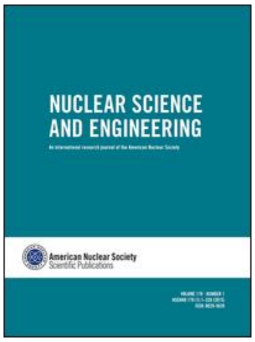

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Transient Two-Phase Two-Component Water-Steam-Air Flow

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Transient Two-Phase Two-Component Water-Steam-Air Flow

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A mathematical model of a transient one-dimensional two-phase flow consisting of water, steam, and air is described. In the general case, a system of four partial differential equations of hyperbolic type was derived. The expression obtained for the velocity of sound is in good agreement with the results of the experiment. There is also good agreement with existing theories for the limiting cases of a missing phase or component. A numerical example demonstrating the application of the theory is shown.

I. INTRODUCTION

In analyzing a loss-of-coolant accident (LOCA) in nuclear power stations with water-cooled reactors, one comes to the problem of mathematically describing the spreading of this accident into the containment. During the last two decades, some programs were developed, such as CONTEMPT (Ref. 1), ZOCO (Ref. 2), SONJA (Refs. 3 and 4), etc. In these programs each area is considered as a separate volume filled with water, steam, and air. The thermodynamic parameters of this volume are those of its center. All volumes in the particular application have finite dimensions and the perturbation propagates with finite velocity. It is possible to have a significant difference in pressure and temperature at two different points in one and the same volume at the same time (see Fig. 1). Especially for large volumes, the analysis with the type of programs mentioned above is neither the most precise nor the most conservative. The division of the large volumes into a number of zones and the use of these programs is limited by their basic conception and turns out to be ineffective. Kanzleiter[5] points out the necessity of developing programs that take into account the spatial distribution of the parameters of the state. The best example of such a program is BEACON (Ref. 6). A transient two-phase two-component one-, two-, and three-dimensional flow is described. It is allowed to have one continuous phase with the other discretely distributed inside it. The following system of field equations is used[7] (a Nomenclature appears on p. 219):

[1]C. F. CARMICHEL and S. A. MARKO, "CONTEMPT-PS—A Digital Computer Code for Predicting the Pressure-Temperature History Within a Pressure-Suppression Containment Vessel in Response to a Loss-of-Coolant Accident," IDO-17252, Phillips Petroleum Company (1969).

[2]G. MANSFELD, "ZOCO-VI—Ein Rechenprogramm zur Berechnung von Zeitlichen und örtlichen Druckverteilungen in Volldrucksicherheitsbehältern wassergekülter Kernreaktoren," MRR-P-14, Laboratorium für Reaktorregelung und Anlagensicherung, Garching; Lehrstul für Reaktordynamik und Reaktorsicherheit, Technische Universität Munchen, Germany (1974).

[3]N. I. KOLEV, *Nucl. Energy*, **13**, 21 (1981).

[4]N. I. KOLEV, *Nucl. Energy*, **11**, 53 (1980).

[5]T. KANZLEITER, "Experimentelle Ergebnisse zur Ausbreitung von Druckwellen und Dampffronten in einem DWR-Containment bei einem Kuhlmittelverlustunfall," *Reaktortagung Mannheim*, **1**, *4*, 171 (1977).

[6]C. R. BROADUS, S. W. JAMES, W. H. LEE, J. F. LIME, and R. A. PATE, "BEACON/MOD2, A CDC 7600 Computer Program for Analyzing the Flow of Mixed Air, Steam, and Water in a Containment System," CDAP TR 002, EG&G Idaho (1977).

[7]F. H. HARLOW and A. A. AMSDEN, *J. Comp. Phys.*, **17**, 19 (1975).

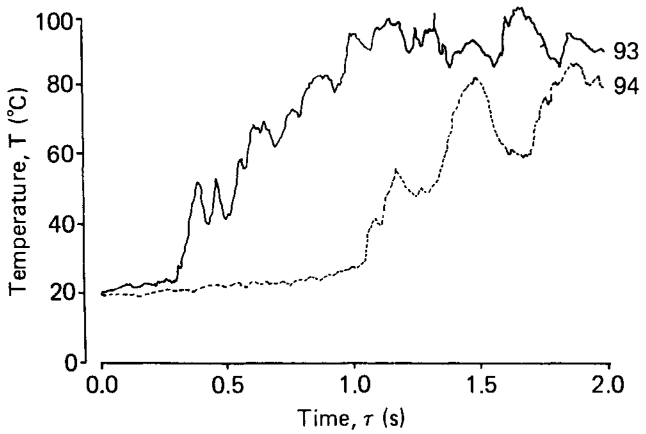

Fig. 1. The temperature T, as a function of time τ at points 93 and 94 measured by Kanzleiter (Ref. 5) during LOCA experiments.

$$\frac{\partial}{\partial \tau}(\alpha\rho_{1g}) + \nabla(\alpha\rho_{1g}w_g) = \mu_{1fg} - \mu_{1gf} \tag{1}$$

$$\frac{\partial}{\partial \tau}[(1-\alpha)\rho_{1f}] + \nabla[(1-\alpha)\rho_{1f}w_f] = \mu_{1gf} - \mu_{1fg} \tag{2}$$

$$\frac{\partial}{\partial \tau}(\alpha\rho_{2g}) + \nabla(\alpha\rho_{2g}w_g) = \mu_{2fg} - \mu_{2gf} \tag{3}$$

$$\frac{\partial}{\partial \tau}[(1-\alpha)\rho_{2f}] + \nabla[(1-\alpha)\rho_{2f}w_f] = \mu_{2gf} - \mu_{2fg} \tag{4}$$

$$\begin{aligned}\frac{\partial}{\partial \tau}(\alpha\rho_g w_g) &+ \nabla(\alpha\rho_g w_g^2) + \alpha\nabla p \\ &= f_g + k(w_f - w_g) + \alpha\rho_g g\cos\varphi \\ &\quad + \mu_{fg}w_f - \mu_{gf}w_g \end{aligned} \tag{5}$$

$$\begin{aligned}\frac{\partial}{\partial \tau}[(1-\alpha)\rho_f w_f] &+ \nabla[(1-\alpha)\rho_f w_f^2] + (1-\alpha)\nabla p \\ &= f_f + k(w_g - w_f) + (1-\alpha)\rho_f g\cos\varphi \\ &\quad + \mu_{gf}w_g - \mu_{fg}w_f \end{aligned} \tag{6}$$

$$\begin{aligned}\alpha\rho_g&\left[\frac{\partial u_g}{\partial \tau} + \nabla(w_g u_g) - u_g\nabla w_g\right] \\ &= \dot q'''_{g,e} - \dot q'''_{g,c} + C(T_f - T_g) + k(w_f - w_g)^2 \\ &\quad + \nabla(\lambda_g\alpha\nabla T_g) + Ef_g - p\nabla[\alpha w_g + (1-\alpha)w_f] \end{aligned} \tag{7}$$

$$\begin{aligned}(1-\alpha)\rho_f&\left[\frac{\partial u_f}{\partial \tau} + \nabla(w_f u_f) - u_f\nabla w_f\right] \\ &= \dot q'''_{f,c} - \dot q'''_{f,e} + \dot q'''_f + C(T_g - T_f) \\ &\quad + \nabla[\lambda_f(1-\alpha)\nabla T_f] + Ef_f \ . \end{aligned} \tag{8}$$

The system is used in this form for numerical integration. For this system of equations, the type of eigenvalues of the characteristic matrix and the corresponding theory for the critical flow or the speed of sound in the mixture have not been studied. These are elements of the general model, which are important for adequate understanding of the physics of the process as well as for the numerical integration.

The object of the study reported in this paper is the two-phase two-component flow of water-steam-air. Our purpose is to obtain a hyperbolic system of differential equations describing the process and transform it into a canonical one, so that the well-known method of characteristics can be used in any of its modifications. We look for a theory, corresponding to the simplifying assumptions used, which will describe the critical flow and the speed of sound in the mixture. The model must be able to describe the processes in the sections of a nuclear power station with water-cooled reactors during the blowdown phase of the LOCA.

II. THE MODEL

The model is derived with the following basic simplifying assumptions.

a. Equal pressures exist in the two phases.

b. The two components of the gas phase (air/steam) obey Dalton's law.

c. There are equal temperatures in the two phases.

d. Equal velocities exist in the two phases.

e. In the presence of water, the steam is always saturated with respect to the corresponding temperature of the system.

f. The air is assumed to be a single noncondensable ideal gas.

g. The cross section of the channel is constant.

Some consequences of the assumptions are:

1. It follows from assumptions a through g that the heat and mass exchange processes between the two phases take place instantly.

2. The liquid phase is always supercooled in the presence of air with respect to the saturation temperature corresponding to the total pressure of the system. This follows from assumptions a, c, and e.

3. The supercooling decreases with decreasing air content due to assumption c. In the absence of air, there is no supercooling as the partial steam pressure becomes equal to the pressure of the system; i.e., the steam and the liquid phase are on the saturation line. Because of this, the expression obtained defining the speed of sound in the limiting case of absence of air gives values typical of a homogeneous equilibrium two-phase flow; in the case of a no-steam phase, it gives values typical of a homogeneous nonequilibrium two-phase flow. The general model has the same

properties in the case of these simplifying assumptions.

The following considerations lead us to these simplifying assumptions.

1. As in the blowdown phase, the flow is critical; the liquid phase of the two-phase system flowing out of the first circulatory pipeline is superheated, compared to the saturation temperature, corresponding to the pressure in the area where the discharge occurs. The characteristic time for restoring equilibrium in the neighborhood of the discharge is $\sim 10^{-3}$ to 10^{-2} s. The processes in the areas are considerably slower than those in the first cycle in the sense of gradients and eigenfrequencies. Thus the superheating of the liquid phase compared to the gaseous phase is unlikely. Since, in the blowdown phase, pressure builds up in the areas due to the energy release from the coolant, which has a higher specific energy, supercooling is also unlikely. Then the most probable assumption is equal temperatures of the two phases. A good reason for this is the coincidence of equilibrium theory with the experiment reported by Brosche.[8] After the blowdown phase and startup of the spray condensing system, the liquid phase can be locally supercooled. Since the processes are much slower compared to the blowdown phase, the transient analysis of this phase of the LOCA in the containment is considerably simpler and is not discussed here.

2. The processes in the case of an abrupt change of the cross section in its neighborhood can be approximately considered as quasi-stationary; i.e., the convective changes in the dependent variables due to the changing cross section are considerably greater than the local changes with time. Solving the quasi-stationary problem is not complicated so we shall not discuss it. This is the reason for the presumption of a constant cross section.

We choose

$$\boldsymbol{U}^T = (T,p,w,\rho) \tag{9}$$

as the vector of the dependent variables. This vector and the simplifying assumptions completely determine the state of the flow. Taking into account assumptions a through e, we can write for the thermodynamic state of the flow:

$$\rho_L = p_L/(R_L T) \ ; \quad d\rho_L = \frac{1}{R_L T}dp_L - \frac{\rho_L}{T}dT = \frac{1}{R_L T}dp - \left(\frac{\rho_L}{T} + \frac{1}{R_L T}\frac{dp_D''}{dT}\right)dT \tag{10}$$

$R_L = 287.1$ [J/(kg·K)]

$$\rho_D'' = \rho_D''(T) \ ; \qquad d\rho_D = \frac{d\rho_D''}{dT}dT \tag{11}$$

$$\rho_f = \rho_f(T,p) \ ; \qquad d\rho_f \approx \frac{\partial \rho_f}{\partial T}dT \tag{12}$$

$$p_D = p_D''(T) \ ; \qquad dp_D = \frac{dp_D''}{dT}dT \tag{13}$$

$$p_L = p - p_D \ ; \qquad dp_L = dp - \frac{dp_D''}{dT}dT \tag{14}$$

$$h_L = h_L(p_L,T) \ ; \quad dh_L = \frac{\partial h_L}{\partial p_L}dp_L + \frac{\partial h_L}{\partial T}dT = \frac{\partial h_L}{\partial p_L}dp + \left(\frac{\partial h_L}{\partial T} - \frac{\partial h_L}{\partial p_L}\frac{dp_D''}{dT}\right)dT \tag{15}$$

$$h_D = h_D''(T) \ ; \qquad dh_D = \frac{dh_D''}{dT}dT \tag{16}$$

$$h_f = h_f(p,T) \ ; \qquad dh_f = \frac{\partial h_f}{\partial p}dp + \frac{\partial h_f}{\partial T}dT \ . \tag{17}$$

Assumption b yields

$$\alpha_L = \alpha_D = \alpha \ . \tag{18}$$

The equations for the conservation of mass, momentum, and energy for the air, steam, and water will be written separately. The conservation equations for the mass and energy are analogous to those used in Ref. 7. The energy conservation equations differ from those in Ref. 7. They have been derived[9] to be

$$\frac{\partial}{\partial\tau}(\alpha\rho_L A) + \frac{\partial}{\partial z}(\alpha\rho_L w_g A) = 0 \tag{19}$$

$$\frac{\partial}{\partial\tau}(\alpha\rho_D A) + \frac{\partial}{\partial z}(\alpha\rho_D w_g A) = \mu \tag{20}$$

$$\frac{\partial}{\partial\tau}[(1-\alpha)\rho_f A] + \frac{\partial}{\partial z}[(1-\alpha)\rho_f w_f A] = -\mu \tag{21}$$

$$\frac{\partial}{\partial\tau}(\alpha\rho_L w_g A) + \frac{\partial}{\partial z}(\alpha\rho_L w_g^2 A) + \alpha A\frac{\partial p_L}{\partial z} + \alpha\rho_L g A\cos\varphi + \frac{F_{RL}}{\Delta z} = 0 \tag{22}$$

$$\frac{\partial}{\partial\tau}(\alpha\rho_D w_g A) + \frac{\partial}{\partial z}(\alpha\rho_D w_g^2 A) + \alpha A\frac{\partial p_D}{\partial z} + \alpha\rho_D g A\cos\varphi + \frac{F_{RD}}{\Delta z} = \mu w_{ex} A \tag{23}$$

[8] D. BROSCHE, *Nucl. Eng. Des.*, **23**, 239 (1972).

[9] N. I. KOLEV, "Zweiphasen- Zweikomponentenströmung (Luft/Wasserdampf/Wasser) zwischen den Sicherheitsräumen der KKW mit wassergekühlten Reaktoren bei Kühlmittelverlusthavarie," Dissertation, Technische Universität, Dresden (1977).

$$\frac{\partial}{\partial \tau}[(1-\alpha)\rho_f w_f A] + \frac{\partial}{\partial z}[(1-\alpha)\rho_f w_f^2 A] + (1-\alpha)A\frac{\partial p}{\partial z} + (1-\alpha)\rho_f A g \cos\varphi + \frac{F_{Rf}}{\Delta z} = -\mu w_{ex} A \tag{24}$$

$$\frac{\partial}{\partial z}\left[\alpha\rho_L\left(u_L + \frac{w_g^2}{2}\right)A\right] + \frac{\partial}{\partial z}\left[\alpha\rho_L w_g\left(h_L + \frac{w_g^2}{2}\right)A\right] + \alpha\rho_L w_g A g \cos\varphi + \frac{F_{RL}}{\Delta z} w_g = \frac{\dot{Q}_L}{\Delta z} \tag{25}$$

$$\frac{\partial}{\partial \tau}\left[\alpha\rho_D\left(u_D + \frac{w_g^2}{2}\right)A\right] + \frac{\partial}{\partial z}\left[\alpha\rho_D w_g\left(h_D + \frac{w_g^2}{2}\right)A\right] + \alpha\rho_D w_g A g \cos\varphi + \frac{F_{RD}}{\Delta z} w_g = \frac{\dot{Q}_D}{\Delta z} + \mu\left(h_{ex} + \frac{w_{ex}^2}{2}\right)A \tag{26}$$

$$\frac{\partial}{\partial \tau}\left[(1-\alpha)\rho_f\left(u_f + \frac{w_f^2}{2}\right)A\right] + \frac{\partial}{\partial z}\left[(1-\alpha)\rho_f w_f\left(h_f + \frac{w_f^2}{2}\right)A\right] + (1-\alpha)\rho_f w_f A g \cos\varphi + \frac{F_{Rf}}{\Delta z} w_f = \frac{\dot{Q}}{\Delta z} - \mu\left(h_{ex} + \frac{w_{ex}^2}{2}\right)A \ . \tag{27}$$

After some transformations shown in Appendix A, we come to the system

$$\frac{\partial}{\partial \tau}(\alpha\rho_L) + \frac{\partial}{\partial z}(\alpha\rho_L w) = 0 \tag{28}$$

$$\frac{\partial}{\partial \tau}(\alpha\rho_D'') + \frac{\partial}{\partial z}(\alpha\rho_D'' w) = \mu \tag{29}$$

$$\frac{\partial}{\partial \tau}[(1-\alpha)\rho_f] + \frac{\partial}{\partial z}[(1-\alpha)\rho_f w] = -\mu \tag{30}$$

$$\frac{\partial w}{\partial \tau} + w\frac{\partial w}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} = -\frac{Z}{\rho} \tag{31}$$

$$\alpha\rho_L\left(\frac{\partial h_L}{\partial \tau} + w\frac{\partial h_L}{\partial z}\right) + \alpha\rho_D''\left(\frac{\partial h_D''}{\partial \tau} + w\frac{\partial h_D''}{\partial z}\right) + (1-\alpha)\rho_f\left(\frac{\partial h_f}{\partial \tau} + w_f\frac{\partial h_f}{\partial z}\right) - \left(\frac{\partial p}{\partial \tau} + w\frac{\partial p}{\partial z}\right) = \dot{q}''' - \mu(h_D'' - h_f) \ , \tag{32}$$

where

$$\rho = \alpha(\rho_L + \rho_D'') + (1-\alpha)\rho_f \ . \tag{33}$$

Using the transformations in Appendix B, we eliminate $\partial\alpha/\partial\tau + w\partial\alpha/\partial z$ and μ so that

$$\alpha\frac{\rho_D''}{\rho_L}\dot{\rho}_L + \alpha\left(\frac{1}{1-\frac{\rho_D''}{\rho_f}} - 1\right)\dot{\rho}_D + \frac{(1-\alpha)\rho_D''}{\rho_f - \rho_D''}\dot{\rho}_f + \frac{\rho_D''}{1-\frac{\rho_D''}{\rho_f}}\frac{\partial w}{\partial z} = 0 \tag{34}$$

$$\dot{\rho} + \rho\frac{\partial w}{\partial z} = 0 \tag{35}$$

$$\dot{w} + \frac{1}{\rho}\frac{\partial p}{\partial z} = -\frac{Z}{\rho} \tag{36}$$

$$\alpha(\rho_L\dot{h}_L + \rho_D''\dot{h}_D'') + (1-\alpha)\rho_f\dot{h}_f - \dot{p} + \frac{h_D'' - h_f}{v_D'' - v_f} \times \left(\frac{\alpha}{\rho_D''}\dot{\rho}_D'' + \frac{1-\alpha}{\rho_f}\dot{\rho}_f\right) + \frac{h_D'' - h_f}{v_D'' - v_f}\frac{\partial w}{\partial z} = \dot{q}''' \ . \tag{37}$$

In this case we use the abbreviation

$$\dot{X} = \frac{\partial X}{\partial \tau} + w\frac{\partial X}{\partial z} \ . \tag{38}$$

As $\rho_f \gg \rho_D''$, $\rho_D''/\rho_f \ll 1$, and $\rho_f - \rho_D'' \approx \rho_f$, Eq. (34) is transformed into

$$\frac{\alpha}{\rho_L}\dot{\rho}_L + \frac{1-\alpha}{\rho_f}\dot{\rho}_f + \frac{\partial w}{\partial z} = 0 \ . \tag{39}$$

Substituting $d\rho$ and dh in Eqs. (35) through (37) and (39) for their equals from the equations of state, Eqs. (10) through (12) and Eqs. (15) through (17), we set

$$C = p_L\left(\frac{1}{T} + \frac{1}{p_L}\frac{dp_D''}{dT} - \frac{1-\alpha}{\alpha}\frac{1}{\rho_f}\frac{\partial\rho_f}{\partial T}\right) \tag{40}$$

$$D = \alpha\left[\rho_L\left(\frac{\partial h_L}{\partial T} - \frac{\partial h_L}{\partial p_L}\cdot\frac{dp_D''}{dT}\right) + \rho_D''\frac{dh_D''}{dT}\right] + (1-\alpha)\rho_f\frac{\partial h_f}{\partial T} \tag{41}$$

$$E = \alpha\rho_L\frac{\partial h_L}{\partial p_L} + (1-\alpha)\rho_f\frac{\partial h_f}{\partial p} - 1 \tag{42}$$

$$A = CE + D \tag{43}$$

$$B = \frac{h_D'' - h_f}{v_D'' - v_f} - \frac{p_I}{\alpha}E \tag{44}$$

$$\rho a^2 = \frac{CB}{A} + \frac{p_L}{\alpha} \tag{45}$$

thus obtaining

$$\frac{\partial T}{\partial \tau} + w\frac{\partial T}{\partial z} + \frac{B}{A}\frac{\partial w}{\partial z} = \frac{\dot{q}'''}{A} \tag{46}$$

$$\frac{\partial p}{\partial \tau} + w\frac{\partial p}{\partial z} + \rho a^2 \frac{\partial w}{\partial z} = \frac{C}{A}\dot{q}''' \quad (47)$$

$$\frac{\partial w}{\partial \tau} + w\frac{\partial w}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} = -\frac{Z}{\rho} \quad (48)$$

$$\frac{\partial \rho}{\partial \tau} + w\frac{\partial \rho}{\partial z} + \rho\frac{\partial w}{\partial z} = 0 \ . \quad (49)$$

The eigenvalues of the characteristic matrix

$$\begin{vmatrix} w-\lambda & 0 & B/A & 0 \\ 0 & w-\lambda & \rho a^2 & 0 \\ 0 & 1 & w-\lambda & 0 \\ 0 & 0 & \rho & w-\lambda \end{vmatrix} = 0 \quad (50)$$

of this quasi-linear inhomogeneous system of partial differential equations are

$$\lambda_{1,2} = w \ ; \quad \lambda_{3,4} = w \pm a \ . \quad (51\text{-}54)$$

The following linearly independent vectors are obtained as eigenvectors of the transposed characteristic matrix corresponding to each eigenvalue

$$\boldsymbol{h}_1^T = \left(1,0,0,-\frac{B}{A\rho}\right) \quad (55)$$

$$\boldsymbol{h}_2^T = (0,1,0,-a^2) \quad (56)$$

$$\boldsymbol{h}_3^T = (0,1,\rho a,0) \quad (57)$$

$$\boldsymbol{h}_4^T = (0,1,-\rho a,0) \ . \quad (58)$$

The system [Eqs. (46) through (49)] has three different real eigenvalues of the characteristic matrix and four linearly independent eigenvectors of the transposed characteristic matrix corresponding to each eigenvalue. That is why, in accordance with Ref. 10, this system is of a hyperbolic type and can be transformed into a completely equivalent canonic form:

$$\frac{dz}{d\tau} = w \qquad \frac{dT}{d\tau} - \frac{B}{A\rho}\frac{d\rho}{d\tau} = \frac{\dot{q}'''}{A} \quad (59,60)$$

$$\frac{dz}{d\tau} = w \qquad \frac{dp}{d\tau} - a^2\frac{d\rho}{d\tau} = \frac{C}{A}\dot{q}''' \quad (61,62)$$

$$\frac{dz}{d\tau} = w + a \qquad \frac{dw}{d\tau} + \frac{1}{\rho a}\frac{dp}{d\tau} + f\Phi_{Tph}\frac{w|w|}{2D_h} + g\cos\varphi = \frac{C}{\rho a A}\dot{q}''' \quad (63,64)$$

$$\frac{dz}{d\tau} = w - a \qquad \frac{dw}{d\tau} - \frac{1}{\rho a}\frac{dp}{d\tau} + f\Phi_{Tph}\frac{w|w|}{2D_h} + g\cos\varphi = -\frac{C}{\rho a A}\dot{q}''' \ . \quad (65,66)$$

Comparing this system with that describing a homogeneous equilibrated single-component two-phase flow, we see an extra equation (60) that contains the information about the temperature of the system.

The stationary state of the flow is described by the steady-state part of the system [Eqs. (46) through (49)]. The derivatives by the linear coordinates are obtained as follows:

$$\frac{dp}{dz} = -\frac{Z + \frac{C}{A}\frac{w}{a^2}\dot{q}'''}{1 - \frac{w^2}{a^2}} \quad (67)$$

$$\frac{dw}{dz} = \frac{1}{\rho a^2}\frac{wZ + \frac{C}{A}\dot{q}'''}{1 - \frac{w^2}{a^2}} \quad (68)$$

$$\frac{d\rho}{dz} = \frac{\rho}{w}\frac{dw}{dz} \quad (69)$$

$$\frac{dT}{dz} = \frac{1}{wA}\left(\dot{q}''' - B\frac{dw}{dz}\right) \ . \quad (70)$$

We see that if the velocity of the flow, w, reaches the local value of the sound speed, a, the gradients of pressure and the velocity tend to $-\infty$ and $+\infty$, respectively; i.e., the critical state of the flow is reached. This is a well-known fact about a compressible flow.

The limiting case of no air in the flow is shown in Appendix C. The same case without a steam or liquid phase is shown in Appendixes D and E, respectively.

III. SOME NUMERICAL EXAMPLES

Figure 2 illustrates the third consequence of the simplifying assumptions in deriving the model discussed in Sec. II. The speed of sound is shown, calculated from Eq. (45), as a function of the void fraction. The parameter is the air mass content in the

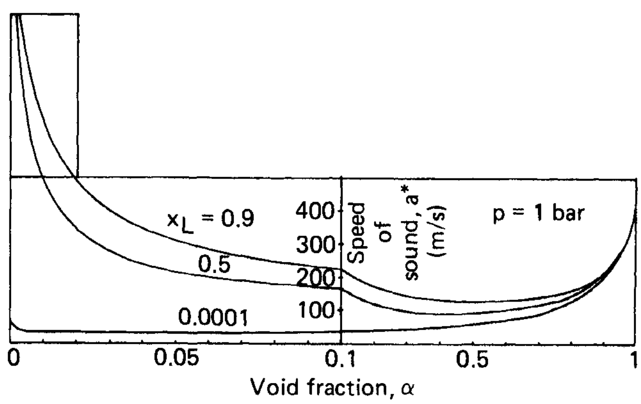

Fig. 2. The speed of sound, a^*, in a homogeneous two-component equilibrium mixture predicted by Eq. (45) as a function of the void fraction α with the air mass content in the gas phase x_L.

[10] R. U. COURANT and D. HILBERT, *Methods of Mathematical Physics*, Vol. II, New York (1962).

gas phase. As the air mass content increases from 0 to 1, the values of the speed of sound change from values typical of a homogeneous equilibrium two-phase flow to values typical of a homogeneous nonequilibrium two-phase flow. This is caused by the decrease of spontaneous mass exchange with the increase of air content in the gas phase, as is simply illustrated in the following:

$$\mu = \alpha\left(\frac{\partial \rho_D''}{\partial \tau} + w\,\frac{\partial \rho_D''}{\partial z}\right) + \rho_D''\left(\frac{\partial \alpha}{\partial \tau} + w\,\frac{\partial \alpha}{\partial z} + \alpha\,\frac{\partial w}{\partial z}\right)$$

with $\rho_D'' = 0$, $d\rho_D'' = 0$, and $\mu = 0$. This is a typical property of the present model, and since the spontaneous mass exchange is the upper limit of a possible mass exchange, this also is a typical property of the system of water-steam-air. In other words, if air is predominating in the gas phase, the properties of the flow are close to those of the nonequilibrium two-phase flow. If the steam is predominating, then the properties are near those of the homogeneous equilibrium two-phase flow.

Experimental data have been obtained[11] at a pressure of 10^5 Pa for the speed of sound in a water/air mixture flow as a function of the void fraction. Two gauges for pressure measurements are mounted 1 m apart in a horizontal tube with constant cross section. The gauges are connected to an oscilloscope. A pneumatic pulse is applied to one end of the tube and the time of travel of the pulse from one gauge to the other is measured determining the speed of propagation. The volumetric gas content is measured by gamma radiography. Unfortunately, the quantity of steam in the mixture was not measured. Figure 3 shows the experimental results[1] and the calculations by means of Eq. (45) with 90% of the mass content of air in the gas phase. Taking into account the behavior of this function, which is shown in Fig. 1, the calculated values approximate the experimental if the fraction of air in the gas phase is very large (tending to unity). This is an indication of the ability of the model to describe the behavior of such systems.

Similar experimental data for water/air have been reported[12] for a pressure of 1.75 X 10^5 Pa, with no details on how they were obtained. Figure 4 shows a comparison between these data and our theory. Once again we see that what was said about Fig. 3 is also valid in this case.

Let us illustrate the application of the theory in solving the following problem. In a volume of 1000 m^3 shaped as shown in Fig. 5, a coolant blowdown occurs from the primary pipeline of a power station with a WWER-440 reactor. We analyze the

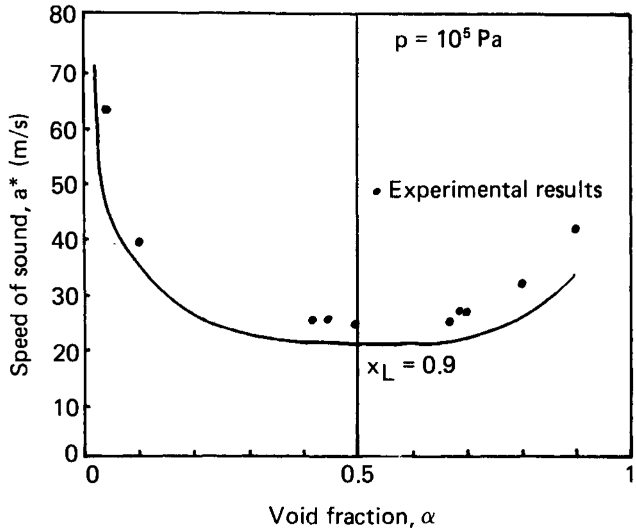

Fig. 3. The speed of sound, a^*, as a function of the void fraction α. Comparison between theoretical Eq. (45) (x_L = 0.9) and experiment (Ref. 11). Pressure $p = 10^5$ Pa.

shock loads in case the volume remains airtight and retains its geometry until after the first second of the accident. The cross section of the volume is 100 m^2, and its mean circumference is 100 m. The results from the new theory are to be compared with those obtained from a similar analysis with lumped parameters. The initial conditions are: The volume

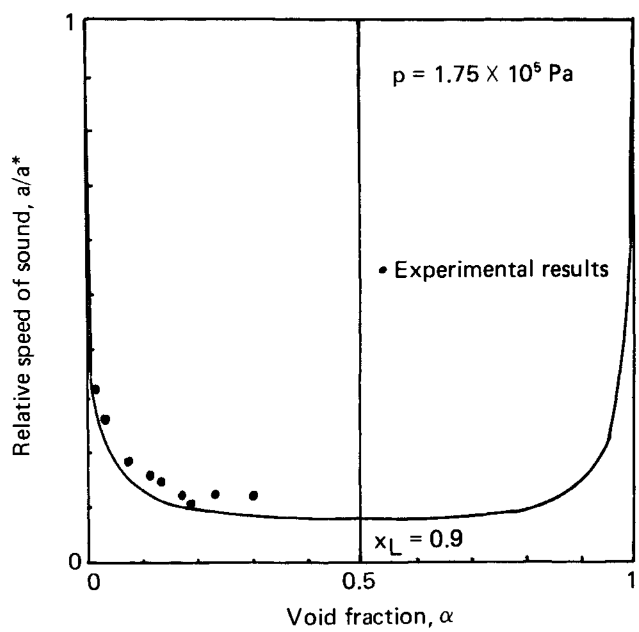

Fig. 4. The relative speed of sound a^*/a_g^* as a function of the void fraction α. Comparison between theoretical Eq. (45) and experiment (Ref. 12). Pressure $p = 1.75 \times 10^5$ Pa.

[11]P. BÖCK and J. M. CHAWLA, "Ausbreitungsgeschwindigkeit einer Druckstörung in Flüssigkeit/Gas-Gemischen," *Brenstoff Wärme Kraft*, **26**, *2*, 65 (1974).

[12]F. J. MOODY, *J. Heat Transfer*, *3*, 98 (1969).

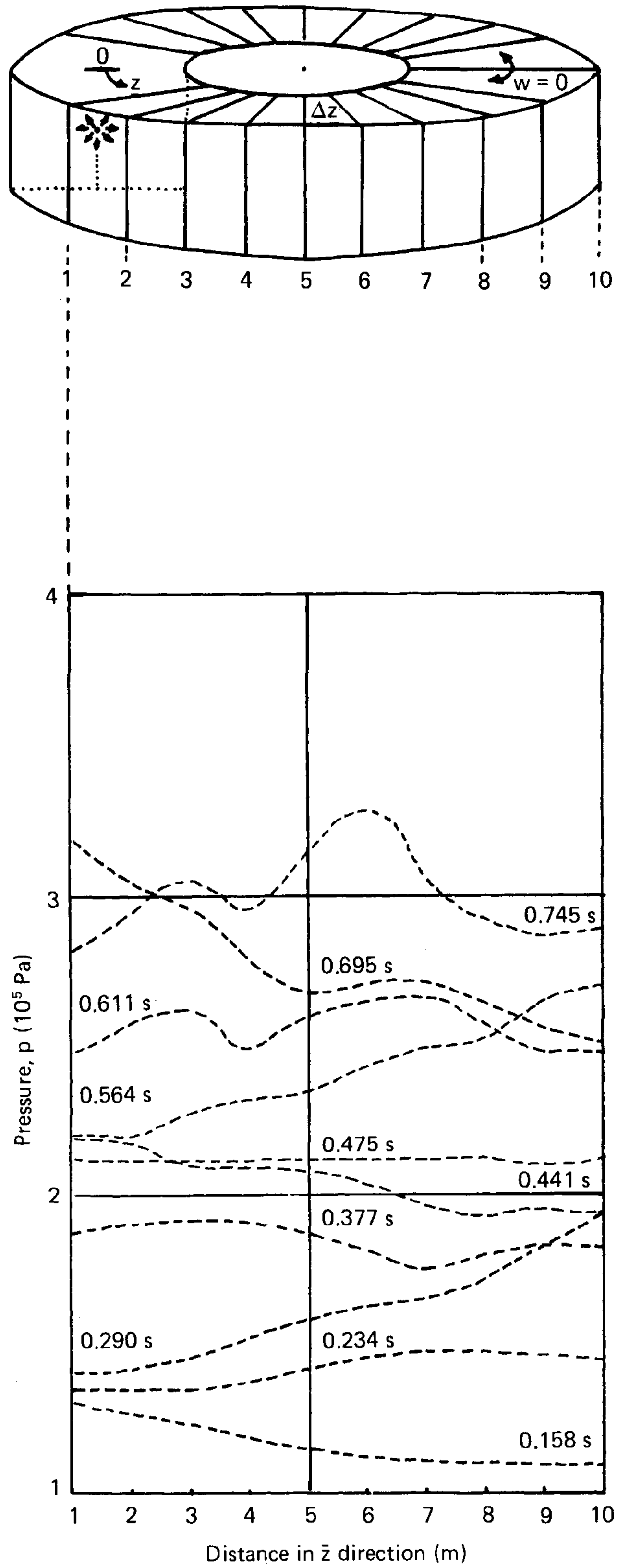

Fig. 5. The pressure p as a function of the space coordinate during a LOCA at various times from the beginning of the accident.

is filled with air at 80°C and at a pressure of 10^5 Pa. The mass flow escaping from the main pipeline (cold) and its enthalpy are calculated by means of the BRUCH D 06 program.[12] The results of this calculation are shown in Ref. 13.

We assume that the thermodynamic equilibrium of the escaping coolant from the main pipeline is restored symmetrically about the rupture in one-tenth of the total volume. We simulate this volume in lumped parameters using the model[4] for which the simplifying assumptions of the proposed theory are valid. Thus we calculate the left boundary conditions p_1, T_1, and ρ_1 in the course of the accident.

In order to decrease the number of calculations, we assume a symmetrical propagation of the perturbation. That is why only half of the whole volume will be analyzed. We divided that half of the total volume into nine parts. The values of the dependent variables will be obtained at ten points along the linear coordinate z, as is shown in Fig. 5. The right boundary condition is the velocity of the fluid at point $z = z_{10}$ is equal to zero: $w(z = z_{10}) = 0$.

In this analysis we neglect friction. The system of Eqs. (59) through (66) is integrated numerically by means of the method of characteristics.[14] The results are shown in Figs. 5, 6, and 7.

It is clear that, with increasing pressure in the left volume, the fluid is accelerated, reaches the right boundary, and transforms its kinetic energy into internal energy. This is manifested in an increase of temperature and pressure at point z_{10}. In case of sufficient pressure increase at point z_{10}, the kinetic energy in the whole volume decreases in favor of the internal energy. The reflection of the wave starts at the next moment. The wave interacts continuously with the fluid coming from the left. The pressure on the left increases again to give rise to a new wave, propagating to the right. This process is repeated with a frequency that depends on the particular geometry, 7 Hz in our case. The superposition of the reflected waves on the new ones after some time makes the pressure distribution along z more complicated than in the first few cycles. Figure 6 shows the pressures in points z_1 and z_{10}. A quantitative difference can be seen in each point at the same instant, as well as the divergence from the lumped parameter analysis. This difference grows with the pressure increasing as the

[13] N. I. KOLEV and L. SABOTINOV, "Analis na fasata na istitzane pri awarija s raskaswane na glawen zirkulazionen traboprowod sa AEZ s WWER-440 pri nominalna i powischena mostnost," *5th Nat. Conf. Heat and Nuclear Engineering Problems of Peoples Republic of Bulgaria*, Varna, Bulgaria, May 21-23, 1981, p. 73.

[14] K. KÖBERLEIN, "Die verzögerte Einstellung des thermodynamischen Gleichgewichts als Grundlage eines Rechenmodells für die Druckwellenausbreitung in der Zweiphasenströmung von Wasser," Dissertation (1972).

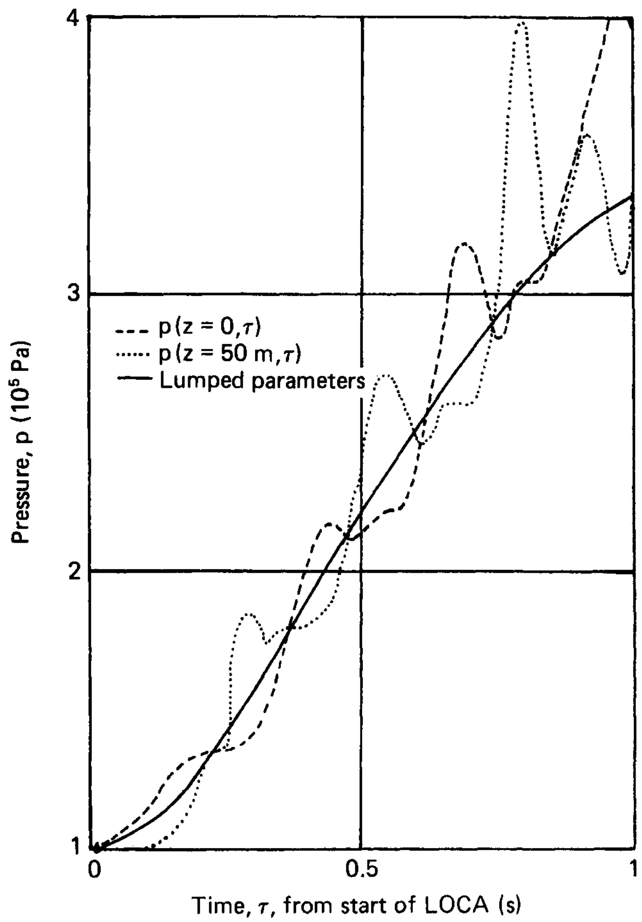

Fig. 6. The pressure p in the two opposite points z_1 $(=0)$ and z_{10} $(= 50$ m$)$ of the volume shown in Fig. 5 as a function of the time during LOCA. Comparison of the prediction given by the present theory and by the lumped parameter analysis.

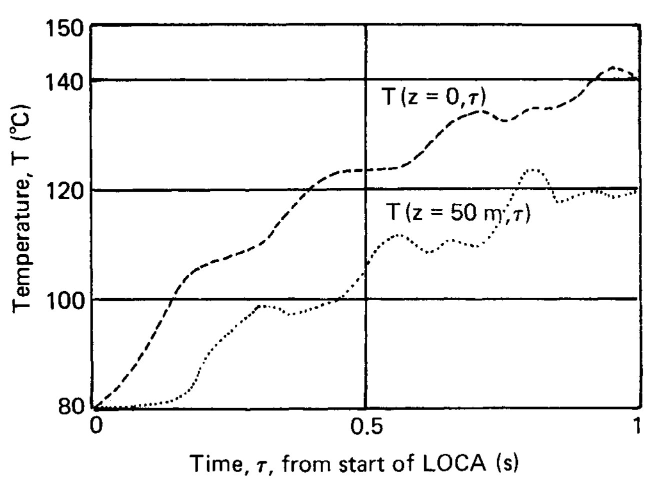

Fig. 7. The temperature T at the two opposite points $z_1 = 0$ and $z_{10} = 50$ m as a function of the time.

system becomes denser and, therefore, less elastic. Figure 7 shows the temperatures at z_1 and z_{10}. A comparison with Fig. 1 shows a qualitative coincidence. (We cannot make a quantitative comparison with Fig. 1 because of a lack of information about the experiment.[5])

IV. CONCLUSION

The conditional division of two-phase flow into four groups, homogeneous equilibrium, nonhomogeneous equilibrium, homogeneous nonequilibrium, and nonhomogeneous nonequilibrium, allows, in case of two-component flow, the step-by-step development of a hyperbolic systems of partial differential equations describing the flow and taking into account the thermodynamic peculiarity of the system caused by the presence of a noncondensable gas. In the present work, the first step is the development of a hyperbolic model for a homogeneous equilibrium flow consisting of water-steam-air. The flow was described by four dependent variables. The system of four partial differential equations obtained is of a hyperbolic type. The system was transformed into the canonical form. The expression defining the speed of sound in a homogeneous equilibrium mixture of water-steam-air is obtained. The system describing the limiting case of a missing phase or component is also obtained. The properties of the flow model are approximately those of the nonequilibrium two-phase flow model when air is predominating. If steam is predominating, the properties are approximately those of the homogeneous equilibrium two-phase flow model. The speed of sound predicted by the present theory is compared with experimental data for the gas phase with low steam content. Good agreement is observed. Using the method of characteristics, the system obtained is numerically integrated with the initial and the boundary conditions of a LOCA in a nuclear power station utilizing a pressurized water reactor. The very interesting phenomenon of the propagation of a pressure wave in a toroidal ring having a 1000-m^3 volume and a 100-m^2 cross section is shown. The results of this analysis are compared to the results of a lumped parameter analysis performed with the same thermodynamic assumptions for the state of the system. The differences between these two approaches are shown.

In conclusion, we add that, for phenomena with separation predominating, the nonhomogeneity must be introduced in the flow model. No principal problems will arise in the development of a model of more dimensions by the same assumptions. Such a model will have the same thermodynamic properties. The present model may have interesting applications in addition to the description of the propagation of a LOCA into the low-pressure system of the nuclear station. The following are two examples:

1. the flow of water in piping networks naturally containing dissolved noncondensable gases
2. pressure-wave propagation through a rainy atmosphere.

APPENDIX A

We differentiate the first two terms of the momentum equations [Eqs. (22), (23), and (24)] in order to obtain the left sides of the mass conservation equations and replace them by the corresponding right sides of Eqs. (19), (20), and (21). A similar transformation is applied to the energy equations. Then we divide all equations by the cross section and transfer all the terms with the change of the cross section with the linear coordinates in the right sides. Thus we obtain

$$\frac{\partial}{\partial\tau}(\alpha\rho_L) + \frac{\partial}{\partial z}(\alpha\rho_L w_g) = -\alpha\rho_L w_g \frac{d}{dz}\ln A \tag{A.1}$$

$$\frac{\partial}{\partial\tau}(\alpha\rho_D) + \frac{\partial}{\partial z}(\alpha\rho_D w_g) = \mu - \alpha\rho_D w_g \frac{d}{dz}\ln A \tag{A.2}$$

$$\frac{\partial}{\partial\tau}[(1-\alpha)\rho_f] + \frac{\partial}{\partial z}[(1-\alpha)\rho_f w_f] = -\mu - (1-\alpha)\rho_f w_f \frac{d}{dz}\ln A \tag{A.3}$$

$$\alpha\rho_L\left(\frac{\partial w_g}{\partial\tau} + w_g\frac{\partial w_g}{\partial z}\right) + \alpha\frac{\partial p_L}{\partial z} + \alpha\rho_L g\cos\varphi + f_{RL} = 0 \tag{A.4}$$

$$\alpha\rho_D\left(\frac{\partial w_g}{\partial\tau} + w_g\frac{\partial w_g}{\partial z}\right) + \alpha\frac{\partial p_D}{\partial z} + \alpha\rho_D g\cos\varphi + f_{RD} = \mu(w_{ex} - w_g) \tag{A.5}$$

$$(1-\alpha)\rho_f\left(\frac{\partial w_f}{\partial\tau} + w_f\frac{\partial w_f}{\partial z}\right) + (1-\alpha)\frac{\partial p}{\partial z} + (1-\alpha)\rho_f g\cos\varphi + f_{Rf} = -\mu(w_{ex} - w_f) \tag{A.6}$$

$$\alpha\rho_L\left(\frac{\partial h_L}{\partial\tau} + w_g\frac{\partial h_L}{\partial z}\right) + \alpha\rho_L w_g\left(\frac{\partial w_g}{\partial z} + w_g\frac{\partial w_g}{\partial z}\right) - p_L\frac{\partial\alpha}{\partial\tau} - \alpha\frac{\partial p_L}{\partial\tau} + w_g(\alpha\rho_L g\cos\varphi + f_{RL}) = \dot{q}_L''' \tag{A.7}$$

$$\alpha\rho_D\left(\frac{\partial h_D}{\partial\tau} + w_g\frac{\partial h_D}{\partial z}\right) + \alpha\rho_D w_g\left(\frac{\partial w_g}{\partial\tau} + w_g\frac{\partial w_g}{\partial z}\right) - p_D\frac{\partial\alpha}{\partial\tau} - \alpha\frac{\partial p_D}{\partial\tau} + w_g(\alpha\rho_D g\cos\varphi + f_{RD}) = \dot{q}_D''' + \mu\left(h_{ex} + \frac{w_{ex}^2}{2} - h_D - \frac{w_g^2}{2}\right) \tag{A.8}$$

$$(1-\alpha)\rho_f\left(\frac{\partial h_f}{\partial\tau} + w_f\frac{\partial h_f}{\partial z}\right) + (1-\alpha)\rho_f w_f\left(\frac{\partial w_f}{\partial\tau} + w_f\frac{\partial w_f}{\partial z}\right) + p\frac{\partial\alpha}{\partial\tau} - (1-\alpha)\frac{\partial p}{\partial\tau} + w_f[(1-\alpha)\rho_f g\cos\varphi + f_{Rf}] = \dot{q}_f''' - \mu\left(h_{ex} + \frac{w_{ex}^2}{2} - h_f - \frac{w_f^2}{2}\right) . \tag{A.9}$$

Each momentum equation [Eqs. (A.4), (A.5), and (A.6)] is multiplied by w_g, w_g, and w_f, respectively, and subtracted from the corresponding energy equations [Eqs. (A.7), (A.8), and (A.9)]. The following system is the result:

$$\alpha\rho_L\left(\frac{\partial h_L}{\partial\tau} + w_g\frac{\partial h_L}{\partial z}\right) - \frac{\partial}{\partial\tau}(\alpha p_L) = \dot{q}_L''' \tag{A.10}$$

$$\alpha\rho_D\left(\frac{\partial h_D}{\partial\tau} + w_g\frac{\partial h_D}{\partial z}\right) - \frac{\partial}{\partial\tau}(\alpha p_D) = \dot{q}_D''' + \left(h_{ex} + \frac{w_{ex}^2}{2} - h_D - \frac{w_g^2}{2}\right) - \mu w_g(w_{ex} - w_g) \tag{A.11}$$

$$(1-\alpha)\rho_f\left(\frac{\partial h_f}{\partial\tau} + w_f\frac{\partial h_f}{\partial z}\right) - \frac{\partial}{\partial\tau}[(1-\alpha)p] = \dot{q}_f''' - \left(h_{ex} + \frac{w_{ex}^2}{2} - h_f - \frac{w_f^2}{2}\right) + \mu w_f(w_{ex} - w_f) . \tag{A.12}$$

Further, using assumptions a through g, we add the three momentum equations and the three energy equations and obtain Eqs. (28) through (32).

APPENDIX B

Adding Eqs. (28), (29), and (30), we obtain Eq. (35)

$$\frac{\partial\rho}{\partial\tau} + w\frac{\partial\rho}{\partial z} + \rho\frac{\partial w}{\partial z} = 0 . \tag{35}$$

Differentiating Eqs. (28), (29), and (30) term by term and dividing by ρ_L, ρ_D'', and ρ_f, respectively,

$$\frac{\alpha}{\rho_L}\left(\frac{\partial\rho_L}{\partial\tau} + w\frac{\partial\rho_L}{\partial z}\right) + \frac{\partial\alpha}{\partial\tau} + w\frac{\partial\alpha}{\partial z} + \alpha\frac{\partial w}{\partial z} = 0 \tag{B.1}$$

$$\frac{\alpha}{\rho_D''}\left(\frac{\partial\rho_D''}{\partial\tau} + w\frac{\partial\rho_D''}{\partial z}\right) + \frac{\partial\alpha}{\partial\tau} + w\frac{\partial\alpha}{\partial z} + \alpha\frac{\partial w}{\partial z} = \frac{\mu}{\rho_D''} \tag{B.2}$$

$$\frac{1-\alpha}{\rho_f}\left(\frac{\partial\rho_f}{\partial\tau} + w\frac{\partial\rho_f}{\partial z}\right) - \left(\frac{\partial\alpha}{\partial\tau} + w\frac{\partial\alpha}{\partial z}\right) + (1-\alpha)\frac{\partial w}{\partial z} = -\frac{\mu}{\rho_f} . \tag{B.3}$$

Subtracting Eq. (B.1) from (B.2), and adding Eq. (B.2) to (B.3), we obtain

$$\frac{1}{\rho_L}\left(\frac{\partial\rho_L}{\partial\tau}+w\,\frac{\partial\rho_L}{\partial z}\right)-\frac{1}{\rho_D''}\left(\frac{\partial\rho_D''}{\partial\tau}+w\,\frac{\partial\rho_D''}{\partial z}\right)=-\frac{\mu}{\alpha\rho_D''} \quad (B.4)$$

and

$$\frac{\alpha}{\rho_D''}\left(\frac{\partial\rho_D''}{\partial\tau}+w\,\frac{\partial\rho_D''}{\partial z}\right)+\frac{1-\alpha}{\rho_f}\left(\frac{\partial\rho_f}{\partial\tau}+w\,\frac{\partial\rho_f}{\partial z}\right)+\frac{\partial w}{\partial z}$$
$$=\mu(v_D''-v_f) \ . \quad (B.5)$$

Equation (B.4) is multiplied by $\alpha\rho_D''$, Eq. (B.5) is divided by $(v_D''-v_f)$, and Eq. (32) is divided by $h_D''-h_f$. Introducing an abbreviation

$$\dot{X}=\frac{\partial X}{\partial\tau}+w\,\frac{\partial X}{\partial z} \ , \quad (B.6)$$

we come to

$$\alpha\frac{\rho_D''}{\rho_L}\dot{\rho}_L-\alpha\dot{\rho}_D=-\mu \ , \quad (B.7)$$

$$\frac{1}{v_D''-v_f}\left(\frac{\alpha}{\rho_D''}\dot{\rho}_D''+\frac{1-\alpha}{\rho_f}\dot{\rho}_f\right)+\frac{1}{v_D''-v_f}\frac{\partial w}{\partial z}=\mu \ , \quad (B.8)$$

and

$$\frac{1}{h_D''-h_f}\left[\alpha\rho_L\dot{h}_L+\alpha\rho_D''\dot{h}_D''+(1-\alpha)\rho_f\dot{h}_f-\dot{p}\right]$$
$$=\frac{\dot{q}'''}{h_D''-h_f}-\mu \ . \quad (B.9)$$

Adding Eq. (B.7) to (B.8) and Eq. (B.8) to (B.9), we obtain Eqs. (34) and (37).

APPENDIX C

In the limiting case of no air ($p_L=0$) in the two-phase system, we have

$$\frac{\partial\rho}{\partial\tau}+w\,\frac{\partial\rho}{\partial z}+\rho\,\frac{\partial w}{\partial z}=0 \ , \quad (C.1)$$

$$\frac{\partial w}{\partial\tau}+w\,\frac{\partial w}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial z}=-\frac{Z}{\rho} \ , \quad (C.2)$$

and

$$\rho\left(\frac{\partial h}{\partial\tau}+w\,\frac{\partial h}{\partial z}\right)-\left(\frac{\partial p}{\partial\tau}+w\,\frac{\partial p}{\partial z}\right)=\dot{q}''' \ . \quad (C.3)$$

After some transformations, and denoting

$$a^2=\frac{\partial h}{\partial\rho}\bigg/\left(\frac{1}{\rho}-\frac{\partial h}{\partial p}\right) \ , \quad (C.4)$$

we come to

$$\frac{\partial p}{\partial\tau}+w\,\frac{\partial p}{\partial z}+\rho a^2\,\frac{\partial w}{\partial z}=-\dot{q}'''\,\frac{a^2}{\rho}\frac{\partial\rho}{\partial h} \ , \quad (C.5)$$

$$\frac{\partial\rho}{\partial\tau}+w\,\frac{\partial\rho}{\partial z}+\rho\,\frac{\partial w}{\partial z}=0 \ , \quad (C.6)$$

and

$$\frac{\partial w}{\partial\tau}+w\,\frac{\partial w}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial z}=-\frac{Z}{\rho} \ , \quad (C.7)$$

or, in a canonical form,

$$\frac{dz}{d\tau}=w \qquad \frac{dp}{d\tau}-a^2\frac{d\rho}{d\tau}=\dot{q}'''\,\frac{a^2}{\rho}\frac{\partial\rho}{\partial h} \ , \quad (C.8,9)$$

$$\frac{dz}{d\tau}=w+a \qquad \frac{dw}{d\tau}+\frac{1}{\rho a}\frac{dp}{d\tau}+g\cos\varphi+f\Phi_{Tph}\,\frac{w|w|}{2D_h}$$
$$=-\dot{q}'''\,\frac{a}{\rho^2}\frac{\partial\rho}{\partial h} \ , \quad (C.10,11)$$

and

$$\frac{dz}{d\tau}=w-a \qquad \frac{dw}{d\tau}-\frac{1}{\rho a}\frac{dp}{d\tau}+g\cos\varphi+f\Phi_{Tph}\,\frac{w|w|}{2D_h}$$
$$=\dot{q}'''\,\frac{a}{\rho^2}\frac{\partial\rho}{\partial h} \ . \quad (C.12,13)$$

APPENDIX D

In the case of no steam, i.e., a flow consisting of supercooled water and air, we obtain

$$C=p\left(\frac{1}{T}-\frac{1-\alpha}{\alpha}\frac{1}{\rho_f}\frac{\partial\rho_f}{\partial T}\right) \ , \quad (D.1)$$

$$E=\alpha\rho_L\,\frac{\partial h_L}{\partial p}+(1-\alpha)\rho_f\,\frac{\partial h_f}{\partial p}-1 \ , \quad (D.2)$$

$$D=\alpha\rho_L\,\frac{\partial h_L}{\partial T}+(1-\alpha)\rho_f\,\frac{\partial h_f}{\partial T} \ , \quad (D.3)$$

$$A=CE+D$$

$$B=-\frac{p}{\alpha}E \ , \quad (D.4)$$

and

$$\rho a^2=\frac{CB}{A}+\frac{p}{\alpha} \ . \quad (D.5)$$

For the limiting case of no liquid, i.e., $\alpha=1$, we get

$$a^2=\frac{c_{pL}}{c_{pL}-R_L}R_LT \ , \quad (D.6)$$

the well-known expression from gas dynamics.

APPENDIX E

Using the same vector of the dependent variables $\boldsymbol{U}^T=(T,p,w,\rho)$, we can also treat the limiting case of missing water (i.e., $\alpha=1$). Assuming that the superheated steam can be considered as an ideal gas, we get, for the equations of state,

$$T_L = T_D = T \ , \tag{E.1}$$

$$p = p_D + p_L \ , \tag{E.2}$$

$$\rho = \rho_D + \rho_L \ \text{(Dalton's law)} \ , \tag{E.3}$$

$$p_D = \rho_D R_D T \ , \tag{E.4}$$

and

$$p_L = \rho_L R_L T \ . \tag{E.5}$$

From Eqs. (E.1) through (E.5), it follows that

$$\rho_D = \frac{1}{R_D - R_L}\left(\frac{p}{T} - \rho R_L\right) \tag{E.6}$$

or

$$d\rho_D = \frac{\partial \rho_D}{\partial p} dp + \frac{\partial \rho_D}{\partial T} dT + \frac{\partial \rho_D}{\partial \rho} d\rho \ , \tag{E.7}$$

where

$$\frac{\partial \rho_D}{\partial p} = \frac{1}{(R_D - R_L)T} \ ; \quad \frac{\partial \rho_D}{\partial T} = -\frac{p}{(R_D - R_L)T^2} \ ;$$

$$\frac{\partial \rho_D}{\partial \rho} = -\frac{R_L}{R_D - R_L} \tag{E.8-10}$$

$$dh_D = c_{pD} dT \ ; \quad dh_L = c_{pL} dT \ . \tag{E.11,12}$$

We define c_p, R, and κ for the gas phase in the following way:

$$c_p = \rho_D c_{pD} + \rho_L c_{pL} \ ; \quad R = \frac{p}{\rho T} \ ; \ \kappa = \frac{c_p}{c_p - R} \ . \tag{E.13-15}$$

We use the following system for our considerations:

$$\frac{\partial \rho_D}{\partial \tau} + w \frac{\partial \rho_D}{\partial z} + \rho_D \frac{\partial w}{\partial z} = 0 \ , \tag{E.16}$$

$$\frac{\partial \rho}{\partial \tau} + w \frac{\partial \rho}{\partial z} + \rho \frac{\partial w}{\partial z} = 0 \ , \tag{E.17}$$

$$\frac{\partial w}{\partial \tau} + w \frac{\partial w}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} = -\frac{Z}{\rho} \ , \tag{E.18}$$

and

$$\rho_L \left(\frac{\partial h_L}{\partial \tau} + w \frac{\partial h_L}{\partial z}\right) + \rho_D \left(\frac{\partial h_D}{\partial \tau} + w \frac{\partial h_D}{\partial z}\right) - \left(\frac{\partial p}{\partial z} + w \frac{\partial p}{\partial z}\right) = \dot{q}''' \ . \tag{E.19}$$

After several transformations, setting

$$A = \rho(c_p - R) \ , \tag{E.20}$$

$$B = p \ , \tag{E.21}$$

and

$$a^2 = \kappa \frac{p}{\rho} \tag{E.22}$$

we obtain the hyperbolic system

$$\frac{\partial T}{\partial \tau} + w \frac{\partial T}{\partial z} + \frac{a^2}{c_p}\frac{\partial w}{\partial z} = \dot{q}''' \frac{a^2}{pc_p} \ , \tag{E.23}$$

$$\frac{\partial p}{\partial \tau} + w \frac{\partial p}{\partial z} + \rho a^2 \frac{\partial w}{\partial z} = \dot{q}'''(\kappa - 1) \ , \tag{E.24}$$

$$\frac{\partial w}{\partial \tau} + w \frac{\partial w}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} = -\frac{Z}{\rho} \ , \tag{E.25}$$

and

$$\frac{\partial \rho}{\partial \tau} + w \frac{\partial \rho}{\partial z} + \rho \frac{\partial w}{\partial z} = 0 \ , \tag{E.26}$$

or, in canonical form,

$$\frac{dz}{d\tau} = w \ ; \qquad \frac{dT}{d\tau} + \frac{a^2}{c_p \rho} \cdot \frac{d\rho}{d\tau} = \dot{q}''' \frac{a^2}{pc_p} \tag{E.27}$$

$$\frac{dz}{d\tau} = w \ ; \qquad \frac{dp}{d\tau} - a^2 \frac{d\rho}{d\tau} = \dot{q}'''(\kappa - 1) \tag{E.28}$$

$$\frac{dz}{d\tau} = w + a \ ; \qquad \frac{dw}{d\tau} + \frac{1}{\rho a}\frac{dp}{d\tau} + f\frac{w|w|}{2D_h} + g\cos\varphi = \dot{q}''' \frac{\kappa - 1}{a} \tag{E.29}$$

$$\frac{dz}{d\tau} = w - a \ ; \qquad \frac{dw}{d\tau} - \frac{1}{\rho a}\frac{dp}{dz} + f\frac{w|w|}{2D_h} + g\cos\varphi = -\dot{q}''' \frac{\kappa - 1}{a} \ . \tag{E.30}$$

The system describing the steady state can be obtained from Eqs. (E.23) through (E.26) so that

$$\frac{dp}{dz} = -\frac{Z + \frac{w}{a^2}\dot{q}'''(\kappa - 1)}{1 - \frac{w^2}{a^2}} \tag{E.31}$$

$$\frac{dw}{dz} = \frac{1}{\rho a^2} \frac{\dot{q}'''(\kappa - 1) + Zw}{1 - \frac{w^2}{a^2}} \tag{E.32}$$

$$\frac{dT}{dz} = \frac{a^2}{wc_p}\left(\frac{\dot{q}'''}{p} - \frac{dw}{dz}\right) \tag{E.33}$$

$$\frac{d\rho}{dz} = -\frac{\rho}{w}\frac{dw}{dz} \ . \tag{E.34}$$

NOMENCLATURE

A = cross section

a = speed of sound

C = exchange function describing the transfer of heat between fields

c_p = specific heat at constant pressure

D_h = hydraulic diameter

E_f = source of internal energy density from viscous dissipation

f_R = frictional force per unit volume of the phase or component

f = friction coefficient

g = gravitational acceleration

h = specific enthalpy

h_i = eigenvector

k = drag function, related to the exchange of momentum between phases

p = pressure

$\dot{q}'''$ = thermal power supplied to unit volume of flow

R = gas constant

T = absolute temperature

u = specific internal energy

w = velocity

z = linear coordinate

Greek Letters

α = void fraction

Δ = finite differential

∂ = partial differential

Φ_{Tph} = two-phase friction multiplier

κ = isentropic exponent

λ = eigenvalue

λ = heat conduction coefficient

μ = amount of steam generated per unit time per unit volume of the flow

μ_{fg},μ_{gf} = mass evaporating or condensing per unit time per unit volume and therefore interchanging between phases

ρ = density

τ = time

φ = angle between the upward directed vertical and the direction of flow

Indices

1,2 = component 1 or 2 in Eqs. (1) through (4)

L = air

D = steam

f = water

$''$ = saturated steam

$'$ = saturated water

ex = phase which yields mass

p = constant pressure

fr = friction

Tph = two phase

e = evaporation

c = condensation

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