Incentivising Market Making in Financial Markets

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ABSTRACT

In their pursue for profit, market makers contribute liquidity and thus play a fundamental role for the health of financial markets. The mechanism used to rank bids and asks in order-driven markets can influence trader behaviour and discourage market making, with obvious consequences on market fundamentals. This is the rationale behind market trading mechanisms, which assign weight to both the spread of two-sided orders and order prices.

In this work, we assess the effectiveness of this proposal from a game-theoretic standpoint. We use strategic agents and explicitly define a utility function that treats the probability of a trader becoming a market maker as a pure strategy. We then employ empirical game-theoretic analysis to analyse the market at equilibrium; we illustrate the strategic responses to different setups of the matching mechanisms, how agents are incentivised to become market makers, agent behaviour and market states. Our analysis shows that this spread-based priority works well to reduce market volatility and maintain trading volume, provided that an appropriate setting is used, which weighs spread ranking 30% and price ranking 70%.

CCS CONCEPTS

• **Computing methodologies → Agent / discrete models**.

KEYWORDS

Market Mechanism Design; Empirical Game-Theoretic Analysis; Market Making

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1 INTRODUCTION

Market makers (MMs) plays a significant role in (order-driven) financial markets, facilitating trade. A MM is a liquidity provider with quotes to buy and sell simultaneously a certain asset, in the hope to profit from the bid-ask spread. MMs can be officially "registered", acting as dealers with contractual obligations to offer a reasonable spread maintaining the market orderly and fair. [1] Nevertheless, in any limit order book (LOB) market, investors can submit two-sided orders simultaneously, the quotes being stored in the LOB until the orders are traded or cancelled [15]. These investors are effectively MMs, providing liquidity to the market without contractual limitations. In our research, we focus on the latter kind of market making; any (high-frequency) electronic trading system would benefit from the liquidity guaranteed by these traders.

MMs carry some inventory to adapt to temporary order imbalances and stabilise the price with profit [21]. However, this brings high risk; for example, the MM may build up a large inventory over time. Although MMs can effectively hedge the risk by future contracts, this inventory risk can affect liquidity supply [29]. Overall, the impact of market-making on market stability and liquidity is still unclear. Prior research suggested that the effect of MMs is expected to be positive to contribute to liquidity, price discovery and market stability [2, 5, 24, 29]. Since the market benefits from MMs, a vast literature on optimal market making [1, 7, 8, 10, 14, 25, 28] has emerged.

Currently, high-frequency trading is dominated in continuous double auctions (CDAs). The widely used price-time priority to rank orders encourages high-frequency traders to submit orders with very low latency and near the best ask/best bid since the best bid/ask price received first in the LOB will be executed first. Whilst some argue that this brings some form of liquidity, price-time priority also raises concerns about significant price movements (as in the cases of the 2008 Black Swan [20] and the 2010 flash crash [16]) due to the sudden execution of large numbers of one-sided orders (i.e., a large order imbalance).

This problem can be cast in a mechanism design framework: can we formulate game (i.e., priority) rules to encourage traders to participate in the game while avoiding this imbalance? This is the approach taken by [9], which proposed a novel mechanism called spread/price-time priority to overcome the misaligned incentives of price-time priority. The idea is to consider three indicators – spread and price, opportunely weighted, and time as a tie-breaker – to determine order execution priorities. The price ranking remains in place whilst spread ranking encourages the agents to become MMs by submitting narrow spreads that balance out skewed one-sided orders. The weights of spread and price are set to determine the relative strength between them. The authors of [9] confirmed – via agent-based modeling (ABM) – that this novel mechanism could indeed achieve the objectives of reducing market volatility whilst maintaining trading volumes (i.e., MMs can offer liquidity, and a narrow spread restriction, in turn, contributes to market stability). Their ABM and simulations actually ignore traders' incentives.

[1]For example, the Regulation National Market System requires MMs to follow the National Best Bid and Offer (NBBO) quoting rules to its customers in the USA [11].

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Strong insights about the impact of matching paradigms in financial markets cannot be obtained from ABMs ignoring agent incentive.[2]

Against this background, we define a a game built on the ABM of [9], where the probability of being a market maker is a strategic choice, and the traders (players) behave in the market with the aim to maximize their profits. Players choose the strategy in response to the different matching mechanisms (and the actions of the others). We adopt a simulation-based method and the empirical game-theoretic analysis (EGTA) technique to search for an evolutionary stable state, where the population is stable and no trader has an incentive to imitate others. We analyse the market behaviour at equilibrium for both price-time and spread/price-time priority. Our results qualitatively change the conclusions of [9] in that they show that spread/price-time priority does not always perform better than price-time priority and suggest that a 30% – 70% split for spread and price weights, respectively, can work well.

The remainder of the paper is organised as follows. Section 2 sets the scene by introducing the necessary background and our specific research questions. Section 3 introduces our approach including the definition of the game, EGTA procedures and design, and the experimental settings. Results and discussion will be shown in the Section 4 and Section 5 respectively.

2 BACKGROUND

Our research starts from [9] and we want to understand whether their conclusions are robust to agent incentives. Our approach is motivated by the novel findings on strategic response of the agents in ABMs, see, e.g., [6, 18, 22, 23, 26]. Firstly, we need to adopt the agent-based model of LOB dynamics [3] in the context of the previous experimental settings for a fair comparison.

An Agent-Based Model (ABM) of LOB Dynamics. Our study will build upon on the initial agent-based model of limit orders books proposed by [3], which is able to reproduce several empirical features of the high-frequency dynamics of the stock market. The ABM is based on discrete time steps where agents decide whether to take a certain action or wait for a better opportunity by integrating private information (private information is sampled from a Gaussian process) and public information (information available to the agent at the time of placing a limit order: e.g. the mid-price, the best bid, the best offer or market volatility). Possible actions include cancelling an submitted limit order or entering the market to submit a limit order or market order. In addition, orders in the limit order book (LOB) may also be cancelled due to a simple timeout. Orders are therefore traded continuously according to matching rules (price-time priority) and LOB is updated over time, producing a time series which is able to reproduce the so-called stylized facts of the high-frequency trading dynamics [3].

Limitations. The above original model for LOB dynamics [3] is limited in that traders can only submit one-sided orders. The subsequent study [9] proposes an upgraded model to compensate for this limitation, where agents can be MMs to place two-sided limited orders. In addition, a new spread/price-time priority ranking of limit orders mechanism is adopted. Therefore, the model has two degrees of freedom, i.e. the agent has a certain probability (denoted as p) of placing a two-side order, and the spread weight (denoted as α) will be a significant indicator (together with time and price) to be considered in the new matching system. Although a parameter p is introduced to allow participants to become market makers, unlike in the actual trading market, the probability of all agents becoming market makers is fixed in each simulation [9]. In fact, each agent should be able to choose her probability of becoming a market maker based on investment preferences or market conditions (e.g., risk aversion, trading strategy or market volatility). Because the probability p is related to many factors, it is difficult for the model to allow each participant to intelligently and selfishly choose p.

Spread/price-time priority. The spread/price-time priority ranks orders by two attributes of the order (price and spread) and $\alpha \in [0, 1]$ determines the weight between spread and price. The lowest ranking will receive the highest priority for execution.

$$rank(\alpha) = \alpha \cdot price + (1 - \alpha) \cdot spread, \tag{1}$$

where *price* and *spread* ranking are defined next. Price ranking measures the distance between the order and the best ask/bid.

$$price = \begin{cases} ask - ask^{best}, & \text{sell limit order} \\ bid^{best} - bid, & \text{buy limit order} \\ 0, & \text{new best buy/sell price} \end{cases}$$

Spread ranking measures how narrow the order is, and considers its two "sides".

$$spread = \begin{cases} ask - bid, & \text{two-sided limited order} \\ spread^{max}, & \text{one-sided limit order} \end{cases}$$

The authors of [9] evaluate the market liquidity and market volatility under the two different priority regimes (price-time priority and new spread/price-time priority). The results only show that the probability p of submitting two-side orders is related to the total trading volume, where an increase p can also lead to an increase in the total trading volume. However, the paper does not discuss changes in participant behaviour affected by the different mechanisms, and therefore the following are natural follow-up research questions on which we focus:

- Does the new setting of spread effectively encourage participants to submit two-side orders?
- Does continuously increasing the weight of spread in the algorithm lead to the expected trading behaviour (higher probability of submitting two-side orders)?
- What is the relationship between the traders' behaviour and the market state under the incentives of different matching mechanisms?
- Which setting of α is the best for maximising trading volume and minimising market volatility? (α is the weight parameter of spread/price-time priority in (1).)

3 MODELLING APPROACH

Our research follows the previous model, but some parameters are slightly modified to account for the large computational time; the modified model still guarantees the properties of the original model. We propose an approach for evaluating the matching mechanisms when traders are given the opportunity to adapt strategically toward making a profit. We will evaluate the impact of different algorithms in equilibrium, where agents choose the best strategy in a given

[2]For instance, in the context of Basel regulations, agents' strategic responses are shown to be qualitatively different [6] than those produced on simple ABM simulations [19].

environment. Our analysis will adopt the following three steps: construct a game, define a meta game and obtain a heuristic payoff matrix, and the EGTA process.

Step 1 : Construct a Game. In the original experiment, [3] assumed that traders could only submit one-side orders in the model. The authors of [9] gave each participant a probability of submitting two-sided orders in the simulation; the probability of being a market maker was the same for each trader. This assumption is problematic for several reasons, but most fundamentally, it limits the ability of trader behaviour to respond to environmental conditions (e.g., to the corresponding return/loss or to the market volatility). We are particularly concerned about this assumption because the spread/price-time priority appears to be sensitive to whether participants will choose to become market makers. For example, if all traders would not submit two-sided orders, i.e., $p = 0$, then the new matching mechanism in [9] would become equivalent to the price-time matching mechanism since the spread for all the orders. This is also the reason for which, in our work, we do not perform the simulation for $p = 0$. This is not an extreme point of view since, in reality, there would not be an equal probability of all traders becoming market makers. In fact, a better hypothesis could be that the probability of a trader becoming a market maker is a payoff-dependent strategic choice, which could be regarded as an endogenous characteristic of the trader. This character could be related to her own risk aversion, to the volatility of the market, or to the behaviour of other traders. In the market, these complex relationships can be reflected in the trader's strategy and the returns that depend on it, while the trader can adjust her strategy guided by the return profile. Therefore, the probability of becoming a market maker in our study will be considered as the trader's strategic choice.

The first step in our study is to extend the above scenario to a game where the traders are the players, the strategy is the probability of submitting a two-sided order, and the payoff is the profit from the trade. In other words, taking into account all the other players, all probability distributions is the strategy profile in the game. Our goal is to find the equilibrium through all possible distributions where traders find the best strategy and do not have an incentive to change strategies unilaterally. Let $\lambda \in (0, 1)$ represent the strategy, which is the probability of submitting two-sided orders. We define the wealth $W_i(t)$ of a player i at time t as $W_i(t) = E_i(t) \cdot p(t) + C_i(t)$, where $p(t)$ is the mid-price in the LOB at time t, $E_i(t)$ is the amount of the asset that the player i holds at t and $C_i(t)$ is the cash position of the player. The payoff of player i at time $t+1$ adopting strategy λ is defined as $\pi_i^{\lambda}(t) = W_i^{\lambda}(t) - W_i(t-1)$, where $W_i^{\lambda}(t)$ is the wealth of the player at time t after having played strategy λ.

If we want to fully analyse this normal-form game, the payoff table should be obtained. There are two issues to overcome. One is that the number of players should be sufficient to run the agent-based model. The other is the computation of the strategy profile of this normal-form game, which depends on the number of strategies and players. However, the number of strategies (λ) is actually infinite, and the number of players for high-frequency trading must be large. Due to its complexity, it is not feasible to evaluate this complete game even for discrete λs. Furthermore, the different parameter settings in spread/price-time priority result in an exponential increase in the number of experiments.

Step 2 : Define a Meta Game and Compute Heuristic Payoff Matrix. The scenario above is a symmetric game [22], in which players are trading in the market, and each player has the same strategy set to choose a probability of being MM. Let A denote the number of agents and S denote the number of strategies. For a normal-form game, the number of entries in the payoff table will be S^A. Even with a moderate number of S and A, the payoff matrix can be very large. However, our analysis is restricted to symmetric games and does not have to concern itself with the individual identities of these agents, which also greatly reduces the complexity of the study.

Meta Game. Firstly, to simplify the analysis, we assume different types of agents, and we also assume that agents choose their strategies depending on their type. We do, however, assume that the type of agent and the distribution of types are available for the strategy itself. We call the type of each agent her meta-strategy. In our study, we classify agents into three types, high probability of being MMs (*HP*), medium probability of being MMs (*MP*) and low probability of being MMs (*LP*). The atomic strategies $\lambda \in (0, 1)$ will be assigned to above categories as follows: $\lambda \in (0, 0.33]$ means *LP*, $\lambda \in (0.33, 0.66]$ means *MP* and $\lambda \in (0.66, 1]$ means *HP*. Thus, we convert the original large game to a meta game with 3-meta strategies. In this way, we simply change the game model from complex iterative games to a one-short normal-form game (where players cannot change their strategies during a single simulation). Meta strategies determine heuristic strategy choices rather than basic actions. This approach makes it possible to translate complex scenarios into the standard game analysis.

Heuristic Payoff Table. A key step in our approach is the calculation of the heuristic payoff table. According to Figure 2, the meta game payoff table reduces the entries of the original game from S^A to $(A+2) \cdot (A+1)/2$ entries. Firstly, we easily get the full meta-strategy profile and then fill in the corresponding payoffs in the table. The idea behind this payoff table is to obtain the fitness of different strategies for different species in a given environment. With this heuristic payoff matrix, standard game-theoretic analysis can be applied just as we apply it to a normal-form game with simple actions.

To fill in each row of the heuristic payoff, firstly, we need to track each player's wealth updates in the simulation to compute the profit or loss incurred when choosing a strategy. Secondly, we need to obtain the fitness of the corresponding species based on the payoffs of each player. For each player i, the pure strategy set of the meta-game is $S = \{LP, MP, HP\}$. More specifically, for $s \in S$, we let $N_s(t)$ denote the set of players choosing meta-strategy s at time t and $\lambda_j(s)$ denote the atomic strategy chosen by player $j \in N_s(t)$. We define the utility associated to s as

$$U_s(|N_s(t)|) = \frac{\sum_{j \in N_s(t)} \pi_j^{\lambda_j(s)}(t)}{|N_s(t)|} \text{ if } |N_s(t)| \geq 0 \text{ and } 0 \text{ oth.} \quad (2)$$

Step 3 : EGTA Analysis. We evaluate the market performance under two conditions (with/without participants' incentives) of 6 parametric environment. For each environment, we assign a level of spread/price-time priority by α ($\alpha \in \{0.1, 0.3, 0.5, 0.7, 0.9, 1\}$)

which denotes price ranking contribution. We slightly modified some parameter settings aiming to maintain all the features of the ABM in [3, 9] of the high-frequency market with manageable time computation. Specifically, in all settings, there are a reduced number of $n = 1,000$ traders, $T = 500$ time steps, $T^{max} = 100$ time steps, reduced from $n = 10000$, $T = 1000$ and $T^{max} = 200$ in previous research. In addition, although the number of agents was set to be $1,000$, the agents consisted of both inactive and active traders. To ensure that there are sufficient orders in the simulation, the probability of a trader entering the market increases by 10% and the probability of order cancellation is decreased by 10%. Two experiments will be conducted under each value of α; one is to replicate experiments since we modified some parameters and the other experiment is to obtain the market state in equilibrium. The equilibrium is computed by the above heuristic payoff matrix. Finally, we will compare the results to evaluate the impact on market liquidity and volatility of spread/price-time and price-time priority (with incentive).

EGTA Procedures. There will be $501,501$ entries in the payoff table in our settings, which means $501,501$ simulations are needed for each α. The heuristic payoff matrix is still very large. It will take a long time to generate a payoff table and compute the equilibrium. To solve this problem, we use the method of EGTA as shown in Figure 1. Therefore, firstly, we will generate an empirical heuristic payoff table to solve this computation problem and adopt the setting of at least 100 runs. A random strategy will be chosen from the meta-strategy profile, and the player will choose the corresponding atomic strategy. A single run will fill in a row of the payoff table; 100 runs in the simulator will determine an empirical game of 100-row heuristic payoff table. Secondly, the replicator function will be applied for computing the evolutionary stable state (ESS). This equilibrium is an approximated Nash equilibrium. The authors of [27] concluded that the equilibrium in an empirical game is likely to be relatively stable in a full game. In addition, the authors of [22] posit that a Nash equilibrium of the meta-game is an approximate Nash equilibrium of the true underlying game. If a hundred simulations are not enough to get an equilibrium, then we increase the number of simulations until we get an approximate equilibrium. Thirdly, after we have obtained the equilibrium and an ESS for the above empirical game, one more simulation needs to be run to analyse the evolutionary stable state. Finally, we evaluate the market performance in equilibrium for price-time priority and spread/price-time priority.

Dynamic Analysis. Nash equilibrium provides a theoretical point of view, that is, the ideal static state of a multi-agent system. However, dynamic properties may be more of interest. When multiple Nash equilibrium exists, we still have to address the question: which equilibrium will be chosen? What are the dynamics from an initial state to reach that equilibrium? The evolutionary game theory model is applied to generate many strategy trajectories via heuristic payoff tables and replicator equations. Replicator equations are adopted to compute each dynamics trajectory of adjustment towards NE in competing situation ,and an ESS is a terminal strategy of an evolutionary process where populations are evolutionary stable [13]. We record the evolutionary trajectories of the three populations using two-dimensional triangles (see details in the next section). Any random set of initial strategies will evolve until an evolutionary stable state is reached. There may be multiple equilibria (attractors) in the simplex and we visualise the attraction of different equilibrium; it is clear that the equilibrium with a larger basin of attraction is more reasonable. In addition to this, we can observe that different initial populations lead to different evolutionary trajectories. Our study can observe the relationship between algorithm setup and population behaviour by visually seeing how populations coordinate to reach that equilibrium in the simplex [4, 12].

4 RESULTS AND EVALUATIONS

We will run at least 636 simulations compared to 81 simulations in [9]. As noted above, some parameters have been modified to reduce the time consumption. For a fair comparison, the first step is to replicate the experiments as done in [9]. The results of the replicated experiments without incentives will be used as a benchmark compared with the results of EGTA-based analysis with incentives. The evolutionary game model allows for observing the evolutionary dynamics of the populations under each setting of α. It visualizes the impact of the algorithm on evolutionary population changes. Finally, we analyze the market state under ESS, which gives us the insight to qualify the market impact of spread/price-time priority and further account for the relationship between market-making and market stability.

Replication Experiments Analysis. Figure 3a and Table 1 report the results of the replicated experiments with a new parametric setting of 1000 non-strategic players, 1000 time-steps and $T_{max} = 100$ (T_{max} represents a simple time-out that the order will be cancelled if it has not been executed over a max number of $T_{max} = 100$ time-steps). We replicated the experiments of 30 pairs of (α, p) where $\alpha \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ and $p \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$. When α equals 0.9, the traditional price-time priority is approximated, since price accounts for 90% of the weight given to performing the ranking, and conversely, when *alpha* equals 0.1, it means that price accounts for only 10% of the weight given to performing the ranking. Following previous evaluation in [9], Table 1 gives a whole picture of the cumulative trading volume and average market volatility under different pairs of (α, p) and Figure 3a visualises the trends in volatility and trading volume. The replicated experiment results are similar to those in [9] in that spread/price time priority indeed reduces market volatility, as well as maintain market liquidity.

In detail, the same finding is that, with α fixed, trading volume increases with the probability of being MM. This can be explained as follows; p refers to the probability of submitting a two-sided order, which means that as p increases, more orders will enter the market. Ultimately, more orders will result in more opportunities to be matched, which is shown in the results as an increase in trading volume. On the other hand, evidently, spread/ price-time priority has not reduced the trading volume compared with the traditional price-time priority while p is fixed.

In addition, we can obviously see spread/price-time priority are effective to reduce market volatility where the volatility for price-time priority ($\alpha = 1$) or a small weight (10%) of spread of spread/price-time priority always be highest for a certain value of p. Table 1 shows a more detailed description for volatility changes

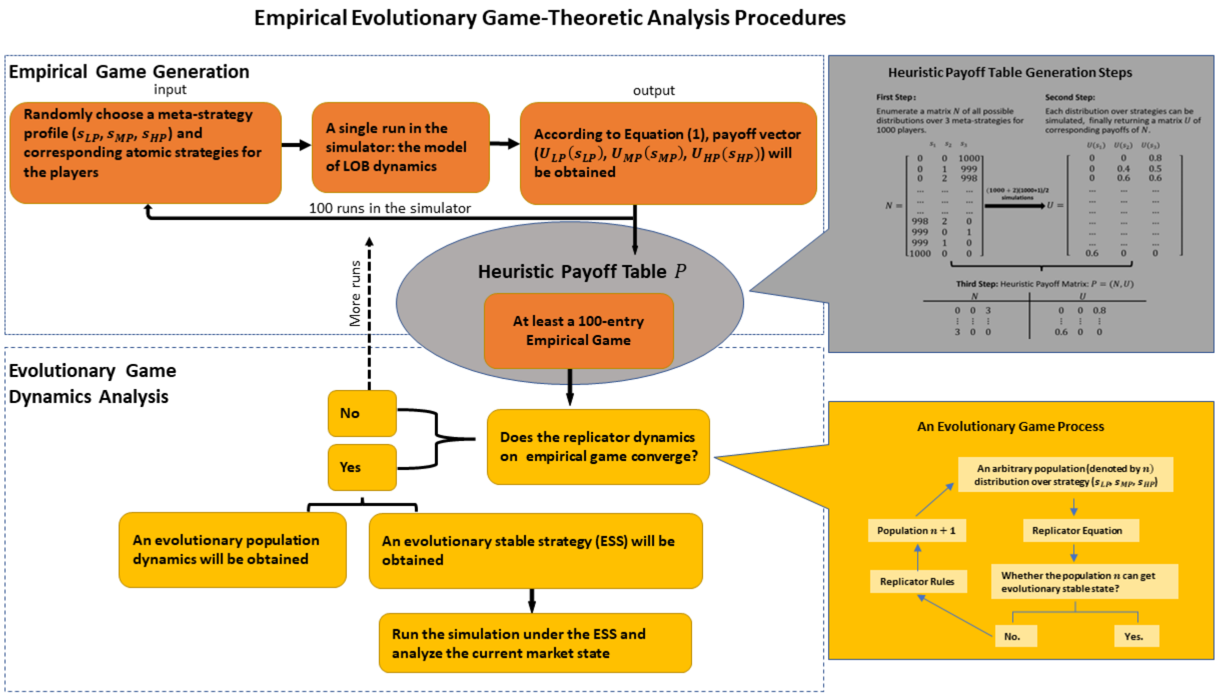


Figure 1: Empirical evolutionary game-theoretic analysis

Table 1: Average (over 100 runs) market data of ABM simulations without incentives

α	*Volume* $p=0.1$	$p=0.3$	$p=0.5$	$p=0.7$	$p=0.9$		*Volatility* $p=0.1$	$p=0.3$	$p=0.5$	$p=0.7$	$p=0.9$
	Volume increases $\longrightarrow$						$\longleftarrow$ *Volatility increases*				
1	52048	49736	56913	57290	63535		216456	202091	206242	171387	127445
0.9	48497	58616	59377	60723	64067	↑	243375	184892	113681	140611	165411
0.7	53012	59762	60554	66167	67646		191478	159842	190353	94013	84460
0.5	54602	52646	53653	54248	66287		2856	1374	1253	1243	1334
0.3	50634	52564	54475	64666	59174	↓	3375	2386	2648	3781	2123
0.1	46134	52171	53346	54052	55625		3733	24934	3592	4003	2706

where the volatility will experience two periods of changes while increasing the weight of spread contribution of the mechanism. Initially, for any fixed p, the spread with an increasing weight of the new algorithm will lead to an obvious reduction of the volatility until reaching the minimum value ($\alpha = 0.5$). Afterwards, a continuous decrease in α will conversely lead to an slight increase in market volatility. The interpretation of volatility is more complex, as there are two indicators both designed for ranking priorities to balance the market. In the traditional price-time priority, the price ranking for execution encourages traders to submit orders near mid-price for stabilizing the market. However, there is still a risk that high-frequency traders carry a large number of one-sided orders, which could lead to a significant fluctuation. The new contribution of spread hedges this risk by encouraging traders to post two-sided orders with minimum spread. For instance, two-sided orders with small spreads can balance themselves out compared to the huge price tilt of a large number of one-way orders. It can be seen from Table 1 that for a fixed value of p, the volatility of spread/price-time priority ($\alpha \neq 1$) is generally lower than that of price/time priority ($\alpha = 1$). Another finding is that consistently increasing the weight of the spread in the algorithm does not consistently reduce volatility (when the spread weight exceeds 50%). The market will be volatile instead once we only continuously increase the ratio of the spread while ignoring the price indicator. For the extreme case ($\alpha = 0$), the spread is the only ranking indicator, which also can increase the volatility of the market since the orders far from the mid-price with a small spread are often executed.

Overall, we conclude that an appropriate setting of mechanism is needed. Setting $\alpha = 0.5$ leads to the lowest volatility while maintaining the trading volume.

Evolutionary Dynamics. In the original study [9], there are two degrees of freedoms in setting spread/price-time priority, p and α. Since p is treated as a strategic choice, only six possible settings of α should be considered. Each α requires a series of simulations for generating a heuristic meta-game payoff table (see Figure 1). The replicator model is used to compute all possible trajectories, evolving from any random set of initial mixed strategy until an evolutionary stable strategy is reached. Finally, all evolutionary trajectories are summarised in the triangle coordinate containing the dynamics of evolution. The replicator dynamics for three categories of traders (LP, MP, HP) are shown in Figure 2. The points in the triangle represent the strategy space in a two-dimensional unit simplex. The vertices represent the pure strategies; for instance, the vertex (0, 1, 0) represents that all traders are with high probability (HP) of being MMs. The open circle in the triangle represents a Nash equilibrium. Each line represents a trajectory that starts from an initial mixed strategy and then applies the replicator function repeatedly until an equilibrium is reached (an open circle).

The resulting evolutionary dynamics for 3 types of traders under different levels of spread/price-time priority are presented in Figure 2. Two extreme conditions for the algorithm are price-time priority ($\alpha = 1$) and essentially spread-time priority ($\alpha = 0.1$). Firstly, in the absence of spread ranking ($\alpha = 1$), the best strategy terminates at the right bottom (MP) with high frequencies and left bottom (LP). These results confirm the intuition that traders will not be MMs with high probability since price-time priority does not reward two-side orders; the HP population has not survived due to its low fitness. Secondly, gradually increasing the proportion of spread (from 0 to 30%) in the algorithm, an obvious trend for the dominant ESS starts from pure strategy MP and moves towards pure strategy HP. Afterwards, we can find the ESS becomes more random in the triangle while spread and price account for 50%, respectively. Then, by continuously putting more weight to the spread, another obvious trend can be found in that the mainstream ESS will move towards LP first and lead to a domination of HP with spread-time priority. These findings can be explained by looking at incentives since payoffs guide traders' strategic choices toward being more profitable. In the model, the primary condition for an agent to be profitable is that the order needs to be executed first. According to the rules for order execution, from $\alpha = 0$ to $\alpha = 0.3$, the criteria for order execution are gradually changing from measuring only price to measuring not only price but also spread. Therefore, when the spread is introduced, it increases the probability of an agent submitting a two-sided order, which also leads to a higher probability of execution. In the population dynamics, the changes visualised show that the equilibrium point at the right bottom gradually moves towards the pure strategy HP. Likewise, the equilibrium at the left bottom follows the same trend moving towards the top. What both these shifts have in common is that more people choose HP strategy at equilibrium, meaning that spread introduction indeed has the effect of encouraging participants to become market makers.

When spread and prices each have a 50% weighting in the matching system, the equilibrium points are randomly distributed in the triangle. In the case in which the spread dominates the matching rule (setting spread weight at 70% and 90%), we can see an obvious tendency for the equilibrium point to move towards the top. In both cases, the top is strongly attractive and the larger the spread weight is, the closer the equilibrium point is to the top. This phenomenon can be explained by the fact that the role of price has been diminished when spread dominates the matching algorithm. The criteria for trading are mainly based on spread ranking, which results in many extreme cases. For example, orders that are far from the mid-price but with a relatively small spread will frequently be executed thereby generating profit. For the same reason, some orders with a small spread close to the mid-price are executed frequently with losing money instead. Another extreme condition will be that one-sided order with a maximum spread will not be executed. Therefore, the orders are executed primarily based on the spread, and the profitability of the order becomes more random. The agents adapt their strategies to be as profitable as possible in the above scenario. In the first case, the agent wants to ensure that orders are executed frequently enough to make a profit, as shown by the equilibrium concentration on HP. In the second case, the agent wishes to ensure that orders are not executed and that the agent will not lose money, as shown by the equilibrium concentration on LP.

In summary, the dynamics of population evolution were obtained under different algorithmic settings, depending on the strategic interactions in the game. We find that spread/price time priority does effectively incentivise participants to become market makers. However, although the example given above shows that consistently increasing the spread weight in the algorithm elevates the probability of the agent becoming a market maker, the liquidity brought to the market may be ineffective while $\alpha = 0.1$. According to the analysis above, setting the spread at a smaller proportion in the algorithm can have a benign effect on the market. Furthermore, properly setting for α is essential for a well-functioning market.

Market Behaviour at Equilibrium. In this section, we report the state of the market at equilibrium under each setting of α. The equilibrium adopted is determined based on the most attractive open circle in the triangle (equilibrium with the highest frequency of the trajectories' converging to it). Table 2 and Figure 3b summarise the trading volume and market volatility at equilibrium. Figure 3b indicates that a new trend for trading volume is obtained while p is regarded as the agents' strategic choice. Firstly, we observed and analysed the change in trading volume. When the traditional price-time priority mechanism was applied to the system, the volume was 43,730. Subsequently, when the spread is introduced to the ranking system, the volume firstly experiences an increase (a slight increase to $45,878$ and a sustained increase to $55,689$ when the spread weight in the algorithm is 10% and 30%). However, when the spread weight continues to increase to the point where both the spread and price are equal at 50%, the trading volume drops to $42,343$. With putting spread weights over 50%, in this case the pattern can be found under the spread-dominated algorithm that Volume (spread weight=0.9) >Volume (spread weight=0.7) >Volume (spread weight=0.5). Another finding was that Volume (spread weight >50%) >Volume (spread weight <50%).

This section explains the above features in the context of an evolutionary dynamics diagram. Firstly, the introduction of the spread does succeed in encouraging agents to submit two-sided orders, as shown by the increase in trading volume when the spread is initially introduced. However, the equilibrium point is derived from the trading profit. When the spread and price both take an

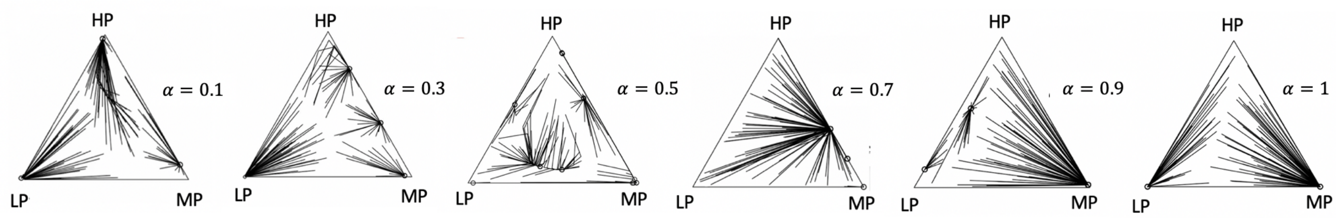
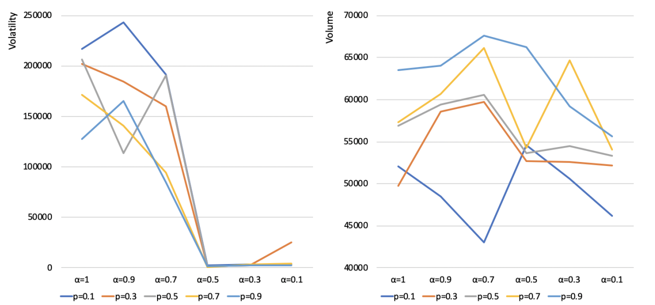


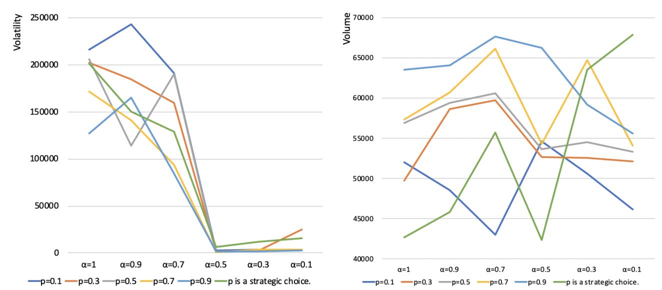
Figure 2: Evolutionary dynamics of different α

Table 2: Average (over 100 runs) market state of ABM simulations with incentives

$Market\ State$	$\alpha = 0.1$	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$	$\alpha = 1\ (Price - time)$
$Volatility$	15902	11921	6340	128789	149864	201388
$Volume$	67828	63565	42343	55689	45878	42730



(a) Average (over 100 runs) volatility and volume for ABM simulations without incentives



(b) Average (over 100 runs for each α) volatility/volume for ABM simulations with incentives

Figure 3: Results on volatility and volume without (left) and with (right) incentives

equal weight of 50%, the equilibrium point in the triangle becomes more random, and the profit-pulling equilibrium shows a lower trading volume at this point, which can be explained by the fact that more traded orders may not lead to more profit either under this setting. Conversely, more spread weight in the algorithm leads to more volume in equilibrium; the phenomenon can be explained that more execution opportunities lead to more opportunities to make a profit. Since the order execution criteria mainly depend on the spread and the profitability depends on order price, agents require more opportunities to gamble on profitability which is shown in the Table 2 as an increase in volume.

The second significant indicator of market performance is market volatility which is summarised in Table 2. Figure 3b also visualises the trend for volatility changes which is similar to [9]. The results confirm the findings of the previous study that spread/price time priority does reduce market volatility even with the consideration of incentives. The volatility under price-time priority is always greater than any of the settings in spread/price-time priority. In detail, from $\alpha = 10\%$ to a continuous increase to $\alpha = 50\%$ in the algorithm, the volatility experiences a decrease until it reaches a minimum value of 6340 ($\alpha = 0.5$). Continuously putting more weight on the spread will lead to an increase in volatility. Although the volatility increases, it is still smaller compared to the condition under the original price-time priority, e.g. 15902 ($\alpha = 0.1$) vs. 201388 (price-time priority). The above scenario shows that spread/price-time priority can avoid a large number of unilaterally skewed orders being executed thereby reducing the volatility of the market. Table 2 indicates that the volatility bounces from $\alpha = 0.5$ to $\alpha = 0.9$, this can be explained in relation to the evolutionary dynamics as follows. This process, based on the movement of the equilibrium point in the triangle, finds that agents in the market increase the probability of submitting two-sided orders meaning that there are more two-sided orders than before. Due to the fact that spread mainly determines order execution, some orders that are further away from the mid-price but with a smaller spread are executed first and then the market volatility increases. However, the volatility does not increase dramatically as the two-sided orders have the ability to self-balance.

Overall, the parameter settings of the algorithm are of particular concern. The experimental results above prove that the suggested setting $\alpha = 0.3$ not only reduces market volatility but also maintains trading volume. However, with the discussion of the agents' behaviour in the evolutionary dynamics, we find that when spread dominates the order ranking, the trading volume increases and then more liquidity comes to the market, but this liquidity is ineffective in practice, as explained above. This suggests that the results of our experiments are limited to the number of agents set. Once the spread-dominant matching system is applied to the real financial market, the large number of participants amplifies market instability and inefficient liquidity. Therefore, we rule out the settings of $\alpha = 0.1$ and $\alpha = 0.3$. In addition to this, we also find that the volatility is reduced while $\alpha = 0.5$; however, the evolutionary stable strategy in the triangle becomes random and the volume of trading is reduced. Finally, this study has identified the appropriate setting of $\alpha = 0.7$ with lower volatility and maximum market liquidity.

5 DISCUSSION AND CONCLUSIONS

The behaviours of agents in financial markets are highly complex and strategic. It is necessary to consider the impact of incentives on the behaviours of financial market agents. Our research builds on a novel approach that applies the empirical game-theoretic analysis method to the financial market. Firstly, a reasonable game scenario is constructed where the agents' behaviour are actually guided by profits. In this case, we quantified how spread/price-time priority motivates agents' behaviour and the corresponding market state. We adopt the proposed meta-game model and generate a meta-game-inspired payoff table, which we achieve by considering high-level strategic actions rather than the atomic actions of a normal game for the analysis of a complex game. This approach allows for the study of multi-agent interactions in the real world. Our analysis begins with an agent-based model that captures the microstructure of the financial market, and the meta-game model employed can capture the trading strategies of interest (strategies at equilibrium) to agents. We analyse the market characteristics in equilibrium under different parameter settings. The results indicate that the spread/price-time priority does effectively incentivise agents to market making with appropriate parameter settings. This study has provided a deeper insight into the combination of agents' behaviour and market state. The proposed setting of $\alpha = 0.7$ ensures both low market volatility and a fairly large trading volume.

Our work suggests a number of follow-up research questions. A possible research direction is that we analyse traders' behaviour and the state of the market by using the aggression levels as an endogenous variable in strategy choice. The aggression levels will determines the ways in which the quotes are made. Furthermore, the aggression levels related to the quotes in the algorithm, while the market-making strategy is mainly related to the spread in the algorithm. Therefore, another direction can be that we extend the symmetric game to an asymmetric one in order to include both factors. The asymmetric game allows for the study of both market-making behaviour and aggressive strategies, treating market makers as agents' characters and the aggression levels as endogenous strategies, a setup that corresponds to the behaviour of agents in the actual market. A related research [22] can serve as a guide for this direction. We can also consider dynamically changing α. For instance, α can occasionally be set higher (in high-volatility periods) and occasionally be set lower (in low-volatility periods). We can complete our dynamic setup with an alternative game setting. The order matching mechanism can be regarded as a living and adaptive system with strategic choices (α) for a certain time period-the payoffs generated according to different α are represented by the volatility of the market. The equilibrium state of the game can be considered the most stable time in the market during all of our study time periods. We would like to create an adaptive, dynamical setup order matching system. A similar idea has been considered for so-called call markets [17].

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