Journal Title: The International journal of mechanical engineering education.

Volume: 7 **Issue:** 3
Month/Year: / 1979
Pages: 131-6

Article Author: Bober, W., Heinzman, J.D., and Laptchak, D.M.
Article Title: Non-Steady Nozzle Flow: An Educational Experiment

---

NON-STEADY NOZZLE FLOW: AN EDUCATIONAL EXPERIMENT

W BOBER *Associate Professor, Department of Mechanical Engineering, Rochester Institute of Technology, Rochester, New York*

J D HEINZMANN *Former Undergraduate Student, Department of Mechanical Engineering, Rochester Institute of Technology, Rochester, New York*

D M LAPYCHAK *Former Undergraduate Student, Department of Mechanical Engineering, Rochester Institute of Technology, Rochester, New York*

Received 16 October 1978

A non-steady nozzle flow experiment was carried out at RIT for the purpose of student education. A small tank was pressurized with air to 50 lbf/in^{2}a. Then the air was allowed to discharge through a converging nozzle drilled in the end wall of the tank. Three runs were made, each with a different size nozzle. The nozzle diameters ranged from 0·11 in to 0·05 in. During the run with the largest diameter nozzle, the air temperature inside the tank dropped as much as 150°F in approximately two seconds. After twenty seconds the air temperature in the tank returned nearly completely back to its initial value. This phenomenon was also mathematically modelled. Concepts from thermodynamics, gas dynamics and heat transfer were used in the model. The mathematical development was sufficiently complex that a computer solution was required. The nozzle and the accompanying apparatus was fabricated by an RIT student utilizing concepts from material processing. The instrumentation for the experiment included a piezoelectric transducer, a charge amplifier, a thermocouple made from very fine wires and two x, y plotters. The mathematical solution agreed reasonably well with the experimental results.

NOTATION

A_e	cross-sectional area of nozzle at exit
A_s	surface area of tank
$c_{p,a}$	specific heat of air at constant pressure
$c_{v,a}$	specific heat of air at constant volume
c_w	specific heat of the tank wall material
$\left(\frac{d'Q}{dt}\right)_{cv}$	rate heat flows out of control volume
$\left(\frac{dU}{dt}\right)_{cv}$	rate of change of the total internal energy in the control volume
$\left(\frac{d'W}{dt}\right)_{cv}$	rate work is done on control volume
$\frac{dm_a}{dt}$	rate of change of air mass inside tank
$\frac{dT_a}{dt}$	rate of change of air temperature
$\frac{dT_w}{dt}$	rate of change of wall temperature
g_c	proportionality constant appearing in Newton's Second Law
g	gravitational constant
Gr	Grashof number
h_e	enthalpy of air at nozzle exit
$\bar{h}_i$	average convective heat transfer coefficient inside tank
$\tilde{h}_i$	local convective heat transfer coefficient
$\bar{h}_0$	average convective heat transfer coefficient outside tank
J	Joules constant
k	c_p/c_v = ratio of specific heats
k_t	thermal conductivity
L	length of tank
m_a	air mass inside tank

INTRODUCTION

The first named author, as an instructor in thermodynamics, fluid mechanics and heat transfer, is continuously in search of simple experiments to illustrate concepts in the thermal fluid sciences. An experiment has recently been devised which can be analysed with concepts from thermodynamics, gas dynamics and heat transfer. Interest in this experiment arose when several students in thermodynamics attempted, as a group project, to verify experimentally a solution to a homework problem obtained from a standard introductory thermodynamics textbook. The problem was to determine the final temperature in an air tank after it was connected to a high pressure air line at 100°F and 300 lbf/in^{2}a. The tank volume was specified as well as the conditions of the air inside the tank just prior to the time when the high pressure line valve was opened. The students obtained a small tank made from a structural plastic material, connected it to the air line in the building, instrumented the tank with a pressure gauge and a thermocouple and proceeded to carry out the experiment. To their disap-

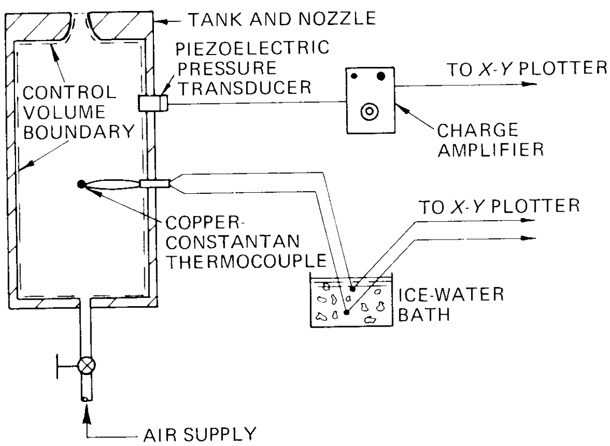

Fig 1. Schematic of experimental apparatus and instrumentation

pointment the experimental results did not agree with the analysis of the homework problem. The difficulty with the experiment is that the heat capacity of the solid walls of the tank is so much greater than that of air that heat transfer from the walls cannot be neglected, despite the fact that the structural plastic is not a good thermal conductor. A second difficulty in analysing the problem is the unsteady conditions at the inlet. As a result, we redesigned the experimental arrangement and proceeded with a more careful analysis. The redesigned experiment and its mathematical modelling are described in the following sections.

EXPERIMENTAL APPARATUS AND NOZZLE FABRICATION

The experimental apparatus consists of a cylindrical tank, 0·854 ft long and 0·220 ft in diameter. The walls are made from a polycarbonate plastic material. A nozzle opening was drilled at one end of the tank and the other end was connected to an air line in the building through an inlet valve (see Figs 1 and 2). Two portholes were drilled in the side walls of the tank to provide the means for the installation of a thermocouple and a piezoelectric pressure transducer. The thermocouple was made from copper-constantan wires, 0·001 in in diameter. The piezoelectric transducer was made from crystalline quartz material which has little temperature dependency. The transducer sensitivity remains almost constant from −400°F to 400°F. A Kistler, model 504, charge amplifier was employed with the piezoelectric transducer.

THE EXPERIMENT

The experimental apparatus was supported in a chemistry stand with the tank axis in a vertical position, and with the end containing the nozzle on top. The tank inlet was connected to the building air supply line through an inlet valve. The thermocouple leads were then inserted into an ice-water bath and connected to the x-y plotter through copper wire leads as shown in Fig 1. The piezoelectric transducer was connected to the charge amplifier. A second x-y plotter was then connected to the output of the charge amplifier. The settings on the x-y plotters and the charge amplifier were adjusted so that temperature and pressure readings were in °F and lbf/in^{2}g respectively. The x positions on the plotters were set to the internal time base position. To establish the initial conditions for the experiment, the nozzle exit was plugged by pressing a pencil eraser against the nozzle outlet while air was bled into the tank from the air supply line. When the tank pressure reached 50 lbf/in^{2}g the inlet valve was closed. Since the pressurization of the tank caused the air temperature inside the tank to rise, a few minutes had to be allowed for thermal equilibrium to be re-established before the unplugging of the nozzle took place. This was done as a matter of

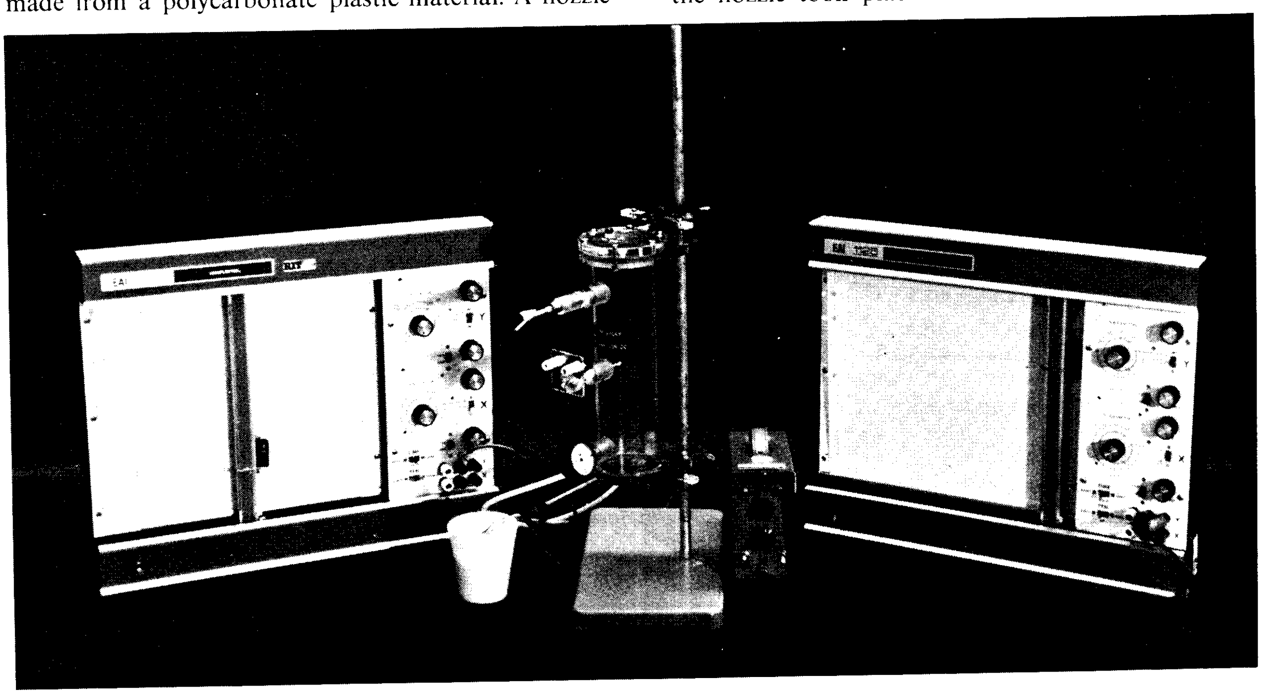

Fig 2. Photograph of experimental apparatus and instrumentation

convenience, and not as a necessary experimental procedure. When the air temperature inside the tank was sufficiently close to the ambient temperature, the x-y plotters were set into operation and the nozzle was unplugged. Air temperature and pressure variations inside the tank were recorded by the x-y plotters as the gas escaped through the nozzle. Experimental curves were obtained for several different runs with different size nozzles. Typical curves are shown in Figs 3 and 4. Several runs were also made with each nozzle to ensure the repeatability of the data.

THE MATHEMATICAL MODELLING OF THE EXPERIMENT

Select a control volume as shown by the dashed line in Fig 1. If $(\mathrm{d}U_a/\mathrm{d}t)_{cv}$ is the rate of increase in the internal energy of the air within the control volume, $(\mathrm{d}'W/\mathrm{d}t)_{cv}$ is the rate work is done on the fluid inside the control volume, $(\mathrm{d}'Q/\mathrm{d}t)_{cv}$ is the rate heat is added to the control volume, $\dot{m}_e$ is the rate mass flows out of the tank through the nozzle, h_e and v_e are, respectively, the enthalpy per unit mass and velocity magnitude of the air at the exit of nozzle, J is Joules constant and g_c is the proportionality constant appearing in Newton's second law, then the energy equation (**1**) for the control volume is

$$\left(\frac{\mathrm{d}U_a}{\mathrm{d}t}\right)_{cv} = \left(\frac{\mathrm{d}'W}{\mathrm{d}t}\right)_{cv} + \left(\frac{\mathrm{d}'Q}{\mathrm{d}t}\right)_{cv} - \dot{m}_e\left(h_e + \frac{1}{J}\frac{v_e^2}{2g_c}\right) \tag{1}$$

Here the terms representing the kinetic and potential energy of the air inside the tank have been neglected. Since the flow work term at the exit of the nozzle is contained in the h_e term, and there is no other work done on the fluid in the control volume

$$\left(\frac{\mathrm{d}'W}{\mathrm{d}t}\right)_{cv} = 0 \tag{2}$$

The internal energy, U_a, of the air inside the control volume can be expressed in terms of the air mass inside the control volume, m_a, its specific heat at constant volume, $c_{v,a}$ and the air temperature, T_a, inside the tank. We have assumed that the air temperature inside the

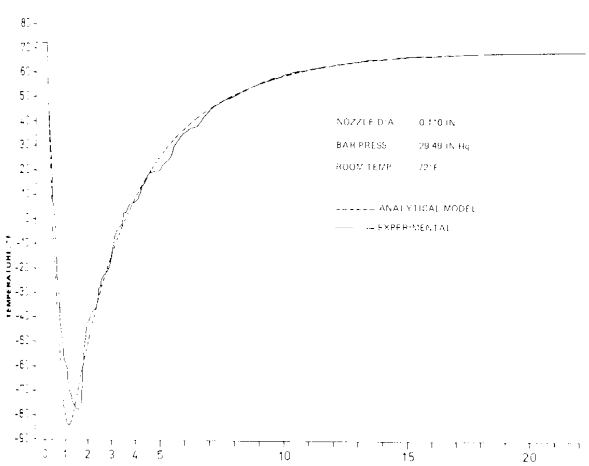

Fig 3. Tank air temperature time history for 0·11 in nozzle

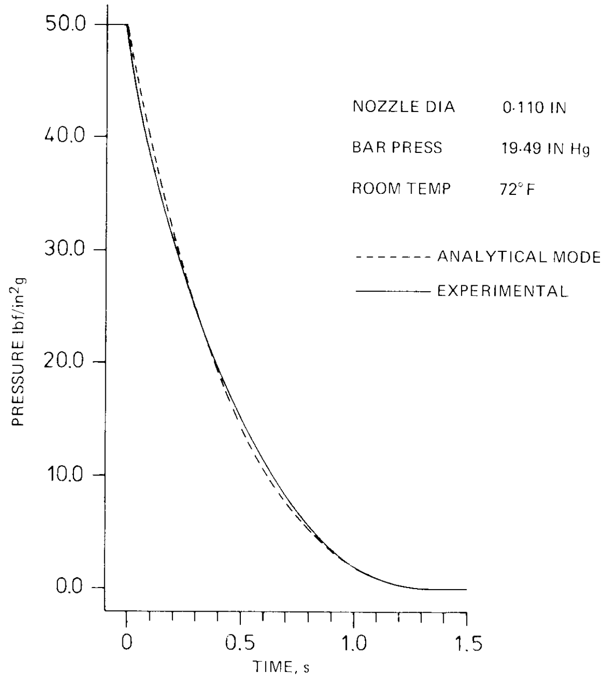

Fig 4. Tank air pressure time history for 0·11 in nozzle

tank is uniform, except for a very small boundary layer; i.e.

$$U_a = m_a c_{v,a} T_a \tag{3}$$

The rate heat flows into the control volume is given by the empirical relation (**2**) containing the inside average convective heat transfer coefficient, $\bar{h}_i$; i.e.

$$\left(\frac{\mathrm{d}'Q}{\mathrm{d}t}\right)_{cv} = \bar{h}_i A_s(T_w - T_a) \tag{4}$$

where A_s is the surface area of the walls of the tank and T_w is the wall temperature. This empirical relation is introduced to uncouple the heat transfer problem in the fluid from the heat transfer problem in the solid wall. There exist extensive experimental data, as well as analytical techniques, for determining the value of the convective heat transfer coefficient under a variety of conditions. The temperature variation through the wall is expected to be insignificant. Thus we may take $T_w = T_w(t)$. The enthalpy per unit mass of air at the exit can be expressed in terms of specific heat at constant pressure, $c_{p,a}$, and the air temperature, T_e, at the exit of the nozzle; i.e.

$$h_e = c_{p,a} T_e \tag{5}$$

Substituting equations (2), (3), (4) and (5) into equation (1), we obtain

$$c_{v,a}\left[\dot{m}_a T_a + m_a \frac{\mathrm{d}T_a}{\mathrm{d}t}\right] = h_i A_s(T_w - T_a) - \dot{m}_e\left(c_{p,a} T_e + \frac{1}{J}\frac{v_e^2}{2g_c}\right) \tag{6}$$

where $\dot{m}_a$ is the rate of change of mass within the control volume.

In a similar manner we can write an energy equation for the walls of the tank; i.e.

$$\frac{d}{dt}(m_w c_w T_w) = A_s \bar{h}_0 (T_\infty - T_w) - A_s \bar{h}_i (T_w - T_a) \quad (7)$$

where m_w is the mass of the wall, $\bar{h}_0$ is the outside convective heat transfer coefficient, c_w is the specific heat of the wall and T_∞ is the room temperature. In this equation, we have taken the temperature of the wall to be uniform. Thus, $T_w(t)$ really represents an average wall temperature. This is justified because the temperature drop across the wall is small. Finally, the time rate of change of mass within the tank is equal to the negative of the rate mass flows out of the tank through the nozzle, i.e.

$$\dot{m}_a = -\dot{m}_e \quad (8)$$

Taking m_w and c_w as constants in equation (7) and rearranging terms in equations (6) and (7) we obtain

$$\begin{aligned} \frac{dT_a}{dt} &= \frac{\dot{m}_e}{m_a} T_a + \frac{A_s \bar{h}_i}{c_{v,a} m_a}(T_w - T_a) \\ &\quad - \frac{\dot{m}_e}{m_a}\frac{c_{p,a}}{c_{v,a}} T_e - \frac{\dot{m}_e v_e^2}{2Jg_c c_{v,a} m_a} \\ &= f_1(t;\, T_a,\, T_w,\, m_a) \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{dT_w}{dt} &= \frac{A_s \bar{h}_0}{m_w c_w}(T_\infty - T_w) - \frac{A_s \bar{h}_i}{m_w c_w}(T_w - T_a) \\ &= f_2(t;\, T_a,\, T_w,\, m_a) \end{aligned} \quad (10)$$

$$\frac{dm_a}{dt} = -\dot{m}_e = f_3(t;\, T_a,\, T_w,\, m_a) \quad (11)$$

Equations (9), (10) and (11) represent three coupled ordinary differential equations. A fourth equation, which is the equation of state for the air, relates the air pressure inside the tank to the other state variables. In this temperature and pressure range, the ideal gas law is valid, i.e.

$$p_a = \frac{m_a R_a T_a}{V} \quad (12)$$

where V is the volume inside the tank and R_a is the gas constant for air.

The functional relation for $\dot{m}_e$ in terms of the other variables is obtained from one-dimensional compressible flow (**3**). Two possible cases exist, depending on the ratio p_b/p_a where p_b is the pressure of the region in which the nozzle empties, which in this case, is the room pressure; p_b is often called the back pressure. The two cases are:

Case 1

$$p_b/p_a > 0{\cdot}528$$

then

$$\begin{aligned} p_e &= p_b \\ M_e &= \frac{2}{k-1}\left[1 - \left(\frac{p_e}{p_a}\right)^{(k-1)/k}\right] \\ T_e &= \frac{T_a}{1 + \dfrac{k-1}{2} M_e^2} \\ c_e &= (kR_a g_c T_e)^{1/2} \\ v_e &= M_e c_e \\ \dot{m}_e &= \frac{p_e}{R_a T_e} A_e v_e \end{aligned} \quad (13)$$

where the subscript e designates that the fluid property is to be evaluated at the nozzle exit, M is the mach number, c is the speed of sound, k is the ratio of specific heats and A is the cross-sectional area in the nozzle.

Case 2

$$p_b/p_a \le 0{\cdot}528 \qquad \text{(flow is choked)}$$

then

$$\begin{aligned} p_e &= 0{\cdot}528 p_a \\ M_e &= 1{\cdot}0 \\ T_e &= \frac{T_a}{1 + (k-1)/2} \\ v_e &= c_e = (kR_a g_c T_e)^{1/2} \\ \dot{m}_e &= \frac{p_e}{R_a T_e} A_e v_e \end{aligned} \quad (14)$$

Finally, an estimate of the convective heat transfer coefficient $\bar{h}_i$ still needs to be made. Since the cross-sectional area of the tank is so much greater than the cross-sectional area of the nozzle, the velocity of the air stream inside the tank will be very small, despite the fact that over part of the run the velocity of the air stream at the exit of the nozzle will be sonic. Thus it is reasonable to assume that heat transfer will take place from the walls to the air inside the tank predominantly by natural convection. The analytical solution (**4**) for natural convection from a vertical plate is applicable to vertical cylindrical walls. The average convective heat transfer coefficient $\bar{h}_i$ is obtained from the local convective heat transfer coefficient, $\bar{h}_i(x)$, which is itself obtained through the local Nusselt number, defined by

$$Nu(x) = \frac{x\bar{h}_i(x)}{k_t} \quad (15)$$

where k_t = thermal conductivity in the fluid. When the flow is laminar, the solution for the local Nusselt number is given by

$$Nu(x) = 0{\cdot}508 Pr^{1/2}(0{\cdot}952 + Pr)^{-1/4}[Gr(x)]^{1/4} \quad (16)$$

where Pr is the Prandtl number and $Gr(x)$ is the local Grashof number, which is defined by

$$Gr(x) = \frac{g\beta(T_w - T_a)x^3}{\nu^2} \quad (17)$$

where β is the coefficient of thermal expansion of the fluid. β is defined by

$$\beta = \frac{1}{v^*}\left(\frac{\partial v^*}{\partial T}\right)_p$$

where v^* is the specific volume of the fluid and the subscript p means that the derivative is to be taken at constant pressure. For an ideal gas

$$\beta = \frac{1}{T}$$

Since T varies within the boundary layer, the fluid properties, except the coefficient of thermal expansion (**4**), are evaluated at the film temperature, T_f, where

$$T_f = \tfrac{1}{2}(T_\infty + T_w) \tag{18}$$

The coefficient of thermal expansion is evaluated at the temperature of the air some distance from the wall; i.e. at T_a. The average heat transfer coefficient, $\bar{h}_i$, along the wall is obtained from

$$\bar{h}_i = \frac{1}{L}\int_0^L \tilde{h}_i(x)\,dx = \tfrac{4}{3}\tilde{h}_i(L) \tag{19}$$

The Grashof number is a dimensionless group which in free convection gives the ratio of buoyancy forces to the viscous forces (**4**). It is the variable used as a criterion for establishing transition from a laminar to a turbulent boundary layer, just as the Reynolds number is used in forced convection. The critical Grashof number (**5**) is approximately 4×10^8. An empirical formula is also available for evaluating the average convective heat transfer coefficient, $\bar{h}_i$. Heat transfer rates determined by this empirical formula are approximately 10 per cent greater than those obtained by the formulae developed from analytical methods (**5**). The empirical relation for $\bar{h}_i$ for natural convection from vertical plates and cylinders is

$$\frac{\bar{h}_i L}{k_t} = C(Gr\,.\,Pr)^m \tag{20}$$

where

$$Gr = \frac{g\beta(T_w - T_a)L^3}{\nu^2}$$

$C = 0{\cdot}59$ and $m = \frac{1}{4}$ for $10^4 < Gr\,.\,Pr \leq 10^9$

and

$C = 0{\cdot}129$ and $m = \frac{1}{3}$ for $10^9 < Gr\,.\,Pr \leq 10^{12}$

Both methods were tried in the computer program and it was found that the empirical formula for $\bar{h}_i$ gave better agreement with the experimental results. Thus, equation (20) was incorporated into the final computer program. However, for values of $Gr\,.\,Pr < 10^4$, the analytical solution for $\bar{h}_i$ was used. This was done because values of C and m for equation (20) were not readily available in this range.

Finally, heat transfer from the horizontal end walls was accounted for by increasing the length of the vertical wall by the length of the radius of the circular plate at each end. This was done because the correction was relatively small and the method was rather convenient. If the heat transfer from the horizontal surfaces was evaluated separately, then the temperatures of the three separate walls would have to be carried in the program, further complicating the problem.

A numerical solution to the problem was obtained on a Sigma IV digital computer using the fourth order Runge-Kutta method for solving a system of coupled first order differential equations. In this method the time axis is sub-divided into N sub-intervals. If the system of equations is written in the form of

$$\frac{dT_a}{dt} = f_1(t;\ T_a,\ T_w,\ m_a) \tag{21}$$

$$\frac{dT_w}{dt} = f_2(t;\ T_z,\ T_w,\ m_a) \tag{22}$$

$$\frac{dm_a}{dt} = f_3(t;\ T_a,\ T_w,\ m_a) \tag{23}$$

and

$$p_a = \frac{R_a}{V} m_a T_a \tag{24}$$

then the Runge-Kutta formula (**6**) for obtaining the values of the dependent variables at the time step, t_{i+1}, is given by

$$T_{a,i+1} = T_{a,i} + \frac{\Delta t}{6}(l_1 + 2l_2 + 2l_3 + l_4) \tag{25}$$

$$T_{w,i+1} = T_{w,i} + \frac{\Delta t}{6}(q_1 + 2q_2 + 2q_3 + q_4) \tag{26}$$

$$m_{a,i+1} = m_{a,i} + \frac{\Delta t}{6}(r_1 + 2r_2 + 2r_3 + r_4) \tag{27}$$

$$p_{a,i+1} = \frac{R_a}{V} m_{a,i+1} T_{a,i+1} \tag{28}$$

where

$$l_1 = f_1(t_i;\ T_{a,i},\ T_{w,i},\ m_{a,i}) \tag{29}$$

$$l_2 = f_1\left(t_i + \frac{\Delta t}{2};\ T_{a,i} + \frac{\Delta t}{2} l_1,\ T_{w,i} + \frac{\Delta t}{2} q_1,\ m_{a,i} + \frac{\Delta t}{2} r_1\right) \tag{30}$$

$$l_3 = f_1\left(t_i + \frac{\Delta t}{2};\ T_{a,i} + \frac{\Delta t}{2} l_2,\ T_{w,i} + \frac{\Delta t}{2} q_2,\ m_{a,i} + \frac{\Delta t}{2} r_2\right) \tag{31}$$

$$l_4 = f_1(t_i + \Delta t;\ T_{a,i} + \Delta t l_3,\ T_{w,i} + \Delta t q_3,\ m_{a,i} + \Delta t r_3) \tag{32}$$

r_1, r_2, etc. are analogous to l_1, l_2, etc. The initial conditions, $T_a(0)$, $T_w(0)$, $m_a(0)$ and $p_a(0)$ must also be specified. In the computer program the variations of the thermal properties of air with respect to temperature were taken into account. Halving the step size for the run involving the 0·110 in nozzle did not significantly affect the analytical solution. Thus, the original step size was kept for all three runs.

EXPERIMENTAL AND ANALYTICAL RESULTS

The experiment was run for three different nozzle sizes; these being, 0·050, 0·080 and 0·110 in diameter. The initial conditions of each experiment were read into the computer program. A step size, Δt, equal to 0·01 s was used in the program. Typical plots of T_a vs t and p_a vs t obtained both analytically and experimentally are shown in Figs 3 and 4. As can be seen, there is good agreement between the analytical solution and the experimental results. The largest air temperature drop occurred for the largest diameter nozzle. The temperature drop for this case was approximately 150°F, which took place in approximately 2 s. After 20 s the air temperature in the tank returned nearly completely back to its initial value. This comes about because of the heat transfer from the tank walls. It can also be seen that pressure equalization occurs much quicker than temperature equalization. The pressure equalization occurred in approximately 1·4 s.

From Fig 3, it can be seen that the experimental temperature time history curve indicates some mild fluctuations in the signal from the thermocouple. Temperature curves obtained earlier, showed much larger fluctuations than these temperature curves. Two possible explanations were postulated, these being: (a) water vapour present in the air condensing and freezing on the thermocouple, (b) the air velocity inside the tank causing the thermocouple wires to vibrate and thus affecting the convective heat transfer coefficient from air to thermocouple. Steps were taken to reduce or eliminate both effects. First, bottled, pressurized, dry air was used instead of the building air supply. This resulted in only a minor improvement. Next, the thermocouple support was changed to make the thermocouple more rigid. This had a more significant effect, giving the experimental curves shown in Fig 3.

CONCLUSIONS

This project provided an excellent educational experience for our RIT students. They had the opportunity to develop both their experimental and analytical capabilities. The mathematical modelling required them to draw on the knowledge they obtained in several courses in the thermal fluid sciences, as well as their course in computer techniques. The good agreement between experiment and theory has given them confidence in the subject matter that they have learned in their courses.

REFERENCES

(1) HOLMAN, J P, *Thermodynamics*, McGraw-Hill, New York, 1974
(2) KREITH, F, *Principles of heat transfer*, Intext, New York, 1973
(3) JOHN, J E A, *Gas dynamics*, Allyn and Bacon, Boston, 1969
(4) PARKER, J D, BOGGS, J H, and BLICK, E F, *Introduction to fluid mechanics and heat transfer*, Addison-Wesley, Reading, Massachusetts, 1969
(5) CHAPMAN, A J, *Heat transfer*, Macmillan, New York, 1967
(6) JAMES, M L, SMITH, G M, and WOLFORD, J C, *Applied numerical methods for digital computation with Fortran*, International, Scranton, Pennsylvania, 1970