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### ABSTRACT

We create a ranking algorithm, the naive Bayes asset ranker. Our algorithm computes the posterior probability that individual assets will be ranked higher than other portfolio constituents. Unlike earlier algorithms, such as the weighted majority, our algorithm allows poor-performing experts to have increased weight when they start performing well. We outperform the long-only holding of the S&P 500 index and a regress-then-rank baseline.

### 1. Introduction

Our particular modelling interest is in financial time series, which are typically nonstationary. Nonstationarity implies statistical distributions that adapt over time and violates the independent and identically distributed (iid) random variables assumption of most regression and classification models. We require approaches that adopt sequential optimisation methods, preferably methods that make little or no assumptions about the data-generating process. The main result of this paper is our novel ranking algorithm, the naive Bayes asset ranker, which we use to select subsets of assets to trade from the S&P 500 index in either a longonly or a long/short (cross-sectional momentum) capacity. We achieve higher risk-adjusted and total returns than a strategy that would hold the long-only S&P 500 index with hindsight, despite the index appreciating by 205\% during the test period. We also outperform a regress-then-rank baseline, a sequentially fitted curds and whey (Breiman and Friedman, 1997) multivariate regression model.

# 2. The naive Bayes asset ranker

Our ranking algorithm is the naive Bayes asset ranker (nbar). The nbar sequentially ranks a set of experts, estimating the one-step-ahead posterior probability that individual experts will be ranked higher than the remaining experts. In the context of the experiment described in section 3, each expert is a forecasted return for an individual portfolio constituent of the S&P 500. The forecasted returns come from the curds and whey (caw) multivariate regression model, which utilises feature representation transfer from the constituent S&P 500 returns to radial basis function networks (rbfnets) (Moody and Darken, 1989) whose kmeans++ (Arthur and Vassilvitskii, 2007) clusters form hidden units. Assume that the nbar is presented with a set of q forecasts. The goal is to select a subset of experts  $1 \le$  $k \leq q$  such that the reward of the k experts is expected to be the highest; this is achieved by estimating the sequential posterior probability that expert  $j \in 1, ..., q$  is ranked higher than each of the remaining q-1 experts. This posterior

probability is computed with exponential decay, allowing experts who performed poorly and now perform well to be selected with greater weight than previously.

# 3. The research experiment

Our research experiment aims to assess the benefits of sequentially optimised ranking algorithms to select subsets or portfolios of financial assets to hold in either a long-only or long/short (cross-sectional momentum) capacity. More concretely, we experiment with the constituents of the S&P 500 index. We use the nbar as our portfolio selection algorithm. In order to assess the benefits of our ranking metamodel, we adopt a baseline, the long-only holding of the S&P 500 constituents with equal weighting. This baseline replicates a passive, index-tracking investment strategy.

## **3.1.** The S&P 500 dataset

We conduct this research experiment using the daily closing constituent prices for the S&P 500 index, which we extract from Refinitiv. Due to their relatively new trade history, some time series have little data. Therefore, we select a subset of the S&P 500 index, where each constituent contains a trade count greater than or equal to the 25'th percentile of trade counts; this leaves us with a subset of 378 Refinitiv information codes (rics). The dataset begins on 2001-01-26 and ends on 2022-03-25, 5326 days.

### 3.2. Experiment design

We use the first 25% of the data as a training set and the remaining data as a test set. The caw and nbar models are initialised and fitted in the training set. These models are also sequentially optimised without forward-looking bias in the test set. Once the training data are assigned to their nearest cluster centres, the cluster-conditional covariance matrices and their inverses are estimated. Cluster centres with few training data vectors assigned to them are regularised to a diagonal variance prior. Thus, we are adopting a Bayesian maximum a posteriori procedure here.

We use the forecasts of the caw model as the basis for taking risk in a subset of constituents in the S&P 500 index. Specifically, the long-only caw model buys the expected top five per cent of performing assets with equal weight. The long/short caw model works similarly, except that it

Page 1 of 3

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Table 1

Relative transaction costs incurred by each model in the test set. A buy-and-hold strategy on the S&P 500 achieves the lowest transaction costs. However, from the perspective of a more active portfolio management standpoint, our ranking algorithm incurs far lower transaction costs than the regress-then-rank baseline.

	transaction costs
long S&P 500	-0.003
long caw	-0.933
long/short caw	-1.966
long nbar	-0.050
long/short nbar	-0.104

includes the short-selling of the bottom five per cent of most negative forecasts. A second forecaster we consider is the nbar algorithm, applied to the one-step-ahead forecasts of the caw model. The nbar selects portfolio constituents with weights determined by the posterior ranking probabilities. We must also consider execution costs. We force the caw and nbar models to trade as price takers, meaning that the models incur a cost equal to half the bid/ask spread times the change in absolute position. Furthermore, as these data are sampled daily, any portfolio rebalancing is applied at most once a day, at the close of trading circa 4 pm EST.

# 3.3. Results

The passive index tracking baseline purchases each constituent with equal weighting at t=0 and holds them till the end of the experiment. This strategy pays transaction costs once and therefore has the least fees, as shown in table 1. Table 2 and figure 1 also show that the cumulative returns generated by this strategy are 205%, the compound annual growth rate (cagr) is 7.3% and the risk-adjusted annualised Sharpe ratio (sr) is a little under 0.8. Assuming normally distributed returns, the Sharpe ratio implies a probability of positive annual returns of 71%. The largest peak-to-trough drawdown for the strategy is just under 72%, and the total return to maximum drawdown is around 2.9. Finally, by simply holding the index, the percentage of days with positive returns is 55%.

The same performance metrics are available for the caw and nbar models. Both long-only and long/short caw and nbar models outperform the passive index tracking baseline, with the long/short models showing higher risk-adjusted performance measures indicated by the Sharpe ratios. The nbar performs best, with the long/short nbar showing the highest total and risk-adjusted returns. Table 1 shows that despite the caw and nbar models being actively managed strategies that rebalance the portfolios daily, only the caw models show high transaction costs. The nbar models rebalance less often and do a better job of picking portfolio constituents.



**Figure 1:** Total return by each model in the test set where the maximum selection percentile is set to 5% of the total number of portfolio constituents. The naive Bayes asset ranker performs best, particularly the cross-sectional momentum version.

**Table 2**Summary returns statistics are shown in relation to the experiment, shown visually in figure 1. The cross-sectional momentum naive Bayes asset ranker has the highest total and risk-adjusted returns.

	long S&P 500	long caw	long nbar	long short caw	long short nbar
mean	0.0005	0.001	0.0013	0.0009	0.0015
std	0.012	0.016	0.016	0.010	0.010
total ret	2.047	4.113	5.372	3.397	5.806
cagr	0.073	0.108	0.124	0.098	0.128
sr	0.798	1.243	1.636	1.624	2.879
pr(ann. ret > 0)	0.71	0.816	0.895	0.893	0.994
max dd	0.717	0.64	0.646	0.942	0.202
total ret / max dd	2.853	6.423	8.311	3.607	28.7
win ratio %	0.549	0.553	0.563	0.547	0.575

# 4. Discussion

The shortcomings of regression models over classification ones in financial time series modelling are wellunderstood. Satchell and Timmermann (1995) show that regression models that typically minimise prediction meansquare error (mse) obtain worse performance than a randomwalk model when forecasting daily foreign exchange (fx) returns. Furthermore, they show that the probability of correctly predicting the sign of the change in daily fx rates is higher for the regression models than the random-walk baseline, even though the mse of the regression models exceeds that of the random-walk model. They conclude that mse is only sometimes an appropriate performance measure for evaluating predictive performance. More recently, Amjad and Shah (2017) find that classical time series regression algorithms, such as arima models, have poor performance when forecasting Bitcoin returns. However, they find that the probability distribution of the sign of future price changes is

adequately approximated from finite data, specifically classification algorithms that estimate this conditional probability distribution.

# 5. Conclusions

We extend the research into cross-sectional momentum trading strategies. Our main result is our novel ranking algorithm, the naive Bayes asset ranker (nbar), which we use to select subsets of assets to trade from the S&P 500 index. We perform feature representation transfer from radial basis function networks to a curds and whey (caw) multivariate regression model that takes advantage of the correlations between the response variables to improve predictive accuracy. The nbar ranks this regression output by forecasting the onestep-ahead sequential posterior probability that individual assets will be ranked higher than other portfolio constituents. Earlier algorithms, such as the weighted majority, deal with nonstationarity by ensuring the weights assigned to each expert never dip below a minimum threshold without ever increasing weights again. Our ranking algorithm allows experts who previously performed poorly to have increased weights when they start performing well. Our algorithm outperforms a strategy that would hold the long-only S&P 500 index with hindsight, despite the index appreciating by 205% during the test period. It also outperforms a regressthen-rank baseline, the caw model.

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Borrageiro et al.: preprint Page 3 of 3