}$	$-4,154 \cdot 10^{-5} \pm$	$-21,689 \cdot 10^{-5}$
II			$F_I = +242,406,855$	21,492,805,024			
	1	$+18,056 \cdot 10^{-5}$	$+614,087,396$	6,189,913,452	$+5,572 \cdot 10^{-5}$	–	$+23,628 \cdot 10^{-5}$
	2	$-37,500 \cdot 10^{-5}$	$-346,390,930$	1,681,162,091	$+5,572 \cdot 10^{-5}$	–	$-31,927 \cdot 10^{-5}$
	4	$-25,767 \cdot 10^{-5}$	$-1,173,109,335$	8,285,863,414	$+5,572 \cdot 10^{-5}$	$+1,128 \cdot 10^{-5} \ddagger$	$-19,067 \cdot 10^{-5}$
	5	$-6,063 \cdot 10^{-5f}$	$-7,569,671$	227,216,622.4	$+5,572 \cdot 10^{-5}$	$-4,154 \cdot 10^{-5} \pm$	$-4,645 \cdot 10^{-5}$
III			$F_{II} = -912,982,541$	16,384,155,579			
	5	$+6,063 \cdot 10^{-5f}$	$+7,569,671$	227,216,622	$+4,154 \cdot 10^{-5}$	$-5,572 \cdot 10^{-5} =$	$+4,645 \cdot 10^{-5}$
	6	$+1,897 \cdot 10^{-5}$	$+32,767,180$	3,143,915,857	$+4,154 \cdot 10^{-5}$	–	$+6,051 \cdot 10^{-5}$
	7	$+16,407 \cdot 10^{-5}$	$+773,797,561$	8,583,648,143	$+4,154 \cdot 10^{-5}$	$+1,128 \cdot 10^{-5} \mp$	$+21,689 \cdot 10^{-5}$
	8	$-17,548 \cdot 10^{-5}$	$-2,302,927,589$	23,885,524,961	$+4,154 \cdot 10^{-5}$	–	$-13,394 \cdot 10^{-5}$
			$F_{III} = -1,488,793,175$	35,840,305,583			

^aPipe lengths and diameters are shown in Fig. 1 and initial flow pattern in Fig. 2; six iterations are enough to achieve results shown in Table 9 (this is for $\Sigma F \rightarrow 0$)

^bThis is F calculated using (1)

^cAlso using (9)

^d ΔQ_2 is ΔQ_1 from adjacent loop

^eFinal calculated flow in the first iteration is used for the calculation in the second iteration

^fOpposite flow direction than in previous iteration

Table 2 The Hardy Cross calculation for example water network

Loop	Pipe	Iteration 1				
		Q^a	$\Delta p = p_1 - p_2^b$	$F' = \left \frac{\partial \Delta p(Q)}{\partial Q} \right $	$\Delta Q_1 = \frac{F^c}{F'}$	$Q_1 = Q^e$
I	3	$-19,444 \cdot 10^{-5}$	-5,279,095	54,299,272	$+12,306 \cdot 10^{-5}$	$-7,138 \cdot 10^{-5}$
	4	$+2,778 \cdot 10^{-5}$	+69,146	4,978,573	$+12,306 \cdot 10^{-5}$	$+24,433 \cdot 10^{-5}$
	7	$-30,556 \cdot 10^{-5}$	-10,718,549	70,157,780	$+12,306 \cdot 10^{-5}$	$-17,181 \cdot 10^{-5}$
II	Σ		$F_I = -15,928,498$	129,435,625		
	1	$+27,778 \cdot 10^{-5}$	+5,919,850	42,622,924	$-9,349 \cdot 10^{-5}$	$+18,429 \cdot 10^{-5}$
	2	$-27,778 \cdot 10^{-5}$	-816,585	5,879,413	$-9,349 \cdot 10^{-5}$	$-37,127 \cdot 10^{-5}$
	4	$-2,778 \cdot 10^{-5}$	-69,146	4,978,573	$-9,349 \cdot 10^{-5}$	$-24,433 \cdot 10^{-5}$
	5	$+2,778 \cdot 10^{-5}$	+5,967	429,677	$-9,349 \cdot 10^{-5}$	$-5,503 \cdot 10^{-5f}$
III	Σ		$F_{II} = +5,040,086$	53,910,587		
	5	$-2,778 \cdot 10^{-5}$	-5,967	429,677	$-1,068 \cdot 10^{-5}$	$+5,503 \cdot 10^{-5f}$
	6	$+2,778 \cdot 10^{-5}$	+232,186	16,717,415	$-1,068 \cdot 10^{-5}$	$+1,709 \cdot 10^{-5}$
	7	$+30,556 \cdot 10^{-5}$	+10,718,549	70,157,780	$-1,068 \cdot 10^{-5}$	$+17,181 \cdot 10^{-5}$
	8	$-16,667 \cdot 10^{-5}$	-8,874,257	106,491,089	$-1,068 \cdot 10^{-5}$	$-17,735 \cdot 10^{-5}$
	Σ		$F_{III} = +2,070,510$	193,795,963		

Table 2 (continued)

Loop	Pipe	Iteration 2	$\Delta p = p_I - p_2^b$	$F' = \left \frac{\partial \Delta p(Q)}{\partial Q} \right $	$\Delta Q_1 = \frac{F^c}{F'}$	ΔQ_2^d	$Q_2 = Q$
I	3	$-7,138 \cdot 10^{-5}$	-750,378	21,023,936	$-400 \cdot 10^{-5}$	-	$-7,538 \cdot 10^{-5}$
	4	$+24,433 \cdot 10^{-5}$	+4,596,449	37,625,131	$-400 \cdot 10^{-5}$	$-4,573 \cdot 10^{-5} =$	$+19,459 \cdot 10^{-5}$
	7	$-17,181 \cdot 10^{-5}$	-3,450,640	40,167,998	$-400 \cdot 10^{-5}$	$-3,920 \cdot 10^{-5} \pm$	$-21,501 \cdot 10^{-5}$
II			$F_I = +395,430$	98,817,066			
	1	$+18,429 \cdot 10^{-5}$	+2,639,603	28,646,495	$+4,573 \cdot 10^{-5}$	-	$+23,002 \cdot 10^{-5}$
	2	$-37,127 \cdot 10^{-5}$	-1,443,784	7,777,597	$+4,573 \cdot 10^{-5}$	-	$-32,553 \cdot 10^{-5}$
	4	$-24,433 \cdot 10^{-5}$	-4,596,449	37,625,131	$+4,573 \cdot 10^{-5}$	$+400 \cdot 10^{-5} \ddagger$	$-19,459 \cdot 10^{-5}$
	5	$-5,503 \cdot 10^{-5f}$	-21,346	775,842	$+4,573 \cdot 10^{-5}$	$-0.03920 \pm$	$-4,850 \cdot 10^{-5}$
III			$F_{II} = -3,421,977$	74,825,067			
	5	$+5,503 \cdot 10^{-5f}$	+21,346	775,842	$+3,920 \cdot 10^{-5}$	$-4,573 \cdot 10^{-5} =$	$+4,850 \cdot 10^{-5}$
	6	$+1,709 \cdot 10^{-5}$	+92,718	10,848,205	$+3,920 \cdot 10^{-5}$	-	$+5,629 \cdot 10^{-5}$
	7	$+17,181 \cdot 10^{-5}$	+3,450,640	40,167,998	$+3,920 \cdot 10^{-5}$	$+400 \cdot 10^{-5} \mp$	$+21,501 \cdot 10^{-5}$
	8	$-17,735 \cdot 10^{-5}$	-10,028,129	113,088,168	$+3,920 \cdot 10^{-5}$	-	$-13,815 \cdot 10^{-5}$
			$F_{III} = -6,463,424$	164,880,214			

^aPipe lengths and diameters are shown in Fig. 1 and initial flow pattern in Fig. 2; six iterations are enough to achieve results shown in Table 9 (this is for $\Sigma F \rightarrow 0$)

^bThis is F calculated using (3)

^cAlso using (9) or (10)

^d ΔQ_2 is ΔQ_1 from adjacent loop

^eFinal calculated flow in the first iteration is used for the calculation in the second iteration

^fOpposite flow direction than in previous iteration

Table 3 Calculation after the modified Hardy Cross method for example gas network

Loop	Pipe	Iteration 1					
		Q^a	$C=p_1^2-p_2^{2b}$	$F'=\left \frac{\partial C(Q)}{\partial Q}\right $	ΔQ_1^c	ΔQ_2^d	$Q_1=Q^e$
I	3	$-19,444 \cdot 10^{-5}$	-1,264,933,339	11,839,776,055	$+15,114 \cdot 10^{-5}$	-	$-4,331 \cdot 10^{-5}$
	4	$+2,778 \cdot 10^{-5}$	+20,357,137	1,333,799,622	$+15,114 \cdot 10^{-5}$	$+7,937 \cdot 10^{-5} \mp$	$+25,828 \cdot 10^{-5}$
	7	$-30,556 \cdot 10^{-5}$	-2,399,620,963	14,293,015,047	$+15,114 \cdot 10^{-5}$	$-4,288 \cdot 10^{-5} \pm$	$-19,730 \cdot 10^{-5}$
II	Σ		$F_I = -3,644,197,166$	27,466,590,725			
	1	$+27,778 \cdot 10^{-5}$	+1,344,982,710	8,812,326,713	$-7,937 \cdot 10^{-5}$	-	$+19,841 \cdot 10^{-5}$
	2	$-27,778 \cdot 10^{-5}$	-200,615,476	1,314,432,601	$-7,937 \cdot 10^{-5}$	-	$-35,715 \cdot 10^{-5}$
	4	$-2,778 \cdot 10^{-5}$	-20,357,137	1,333,799,622	$-7,937 \cdot 10^{-5}$	$-15,114 \cdot 10^{-5} \pm$	$-25,828 \cdot 10^{-5}$
	5	$+2,778 \cdot 10^{-5}$	+1,828,426	119,798,452	$-7,937 \cdot 10^{-5}$	$-4,288 \cdot 10^{-5} =$	$-9,447 \cdot 10^{-5f}$
III	Σ		$F_{II} = +1,125,838,522$	11,580,357,388			
	5	$-2,778 \cdot 10^{-5}$	-1,828,425	119,798,452	$+4,288 \cdot 10^{-5}$	$+7,937 \cdot 10^{-5} \mp$	$+9,447 \cdot 10^{-5f}$
	6	$+2,778 \cdot 10^{-5}$	+65,604,940	4,298,435,730	$+4,288 \cdot 10^{-5}$	-	$+7,066 \cdot 10^{-5}$
	7	$+30,556 \cdot 10^{-5}$	+2,399,620,963	14,293,015,047	$+4,288 \cdot 10^{-5}$	$-15,114 \cdot 10^{-5} =$	$+19,730 \cdot 10^{-5}$
	8	$-16,667 \cdot 10^{-5}$	-2,096,864,105	22,897,756,035	$+4,288 \cdot 10^{-5}$	-	$-12,379 \cdot 10^{-5}$
	Σ		$F_{III} = +366,533,372$	41,609,005,264			

Table 3 (continued)

Loop	Pipe	Iteration 2	$C = p_1^2 - p_2^{2b}$	$F' = \left \frac{\partial C(Q)}{\partial Q} \right $	ΔQ_1^c	ΔQ_2^d	$Q_2 = Q$
I	3	$-4,331 \cdot 10^{-5}$	-82,226,369	3,455,339,181	$+2,078 \cdot 10^{-5}$	-	$-2,253 \cdot 10^{-5}$
	4	$+25,828 \cdot 10^{-5}$	+1,178,151,884	8,301,891,430	$+2,078 \cdot 10^{-5}$	$-5,572 \cdot 10^{-5} =$	$+22,263 \cdot 10^{-5}$
	7	$-19,730 \cdot 10^{-5}$	-1,082,435,403	9,985,057,887	$+2,078 \cdot 10^{-5}$	$-4,154 \cdot 10^{-5} \ddagger$	$-17,620 \cdot 10^{-5}$
II			$F_I = +13,490,112$	21,742,488,499			
	1	$+19,841 \cdot 10^{-5}$	+729,037,716	6,687,432,899	$+5,643 \cdot 10^{-5}$	-	$+25,484 \cdot 10^{-5}$
	2	$-35,715 \cdot 10^{-5}$	-316,967,670	1,615,251,716	$+5,643 \cdot 10^{-5}$	-	$-30,072 \cdot 10^{-5}$
	4	$-25,828 \cdot 10^{-5}$	-1,178,151,884	8,301,891,430	$+5,643 \cdot 10^{-5}$	$+1,128 \cdot 10^{-5} \pm$	$-22,263 \cdot 10^{-5}$
	5	$-9,447 \cdot 10^{-5f}$	-16,966,070	326,858,346.3	$+5,643 \cdot 10^{-5}$	$-4,154 \cdot 10^{-5} \ddagger$	$-3,772 \cdot 10^{-5}$
III			$F_{II} = -783,047,908$	16,931,434,392			
	5	$+9,447 \cdot 10^{-5f}$	+16,966,070	326,858,346.3	$-32 \cdot 10^{-5}$	$-5,572 \cdot 10^{-5} =$	$+3,772 \cdot 10^{-5}$
	6	$+7,066 \cdot 10^{-5}$	+358,812,846	9,242,405,949	$-32 \cdot 10^{-5}$	-	$+7,034 \cdot 10^{-5}$
	7	$+19,730 \cdot 10^{-5}$	+1,082,435,403	9,985,057,887	$-32 \cdot 10^{-5}$	$+1,128 \cdot 10^{-5} =$	$+17,620 \cdot 10^{-5}$
	8	$-12,379 \cdot 10^{-5}$	-1,220,331,619	17,942,054,130	$-32 \cdot 10^{-5}$	-	$-12,411 \cdot 10^{-5}$
			$F_{III} = +237,882,701$	37,496,376,312			

^aPipe lengths and diameters are shown in Fig. 1 and initial flow pattern in Fig. 2; three iterations are enough to achieve results shown in Table 9 (this is for $\Sigma F \rightarrow 0$)

^bThis is F calculated using (1)

^cUsing (12) where (Δp) is replaced with (C)

^d ΔQ_2 is ΔQ_1 from adjacent loop

^eFinal calculated flow in the first iteration is used for the calculation in the second iteration

^fOpposite flow direction than in previous iteration

Table 4 Calculation after the modified Hardy Cross method for example water network

Loop	Pipe	Iteration 1					$Q_1 = Q^e$
		Q^a	$\Delta p = p_1 - p_2^b$	$F' = \left \frac{\partial \Delta p(Q)}{\partial Q} \right $	ΔQ_1^c	ΔQ_2^d	
I	3	$-19,444 \cdot 10^{-5}$	-5,279,095	54,299,272	$+14,195 \cdot 10^{-5}$	-	$-5,250 \cdot 10^{-5}$
	4	$+2,778 \cdot 10^{-5}$	+69,146	4,978,573	$+14,195 \cdot 10^{-5}$	$+8,006 \cdot 10^{-5} \mp$	$+24,978 \cdot 10^{-5}$
	7	$-30,556 \cdot 10^{-5}$	-10,718,549	70,157,780	$+14,195 \cdot 10^{-5}$	$-4,053 \cdot 10^{-5} \pm$	$-20,413 \cdot 10^{-5}$
II	Σ		$F_I = -15,928,498$	129,435,625			
	1	$+27,778 \cdot 10^{-5}$	+5,919,850	42,622,924	$-8,006 \cdot 10^{-5}$	-	$+19,772 \cdot 10^{-5}$
	2	$-27,778 \cdot 10^{-5}$	-816,585	5,879,413	$-8,006 \cdot 10^{-5}$	-	$-35,784 \cdot 10^{-5}$
	4	$-2,778 \cdot 10^{-5}$	-69,146	4,978,573	$-8,006 \cdot 10^{-5}$	$-14,195 \cdot 10^{-5} \pm$	$-24,978 \cdot 10^{-5}$
	5	$+2,778 \cdot 10^{-5}$	+5,967	429,677	$-8,006 \cdot 10^{-5}$	$-4,053 \cdot 10^{-5} =$	$-9,281 \cdot 10^{-5}$
III	Σ		$F_{II} = +5,040,086$	53,910,587			
	5	$-2,778 \cdot 10^{-5}$	-5,967	429,677	$+4,053 \cdot 10^{-5}$	$+8,006 \cdot 10^{-5} \pm$	$+9,281 \cdot 10^{-5f}$
	6	$+2,778 \cdot 10^{-5}$	+232,186	16,717,415	$+4,053 \cdot 10^{-5}$	-	$+6,830 \cdot 10^{-5}$
	7	$+30,556 \cdot 10^{-5}$	+10,718,549	70,157,780	$+4,053 \cdot 10^{-5}$	$-14,195 \cdot 10^{-5} \pm$	$+20,413 \cdot 10^{-5}$
	8	$-16,667 \cdot 10^{-5}$	-8,874,257	106,491,089	$+4,053 \cdot 10^{-5}$	-	$-12,614 \cdot 10^{-5}$
	Σ		$F_{III} = +2,070,510$	193,795,963			

Table 4 (continued)

Loop	Pipe	Iteration 2					$Q_1 = Q^e$	$\Delta p = p_1 - p_2^b$	$F' = \left \frac{\partial \Delta p(Q)}{\partial Q} \right $	ΔQ_1^c	ΔQ_2^d	$Q_2 = Q$
I	3						$-5,250 \cdot 10^{-5}$	-416,024	15,849,700	$+2,505 \cdot 10^{-5}$	-	$-2,744 \cdot 10^{-5}$
	4						$+24,978 \cdot 10^{-5}$	$+4,800,902$	38,440,414	$+2,505 \cdot 10^{-5}$	$-5,312 \cdot 10^{-5} =$	$+22,172 \cdot 10^{-5}$
	7						$-20,413 \cdot 10^{-5}$	-4,840,373	47,423,556	$+2,505 \cdot 10^{-5}$	$-107 \cdot 10^{-5} \pm$	$-18,015 \cdot 10^{-5}$
II								$F_I = -455,494$	101,713,670			
	1						$+19,772 \cdot 10^{-5}$	$+3,030,676$	30,656,275	$+5,312 \cdot 10^{-5}$	-	$+25,084 \cdot 10^{-5}$
	2						$-35,784 \cdot 10^{-5}$	-1,342,790	7,505,063	$+5,312 \cdot 10^{-5}$	-	$-30,472 \cdot 10^{-5}$
	4						$-24,978 \cdot 10^{-5}$	-4,800,902	38,440,414	$+5,312 \cdot 10^{-5}$	$-2,505 \cdot 10^{-5} \pm$	$-22,172 \cdot 10^{-5}$
	5						$-9,281 \cdot 10^{-5}$	-57,430	1,237,632	$+5,312 \cdot 10^{-5}$	$-107 \cdot 10^{-5} \pm$	$-4,076 \cdot 10^{-5}$
III								$F_{II} = -3,170,446$	77,839,384			
	5						$+9,281 \cdot 10^{-5f}$	$+57,430$	1,237,632	$+107 \cdot 10^{-5}$	$-5,312 \cdot 10^{-5} =$	$+4,076 \cdot 10^{-5}$
	6						$+6,830 \cdot 10^{-5}$	$+1,312,311$	38,425,447	$+107 \cdot 10^{-5}$	-	$+6,937 \cdot 10^{-5}$
	7						$+20,413 \cdot 10^{-5}$	$+4,840,373$	47,423,556	$+107 \cdot 10^{-5}$	$-2,505 \cdot 10^{-5} =$	$+18,015 \cdot 10^{-5}$
	8						$-12,614 \cdot 10^{-5}$	-5,136,681	81,444,010	$+107 \cdot 10^{-5}$	-	$-12,507 \cdot 10^{-5}$
								$F_{III} = +1,073,434$	168,530,645			

^aPipe lengths and diameters are shown in Fig. 1 and initial flow pattern in Fig. 2; three iterations are enough to achieve results shown in Table 9 (this is for $\Sigma F \rightarrow 0$)

^bThis is F calculated using (3)

^cUsing (12)

^d ΔQ_2 is ΔQ_1 from adjacent loop

^eFinal calculated flow in the first iteration is used for the calculation in the second iteration

^fOpposite flow direction than in previous iteration

Table 5 Calculation after Modified M.M. Andrijašev method for example gas network

Contour ^O	Pipe	Iteration 1	$C = p_1^2 - p_2^{2b}$	$F' = \left \frac{\partial C(Q)}{\partial Q} \right $	ΔQ_1^{oc}	ΔQ_2^{od}	$Q_1 = Q^e$
I ^O	6	$+2,778 \cdot 10^{-5}$	65,604,941	4,298,435,730	$+13,669 \cdot 10^{-5}$	$-9,381 \cdot 10^{-5} \pm$	$+7,066 \cdot 10^{-5}$
	8	$-16,667 \cdot 10^{-5}$	$-2,096,864,106$	22,897,756,035	$+13,669 \cdot 10^{-5}$	$-9,381 \cdot 10^{-5} \mp$	$-12,379 \cdot 10^{-5}$
	3	$-19,444 \cdot 10^{-5}$	$-1,264,933,339$	11,839,776,055	$+13,669 \cdot 10^{-5}$	$+1,444 \cdot 10^{-5} =$	$-4,331 \cdot 10^{-5}$
	4	$+2,778 \cdot 10^{-5}$	20,357,137	1,333,799,622	$+13,669 \cdot 10^{-5}$	$+9,381 \cdot 10^{-5} \mp$	$+25,828 \cdot 10^{-5}$
	5	$-2,778 \cdot 10^{-5}$	$-1,828,426$	119,798,452	$+13,669 \cdot 10^{-5}$	$-1,444 \cdot 10^{-5} \pm$	$+9,447 \cdot 10^{-5f}$
II ^O	Σ		$F^o = -3,277,663,793$	40,489,565,894			
	6	$+2,778 \cdot 10^{-5}$	65,604,941	4,298,435,730	$-9,381 \cdot 10^{-5}$	$+13,669 \cdot 10^{-5} \pm$	$+7,066 \cdot 10^{-5}$
	8	$-16,667 \cdot 10^{-5}$	$-2,096,864,106$	22,897,756,035	$-9,381 \cdot 10^{-5}$	$+13,669 \cdot 10^{-5} =$	$-12,379 \cdot 10^{-5}$
	7	$+30,556 \cdot 10^{-5}$	2,399,620,963	14,293,015,047	$-9,381 \cdot 10^{-5}$	$-1,444 \cdot 10^{-5} =$	$+19,730 \cdot 10^{-5}$
	4	$-2,778 \cdot 10^{-5}$	$-20,357,137$	1,333,799,622	$-9,381 \cdot 10^{-5}$	$-13,669 \cdot 10^{-5} \pm$	$-25,828 \cdot 10^{-5}$
III ^O	2	$-27,778 \cdot 10^{-5}$	$-200,615,476$	1,314,432,601	$-9,381 \cdot 10^{-5}$	$+1,444 \cdot 10^{-5} =$	$-35,715 \cdot 10^{-5}$
	1	$+27,778 \cdot 10^{-5}$	1,344,982,710	8,812,326,713	$-9,381 \cdot 10^{-5}$	$+1,444 \cdot 10^{-5} \pm$	$+19,841 \cdot 10^{-5}$
	Σ		$F^o = 1,492,371,895$	52,949,765,748			
	1	$+27,778 \cdot 10^{-5}$	1,344,982,710	8,812,326,713	$+1,444 \cdot 10^{-5}$	$-9,381 \cdot 10^{-5} \pm$	$+19,841 \cdot 10^{-5}$
	5	$+2,778 \cdot 10^{-5}$	1,828,426	119,798,452	$+1,444 \cdot 10^{-5}$	$-13,669 \cdot 10^{-5} =$	$-9,447 \cdot 10^{-5f}$
	7	$-30,556 \cdot 10^{-5}$	$-2,399,620,963$	14,293,015,047	$+1,444 \cdot 10^{-5}$	$+9,381 \cdot 10^{-5} \pm$	$-19,730 \cdot 10^{-5}$
	3	$-19,444 \cdot 10^{-5}$	$-1,264,933,339$	11,839,776,055	$+1,444 \cdot 10^{-5}$	$+13,669 \cdot 10^{-5} =$	$-4,331 \cdot 10^{-5}$
	2	$-27,778 \cdot 10^{-5}$	$-200,615,476$	1,314,432,601	$+1,444 \cdot 10^{-5}$	$-9,381 \cdot 10^{-5} \mp$	$-35,715 \cdot 10^{-5}$
	Σ		$F^o = -2,518,358,644$	36,379,348,868			

Table 5 (continued)

Contour ^O	Pipe	Iteration 2	$Q_1 = Q^e$	$C = p_1^i - p_2^b$	$F' = \left \frac{\partial C(Q)}{\partial Q} \right $	ΔQ_1^{oc}	ΔQ_2^{od}	$Q_2 = Q$
I ^O	6		$+7,066 \cdot 10^{-5}$	358,812,846	9,242,405,949	$-1,798 \cdot 10^{-5}$	$+1,767 \cdot 10^{-5} \pm$	$+7,034 \cdot 10^{-5}$
	8		$-12,379 \cdot 10^{-5}$	$-1,220,331,619$	17,942,054,130	$-1,798 \cdot 10^{-5}$	$+1,767 \cdot 10^{-5} =$	$-12,411 \cdot 10^{-5}$
	3		$-4,331 \cdot 10^{-5}$	$-82,226,369$	3,455,539,181	$-1,798 \cdot 10^{-5}$	$+3,876 \cdot 10^{-5} =$	$-2,253 \cdot 10^{-5}$
	4		$+25,828 \cdot 10^{-5}$	1,178,151,884	8,301,891,430	$-1,798 \cdot 10^{-5}$	$-1,767 \cdot 10^{-5} =$	$+22,263 \cdot 10^{-5}$
	5		$+9,447 \cdot 10^{-5f}$	16,966,070	326,858,346	$-1,798 \cdot 10^{-5}$	$-3,876 \cdot 10^{-5} =$	$+3,772 \cdot 10^{-5}$
II ^O			$F^o = 251,372,813$		39,268,749,037			
	6		$+7,066 \cdot 10^{-5}$	358,812,846	9,242,405,949	$+1,767 \cdot 10^{-5}$	$-1,798 \cdot 10^{-5} \pm$	$+7,034 \cdot 10^{-5}$
	8		$-12,379 \cdot 10^{-5}$	$-1,220,331,619$	17,942,054,130	$+1,767 \cdot 10^{-5}$	$-1,798 \cdot 10^{-5} \mp$	$-12,411 \cdot 10^{-5}$
	7		$+19,730 \cdot 10^{-5}$	1,082,435,403	9,985,057,887	$+1,767 \cdot 10^{-5}$	$-3,876 \cdot 10^{-5} =$	$+17,620 \cdot 10^{-5}$
	4		$-25,828 \cdot 10^{-5}$	$-1,178,151,884$	8,301,891,430	$+1,767 \cdot 10^{-5}$	$+1,798 \cdot 10^{-5} \pm$	$-22,263 \cdot 10^{-5}$
III ^O	2		$-35,715 \cdot 10^{-5}$	$-316,967,670$	1,615,251,716	$+1,767 \cdot 10^{-5}$	$+3,876 \cdot 10^{-5} =$	$-30,072 \cdot 10^{-5}$
	1		$+19,841 \cdot 10^{-5}$	729,037,716	6,687,432,899	$+1,767 \cdot 10^{-5}$	$+3,876 \cdot 10^{-5} \pm$	$+25,484 \cdot 10^{-5}$
			$F^o = -545,165,207$		53,774,094,012			
	1		$+19,841 \cdot 10^{-5}$	729,037,716	6,687,432,899	$+3,876 \cdot 10^{-5}$	$+1,767 \cdot 10^{-5} \pm$	$+25,484 \cdot 10^{-5}$
	5		$-9,447 \cdot 10^{-5f}$	$-16,966,070$	326,858,346	$+3,876 \cdot 10^{-5}$	$+1,798 \cdot 10^{-5} \pm$	$-3,772 \cdot 10^{-5}$
	7		$-19,730 \cdot 10^{-5}$	$-1,082,435,403$	9,985,057,887	$+3,876 \cdot 10^{-5}$	$-1,767 \cdot 10^{-5} \pm$	$-17,620 \cdot 10^{-5}$
	3		$-4,331 \cdot 10^{-5}$	$-82,226,369$	3,455,539,181	$+3,876 \cdot 10^{-5}$	$-1,798 \cdot 10^{-5} \mp$	$-2,253 \cdot 10^{-5}$
	2		$-35,715 \cdot 10^{-5}$	$-316,967,670$	1,615,251,716	$+3,876 \cdot 10^{-5}$	$+1,767 \cdot 10^{-5} =$	$-30,072 \cdot 10^{-5}$
			$F^o = -769,557,797$		22,070,140,031			

^aPipe lengths and diameters are shown in Fig. 1, initial flow pattern in Fig. 3 and numerical values for initial flows in Fig. 2; three iterations are enough to achieve results shown in Table 9 (this is for $\Sigma F \rightarrow 0$)

^bThis is F calculated using (1)

^cUsing (15)

^d ΔQ_2^o is ΔQ_1^o from adjacent contour

^eFinal calculated flow in the first iteration is used for the calculation in the second iteration

^fOpposite flow direction than in previous iteration

Table 6 Calculation after Modified M.M. Andrijašev method for example water network

Contour ^O	Pipe	Iteration 1	$\Delta p = p_1 - p_2^b$	$F' = \left \frac{\partial \Delta p(Q)}{\partial Q} \right $	ΔQ_1^c	ΔQ_2^{od}	$Q_1 = Q^e$
I ^O	6	$+2,778 \cdot 10^{-5}$	+232,186	16,717,415	$+13,127 \cdot 10^{-5}$	$-9,074 \cdot 10^{-5}$	$+6,830 \cdot 10^{-5}$
	8	$-16,667 \cdot 10^{-5}$	-8,874,257	106,491,089	$+13,127 \cdot 10^{-5}$	$-9,074 \cdot 10^{-5}$	$-12,614 \cdot 10^{-5}$
	3	$-19,444 \cdot 10^{-5}$	-5,279,095	54,299,272	$+13,127 \cdot 10^{-5}$	$+1,068 \cdot 10^{-5}$	$-5,250 \cdot 10^{-5}$
	4	$+2,778 \cdot 10^{-5}$	+69,146	4,978,573	$+13,127 \cdot 10^{-5}$	$+9,074 \cdot 10^{-5}$	$+24,978 \cdot 10^{-5}$
	5	$-2,778 \cdot 10^{-5}$	-5,967	429,677	$+13,127 \cdot 10^{-5}$	$-1,068 \cdot 10^{-5}$	$+9,281 \cdot 10^{-5f}$
		Σ	$F^\circ = -13,857,988$	182,916,028			
II ^O	6	$+2,778 \cdot 10^{-5}$	+232,186	16,717,415	$-9,074 \cdot 10^{-5}$	$+13,127 \cdot 10^{-5}$	$+6,830 \cdot 10^{-5}$
	8	$-16,667 \cdot 10^{-5}$	-8,874,257	106,491,089	$-9,074 \cdot 10^{-5}$	$+13,127 \cdot 10^{-5}$	$-12,614 \cdot 10^{-5}$
	7	$+30,556 \cdot 10^{-5}$	+10,718,549	70,157,780	$-9,074 \cdot 10^{-5}$	$-1,068 \cdot 10^{-5}$	$+20,413 \cdot 10^{-5}$
	4	$-2,778 \cdot 10^{-5}$	-69,146	4,978,573	$-9,074 \cdot 10^{-5}$	$-13,127 \cdot 10^{-5}$	$-24,978 \cdot 10^{-5}$
	2	$-27,778 \cdot 10^{-5}$	-816,585	5,879,413	$-9,074 \cdot 10^{-5}$	$+1,068 \cdot 10^{-5}$	$-35,784 \cdot 10^{-5}$
		Σ	$F^\circ = +7,110,597$	42,622,924	$-9,074 \cdot 10^{-5}$	$+1,068 \cdot 10^{-5}$	$+19,772 \cdot 10^{-5}$
III ^O	1	$+27,778 \cdot 10^{-5}$	+5,919,850	246,847,196	$+1,068 \cdot 10^{-5}$	$-9,074 \cdot 10^{-5}$	$+19,772 \cdot 10^{-5}$
	5	$+2,778 \cdot 10^{-5}$	+5,967	429,677	$+1,068 \cdot 10^{-5}$	$-13,127 \cdot 10^{-5}$	$-9,281 \cdot 10^{-5f}$
	7	$-30,556 \cdot 10^{-5}$	-10,718,549	70,157,780	$+1,068 \cdot 10^{-5}$	$+9,074 \cdot 10^{-5}$	$-20,413 \cdot 10^{-5}$
	3	$-19,444 \cdot 10^{-5}$	-5,279,095	54,299,272	$+1,068 \cdot 10^{-5}$	$+13,127 \cdot 10^{-5}$	$-5,250 \cdot 10^{-5}$
	2	$-27,778 \cdot 10^{-5}$	-816,585	5,879,413	$+1,068 \cdot 10^{-5}$	$-9,074 \cdot 10^{-5}$	$-35,784 \cdot 10^{-5}$
		Σ	$F^\circ = -10,888,413$	173,389,067			

Table 6 (continued)

Contour ^O	Pipe	Iteration 2	$\Delta p = p_1 - p_2^b$	$F' = \frac{\partial \Delta p(Q)}{\partial Q}$	ΔQ_1^{oc}	ΔQ_2^{od}	$Q_2 = Q^e$
I ^O	6	$+6,830 \cdot 10^{-5}$	$+1,312,311$	19,212,723	$-1,350 \cdot 10^{-5}$	$+1,457 \cdot 10^{-5} \pm$	$+6,937 \cdot 10^{-5}$
	8	$-12,614 \cdot 10^{-5}$	$-5,136,681$	40,722,005	$-1,350 \cdot 10^{-5}$	$+1,457 \cdot 10^{-5} =$	$-12,507 \cdot 10^{-5}$
	3	$-5,250 \cdot 10^{-5}$	$-416,024$	7,924,850	$-1,350 \cdot 10^{-5}$	$+3,855 \cdot 10^{-5} =$	$-2,744 \cdot 10^{-5}$
	4	$+24,978 \cdot 10^{-5}$	4,800,902	19,220,207	$-1,350 \cdot 10^{-5}$	$-1,457 \cdot 10^{-5} =$	$+22,172 \cdot 10^{-5}$
	5	$+9,281 \cdot 10^{-5f}$	$+57,430$	618,816	$-1,350 \cdot 10^{-5}$	$-3,855 \cdot 10^{-5} =$	$+4,076 \cdot 10^{-5}$
II ^O			$F^o = 617,939$	87,698,601			
	6	$+6,830 \cdot 10^{-5}$	$1,312,311$	19,212,723	$+1,457 \cdot 10^{-5}$	$-1,350 \cdot 10^{-5} \pm$	$+6,937 \cdot 10^{-5}$
	8	$-12,614 \cdot 10^{-5}$	$-5,136,681$	40,722,005	$+1,457 \cdot 10^{-5}$	$-1,350 \cdot 10^{-5} \mp$	$-12,507 \cdot 10^{-5}$
	7	$+20,413 \cdot 10^{-5}$	4,840,373	23,711,778	$+1,457 \cdot 10^{-5}$	$-3,855 \cdot 10^{-5} =$	$+18,015 \cdot 10^{-5}$
	4	$-24,978 \cdot 10^{-5}$	$-4,800,902$	19,220,207	$+1,457 \cdot 10^{-5}$	$+1,350 \cdot 10^{-5} \pm$	$-22,172 \cdot 10^{-5}$
III ^O	2	$-35,784 \cdot 10^{-5}$	$-1,342,790$	3,752,532	$+1,457 \cdot 10^{-5}$	$+3,855 \cdot 10^{-5} =$	$-30,472 \cdot 10^{-5}$
	1	$+19,772 \cdot 10^{-5}$	3,030,676	15,328,137	$+1,457 \cdot 10^{-5}$	$+3,855 \cdot 10^{-5} \pm$	$+25,084 \cdot 10^{-5}$
			$F^o = -2,097,013$	121,947,382			
	1	$+19,772 \cdot 10^{-5}$	3,030,676	15,328,137	$+3,855 \cdot 10^{-5}$	$+1,457 \cdot 10^{-5} \pm$	$+25,084 \cdot 10^{-5}$
	5	$-9,281 \cdot 10^{-5f}$	$-57,430$	618,816	$+3,855 \cdot 10^{-5}$	$+1,350 \cdot 10^{-5} \pm$	$-4,076 \cdot 10^{-5}$
	7	$-20,413 \cdot 10^{-5}$	$-4,840,373$	23,711,778	$+3,855 \cdot 10^{-5}$	$-1,457 \cdot 10^{-5} \pm$	$-18,015 \cdot 10^{-5}$
	3	$-5,250 \cdot 10^{-5}$	$-416,024$	7,924,850	$+3,855 \cdot 10^{-5}$	$-1,350 \cdot 10^{-5} \mp$	$-2,744 \cdot 10^{-5}$
	2	$-35,784 \cdot 10^{-5}$	$-1,342,790$	3,752,532	$+3,855 \cdot 10^{-5}$	$+1,457 \cdot 10^{-5} =$	$-30,472 \cdot 10^{-5}$
			$F^o = -3,625,941$	51,336,113			

^aPipe lengths and diameters are shown in Fig. 1, initial flow pattern in Fig. 3 and numerical values for initial flows in Fig. 2; three iterations are enough to achieve results shown in Table 9 (this is for $\Sigma F \rightarrow 0$)

^bThis is F calculated using (3)

^cUsing (15)

^d ΔQ_2^{oc} is ΔQ_1^{oc} from adjacent contour

^eFinal calculated flow in the first iteration is used for the calculation in the second iteration

^fOpposite flow direction than in previous iteration

Table 7 Calculation after Modified Node method for example gas network with three loops

Node	Pipe	Iteration 1 C (Pa ²)	Q = f(C) ^a	f ^b	ΔC_1^c	ΔC_2^d	C ₁ = C ₂ ^e
I	2	+0.25 · 10 ¹⁰	+1.1108	24,415 · 10 ⁻¹⁴	-36.1 · 10 ⁸	-	-11.1 · 10 ^{8f}
	3	+0.25 · 10 ¹⁰	+0.2827	6,214 · 10 ⁻¹⁴	-36.1 · 10 ⁸	-6.99 · 10 ⁸ =	-18.1 · 10 ^{8f}
	4	-0.50 · 10 ¹⁰	-0.5715	6,280 · 10 ⁻¹⁴	-36.1 · 10 ⁸	+97.0 · 10 ⁸ ±	+10.9 · 10 ^{8f}
	Constant output flow		-0.0555				
II	3	-0.25 · 10 ¹⁰	Σf = +0.7665	36,909 · 10 ⁻¹⁴	+6.99 · 10 ⁸	+36.1 · 10 ⁸ ±	+18.1 · 10 ^{8f}
	7	-0.75 · 10 ¹⁰	-0.2827	6,214 · 10 ⁻¹⁴	+6.99 · 10 ⁸	+97.0 · 10 ⁸ ±	+29.0 · 10 ^{8f}
	8	-0.75 · 10 ¹⁰	-0.5715	4,187 · 10 ⁻¹⁴	+6.99 · 10 ⁸	+78.0 · 10 ⁸ ±	+9.99 · 10 ^{8f}
	Constant input flow		-0.3357	2,459 · 10 ⁻¹⁴	+6.99 · 10 ⁸		
III	4	+0.50 · 10 ¹⁰	+0.2777	12,860 · 10 ⁻¹⁴	-97.0 · 10 ⁸	+36.1 · 10 ⁸ ±	-10.9 · 10 ^{8f}
	5	+0.50 · 10 ¹⁰	Σf = -0.9121	6,280 · 10 ⁻¹⁴	-97.0 · 10 ⁸	+7.52 · 10 ⁸ ±	-39.5 · 10 ^{8f}
	7	+0.75 · 10 ¹⁰	+0.5715	23,608 · 10 ⁻¹⁴	-97.0 · 10 ⁸	-6.99 · 10 ⁸ =	-29.0 · 10 ^{8f}
	Constant output flow		-0.3611	4,187 · 10 ⁻¹⁴			
IV	1	+0.25 · 10 ¹⁰	Σf = +2.9302	34,075 · 10 ⁻¹⁴	-7.52 · 10 ⁸	-	+17.5 · 10 ⁸
	5	-0.50 · 10 ¹⁰	+0.3905	8,582 · 10 ⁻¹⁴	-7.52 · 10 ⁸	+97.0 · 10 ⁸ ±	+39.5 · 10 ^{8f}
	6	-0.50 · 10 ¹⁰	-2.1483	23,608 · 10 ⁻¹⁴	-7.52 · 10 ⁸	+78.0 · 10 ⁸ ±	+20.5 · 10 ^{8f}
	Constant output flow		-0.3004	3,302 · 10 ⁻¹⁴			
V	6	+0.50 · 10 ¹⁰	-0.2222	35,492 · 10 ⁻¹⁴	-78.0 · 10 ⁸	+7.52 · 10 ⁸ ±	-20.5 · 10 ^{8f}
	8	+0.75 · 10 ¹⁰	Σf = -2.2805	3,302 · 10 ⁻¹⁴	-78.0 · 10 ⁸	-6.99 · 10 ⁸ =	-9.99 · 10 ^{8f}
	Constant output flow		+0.3004	2,459 · 10 ⁻¹⁴			
	Constant output flow		+0.3357				
			-0.1944	5,761 · 10 ⁻¹⁴			
			Σf = +0.4417				

Table 7 (continued)

Node	Pipe	Iteration 2		$Q = f(C)^a$	f^b	ΔC_1	ΔC_2^d	$C_2 = C^e$
I	2	$C_1 = C^e$		-0.7109	$35,204 \cdot 10^{-14}$	$+28.4 \cdot 10^8$	-	$+17.3 \cdot 10^{8f}$
	3			-0.2367	$7,189 \cdot 10^{-14}$	$+28.4 \cdot 10^8$	$+4.57 \cdot 10^{8\ddagger}$	$+14.9 \cdot 10^{8f}$
	4			+0.2475	$12,474 \cdot 10^{-14}$	$+28.4 \cdot 10^8$	$-66.9 \cdot 10^{8\ddagger}$	$-27.6 \cdot 10^{8f}$
	Constant output flow			-0.0555		-0.0555		
II	3			$\Sigma f = +0.7665$	$36,909 \cdot 10^{-14}$	$\Sigma f = -0.7557$	$54,868 \cdot 10^{-14}$	
	7			+0.2367	$7,189 \cdot 10^{-14}$	$-4.57 \cdot 10^8$	$-28.4 \cdot 10^8 =$	$-14.9 \cdot 10^{8f}$
	8			+0.3390	$6,425 \cdot 10^{-14}$	$-4.57 \cdot 10^8$	$-66.9 \cdot 10^8 =$	$-42.5 \cdot 10^{8f}$
	Constant input flow			+0.1109	$6,099 \cdot 10^{-14}$	$-4.57 \cdot 10^8$	$-39.4 \cdot 10^8 =$	$-34.0 \cdot 10^{8f}$
III	4			+0.2777		+0.2777		
	5			$\Sigma f = -0.9121$	$12,860 \cdot 10^{-14}$	$\Sigma f = +0.9644$	$19,713 \cdot 10^{-14}$	
	7			-0.2475	$12,474 \cdot 10^{-14}$	$+66.9 \cdot 10^8$	$-28.4 \cdot 10^{8\pm}$	$+27.6 \cdot 10^{8f}$
	Constant output flow			-1.8867	$26,260 \cdot 10^{-14}$	$+66.9 \cdot 10^8$	$5.29 \cdot 10^{8\ddagger}$	$+32.7 \cdot 10^{8f}$
IV	1			-0.3611	$6,425 \cdot 10^{-14}$	$+66.9 \cdot 10^8$	$4.57 \cdot 10^{8\ddagger}$	$+42.5 \cdot 10^{8f}$
	5			$\Sigma f = -2.8343$	$45,160 \cdot 10^{-14}$	-0.3611		
	6			+0.3208	$10,084 \cdot 10^{-14}$	$-5.29 \cdot 10^8$	-	$+12.2 \cdot 10^8$
	Constant output flow			+1.8867	$26,260 \cdot 10^{-14}$	$-5.29 \cdot 10^8$	$-66.9 \cdot 10^8 =$	$-32.7 \cdot 10^{8f}$
V	6			+0.1840	$4,937 \cdot 10^{-14}$	$-5.29 \cdot 10^8$	$-39.4 \cdot 10^8 =$	$-24.3 \cdot 10^{8f}$
	8			-0.2222		-0.2222		
	Constant output flow			$\Sigma f = 2.1693$	$41,281 \cdot 10^{-14}$			
	Constant output flow			-0.1840	$4,937 \cdot 10^{-14}$	$39.4 \cdot 10^8$	$5.29 \cdot 10^{8\ddagger}$	$+24.3 \cdot 10^{8f}$
VI	8			-0.1109	$6,099 \cdot 10^{-14}$	$39.4 \cdot 10^8$	$4.57 \cdot 10^{8\ddagger}$	$+34.0 \cdot 10^{8f}$
	Constant output flow			-0.1944		-0.1944		
	Constant output flow			$\Sigma f = -0.4893$	$11,036 \cdot 10^{-14}$			
	Constant output flow							

Pipe lengths and diameters are shown in Fig. 1; see Fig. 4 for initial pattern (red letters); 26 iterations are enough to achieve results shown in Table 9 (this is for $\Sigma f \rightarrow 0$)

^aUsing (17); ^b $f^b = |\partial Q(C)/\partial C|$; ^c ΔC_1 is ΔC_1 from adjacent node; ^dFinal calculated pressure function in the first iteration is used for the calculation in the second iteration; ^eOpposite flow direction than in previous iteration

Table 8 Calculation after Modified Node method for example water network with three loops

Node	Pipe	Iteration 1 Δp (Pa)	$Q = f(\Delta p)^a$	f^b	$\Delta_{\Delta p1}^c$	$\Delta_{\Delta p2}^d$	$\Delta p1 = \Delta p^e$
I	2	$+25 \cdot 10^3$	$+4,953 \cdot 10^{-5}$	$9,907 \cdot 10^{-10}$	$+177,553$	–	$+202,553$
	3	$+25 \cdot 10^3$	$+1,360 \cdot 10^{-5}$	$2,720 \cdot 10^{-10}$	$+177,553$	$-167,854=$	$+34,699$
	4	$-50 \cdot 10^3$	$-2,580 \cdot 10^{-5}$	$2,500 \cdot 10^{-10}$	$+177,553$	$-798,781\pm$	$-671,229$
	Constant output flow		$-5,556 \cdot 10^{-5}$				
II		Σf	$-1,823 \cdot 10^{-5}$	$15,207 \cdot 10^{-10}$			
	3	$-25 \cdot 10^3$	$-1,360 \cdot 10^{-5}$	$2,720 \cdot 10^{-10}$	$+167,854$	$-177,553\pm$	$-34,699$
	7	$-75 \cdot 10^3$	$-2,580 \cdot 10^{-5}$	$1,720 \cdot 10^{-10}$	$+167,854$	$-798,781\pm$	$-705,927$
	8	$-75 \cdot 10^3$	$-1,555 \cdot 10^{-5}$	$1,037 \cdot 10^{-10}$	$+167,854$	$-1,244,652\pm$	$-1,151,798$
III	Constant input flow		$27,778 \cdot 10^{-5}$				
		Σf	$22,282 \cdot 10^{-5}$	$5,477 \cdot 10^{-10}$			
	4	$+50 \cdot 10^3$	$+2,580 \cdot 10^{-5}$	$2,580 \cdot 10^{-10}$	$+798,781$	$-177,553=$	$+671,229$
	5	$+50 \cdot 10^3$	$+9,206 \cdot 10^{-5}$	$9,206 \cdot 10^{-10}$	$+798,781$	$-854,621=$	$-5,840^f$
IV	Constant output flow		$+2,580 \cdot 10^{-5}$	$1,720 \cdot 10^{-10}$	$+798,781$	$-167,854=$	$+705,927$
			$-36,111 \cdot 10^{-5}$				
		Σf	$-21,744 \cdot 10^{-5}$	$13,507 \cdot 10^{-10}$			
	1	$+25 \cdot 10^3$	$+1,825 \cdot 10^{-5}$	$3,649 \cdot 10^{-10}$	$+854,621$	–	$+879,621$
V	Constant output flow		$-9,206 \cdot 10^{-5}$	$9,206 \cdot 10^{-10}$	$+854,621$	$-798,781\pm$	$+5,840^f$
			$-1,372 \cdot 10^{-5}$	$1,372 \cdot 10^{-10}$	$+854,621$	$-1,244,652\pm$	$-440,031$
		Σf	$-22,222 \cdot 10^{-5}$				
	6	$+50 \cdot 10^3$	$-30,976 \cdot 10^{-5}$	$14,228 \cdot 10^{-10}$			
VI	Constant output flow		$+1,372 \cdot 10^{-5}$	$1,372 \cdot 10^{-10}$	$+1,244,652$	$-854,621=$	$+440,031$
			$+1,555 \cdot 10^{-5}$	$1,037 \cdot 10^{-10}$	$+1,244,652$	$-167,854=$	$+1,151,798$
		Σf	$-19,444 \cdot 10^{-5}$				
	8	$+75 \cdot 10^3$	$-16,517 \cdot 10^{-5}$	$2,409 \cdot 10^{-10}$			

Table 8 (continued)

Node	Pipe	Iteration 2 $\Delta p_1 = \Delta p^e$	$Q = f(\Delta p)^a$	f^b	$\Delta_{\Delta p_1}^c$	$\Delta_{\Delta p_2}^d$	$\Delta p_2 = \Delta p$
I	2	+202,553	+12,974 · 10 ⁻⁵	3,203 · 10 ⁻¹⁰	+552,198	–	+754,750
	3	+34,699	+1,396 · 10 ⁻⁵	2,010 · 10 ⁻¹⁰	+552,198	–730,668=	–143, 771 ^f
	4	–671,229	–8,616 · 10 ⁻⁵	642 · 10 ⁻¹⁰	+552,198	–2,779,111±	–2,898,142
	Constant output flow		–5,556 · 10 ⁻⁵		–5,556 · 10 ⁻⁵		
II	3	Σf	–1,823 · 10 ⁻⁵	15,207 · 10 ⁻¹⁰	+198 · 10 ⁻⁵	5,856 · 10 ⁻¹⁰	
		–34,699	–1,396 · 10 ⁻⁵	2,010 · 10 ⁻¹⁰	+730,668	–552,198±	+143,771 ^f
	7	–705,927	–7,214 · 10 ⁻⁵	511 · 10 ⁻¹⁰	+730,668	–2,779,111±	–2,754,371
	8	–1,151,798	–5,453 · 10 ⁻⁵	237 · 10 ⁻¹⁰	+730,668	–3,619,538±	–4,040,669
III	Constant input flow		+27,778 · 10 ⁻⁵		+27,778 · 10 ⁻⁵		
		Σf	22,282 · 10 ⁻⁵	5,477 · 10 ⁻¹⁰	+13,715 · 10 ⁻⁵	2,759 · 10 ⁻¹⁰	
	4	+671,229	+8,616 · 10 ⁻⁵	642 · 10 ⁻¹⁰	+2,779,111	–552,198=	+2,898,142
	5	–5,840 ^f	–2,958 · 10 ⁻⁵	25,329 · 10 ⁻¹⁰	+2,779,111	–2,785,111±	–11,840
IV	7	+705,927	+7,214 · 10 ⁻⁵	511 · 10 ⁻¹⁰	+2,779,111	–730,668=	+2,754,371
	Constant output flow		–36,111 · 10 ⁻⁵		–36,111 · 10 ⁻⁵		
		Σf	–23,239 · 10 ⁻⁵	26,482 · 10 ⁻¹⁰			
	1	+879,621	+9,639 · 10 ⁻⁵	548 · 10 ⁻¹⁰	+2,785,111	–	+3,664,732
V	5	+5,840 ^f	+2,958 · 10 ⁻⁵	25,329 · 10 ⁻¹⁰	+2,785,111	–2,779,111=	+11,840
	6	–440,031	–3,672 · 10 ⁻⁵	417 · 10 ⁻¹⁰	+2,785,111	–3,619,538±	–1,274,458
	Constant output flow		–22,222 · 10 ⁻⁵		–22,222 · 10 ⁻⁵		
		Σf	–13,297 · 10 ⁻⁵	26,294 · 10 ⁻¹⁰			
VI	6	+440,031	+3,672 · 10 ⁻⁵	417 · 10 ⁻¹⁰	+3,619,538	–2,785,111=	+1,274,458
	8	+1,151,798	+5,453 · 10 ⁻⁵	237 · 10 ⁻¹⁰	+3,619,538	–730,668=	+4,040,669
	Constant output flow		–19,444 · 10 ⁻⁵		–19,444 · 10 ⁻⁵		
		Σf	–10,320 · 10 ⁻⁵	654 · 10 ⁻¹⁰			

Pipe lengths and diameters are shown in Fig. 1; see Fig. 4 for initial pattern (blue letters); nine iterations are enough to achieve results shown in Table 9 (this is for Σf→0)

^aUsing (18); to calculate friction factor λ , Reynolds number has to be calculated and for this velocity have to be chosen (this velocity does not have effect on final results, here is chosen extremely large velocity 100 m/s); ^b $f^b = |\partial Q(\Delta p)/\partial \Delta p|$; ^c $\Delta_{\Delta p_1}$ after Eq. 20; ^d $\Delta_{\Delta p_2}$ is $\Delta_{\Delta p_1}$ from adjacent node; ^eFinal calculated pressure function in the first iteration is used for the calculation in the second iteration; ^fOpposite flow direction than in previous iteration

correction is first, sign preceding the first iteration from adjacent contour for the conduit taken into consideration). The rules for sign of corrections ΔQ_2 are: (1). the algebraic operation for correction ΔQ_1 should be the opposite of its sign; i.e. add when the sign is minus. (2). the algebraic operation for corrections ΔQ_2 should be the opposite of their lower signs when their upper signs are the same as the sign in front of Q, and as indicated by their lower signs when their upper signs are opposite to the sign in front of Q. For details of sing of corrections consult paper of Brkić (2009) and article Corfield et al. (1974) from Gas Engineers Handbook.

3.1.1 Hardy Cross Method for the Pipeline Calculation

Hardy Cross, American engineer, developed method in 1936, later named after him (Cross 1936). Hardy Cross calculation for gas pipeline network is shown in Table 1 and for water network in Table 2. Only two iterations will be shown in details. Flow Q_1 calculated in first iteration become initial flow Q for second iteration. The plus or minus preceding the flow Q, indicates the direction of the conduit flow for the particular loop (Fig. 2). A plus sign denotes clockwise flow in the conduit within the loop; a minus sign, counter-clockwise. A flow correction ΔQ_1 as shown in Tables 1 and 2 is computed for each loop. This correction must be subtracted algebraically from the assumed gas flow according to specific rules explained previously.

Correction for each loop ΔQ_1 is calculated using first derivative of pressure function C for pipes defined by Renouard's equation for gas networks (5) and derivative of pressure drop Δp for pipes defined by Darcy-Weisbach for water networks (6) where flow Q is treated as variable:

$$F' = \frac{\partial (C)}{\partial Q} = \frac{\partial \left(\frac{4810 \cdot Q_n^{1.82} \cdot L \cdot \rho_r}{D_m^{4.82}} \right)}{\partial Q} = \frac{\partial (R \cdot Q^{1.82})}{\partial Q} \\ = \frac{1.82 \cdot 4810 \cdot Q_n^{0.82} \cdot L \cdot \rho_r}{D_{in}^{4.82}} = 1.82 \cdot R \cdot Q_n^{0.82} \quad (5)$$

$$F' = \frac{\partial (\Delta p)}{\partial Q} = \frac{\partial \left(\frac{8 \cdot \rho \cdot \lambda \cdot L \cdot Q^2}{\pi^2 \cdot D^5} \right)}{\partial Q} = \frac{\partial (R \cdot Q^2)}{\partial Q} = \frac{16 \cdot \rho \cdot \lambda \cdot L \cdot Q}{\pi^2 \cdot D^5} = 2 \cdot R \cdot Q \quad (6)$$

Derivative for loops is calculated using assumed loop shown in the Fig. 2 with no reference to direction. For the loop I this derivative for gas network (7) and for water network (8) can be written:

$$F'_I = \frac{\partial C_I(Q)}{\partial Q_I} = \frac{\partial (|-C_3(Q_3)| + |C_4(Q_4)| + |-C_7(Q_7)|)_I}{\partial Q_I} \\ = 1.82 \cdot 4810 \cdot \rho_r \cdot \left(\frac{Q_3^{0.82} \cdot L_3}{D_3^{4.82}} + \frac{Q_4^{0.82} \cdot L_4}{D_4^{4.82}} + \frac{Q_7^{0.82} \cdot L_7}{D_7^{4.82}} \right)_I \quad (7)$$

$$F'_I = \frac{\partial \Delta p_I(Q)}{\partial Q_I} = \frac{\partial (|-\Delta p_3(Q_3)| + |\Delta p_4(Q_4)| + |-\Delta p_7(Q_7)|)_I}{\partial Q_I} \\ = \frac{16 \cdot \rho}{\pi^2} \cdot \left(\frac{\lambda_3 \cdot L_3 \cdot Q_3}{D_3^5} + \frac{\lambda_4 \cdot L_4 \cdot Q_4}{D_4^5} + \frac{\lambda_7 \cdot L_7 \cdot Q_7}{D_7^5} \right)_I \quad (8)$$

In presented example loop I begins and ends in node II via pipes 3, 4, and 7.

Correction of flow (ΔQ_I) for each loop can be calculated (9):

$$\Delta Q_I = -\frac{F}{|F'|} \quad (9)$$

In the original Hardy Cross method, each loop correction is determined independently of other loops. Original Hardy Cross method is a sort of Newton–Raphson method but used to solve each single loop equation solely, one by one. Hence, Hardy Cross method is also known as the single contour adjustment method. In matrix form, original Hardy Cross method for the example network for water distribution from Fig. 1 can be noted as (10):

$$\begin{bmatrix} \frac{\partial F_I (|-Q_3|, |Q_4|, |-Q_7|)}{\partial (\Delta Q_I)} & 0 & 0 \\ 0 & \frac{\partial F_{II} (|Q_1|, |-Q_2|, |-Q_4|, |Q_5|)}{\partial (\Delta Q_{II})} & 0 \\ 0 & 0 & \frac{\partial F_{III} (|-Q_5|, |Q_6|, |Q_7|, |-Q_8|)}{\partial (\Delta Q_{III})} \end{bmatrix} \times \begin{bmatrix} \Delta Q_I \\ \Delta Q_{II} \\ \Delta Q_{III} \end{bmatrix} = \begin{bmatrix} F_I \\ F_{II} \\ F_{III} \end{bmatrix} \quad (10)$$

Using numerical values from Table 2 for the first iteration of calculation of water network matrix equation can be written (11):

$$\begin{bmatrix} 129435625 & 0 & 0 \\ 0 & 53910587 & 0 \\ 0 & 0 & 193795963 \end{bmatrix} \times \begin{bmatrix} \Delta Q_I \\ \Delta Q_{II} \\ \Delta Q_{III} \end{bmatrix} = \begin{bmatrix} -15928498 \\ 5040086 \\ 2070510 \end{bmatrix} \quad (11)$$

Then set of correction for water network calculation in the first iteration is $[\Delta Q_I, \Delta Q_{II}, \Delta Q_{III}]^T = [-12,306 \cdot 10^{-5}, 9,349 \cdot 10^{-5}, 1,068 \cdot 10^{-5}]^T$ which is identical as in Table 2.

3.1.2 Modified Hardy Cross Method for the Pipeline Calculation

Modified Hardy Cross method (somewhere called improved) is also known as the simultaneous loop adjustment method. As seen in Fig. 1, several loops have common pipes, so corrections to these loops will cause energy losses around more than one loop. In Fig. 1, pipe 4 belongs to two loops (loop I and II), pipe 7 to loop I and III, and finally pipe 5 to II and III. Modified Hardy Cross method is a sort of Newton–Raphson method used to solve unknown flow corrections taking into consideration whole system simultaneously. Epp and Fowler (1970) gave idea for this approach. To increase efficiency of Hardy Cross method zeros from non-diagonal term in matrix Eq. 10 will be replaced to include influence of pipes mutual with adjacent loop (12).

$$\begin{bmatrix} \frac{\partial F_I (|-Q_3|, |Q_4|, |-Q_7|)}{\partial (\Delta Q_I)} & -\left| \frac{\partial F_I (|-Q_4|)}{\partial (\Delta Q_{II})} \right| & -\left| \frac{\partial F_I (|Q_7|)}{\partial (\Delta Q_{III})} \right| \\ -\left| \frac{\partial F_{II} (|-Q_4|)}{\partial (\Delta Q_I)} \right| & \frac{\partial F_{II} (|Q_1|, |-Q_2|, |-Q_4|, |Q_5|)}{\partial (\Delta Q_{II})} & -\left| \frac{\partial F_{II} (|-Q_5|)}{\partial (\Delta Q_{III})} \right| \\ -\left| \frac{\partial F_{III} (|Q_7|)}{\partial (\Delta Q_I)} \right| & -\left| \frac{\partial F_{III} (|-Q_5|)}{\partial (\Delta Q_{II})} \right| & \frac{\partial F_{III} (|-Q_5|, |Q_6|, |Q_7|, |-Q_8|)}{\partial (\Delta Q_{III})} \end{bmatrix} \times \begin{bmatrix} \Delta Q_I \\ \Delta Q_{II} \\ \Delta Q_{III} \end{bmatrix} = \begin{bmatrix} F_I \\ F_{II} \\ F_{III} \end{bmatrix} \quad (12)$$

Presented matrix is symmetrical; for example (13):

$$\frac{\partial F_I(Q_7)}{\partial (\Delta Q_{III})} = \frac{\partial F_{III}(Q_7)}{\partial (\Delta Q_I)} \quad (13)$$

This is because pipe 7 is mutual for two adjacent loops (loop I and loop III). Non-diagonal terms have always opposite sign than diagonal. Spatial networks common for ventilation systems in buildings or mines are exceptions (Brkić 2009).

Using numerical values from Table 2 for the first iteration of calculation of water network matrix equation can be written (14):

$$\begin{bmatrix} 12943562525 & -4978573 & -70157780 \\ -4978573 & 53910587 & -429677 \\ -70157780 & -429677 & 193795963 \end{bmatrix} \times \begin{bmatrix} \Delta Q_I \\ \Delta Q_{II} \\ \Delta Q_{III} \end{bmatrix} = \begin{bmatrix} -159284985 \\ 50400861 \\ 20705102 \end{bmatrix} \quad (14)$$

First two iterations from our example using the modified Hardy Cross method is shown in Table 3 for gas network and in Table 4 for water network.

3.1.3 Modified Andrijašev Method for the Pipeline Calculation

Andrijašev method can be used in the formulation as in the original Hardy Cross method and as in Modified Hardy Cross method. Here it will be given in notation as improved method because this approach shows better convergence performance (example for two iteration of gas network calculation is shown in Table 5 and for water network in Table 6). It can be noted that some pipes in Tables 1, 2, 3 and 4 received only one correction per iteration (for example pipe 3 in contour I). This means that pipe 3 belongs only to one loop. In method of M.M. Andrijašev contours can be defined to include few loops. Thus, contours can be defined in other way and then each pipe in the network belongs to two contours (Fig. 3). Loop is not synonym with contour in M.M. Andrijašev method as in Hardy Cross approach. Andrijašev's contour will be marked with special sign ($^{\circ}$).

Now contour I $^{\circ}$ (red circuit in Fig. 3) starting and ending in node I via pipes 4, 5, 6, 8, 3, contour II $^{\circ}$ (green circuit in Fig. 3) starting and ending in referent node via pipes 1, 6, 8, 7, 4, 2, and finally contour III $^{\circ}$ (blue circuit in Fig. 3) starting and ending in referent node via pipes 1, 5, 7, 3, 2.

Matrix formulation of this method for example gas network can be written as(15):

$$\begin{bmatrix} \frac{\partial F_I^{\circ}(|-Q_3|, |Q_4|, |-Q_5|, |Q_6|, |-Q_8|)}{\partial (\Delta Q_I^{\circ})} & \frac{\partial F_I^{\circ}(|-Q_3|, |Q_6|, |Q_8|)}{\partial (\Delta Q_{II}^{\circ})} & \frac{\partial F_I^{\circ}(|Q_3|, |-Q_5|)}{\partial (\Delta Q_{III}^{\circ})} \\ \frac{\partial F_{II}^{\circ}(|-Q_4|, |Q_6|, |Q_8|)}{\partial (\Delta Q_I^{\circ})} & \frac{\partial F_{II}^{\circ}(|-Q_1|, |-Q_2|, |-Q_4|, |Q_6|, |Q_7|, |Q_{-8}|)}{\partial (\Delta Q_{II}^{\circ})} & \frac{\partial F_{II}^{\circ}(|Q_1|, |Q_2|, |-Q_7|)}{\partial (\Delta Q_{III}^{\circ})} \\ \frac{\partial F_{III}^{\circ}(|Q_3|, |-Q_5|)}{\partial (\Delta Q_I^{\circ})} & \frac{\partial F_{III}^{\circ}(|Q_1|, |Q_2|, |-Q_7|)}{\partial (\Delta Q_{II}^{\circ})} & \frac{\partial F_{III}^{\circ}(|Q_1|, |-Q_2|, |-Q_3|, |Q_5|, |-Q_7|)}{\partial (\Delta Q_{III}^{\circ})} \end{bmatrix} \times \begin{bmatrix} \Delta Q_I^{\circ} \\ \Delta Q_{II}^{\circ} \\ \Delta Q_{III}^{\circ} \end{bmatrix} = \begin{bmatrix} F_I^{\circ} \\ F_{II}^{\circ} \\ F_{III}^{\circ} \end{bmatrix} \quad (15)$$

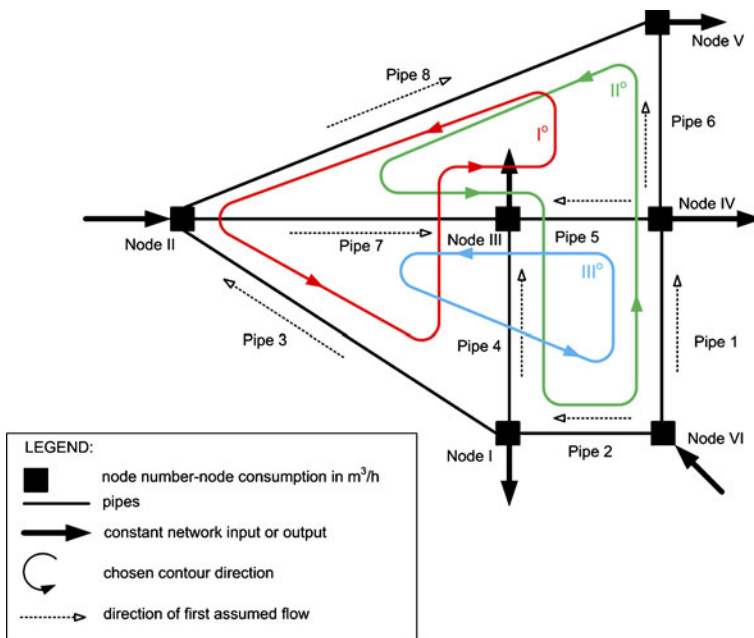


Fig. 3 Contours for method of M.M. Andrijašev calculation

Here has to be very careful because non-diagonal terms are not always negative as in modified Hardy Cross method (16):

$$\begin{bmatrix} 40489565894 & 25862392143 & 11719977603 \\ 25862392143 & 52949765748 & -4166255733 \\ 11719977603 & -4166255733 & 36379348868 \end{bmatrix} \times \begin{bmatrix} \Delta Q_I^o \\ \Delta Q_{II}^o \\ \Delta Q_{III}^o \end{bmatrix} = \begin{bmatrix} -3277663792 \\ 1492371894 \\ -2518358643 \end{bmatrix} \quad (16)$$

For example term in first row and second column from (16) is $25862392143 = 4298435730 + 22897756035 - 1333799622$ according to Table 5. Same value has term in second row, first column, etc. Presented matrix is symmetrical. Similar can be done for water network (Table 6).

M.M. Andrijašev method does not improve convergence properties compared with Modified Hardy Cross method. It only complicates calculation of non-diagonal terms in the matrix which is used for calculation. Furthermore, calculation of single-flat pipeline networks using M.M. Andrijašev method is equally complex as calculation of spatial, multidimensional network, using Modified Hardy Cross method (Brkić 2009).

3.2 Methods Based on Solution of the Node Equations

Nodes are sometimes also referred to as junctions, points or vertices.

Shamir and Howard (1968) introduced first node oriented method. For the node oriented methods second Kirchhoff's law must be satisfied for each loop in all iterations. First Kirchhoff's law for each node must be satisfied with acceptable

tolerance at the end of the calculation. Here will be presented only one method. Improvement of presented method is done according to the same idea as used to improve original Hardy Cross method but here applied to the matrix solution for the node equations.

3.2.1 Node Method

Pipe equations in previous text were expressed as $\Delta p = F(Q)$ for waterworks, or $C = p_1^2 - p_2^2 = F(Q)$ for gas networks. These relations can be rewritten in form as $Q = f(\Delta p)$ for waterworks or $Q = f(C)$ for gas networks. After that, Renouard's Eq. 1 can be rearranged (17):

$$f_g = Q(C) = \left(\frac{(p_1^2 - p_2^2) \cdot D_{in}^{4.82}}{4810 \cdot L \cdot \rho_r} \right)^{\frac{1}{1.82}} = \left(\frac{C \cdot D_{in}^{4.82}}{4810 \cdot L \cdot \rho_r} \right)^{\frac{1}{1.82}} \quad (17)$$

Similar, can be done for Darcy-Weisbach Eq. 18:

$$f_w = Q(\Delta p) = \sqrt{\frac{(p_1 - p_2) \cdot D_{in}^5 \cdot \pi^2}{8 \cdot \lambda \cdot L \cdot \rho}} = \sqrt{\frac{\Delta p \cdot D_{in}^5 \cdot \pi^2}{8 \cdot \lambda \cdot L \cdot \rho}} \quad (18)$$

In Node method, (C) for each pipe in a gas network and (Δp) for each pipe in a water network has to be assumed, not flows. These assumed functions of pressure must be chosen to satisfy second Kirchhoff's law (Fig. 4). First Kirchhoff's law will be fulfilled

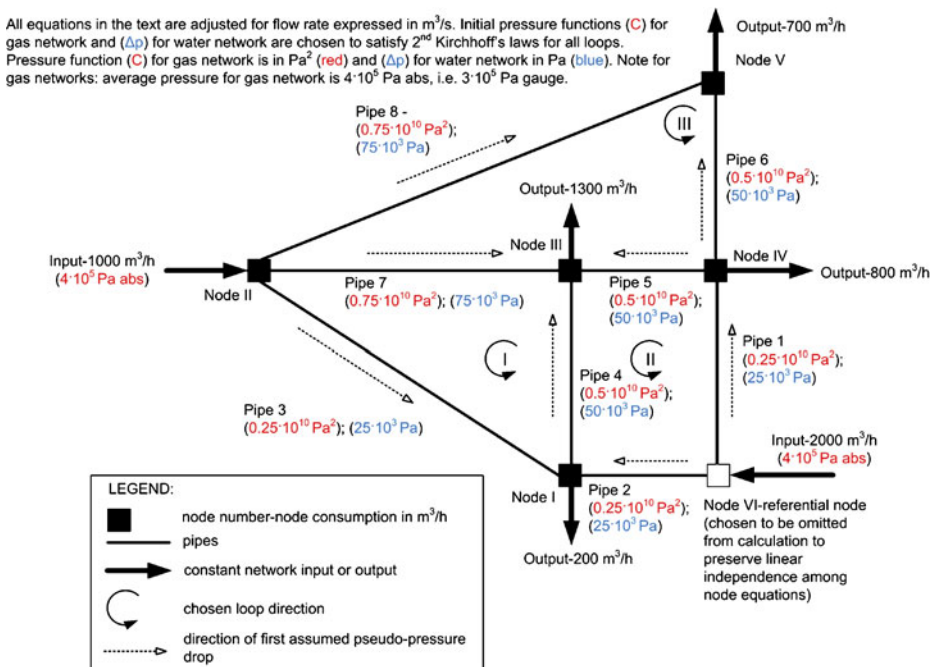


Fig. 4 Example of pipeline network with three loops adjusted for node oriented methods

with demanded tolerance at the end of calculation. For gas network, correction of C , noted as ΔC must be calculated, and for water network, correction of Δp , here noted as $\Delta_{\Delta p}$. Same algebraic rules as for loop oriented methods are valid. Correction ΔC or $\Delta_{\Delta p}$ will be calculated using first derivative of $Q = f(\Delta p)$ for waterworks and $Q = f(C)$ for gas networks, where Δp , i.e. C is treated as variable (19):

$$\Delta x = -\frac{f}{|f'|} \quad (19)$$

In previous equation x is Δp or C . Example network adjusted for node oriented calculation is shown in Fig. 4 (red letters for gas network and blue for water network).

Calculation of the network using non-improved Node method is not shown here.

3.2.2 Modified Node Method

Modified Node method is also referred to as the simultaneous node adjustment method. For the improvement of Node method, same idea as for the improvement of the Hardy Cross method is used. One node must be chosen to be omitted from the calculation shown in Tables 7 and 8, because linear independency among node equations is preserved in that way (Mathews and Köhler 1995; Mah and Shacham 1978). More details about graph theory of networks will be explained in further text (Section 3.3). Gas network calculation after Modified Node method is shown in Table 7.

Corrections in the Tables 7 and 8 are calculated using matrix Eq. 20:

$$\begin{bmatrix} f'_1(|+x_2|, |+x_3|, |-x_4|) & -|f'_1(x_3)| & -|f'_1(x_4)| \\ -|f'_2(x_3)| & f'_2(|-x_3|, |-x_7|, |-x_8|) & -|f'_2(x_7)| \\ -|f'_3(x_4)| & -|f'_3(x_7)| & f'_3(|+x_4|, |+x_5|, |+x_7|) \\ 0 & 0 & -|f'_4(x_5)| \\ 0 & -|f'_5(x_8)| & 0 \\ 0 & 0 & 0 \\ 0 & -|f'_2(x_8)| & 0 \\ -|f'_3(x_5)| & 0 & 0 \\ f'_4(|+x_1|, |-x_5|, |-x_6|) & -|f'_4(x_6)| & 0 \\ -|f'_5(x_6)| & f'_5(|+x_6|, |+x_8|) & 0 \end{bmatrix} \quad (20)$$

$$\times \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \\ \Delta x_5 \end{bmatrix} = \begin{bmatrix} \sum Q_2 + Q_3 - Q_4 - Q_I \\ \sum -Q_3 - Q_7 - Q_8 + Q_{II} \\ \sum Q_4 + Q_5 + Q_7 - Q_{III} \\ \sum Q_1 - Q_5 - Q_6 - Q_{IV} \\ \sum Q_6 + Q_8 - Q_V \end{bmatrix}$$

When non-diagonal terms in the first matrix (21) are equalized with zero, results are equal as for Node method in basic form (19). For the first iteration, numerical values for the matrix equation are extracted from Table 6 (21):

$$\begin{bmatrix} 3.69 \cdot 10^{-10} & -6.21 \cdot 10^{-11} & -6.28 \cdot 10^{-11} & 0 & 0 \\ -6.21 \cdot 10^{-11} & 1.28 \cdot 10^{-10} & -4.19 \cdot 10^{-11} & 0 & -2.46 \cdot 10^{-11} \\ -6.28 \cdot 10^{-11} & -4.18 \cdot 10^{-11} & 3.40 \cdot 10^{-10} & -2.36 \cdot 10^{-10} & 0 \\ 0 & 0 & -2.36 \cdot 10^{-10} & 3.54 \cdot 10^{-10} & -3.30 \cdot 10^{-11} \\ 0 & -2.46 \cdot 10^{-11} & 0 & -3.30 \cdot 10^{-11} & 5.76 \cdot 10^{-11} \end{bmatrix} \times \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \\ \Delta x_5 \end{bmatrix} = \begin{bmatrix} +0.7665 \\ -0.9121 \\ +2.9302 \\ -2.2805 \\ +0.4417 \end{bmatrix} \quad (21)$$

For the node oriented calculation applied for water networks, pressure function (C) must be replaced with pressure drop (Δp). Further analogy is clear (Table 8).

3.3 Node-Loop Method

Conditionally, Node-Loop method can be sorted in the group of methods based on solution of the loop equations according to the previous discussion, but better solution is to be treated as combination of the loop and node oriented methods as its name unambiguously suggest. Node-Loop method is also known as the flow adjustment method.

Wood and Charles (1972) developed the flow adjustment method by coupling the loop equations with the node equations. Wood and Rayes (1981) improved this method. Rather than solve for loop corrections, in this method, conservation of energy around a loop is written directly in the terms of the pipe flow rates. Final result after this method is not flow correction, but even better flow itself.

Pipeline network with three loops and six nodes is used as example in this article. Graph has X branches (pipes) and Y nodes where in our example, $X = 8$ and $Y = 6$. Graph with Y nodes (in our case 6) has $Y - 1$ independent nodes (in our case 5) and $X - Y + 1$ independent loops (in our case 3). Tree is a set of connected branches chosen to connect all nodes, but not to make any closed path (not to form a loop). Branches, which do not belong to a tree, are links (number of links are $X - Y + 1$). Loops in the network are formed using pipes from tree and one more chosen among the link pipes. Number of the loops is determined by number of links. Network from example has six nodes and five independent nodes. One node can be omitted from calculation while no information on the topology in that way will be lost. Rows in the node matrix with all node included are not linearly independent. To obtain linear independence any row of the node matrix has to be omitted. For example, pipe 6 is between node IV and V, and reasonable assumption is that if node IV is output node for flow through pipe 6, then node V must be input node for flow through this pipe.

In our example, node VI will be omitted. First Kirchhoff's law for the initial flow pattern shown in Fig. 2 can be written using set of Eq. 22:

$$\begin{aligned}
 \text{node}_I &\sim -Q_I + Q_2 - Q_3 - Q_4 = 0 \\
 \text{node}_{II} &\sim Q_{II} + Q_3 - Q_7 - Q_8 = 0 \\
 \text{node}_{III} &\sim -Q_{III} + Q_4 + Q_5 + Q_7 = 0 \\
 \text{node}_{IV} &\sim -Q_{IV} + Q_1 - Q_5 - Q_6 = 0 \\
 \text{node}_V &\sim -Q_V + Q_6 + Q_8 = 0 \\
 \text{node}_{VI} &\sim Q_{VI} - Q_1 - Q_2 = 0
 \end{aligned} \tag{22}$$

Node VI will be omitted from the node matrix to assure linear independency of the rows as shown in Fig. 4 (23):

$$[N] = \begin{bmatrix} 0 & +1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & +1 & +1 & 0 & +1 & 0 \\ +1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & +1 \end{bmatrix} \tag{23}$$

First Kirchhoff's law must be fulfilled in all iterations for all nodes. Second Kirchhoff's law for the initial water flow pattern shown in the Fig. 1 can be written using set of Eq. 24:

$$\begin{aligned}
 \text{loop}_I &\sim -F_3 + F_4 - F_7 = F_I \\
 \text{loop}_{II} &\sim F_1 - F_2 - F_4 + F_5 = F_{II} \\
 \text{loop}_{III} &\sim -F_5 + F_6 + F_7 - F_8 = F_{III}
 \end{aligned} \tag{24}$$

Second Kirchhoff's law for the initial flow pattern shown in the Fig. 1 also can be noted in matrix form using loop matrix (25):

$$[L] = \begin{bmatrix} 0 & 0 & -1 & +1 & 0 & 0 & -1 & 0 \\ +1 & -1 & 0 & -1 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & +1 & +1 & -1 \end{bmatrix} \tag{25}$$

Second Kirchhoff's law must be fulfilled for all loops at the end of calculation with demanded accuracy (i.e. $F_I \rightarrow 0$, $F_{II} \rightarrow 0$ and $F_{III} \rightarrow 0$).

In Node-Loop method these two matrices become one with some modifications. The nodes and the loops equations shown in previous text here will be united in one coherent system by coupling these two set of equations. To introduce matrix calculation, the node-loop matrix [NL], matrix of calculated flow in observed iteration [Q], and [V] matrix will be defined (26):

$$[V] = \begin{bmatrix} |Q_I| \\ -|Q_{II}| \\ |Q_{III}| \\ |Q_{IV}| \\ |Q_V| \\ -F_I + (-|F_3| \cdot |F'_3| + |F_4| \cdot |F'_4| - |F_7| \cdot |F'_7|) \\ -F_{II} + (|F_1| \cdot |F'_1| - |F_2| \cdot |F'_2| - |F_4| \cdot |F'_4| + |F_5| \cdot |F'_5|) \\ -F_{III} + (-|F_5| \cdot |F'_5| + |F_6| \cdot |F'_6| + |F_7| \cdot |F'_7| - |F_8| \cdot |F'_8|) \end{bmatrix} \tag{26}$$

Sign minus preceding some of the flows Q in matrix $[V]$ means that this particular Q is not consumption (sing minus represent inlet of fluid). Node-Loop matrix $[NL]$ can be defined using node matrix, loop matrix and first derivative of Renouard's function for gas pipes or of Darcy-Weisbach for water networks, as follows (27):

$$[NL] = \begin{bmatrix} 0 & +1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & +1 & +1 & 0 & +1 & 0 \\ +1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & +1 \\ 0 & 0 & -1 \cdot |F'_3| & +1 \cdot |F'_4| & 0 & 0 & -1 \cdot |F'_7| & 0 \\ +1 \cdot |F'_1| & -1 \cdot |F'_2| & 0 & -1 \cdot |F'_4| & +1 \cdot |F'_5| & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \cdot |F'_5| & +1 \cdot |F'_6| & +1 \cdot |F'_7| & -1 \cdot |F'_8| \end{bmatrix} \quad (27)$$

Further, vector $[Q]$ of the unknown flows can be calculated in the first iteration (28).

$$[Q] = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = inv[NL] \times [V] \quad (28)$$

Possible sign minus in a front of flow Q in the matrix $[Q]$ means that calculated flow direction is opposite compared to shown one in the previous iteration (or in the Fig. 1 in our case for the first calculated values of flows compared with initials flow pattern). If all values of pressure drops sums calculated after (24) are not approximate zero with reasonable accuracy, calculation has to be repeated using values calculated in previous iteration. At the end of calculation calculated set of flows $[Q]$ stays unchanged.

Here will be used values from Table 1 for gas network (29) and from Table 2 for water network (30) as example. These values are valid for the first iteration. First five

rows (first matrix) are from node equation, and next three is from loop equation but multiplied with first derivate marked in tables as F':

$$\begin{bmatrix}
 0 & 1 & 1 & -1 \\
 0 & 0 & -1 & 0 \\
 0 & 0 & 0 & 1 \\
 1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & -11839776055 & 1333799622 \\
 8812326713 & -1314432601 & 0 & -1333799622 \\
 0 & 0 & 0 & 0
 \end{bmatrix}
 \begin{bmatrix}
 0 & 0 & 0 & 0 \\
 0 & 0 & -1 & -1 \\
 1 & 0 & 1 & 0 \\
 -1 & -1 & 0 & 0 \\
 0 & 1 & 0 & 1 \\
 0 & 0 & -14293015047 & 0 \\
 119798452.1 & 0 & 0 & 0 \\
 -119798452.1 & 4298435730 & 14293015047 & -22897756035
 \end{bmatrix} \quad (29)$$

$$\times \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = \begin{bmatrix} 0.055 \\ -0.277 \\ 0.361 \\ 0.222 \\ 0.194 \\ -2988241676 \\ 923187587 \\ 300557365 \end{bmatrix}$$

$$\begin{bmatrix}
 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\
 0 & 0 & -1 & 0 & 0 & 0 & -1 & -1 \\
 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\
 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
 0 & 0 & -54299272 & 4978573 & 0 & 0 & -70157780 & 0 \\
 426229242 & -5879434 & 0 & -4978573 & 429677 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & -429677 & 16717415 & 70157780 & -106491089
 \end{bmatrix}$$

$$\times \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = \begin{bmatrix} 0.055 \\ -0.277 \\ 0.361 \\ 0.222 \\ 0.194 \\ -15928498 \\ 5040086 \\ 2070510 \end{bmatrix}$$

(30)

Table 9 Final flows for network presented in this paper

Flows (m ³ /h)								
Pipe	1	2	3	4	5	6	7	8
Water	902.27	1,097.73	94.86	802.87	-146.23 ^a	248.50	643.36	451.50
Gas	913.72	1,086.28	82.01	804.27	-137.86 ^a	251.58	633.60	448.42

^aSign minus means flow direction opposite than first assumed in Fig. 2

For gas network last three rows are calculated as follows using values from Table 1,

$$\begin{aligned}
 \text{for loop I;} & \quad -2988241676 = -3644197165 + (-0.194 \cdot 11839776055 + 0.027 \cdot \\
 & \quad 1333799622 + (-0.305) \cdot 14293015047) \\
 \text{for loop II;} & \quad 923187587 = 1125838521 + (0.277 \cdot 8812326713 + (-0.277 \cdot \\
 & \quad 1314432601) + (-0.027) \cdot 1333799622) + 0.027 \cdot 119798452) \\
 \text{for loop III;} & \quad 300557365 = 366533372 + (-0.027 \cdot 119798452 + 0.027 \cdot \\
 & \quad 4298435730 + 0.305 \cdot 14293015047 + (-0.166 \cdot 22897756035))
 \end{aligned}$$

After first iteration for gas network vector of flows is $[0.198409265, 0.357146291, 0.043307855, 0.25828288, -0.094469817, 0.07065686, 0.197298048, 0.123787585]^T$. Minus in front of flow in pipe 5 means: change assumed flow direction from previous iteration. After first iteration for water network vector of flows is $[0.197719798, 0.357835758, 0.052496097, 0.249784106, -0.092806697, 0.068304272, 0.204133702, 0.126140172]^T$. Flows are expressed in m³/s.

Excellent book for waterworks calculation by Boulos et al. (2006) can be recommended for further reading. In this book, authors instead to omit one node in the node matrix to preserve linear independency of rows in this matrix introduce one pseudo-loop in loop matrix. This procedure is not practical because at least two nodes with equal pressure must be found in the network. This is not always possible. Further in that way the node-loop matrix has two additional rows which could be avoided. Mathews and Köhler (1995) in his discussion use simplest way, i.e. they omit one row.

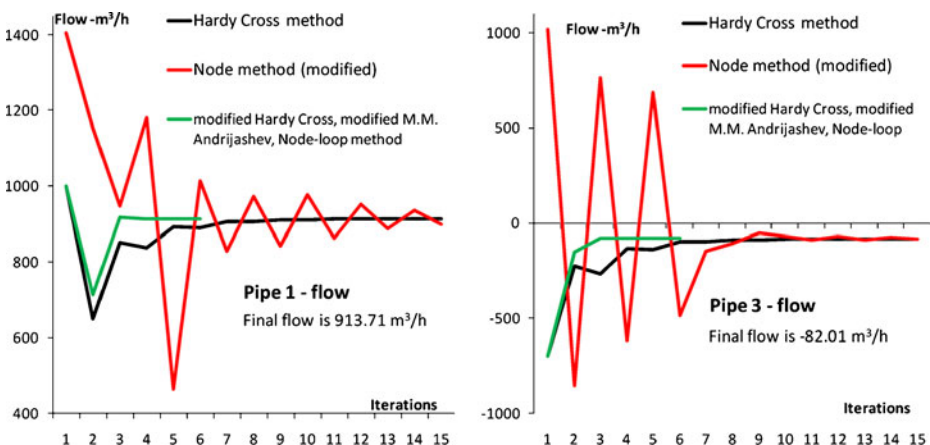
**Fig. 5** Comparisons of convergence for presented methods (gas network)

Table 10 Velocities for water and gas for calculated flows from example network

Velocity (m/s)								
Pipe	1	2	3	4	5	6	7	8
Water	21.0	15.4	2.2	18.6	−1.7 ^a	9.4	14.9	13.2
Gas	5.3	3.8	0.5	4.7	−0.4 ^a	2.4	3.7	3.3

^aSign minus means flow direction opposite than first assumed in Fig. 2

4 Comparison of Solution Techniques for Looped Piping Networks

Final flows are unique after all presented methods, and will be listed in Table 9, both for water and for gas network.

Each method has advantages and shortcomings. Convergence performances will be compared for all presented methods in Fig. 5. Note that Modified Node method cannot be compared literary because initial values cannot be equalized. In all other methods initial patterns are given in the form of flows and equalized, while in Node method initial pattern is in the form of pressures (better to say function of pressure).

Best way to compare water and gas distribution network is to compare velocity of gas and water through pipes. For water, this can be done using (31):

$$v_w = \frac{4 \cdot Q}{D_{in}^2 \cdot \pi} \quad (31)$$

Velocity of gaseous fluids depends on the pressure in pipe since they are compressible (32):

$$v_g = \frac{4 \cdot p_n \cdot Q_n}{p \cdot D_{in}^2 \cdot \pi} \quad (32)$$

Velocities for water and gas for calculated flows through the pipes in our example is listed in Table 10.

5 Conclusion

Comparison between analyzed methods was carried out, taking as a criterion the number of iteration to achieve final results. Modified Hardy Cross method, Modified M.M. Andrijahshev method and Node-Loop method have equal performances according to above adopted criterion. For more complex networks, using Node-Loop method, number of required iterations is smaller even compared with Modified Hardy Cross method. Among these three methods, Node-Loop method is superior because it does not require complex numerical scheme for algebraic addition of corrections in each of iterations. In Node-Loop method, final results of each of the iterations are flows directly and not correction of flows. Modified Andrijašev method is more complex compared with Modified Hardy Cross method but without any improvement in the properties of the convergence. Node method has the worst performance of convergence, but this method is different in its approach compared with all other methods shown in this paper. Node method cannot be rejected easily based only on calculation shown in this paper. Hardy Cross method in original form

has historical value and should be replaced with Modified Hardy Cross method, or even better with Node-Loop method.

Node-Loop method, presented among others in the text, is powerful numerical procedure for calculation of flows in looped fluid distribution networks. Main advantage is that flow in each pipe can be calculated directly, which is not possible after other available methods. In other methods, results of calculation are flow corrections which have to be added to flows calculated in previous iteration using complex algebraic rules. Node-Loop method is recommended to be used.

In the real network, consumers are not concentrated in the nodes. This can cause two-way flow in some pipes (Brkić 2009). This can cause disturbance in convergence properties of certain method. In such case, method should be changed. Some details on convergence properties can be found in the paper of Mah (1974), Mah and Lin (1980) and Altman and Boulos (1995). Simulation problem today can be solved using different software (Huddleston et al. 2004; Lopes 2004) in which can be implemented shown methods. Method for solution of pipe equations proposed by Hamam and Brameller (1971) for gas networks and Todini and Pilati (1988) for water networks is available, but not shown here.

Notations

p	Pressure (Pa)
Δp	Pressure drop (Pa)
C	Pressure function in gas pipelines (Pa^2)
F	Pressure function C or Δp (Pa^2 or Pa)
f	Flow function (m^3/s)
λ	Darcy friction factor (-)
L	Pipe length (m)
ρ	Water density (kg/m^3)
ρ_r	Relative gas density (-)
Q	Flow (m^3/s)
D	Pipe diameter (m)
Re	Reynolds number (-)
ε	Pipe roughness (m)
ΔQ	Correction of flow (m^3/s)
Δx	Correction of pressure function (Pa^2 or Pa)
π	~ 3.1415

subscripts

n	Normal ($\sim 101,325$ Pa, ~ 274 K)
i	Inner
r	Relative
w	Water
g	Gas

Acknowledgements Financial support due scholarship program for PhD research given by the Ministry of Science and Technological Development of Republic of Serbia is gratefully acknowledged.

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