Combining ranking information from different sources in judgment post stratifed samples

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Abstract

This paper constructs estimators that combine ranking information from different methods in a judgment post-stratified (JPS) sample. The JPS sample divides the units in a simple random sample into different ranking groups based on their relative positions (ranks) in a small comparison set. Ranks in the comparison sets can be constructed using different ranking schemes and each ranking scheme leads to different estimator. We provide two weighted estimators that combines ranking information from different sources. The first estimator is constructed using the standard deviation of the estimators for a given set of ranks. The second estimator is constructed from the agreement scores of ranking methods. We show that the new estimators provide substantial amount of improvement over a JPS sample estimator based on a single set of ranks from a single ranking method. The new estimators are applied to a field experiment in agricultural research.

Key words: Agreement scores; meta-analysis; multi-ranker model; ranked set sampling; jackknife estimator

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1 Introduction

Stratified sample controls the total variation better than a simple random sample of the same size by partitioning it as within- and between-stratum variations. In certain settings, partitioning principal may not be applicable due to absence of stratification variable to create disjoint subgroups. In this case, stratification principal can be used at the sample level to create a judgment post-stratified (JPS) sample. JPS sample is introduced in MacEachern et al. (2004) to stratify the sample units in a simple random sample to improve its information content. To construct a JPS sample, one first constructs a simple random sample (SRS) and measures all selected units for the characteristic of interests. For each unit in this sample $H-1$ additional units are selected without measurement from the population to form a comparison set of size H. Relative positions (ranks) of the measured units in comparison sets are determined, and the measured sample units in the original SRS are stratified based on these ranks. Ranking process may be subjective or imprecise. Hence, it may lead to ranking error. Main difference between a stratified and JPS sample is that a stratified sample creates a partitioned sample by selecting simple random samples from disjoint sub-populations while a JPS sample partitions a simple random sample using position information in comparisons sets. In other words, stratified sample uses population level stratification and JPS sample uses sample level stratification. Hence, strata sample sizes are constant in a stratifed sample and random in a JPS sample.

The ranking group h, h = 1, $\cdots$, H, contains all measured observations with judgment rank h. Since the rank of a measured unit in its comparison set provides information about its relative position among the other $H-1$ unmeasured units, observations in judgment class h are stochastically larger then the observations in judgment class h', for $h' < h$. Then , one can view a JPS sample as a stochastically ordered stratified sample with H strata. In this case, the efficiency improvement of a JPS sample can be established from standard theory of stratified sampling in survey sampling designs.

Further details in JPS sampling design can be found in Frey and Feeman (2012, 2013), Frey and Ozturk (2011), Stokes, Wang and Chen(2007), Wang, Lim, Stokes (2008), Wang, Stokes, Lim and Chen (2006), Wang, Wang and Lim(2012), Ozturk (2013, 2014a, 2014b, 2015a) and the references there in.

Recently there have been increased research activities in JPS sampling in finite population setting. Ozturk (2015) constructed estimators for the population mean and total using JPS sample. The JPS sample is constructed either with or without replacement selection. It is shown that the variance estimator of the sample mean requires a finite population correction factor similar to the one used in simple random sample. Ozturk and Bayramoglu Kavlak (2018,2019a,2019b) developed inference to predict the population mean and total using super population model.

In a JPS sample, ranks are assigned post-experimentally after a simple random sample is collected. In certain settings, it is possible to have more than one set of ranks for the same measured units (simple random sample). For example, units in each comparison set may be ranked by K different rankers using visual inspection or K different comparison sets constructed by permuting the unmeasured units may be ranked by the same individual. Both of these ranking schemes will lead to creation of K sets of ranks. Another example can be given from survey sampling studies, where there could be K different auxiliary variables correlated with the variable of interest. Ranking based on these covariates again creates K sets of ranks. Regardless of how the ranks are created, ranking information from different sources needs to be combined to draw statistical inference. Ozturk (2013) and Ozturk and Demirel (2016) used the tie structure in comparison sets and agreement scores among K ranking schemes to construct estimator for the population mean and variance, respectively.

To best of our knowledge, there is no available procedure in the literature to combine ranking information in a JPS sample from different sources. This paper provides a method to construct new estimators to estimate the population mean and total by combining ranking information obtained from different sources. Section 2 provides detailed description for the construction of JPS samples with and without replacement selection. It also provides a brief review of the known results of the distributional properties of the estimators. Section 3 introduces two weighted estimators to combine the ranking information from K different sets of ranks. The weight in the first estimator is a function of the standard error of the JPS estimator based on each ranking source. The weight of the second estimator constructed from the agreement scores of the ranking sources. Section 4 constructs variance estimator for the combined weighted estimator of the population mean. Section 5 investigates the empirical properties of the proposed estimators. Section 6 provides some concluding remark. All proofs are provided in Appendix.

2 Sampling designs

We consider a wide range of population setting where JPS sampling could be applicable. Finite population setting with a replacement selection protocol covers JPS sampling designs in infinite population setting. Therefore, in this paper, we assume a finite population setting unless specified otherwise. We consider JPS samples with two different *without replacement* selection designs, D_1 and D_2. Design D_1 selects the units in the comparison sets without replacement, ranks them from smallest to largest and measure one of the ranked unit. Once the measurement of the ranked unit is completed, all units including the measured one is returned to population before constructing the next comparison set. In this design same unit can appear in the sample more than one time. This design is equivalent to infinite population setting. All observations are independent. The design D_2 is constructed by removing all units in the comparison set from the population including the measured and unmeasured ones before selecting the next comparison set. These designs induce different kind of correlation structure in JPS sample. All observations in design D_1 are independent, the observations in designs D_2 are negatively correlated

The finite population of size N will be denoted with $\mathcal{P}$ where N is the population size. Let Y be the variable of interest. The values of Y on population units will be denoted with $y_1, \cdots, y_N$. Without loss of generality, we assume that the population values of characteristic Y are ordered, $y_1 < \cdots < y_N$.

Design D_1: In this design, We first select a simple random sample with replacement from the populations and measure all of them, $Y_1, \ldots, Y_n$, where $Y_i = Y_{u_i}$. For each Y_i, we then select additional $H-1$ units without replacement from the population to form a comparison set $\{Y_i, Y_1, \ldots, Y_{H-1}\}$ We rank these units from smallest to largest based on the perceived value of the characteristic Y without measurement and identify the rank of Y_i, $R_{i,1}$. All units in the comparison set including the one we measured are returned to the population before the construction of the next comparison set. This process creates the sample

$$D_1 = \{Y_i, R_{i,1}\};\ i = 1, \ldots, n.$$

Design D_2: The JPS sample in design D_2 is constructed in a similar fashion. We first select a simple random sample of size n without replacement and measure all of them, $Y_i, \ldots, Y_n$. Again for each measured value Y_i we construct a comparison set, $\{Y_i, Y_1, \ldots, Y_{H-1}\}$, and identify its ranks $(R_{i,2})$ for $i = 1, \ldots, n$. In this design all unit in comparison set is removed from the population before constructing the next comparison set. Hence, all comparison sets are disjoint. The JPS sample with design D_2 will be represented with

$$D_2 = \{Y_i, R_{i,2}\}; i = 1, \ldots, n.$$

We note that the notation in samples D_1 and D_2 are identical, but they have different distributional properties since sample D_2 is constructed without replacement selection. If the population size is large, pairs in sample D_2 become independent.

The conditional mean and variance of Y_j given $R_{j,r} = h$ and the conditional covariance of Y_j, Y_t given that $R_{j,r} = h, R_{t,r} = h'$ will be denoted by

$$\begin{aligned}
\mu_{[h]} &= E(Y_{[h]}) = E(Y_j | R_{j,r} = h),\ \text{for } r = 1, 2 \\
\sigma^2_{[h]} &= Var(Y_{[h]}) = var(Y_j | R_{j,r} = h) \text{ for } r = 1, 2
\end{aligned}$$

and

$$\sigma_{[h,h']} = cov(Y_{[h]}, Y_{[h]'}) = cov(Y_j, Y_t | R_{j,2} = h, R_{t,2} = h') \text{ for } r = 2.$$

For design D_1, $\sigma^2_{[h,h']} = 0$ since observations are independent. Under perfect ranking, the square brackets in these expressions will be replaced with round parentheses.

Unbiased estimators of the population mean and total based on samples D_1 and D_2 are given by

$$\bar{Y}_{JPS,r} = \sum_{h=1}^{H} \frac{I_{h,r} J_{h,r}}{d_{n,r}} \sum_{i=1}^{n} I(R_{i,r} = h) Y_i, \; r = 1, 2$$

where the subscript r used to denote design D_r and

$$J_{h,r} = \begin{cases} 1/n_{h,r} & n_{h,r} > 0 \\ 0 & n_{h,r} = 0, \end{cases} \qquad n_{h,r} = \sum_{i=1}^{n} I(R_{i,r} = h), \quad I_{h,r} = I(n_{h,r} > 0), \quad d_{n,r} = \sum_{h=1}^{H} I_{h,r}.$$

The estimator $\bar{Y}_r$ can be written in a slightly different form as a weighted average of judgment class means

$$\bar{Y}_r = \sum_{h=1}^{H} w_{h,r} \bar{Y}_h, \; \bar{Y}_h = J_{h,r} \sum_{i=1}^{n} I(R_{i,r} = h) Y_i, \quad w_{h,r} = I_{h,r}/d_{n,r}.$$

where the weights $w_{h,r}$ are used to make the estimator unbiased for any set H and sample size n. In a JPS sample, ranks have a discrete uniform a distribution with a support on integers $1, \ldots, H$. Hence, $n_{h,r}$, $d_{n,r}$, and $w_{h,r}$ are all random variables since they are the functions of ranks $R_{i,r}$, $i = 1, \ldots, n$. The sample size vector of judgment class groups, $\boldsymbol{n}_r = (n_{1,r}, \ldots, n_{H,r})$, has a multinomial distribution with the number of trials n and success probability vector $(1/H, \ldots, 1/H)$. The following theorem provides some useful results on these random variables.

Lemma 1 *The following equalities hold for the distribution of* $\frac{I_{h,r}}{d_{n,r}}, h = 1, \cdots H, r = 1, 2$ *(Ozturk, 2014; Dastbaravarde et.al., 2016):*

- $E(\frac{I_{1,r}}{d_{n,r}}) = 1/H$
- $E(\frac{I_{1,r}}{d_{n,r}^2}) = \frac{1}{H^2} \sum_{k=1}^{H} (\frac{k}{H})^{n-1}$
- $Var(\frac{I_{1,r}}{d_{n,r}}) = \frac{1}{H^2} \sum_{k=1}^{H-1} (\frac{k}{H})^{n-1}$
- $cov\left(\frac{I_{1,r}}{d_{n,r}}, \frac{I_{2,r}}{d_{n,r}}\right) = -\frac{1}{H-1} var\left(\frac{I_1}{d_{n_l}}\right)$
- $E(\frac{I_{1,r}^2}{n_{1,r} d_{n,r}^2}) = \frac{1}{H^n} \left(\frac{1}{n} + \sum_{k=2}^{H} \sum_{j=1}^{k-1} \sum_{t=1}^{n-k+1} \frac{(-1)^{j-1}}{k^2 t} \binom{H-1}{k-1} \binom{k-1}{j-1} \binom{n}{t} (k-j)^{n-t}\right)$

We note that expected values, variances and covariances in Lemma 1 do not depend on the values of Y on population units. They can be computed for given sample and set sizes. We do not assume that the rank $R_{i,r}$ is the same as the actual rank of Y_i in its comparison set. For an unbiased estimator, we use a consistency requirement for the ranking process. The ranking procedure is called consistent if it satisfies the following equality

$$\frac{1}{H}\sum_{h=1}^{H} E\left\{Y_i | I(R_{i,r}=h)\right\} = E(Y_i) \text{ for } r=1,2.$$

The consistency of ranking procedure holds as long as ranks of all units in comparison sets are determined using the same ranking procedure in all comparisons sets.

Ozturk (2016) shows that the estimator $\bar{Y}$ is unbiased for population mean under a consistent ranking scheme and provides a closed form expression for its variance under designs D_1 and D_2. His main results are stated in the following theorem for an easy access in the reminder of the paper.

Theorem 1 *Let $(Y_i, R_i); i = 1, \cdots, n$, be a JPS sample of set size H constructed under a consistent ranking scheme based on either design D_1 or design D_2 from the finite population $\mathcal{P}$. (i) The estimators $\bar{Y}_r$ are unbiased for μ. (ii) the variance of the estimators $\bar{Y}_r$ are*

$$\sigma_1^2 = \frac{H}{H-1} Var(\frac{I_{1,1}}{d_{n,1}}) \sum_{h=1}^{H} (\mu_{[h]} - \mu)^2 + E\left(\frac{I_{1,1}^2}{d_{n,1}^2 n_{1,1}}\right) \sum_{h-1}^{H} \sigma_{[h]}^2$$

for sample D_1 and

$$\sigma_2^2 = C_1(n,H)\left\{\sum_{h=1}^{H}\sigma_{[h]}^2 - \sum_{h=1}^{H}\sigma_{h,h}\right\} + C_2(n,H,N)\frac{H^2\sigma^2}{H-1} \quad (1)$$

$$(2)$$

for sample D_2, where

$$C_1(n,H) = \left\{\frac{1}{H(H-1)} + E(\frac{I_{1,2}^2}{d_{n,2}^2 n_{1,2}}) - \frac{H}{H-1}E(\frac{I_{1,2}^2}{d_{n,2}^2})\right\}$$

$$C_2(n,H,N) = \left\{Var(\frac{I_{1,2}}{d_{n,2}}) - \frac{1}{N-1}\left\{\frac{1}{H} - E(\frac{I_{1,2}^2}{d_{n,2}^2})\right\}\right\}$$

The variance of the estimator $\bar{Y}_r$ can be computed . Let

$$
\begin{aligned}
U_{1,r} &= \frac{1}{E\left(\frac{I_{1,r}I_{2,r}}{d^2_{n,r}}\right)} \sum_{h=1}^{H}\sum_{h\neq h'}^{H} \frac{I_{h,r}I_{h',r}}{n_{h,r}n_{h',r}d^2_{n,r}} \sum_{i=1}^{n}\sum_{j=1}^{n}(X_i - X_j)^2 I(R_{i,r}=h)I(R_{j,r}=h'),\\
U_{2,r} &= \sum_{h=1}^{H} \frac{HI^*_{h,r}}{n_{h,r}d^*_{n,r}(n_{h,r}-1)} \sum_{i=1}^{n}\sum_{j\neq i}^{n}(X_i - X_j)^2 I(R_{i,r}=h)I(R_{j,r}=h),
\end{aligned}
$$

and

$$
\hat{\sigma}^2_{SRS} = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2,
$$

where $I^*_{h,r} = I(n_{h,r} > 1)$, $d^*_{n,r} = \sum_{h=1}^{H} I^*_{h,r}$. From Lemma 1, one can easily establish that $E(I_{1,r}I_{2,r}/d^2_{n,r}) = (1/H - E(I_{1,r}/d_{n,r})^2)/(H-1)$. Hence, $U_{1,r}$ is a statistic that depends only on the data. The following theorem is stated from Ozturk (2016) by adapting it to the notation of this paper.

Theorem 2 *Let $(Y_i, R_{i,r})$, $i = 1, \cdots, n$, be JPS samples of set size H either from design D_r, $r = 1, 2$. Unbiased estimators of σ^2_1 and σ^2_2 are given by, respectively,*

$$
\begin{aligned}
\hat{\sigma}^2_1 &= \frac{Var(I_{1,1}/d_{n,1})}{2(H-1)}U_{1,1} + \left\{E\left(\frac{I^2_{1,1}}{d^2_{n,1}n_{1,1}}\right) - Var(\frac{I_{1,1}}{d_{n,1}})\right\}\frac{U_{2,1}}{2}, && (3)\\
\hat{\sigma}^2_2 &= C_1(n,H)U_{2,2}/2 + C_2(n,H,N)\frac{H^2\hat{\sigma}^2_{SRS}}{H-1}, && (4)\\
\bar{\sigma}^2_2 &= C_1(n,H)U_{2,2}/2 + C_2(n,H,N)\frac{(N-1)(U_{1,2}+U_{2,2})}{2N(H-1)}, && (5)
\end{aligned}
$$

Theorem 2 provides two unbiased estimators $\hat{\sigma}^2_2$ and $\bar{\sigma}^2_2$ for σ^2_2 . Ozturk(2016) shows that for some values of n,H, and N, the coefficient $C_2(n,H,N)$ could be negative. This rarely may lead to a negative value for $\hat{\sigma}^2_2$. When this happens, the following truncated estimator can be used

$$
\tilde{\sigma}^2_2 = \begin{cases} \hat{\sigma}^2_2 & \text{if } \hat{\sigma}^2_2 > 0 \\ C_1(n,H)U_{2,2}/2 & \text{if } \hat{\sigma}^2_2 \leq 0. \end{cases} \qquad (6)
$$

Further development on this estimator can be found in Ozturk(2016).

Ozturk and Bayramoglu Kavlak (2020) constructed estimator for the population mean and total using super population model. Super population model assumes that the population values $y_1, \ldots, y_N$ are just one realization of independent identically distributed ($i.i.d$) random variables $Y_1, \ldots, Y_N$ from a supper population having mean μ and variance σ^2.

$$M : Y_1, \ldots, Y_N \text{ are } i.i.d \text{ from } F \text{ with meand } \mu \text{ and variance } \sigma^2. \tag{7}$$

Using this model, Ozturk and Bayramoglu Kavlak (2020) constructed predictor for the population mean $\bar{Y}_N = \sum_{i=1}^{N} Y_i$ and provided mean square prediction error. Under design D_2, the predictor is given by

$$\bar{Y}_M = \sum_{h=1}^{H} \frac{I_{h,2} J_{h,2}}{d_{n,2}} \sum_{i=1}^{n} I(R_{i,2}) = h) Y_i.$$

Even though $\bar{Y}_M$ has exactly the same form as $\bar{Y}_{JPS,2}$, its distributional properties are different due to super population model assumption. The following result is proved in Ozturk and Bayramoglu Kavlak (2020)

Theorem 3 *Let $(Y_i, R_i), i = 1, \cdots, n$, be a JPS sample from a finite population $\mathcal{P}^N$. Under super population model, the predictor $\bar{Y}_M$ is model unbiased, and the mean square prediction error of $\bar{Y}_M$ is given by*

$$\begin{aligned} \sigma_M^2 &= MSPE(\bar{Y}_M) = \left[\frac{-1}{N} + HE\left(\frac{I_{1,2}^2 J_{1,2}}{d_{n,2}^2}\right)\right]\sigma^2 \\ &+ \left[\frac{H}{H-1} var\left(\frac{I_{1,2}}{d_{n,2}}\right) - E\left(\frac{I_{1,2}^2 J_{1,2}}{d_{n,2}^2}\right)\right] \sum_{h=1}^{H} \left(\mu_{[h]} - \mu\right)^2, \end{aligned} \tag{8}$$

where $\mu_{[h]}$ is the expected value of the h-th judgment order statistics in a comparison set of size H.

For any set and sample sizes unbiased estimator of σ_M^2 is given by

$$\begin{aligned} \hat{\sigma}_M^2 &= \frac{(U_{1,2} + U_{2,2})}{2H^2}\left[\frac{-1}{N} + \frac{H^2}{H-1} var\left(\frac{I_{1,2}}{d_{n,2}}\right)\right] \\ &+ \frac{U_{2,2}}{2}\left[E\left(\frac{I_{1,2}^2 J_{1,2}}{d_{n,2}^2}\right) - \frac{H}{H-1} var\left(\frac{I_{1,2}}{d_{n,2}}\right)\right]. \end{aligned} \tag{9}$$

Readers are referred Ozturk and Bayramoglu Kavlak (2020) for further development on these estimators.

3 Combining ranking Information

In his section, we look at the JPS samples in the previous section from different perspective. We assume that each measured unit is assigned K conditionally independent ranks given the comparison set. The JPS sample still has single measurement (Y_i) for each unit, but it has K different ranks. The definition of the JPS sample in the previous section is extended to cover the source of ranking information

$$D_{r,k} = \{Y_i, R_{i,k,r}\}, i = 1, \ldots, n,\ r = 1, 2,\ k = 1, \ldots, K,\ r = 1, 2, M,$$

where $R_{i,k,r}$ is the rank assigned to Y_i by ranking method k in design D_r. The subscipt $r = M$ is used to denote that the data is genreated from superpopulation model M in equation (7) without replacement selection. Ranks can be constructed using information from different sources. Ranking Information could come from visual inspection of units in the comparison sets by different individuals or set of auxiliary variables correlated with Y or the combination of both. Our objective here is to combine the information contained in these K sets of ranks to construct a better estimator. We consider two different approaches to combine ranking information. The first approach combines the ranking information using standard error of the estimator. The second approach combines the ranking information using agreement scores among K sets of ranks.

Let $\bar{Y}_{k,r}$ and $\hat{\sigma}^2_{k,r}$ be the JPS estimator and its variance estimate, respectively, based on ranker k and design D_r. For the super population model, we use the notation $\bar{Y}_{k,M}$ and $\hat{\sigma^2}_{k,M}$ to define the predictor and mean square prediction error estimates for the ranking method k, $k = 1, \ldots, K$. For each design, we provide two different estimators to combine ranking information at estimator level. For design D_1, we combine the ranking information from different sources by constructing weighted estimators. We use either equal weights (E) or inverse of the standard error weight (S)

$$\bar{Y}_{E,1} = \frac{1}{K}\sum_{k=1}^{K}\bar{Y}_{k,1}, \quad \bar{Y}_{S,1} = \frac{\sum_{k=1}^{K}\frac{\bar{Y}_{k,1}}{\hat{\sigma}^2_{k,1}}}{A_1},$$

where

$$A_1 = \sum_{k=1}^{K}\frac{1}{\hat{\sigma}^2_{k,1}},$$

These two estimators can be considered as a weighted combinations of K different estimators obtained from different ranking method. This first estimator ($\bar{Y}_{E,1}$) gives equal weight to each ranking estimator while the second estimator ($\bar{Y}_{S,1}$) gives more weight to the one having smaller variance.

Similar estimators can be computed for design D_2. For design D_2, there are two unbiased estimators for the variance of $\bar{Y}_2$ (σ^2_2). Our small scale simulation study showed that the variance estimator $\tilde{\sigma}^2_{k,2}$ performs slightly better than $\hat{\sigma}^2_{k,2}$ Therefore, we use $\tilde{\sigma}^2_{k,2}$ to construct the estimator to combine the ranking information from K different ranking methods.

$$\bar{Y}_{E,2} = \frac{1}{K}\sum_{k=1}^{K}\bar{Y}_{k,2}, \quad \bar{Y}_{E,M} = \frac{1}{K}\sum_{k=1}^{K}\bar{Y}_{k,M}$$

$$\bar{Y}_{S,2} = \frac{\sum_{k=1}^{K}\frac{\bar{Y}_{k,2}}{\tilde{\sigma}^2_{k,2}}}{A_2}, \quad \bar{Y}_{E,M} = \frac{\sum_{k=1}^{K}\frac{\bar{Y}_{k,M}}{\tilde{\sigma}^2_{k,M}}}{A_M}$$

where

$$A_2 = \sum_{k=1}^{K}\frac{1}{\tilde{\sigma}^2_{k,2}} \text{ and } A_M = \sum_{k=1}^{K}\frac{1}{\hat{\sigma}^2_{k,M}},$$

where A_M is computed using mean square prediction error estimate $\hat{\sigma}^2_{k,M}$.

All these combined estimators can be put into two categories. The first group gives equal weight to each ranking estimator. The second category consists of the estimators that give more weights to the ones having smaller variances.

The second approach combines the ranking information at the observation level. For each measured observation Y_i, we create agreement weights for each rank value h, $h = 1. \ldots, H$. Let

$A_{i,h,r}$ be the proportion of ranking methods that assign the rank h on measured observation Y_i for design D_r

$$A_{i,h,r} = \sum_{k=1}^{K} I(R_{i,k} = h)/K, \quad h = 1, \ldots, H, \ r = 1, 2, M$$

We again note that subscript $r = M$ denotes the agreement weights are constructed from a sample generated from the super-population model M in equation 7. It is clear that the sum of $A_{i,h,r}$ over h is equal to 1 for each i. The value of $A_{i,h,r}$ represents the strength of agreement among K ranking methods for the rank h on response Y_i in design D_r. For example, for a given design D_r if all K ranking methods assign rank $R_{i,k,r} = 1 (k = 1, \ldots, K)$ for Y_i, then $A_{i,1,r} = 1$ and $A_{i,h,r} = 0$, for $h = 2, \ldots, H$. We now construct our estimator using these agreement weights

$$\bar{Y}_{A,r} = \sum_{h=1}^{H} \frac{I_{h,r}}{d_{w,r}} \sum_{i=1}^{n} A_{i,h,r} Y_i, r = 1, 2, M, \tag{10}$$

where $A_{h,r} = \sum_{i=1}^{n} A_{i,h,r} = \sum_{k=1}^{K} n_{h,k,r}/K$; $I_{h,r} = I(A_{h,r} > 0)$; $d_{w,r} = \sum_{h=1}^{H} I_{h,r}$. One can interpret this type of estimator as portioning (or partitioning) estimator since the value of each observation Y_i is partitioned into H different groups. Then the average of group means yield the combined estimator. The expression $A_{h,r}$ ($A_{h,M}$) can be interpreted as the effective sample size for the h-th partitioned group in design D_r (super population model M) estimator.

4 Variance estimator of the combined estimator

In order to asses the uncertainty of the combined estimators, we need to estimate their variances. Even though it may be possible to construct a large sample approximation for the variance of the combined JPS estimator, required sample size for the approximation would be larger than what the typical sample sizes used in practice. For this reason, we use Jackknife variance estimates for the combined estimators. Jackknife variance estimate is appealing in settings where it may not be clear how to calculate a good estimator or standard deviation of the estimator. For example, there may not be a clear theoretical foundation or there may not be clear functional relationship between the data and the estimator. Therefore, the delta method may not work properly for deriving a reasonable estimator for variance and bias. In our study, the JPS sample has one set of response measurement and K different sets of ranks to obtain the relative position of each measured unit in its comparison set, and ranking methods would be significantly different. For example, the same comparison set may be ranked through visual inspection, auxiliary variable and computer assisted ranking methods. We show in Section **??** that Jackknife variance estimates are not very sensitive to these different ranking methods and do produce reasonable estimates for the standard error of the combined estimator for moderate sample sizes

Let $\bar{Y}_{A,r}^{(-i)}$, $\bar{Y}_{E,r}^{(-i)}$, $\bar{Y}_{S,r}^{(-i)}$, for $r = 1, 2, M$ be the combined estimator after removing the i-th observation along with all its ranks. Jackknife variance estimate for each of the combined estimator is then given by

$$\begin{aligned}
\hat{\sigma}^2_{A,r} &= \sum_{i=1}^{n} (\bar{Y}_{A,r}^{(-i)} - \bar{J}_{A,r})^2((n-1)/n)^2, \;\; r = 1, 2, M, \\
\hat{\sigma}^2_{E,r} &= \sum_{i=1}^{n} (\hat{Y}_{E,r}^{(-i)} - \bar{J}_{E,r})^2((n-1)/n)^2, \;\; r = 1, 2, M, \\
\hat{\sigma}^2_{S,r} &= \sum_{i=1}^{n} (Y_{S,r}^{*(-i)} - \bar{J}_{S,r})^2((n-1)/n)^2, \;\; r = 1, 2, M,
\end{aligned}$$

where $\bar{J}_{A,r} = \sum_{i=1}^{n} \bar{Y}_{A,r}^{(-i)}/n$, $\bar{J}_{E,r} = \sum_{i=1}^{n} \bar{Y}_{E,r}^{(-i)}/n$, and $\bar{J}_{S,r} = \sum_{i=1}^{n} \bar{Y}_{S,r}^{(-i)}/n$ are the averages of the bootstrap replications. We note that we have three different combined estimators and one JPS estimator based on a single ranking method for for a given data set. Since we have jackknife variance estimates for each one of the three combined estimators and an unbiased estimator for the JPS estimator of a single ranking method, we construct the fifth estimator by selecting the one that has the smallest variance estimates among four available estimators

$$\bar{Y}_{Min,r} = \left\{ \bar{Y}_{Min,r} : \min_{A,E,S,JPS} (\hat{\sigma}^2_{\bar{Y}_{JPS,r}}, \hat{\sigma}^2_{\bar{Y}_{A,r}}, \hat{\sigma}^2_{\hat{Y}_{E,r}}, \hat{\sigma}^2_{S,r}) \right\}, r = 1, 2, M,$$

where $\hat{\sigma}^2_{\bar{Y}_{JPS,r}}$ is the variance estimate of a JPS estimator in equations (3), (6), and (9) based on single (first) ranking method for design $r = 1, 2, M$, respectively. Variance estimates allow us to construct approximate $100(1-\alpha)\%$ confidence interval for the population mean and total,

$$\begin{aligned}
\bar{Y}_{A,r} &\pm t_{n-1,1-\alpha/2}\hat{\sigma}_{A,r}, \quad \bar{Y}_{E,r} \pm t_{n-1,1-\alpha/2}\hat{\sigma}_{E,r}, \\
\bar{Y}_{S,r} &\pm t_{n-1,1-\alpha/2}\hat{\sigma}_{S,r}, \quad \bar{Y}_{Min,r} \pm t_{n-1,1-\alpha/2}\hat{\sigma}_{Min,r}, \\
\bar{Y}_{JPS,r} &\pm t_{n-1,1-\alpha/2}\hat{\sigma}_{JPS,r}, \quad \bar{Y}_{SRS,r} \pm t_{n-1,1-\alpha/2}\hat{\sigma}_{SRS,r}
\end{aligned}$$

for $r = 1, 2, M$, where $t_{n-1,a}$ is the a-th upper quantile of t-distribution with $n-1$ degrees of freedom. The last confidence interval is constructed based on a simple random sample under model $r = 1, 2, M$.

5 Empirical comparisons of combined estimator

In this section, we performed a simulation study to investigate the empirical properties of the estimators. For design based inference, we considered a finite population generated by the quantiles of a normal distribution with mean $\mu = 50$ and standard deviation $\sigma = 4$

$$y_i = F^{-1}(i/(N+1), \mu = 50, \sigma = 4), i = 1, \ldots, N,$$

where $N = 400$ is the size of the finite population and F is the cumulative distribution function of the normal distribution. The JPS samples are generated using design D_1 and D_2 with set sizes $H = 2, 3, 4, 5$ and sample sizes $n = 30, 50$. The number of rankers and simulation size are selected to be $K = 12, 24$ and 2000, respectively. The quality of ranking information is modeled through the Dell and Clutter model (1972). This model in each comparison set creates a ranking variable V. Let $\boldsymbol{Y}_i^\top = (Y_i, Y_1, \ldots, Y_{H-1})$ be the comparison set for Y_i generated from a population with variance σ^2. Dell and Clutter model generates another random vector, $\boldsymbol{\epsilon}_i^\top = (\epsilon_i, \epsilon_1, \ldots, \epsilon_{H-1})$, from a normal distribution with mean zero and variance σ_ϵ^2. We then add these two vectors to construct ranking vector $\boldsymbol{V}_i^\top = (V_i, V_1, \ldots, V_{H-1})$. The units in the comparison set are ranked based on the values of V and the rank of V_i is taken as the judgment rank of Y_i. The correlation coefficient between Y_i and V_i is given by $\rho = 1/\sqrt{1 + \sigma_\epsilon^2/\sigma^2}$. The magnitude of σ_ϵ^2 controls the quality of ranking in formation. If $\sigma_\epsilon^2 = 0$, The rank of Y_i is the same as the rank of V_i, Hence, ranking error does not occur. The large values of σ_ϵ^2 leads to random ranking. In our simulation study we considered five different ranking models.

M_1 : All K ranking methods use $\rho = 0.8$.

M_2 : The first half of the ranking methods use $\rho = 0.8$ and the remaining use $\rho = 0.3$.

M_3 : All K ranking methods use $\rho = 0.5$.

M_4 : Bottom, middle and upper one-third of ranking methods use $\rho = 0.8, 0.5, 0.3$, respectively.

M_5 : The first ranking method uses $\rho = 1$ and the remaining $K - 1$ ranking methods use $\rho = 0.8$.

The relative efficiencies are defined as

$$\begin{aligned} RE_{E,r} &= \frac{MSE(\bar{Y}_{E,r})}{MSE(\bar{Y}_{S,2})}, \quad RE_{S,r} = \frac{MSE(\bar{Y}_{S,r})}{MSE(\bar{Y}_{S,2})}, \quad RE_{A,r} = \frac{MSE(\bar{Y}_{A,r})}{MSE(\bar{Y}_{S,2})}, \\ RE_{Min,r} &= \frac{MSE(\bar{Y}_{Min,r})}{MSE(\bar{Y}_{S,2})}, \quad RE_{JPS,r} = \frac{MSE(\bar{Y}_{JPS,r})}{MSE(\bar{Y}_{S,2})}, \quad RE_{SRS,r} = \frac{MSE(\bar{Y}_{SRS,r})}{MSE(\bar{Y}_{S,2})} \end{aligned}$$

for r=1,2. We note that all estimators are compared with the standard error weighted estimator in design D_2 , $\bar{Y}_{S,2}$.

Table 1 provides relative efficiencies when $K = 12$ and $K = 24$. It s clear that the estimator $\bar{Y}_{S,2}$ performs better than all the other estimators. Improvement over regular JPS estimator $\bar{Y}_{JPS,r}$ is substantial. For example the values of $RE_{JPS,1}$ and $RE_{JPS,2}$ are always greater than one and tend to increase with set sizes for all ranking models M_i, $i = 1, \ldots, 5$.

If the ranking correlation ρ is the same for all ranking methods, the estimators $\bar{Y}_{S,2}$ and $\bar{Y}_{A,2}$ have similar efficiency results. On the other hand if ranking methods have different ρ values, estimator weighted with the inverse standard error outperforms the estimator weighted with agreement scores. For example, the values of $RE_{A,2}$ are essentially equal to one for model M_1 and M_3 where $\rho = 0.8$ and $\rho = 0.5$ for all ranking methods. On the other hand, for model M_2 and M_4, where the ranking correlation varies over ranking methods , the efficiency values $RE_{A,2}$ are much greater than one which indicates that the estimator $\bar{Y}_{S,2}$ is more efficient. For model $M = 5$, since $\rho = 1$ for the first ranking method and $\rho = 0.8$ for the other $K - 1$ ranking methods, the contribution of the estimator based on perfect ranking is not strong enough to make a big impact on the efficiency of the precision of Y_2^+. Hence, $RE_{A,2}$ values are slightly bigger than one. As expected efficiencies of all design D_2

(without replacement) estimators are substantially higher than the efficiency of all design D_1 (with replacement) estimators.

Table 2 presents the coverage probabilities of the confidence intervals of population mean for $K = 12, 24$ and sample size $n = 30$. It clear that all coverage probabilities are reasonably close to nominal coverage probability of 0.95.

In this section, we investigate the coverage probabilities of the confidence intervals of population mean and the variance estimates of the combined estimators from K different ranking methods. In this part of the simulation, sample sizes are selected to be $n = 20, 30, 50$ and 100. We considered a finite population of size $N = 600$ generated from the quantiles of a normal distribution with mean $\mu = 30$ standard deviation $\sigma = 5$. All the other simulation parameters, such as set sizes H ($H = 2, 3, 4, 5$) and ranking models M_i (M_1, M_2, M_3, M_4, M_5) are assigned to have the same values as in Section 3. Simulation size taken to be 5000.

Table **??** presents the coverage probabilities of the confidence intervals of the population mean using large sample approximation. It is clear that the coverage probabilities are reasonable close to nominal coverage probability 0.95 for all confidence intervals. Table **??** presents the ratio of the mean variance estimates of asymptotic approximation and simulation. We expect that the ratios to be close to one if the large sample approximation is close to true asymptotic variance. For the sample sizes $n = 50$ and $n = 100$, large sample approximation provides reasonable estimator for the variance of the combined estimators.

Table **??** presents the coverage probabilities of the jackknife confidence intervals for sample sizes $n = 20$ and $n = 30$. It is clear that the coverage probabilities are close to nominal coverage probability 0.95. Table **??** provides the ratio of the average of variance estimates of jackknife and simulation studies. Entries in Table **??** reasonably close to 1 indicating that jackknife variance estimates are reasonably close to the variance estimates obtained from the variance of 5000 combined estimates.

6 Concluding Remarks

Proof of Theorem ??: We define a transformation from $\boldsymbol{T}_n$ to $\bar{Y}_{w,D_1}$

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Table 1: Efficiency of multi-ranker combined estimators wit respect to the estimator $\bar{Y}_{S,r}$, $RE_{E,r} = MSE(\bar{Y}_{E,r})/MSE(\bar{Y}_{S,2})$, r=1,2. The population and sample sizes are $N = 400, n = 30$, respectively. The population values are generated from $y_i = N^{-1}(i/(N+1), 30, 5$, where N is the CDF of normal distirbution. Simulation size is 2000.

			Design D_1						Design D_2				
K	H	M_i	$RE_{E,1}$	$RE_{S,1}$	$RE_{A,1}$	$RE_{JPS,1}$	$RE_{SRS,1}$	$RE_{Min,1}$	$RE_{E,2}$	$R_{A,2}$	$RE_{JPS,2}$	$RE_{SRS,2}$	$RE_{Min,2}$
12	2	M_1	1.057	1.049	1.061	1.284	1.550	1.071	1.007	1.010	1.231	1.462	1.009
12	2	M_2	1.097	1.074	1.143	1.106	1.368	1.086	1.026	1.080	1.051	1.311	1.011
12	2	M_3	1.108	1.102	1.109	1.229	1.272	1.127	1.004	1.006	1.118	1.179	1.010
12	2	M_4	1.124	1.101	1.174	1.083	1.359	1.092	1.021	1.065	1.049	1.238	1.041
12	2	M_5	1.064	1.053	1.074	1.078	1.569	1.049	1.007	1.015	1.026	1.501	1.011
12	3	M_1	1.167	1.154	1.175	1.533	2.029	1.182	1.013	1.021	1.377	1.900	1.023
12	3	M_2	1.123	1.067	1.203	1.153	1.581	1.100	1.059	1.131	1.108	1.521	1.040
12	3	M_3	1.020	1.009	1.018	1.224	1.269	1.029	1.012	1.015	1.229	1.288	1.025
12	3	M_4	1.149	1.103	1.228	1.149	1.537	1.147	1.047	1.122	1.007	1.417	1.019
12	3	M_5	1.193	1.175	1.209	1.231	2.222	1.222	1.021	1.033	1.058	1.990	1.024
12	4	M_1	1.120	1.111	1.120	1.694	2.331	1.141	1.014	1.025	1.625	2.312	1.059
12	4	M_2	1.346	1.253	1.467	1.432	2.046	1.319	1.093	1.197	1.290	1.711	1.124
12	4	M_3	1.133	1.122	1.130	1.427	1.479	1.157	1.012	1.005	1.319	1.336	1.033
12	4	M_4	1.196	1.118	1.285	1.164	1.683	1.164	1.077	1.178	1.160	1.581	1.107
12	4	M_5	1.306	1.272	1.328	1.300	2.825	1.283	1.031	1.037	1.065	2.471	1.056
12	5	M_1	1.211	1.198	1.217	1.890	2.793	1.244	1.008	1.008	1.744	2.488	1.046
12	5	M_2	1.183	1.080	1.304	1.352	1.907	1.181	1.120	1.236	1.248	1.840	1.084
12	5	M_3	1.023	1.015	1.017	1.352	1.380	1.029	1.009	0.991	1.388	1.382	1.030
12	5	M_4	1.147	1.060	1.247	1.157	1.715	1.168	1.099	1.193	1.177	1.649	1.133
12	5	M_5	1.287	1.242	1.283	1.335	3.114	1.311	1.041	1.043	1.079	2.795	1.068
24	2	M_1	1.154	1.146	1.157	1.427	1.660	1.175	1.009	1.014	1.225	1.524	1.017
24	2	M_2	1.085	1.058	1.134	1.109	1.343	1.068	1.025	1.077	1.039	1.300	1.021
24	2	M_3	1.122	1.117	1.124	1.265	1.295	1.142	1.007	1.012	1.159	1.177	1.017
24	2	M_4	1.095	1.075	1.141	1.124	1.319	1.117	1.024	1.074	0.996	1.258	0.995
24	2	M_5	1.154	1.141	1.163	1.152	1.708	1.147	1.011	1.018	1.024	1.539	1.001
24	3	M_1	1.123	1.109	1.135	1.589	1.998	1.153	1.016	1.025	1.437	1.995	1.031
24	3	M_2	1.176	1.114	1.270	1.255	1.680	1.176	1.067	1.161	1.166	1.565	1.047
24	3	M_3	1.107	1.092	1.112	1.390	1.392	1.123	1.013	1.023	1.264	1.316	1.039
24	3	M_4	1.130	1.082	1.202	1.109	1.495	1.112	1.054	1.132	1.038	1.433	1.028
24	3	M_5	1.168	1.150	1.180	1.181	2.191	1.164	1.020	1.032	1.072	1.958	1.034
24	4	M_1	1.219	1.197	1.233	1.865	2.544	1.246	1.020	1.034	1.638	2.377	1.050
24	4	M_2	1.232	1.137	1.348	1.305	1.897	1.207	1.099	1.214	1.229	1.738	1.083
24	4	M_3	1.117	1.103	1.122	1.433	1.504	1.146	1.014	1.014	1.352	1.362	1.037
24	4	M_4	1.218	1.138	1.322	1.212	1.731	1.198	1.083	1.184	1.116	1.597	1.063
24	4	M_5	1.203	1.176	1.221	1.252	2.518	1.228	1.025	1.031	1.046	2.372	1.024
24	5	M_1	1.338	1.320	1.338	2.331	3.101	1.381	1.031	1.040	1.980	2.859	1.071
24	5	M_2	1.266	1.147	1.399	1.489	2.045	1.246	1.137	1.268	1.275	1.922	1.082
24	5	M_3	1.152	1.139	1.155	1.546	1.580	1.168	1.015	1.015	1.444	1.421	1.045
24	5	M_4	1.350	1.236	1.486	1.394	2.033	1.301	1.116	1.231	1.173	1.738	1.088
24	5	M_5	1.264	1.238	1.269	1.314	2.967	1.293	1.037	1.036	1.076	2.746	1.061

Table 2: Coverage probabilities of 95% confidence intervals of population mean. The population and sample sizes are $N = 400, n = 30$, respectively. The population values are generated from $y_i = N^{-1}(i/(N+1), 30, 5$, where N is the CDF of normal distirbution. Simulation size is 2000.

			With replacement, D_1						Without replacement, D_2					
K	H	M_i	$\bar{Y}_{E,1}$	$\bar{Y}_{S,1}$	$\bar{Y}_{A,1}$	$\bar{Y}_{JPS,1}$	$\bar{Y}_{SRS,1}$	$\bar{Y}_{Min,1}$	$\bar{Y}_{E,2}$	$\bar{Y}_{S,2}$	$\bar{Y}_{A,2}$	$\bar{Y}_{JPS,2}$	$\bar{Y}_{SRS,2}$	$\bar{Y}_{Min,2}$
12	2	M_1	0.952	0.952	0.952	0.958	0.302	0.950	0.954	0.952	0.952	0.958	0.293	0.948
12	2	M_2	0.947	0.948	0.945	0.953	0.295	0.944	0.954	0.954	0.949	0.954	0.277	0.950
12	2	M_3	0.943	0.943	0.944	0.948	0.273	0.936	0.948	0.947	0.945	0.959	0.266	0.941
12	2	M_4	0.945	0.946	0.944	0.952	0.270	0.941	0.946	0.945	0.940	0.950	0.276	0.933
12	2	M_5	0.951	0.950	0.949	0.956	0.297	0.945	0.941	0.943	0.942	0.951	0.276	0.939
12	3	M_1	0.946	0.945	0.944	0.944	0.285	0.940	0.953	0.954	0.953	0.958	0.285	0.949
12	3	M_2	0.945	0.948	0.944	0.951	0.288	0.939	0.943	0.943	0.942	0.950	0.287	0.933
12	3	M_3	0.954	0.954	0.949	0.953	0.301	0.949	0.941	0.942	0.936	0.944	0.292	0.931
12	3	M_4	0.949	0.948	0.949	0.951	0.285	0.936	0.954	0.954	0.951	0.967	0.288	0.951
12	3	M_5	0.948	0.945	0.948	0.946	0.278	0.933	0.950	0.953	0.951	0.954	0.286	0.944
12	4	M_1	0.953	0.954	0.956	0.945	0.279	0.947	0.954	0.955	0.951	0.942	0.293	0.939
12	4	M_2	0.945	0.947	0.942	0.946	0.295	0.930	0.957	0.959	0.958	0.948	0.304	0.941
12	4	M_3	0.945	0.946	0.940	0.947	0.279	0.934	0.953	0.953	0.950	0.949	0.300	0.942
12	4	M_4	0.949	0.956	0.946	0.954	0.298	0.937	0.954	0.958	0.953	0.945	0.281	0.934
12	4	M_5	0.948	0.950	0.946	0.946	0.281	0.939	0.964	0.961	0.961	0.954	0.293	0.946
12	5	M_1	0.947	0.946	0.943	0.950	0.270	0.938	0.956	0.955	0.953	0.951	0.300	0.945
12	5	M_2	0.945	0.949	0.941	0.949	0.298	0.931	0.947	0.953	0.944	0.953	0.292	0.937
12	5	M_3	0.950	0.950	0.946	0.954	0.309	0.938	0.946	0.949	0.946	0.942	0.271	0.934
12	5	M_4	0.953	0.954	0.948	0.954	0.282	0.934	0.950	0.954	0.950	0.939	0.278	0.924
12	5	M_5	0.950	0.950	0.946	0.935	0.273	0.923	0.958	0.964	0.955	0.947	0.278	0.941
24	2	M_1	0.942	0.942	0.942	0.945	0.300	0.935	0.954	0.954	0.952	0.966	0.289	0.951
24	2	M_2	0.944	0.944	0.945	0.955	0.308	0.941	0.944	0.945	0.941	0.960	0.312	0.941
24	2	M_3	0.940	0.941	0.939	0.942	0.289	0.935	0.954	0.954	0.953	0.962	0.288	0.949
24	2	M_4	0.945	0.945	0.944	0.947	0.271	0.933	0.937	0.936	0.937	0.955	0.274	0.936
24	2	M_5	0.949	0.949	0.949	0.956	0.291	0.945	0.947	0.947	0.947	0.958	0.293	0.946
24	3	M_1	0.953	0.955	0.951	0.953	0.297	0.947	0.958	0.958	0.959	0.959	0.277	0.955
24	3	M_2	0.948	0.947	0.946	0.945	0.295	0.936	0.952	0.953	0.951	0.959	0.297	0.949
24	3	M_3	0.948	0.948	0.949	0.951	0.287	0.945	0.945	0.945	0.946	0.946	0.291	0.939
24	3	M_4	0.940	0.941	0.943	0.945	0.300	0.927	0.939	0.940	0.936	0.952	0.276	0.933
24	3	M_5	0.952	0.951	0.952	0.950	0.290	0.943	0.954	0.953	0.952	0.950	0.303	0.942
24	4	M_1	0.949	0.946	0.944	0.948	0.296	0.939	0.963	0.963	0.962	0.951	0.299	0.954
24	4	M_2	0.943	0.945	0.943	0.953	0.286	0.934	0.950	0.954	0.949	0.945	0.295	0.938
24	4	M_3	0.951	0.951	0.952	0.949	0.300	0.945	0.953	0.955	0.952	0.955	0.298	0.945
24	4	M_4	0.945	0.949	0.947	0.948	0.294	0.932	0.946	0.948	0.946	0.949	0.286	0.933
24	4	M_5	0.949	0.951	0.949	0.943	0.298	0.936	0.960	0.962	0.962	0.951	0.304	0.941
24	5	M_1	0.945	0.944	0.942	0.934	0.283	0.932	0.964	0.964	0.964	0.949	0.297	0.956
24	5	M_2	0.951	0.950	0.954	0.948	0.297	0.939	0.946	0.950	0.946	0.949	0.284	0.935
24	5	M_3	0.928	0.928	0.930	0.936	0.271	0.921	0.947	0.946	0.947	0.945	0.284	0.935
24	5	M_4	0.943	0.945	0.939	0.951	0.281	0.934	0.960	0.962	0.955	0.948	0.292	0.941
24	5	M_5	0.945	0.947	0.945	0.937	0.303	0.930	0.961	0.961	0.961	0.944	0.309	0.938