Learning to Rank for E-commerce

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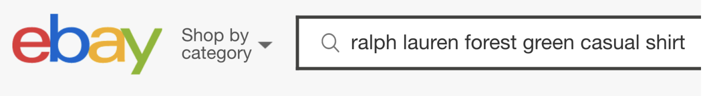
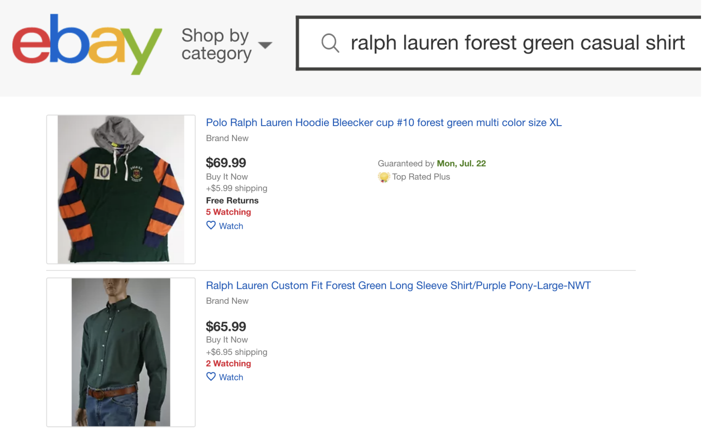
Information Retrieval



User issues a query q

Information Retrieval



Search engine retrieves *relevant items*
Ranker *ranks* items in terms of relevance to q

Query Understanding

Big Assumption!

Disentangle relevant item retrieval problem from relevance ranking

Learning to Rank: Supervised (Batch) Learning

Learn $\sigma : \mathcal{X} \rightarrow \mathcal{Y}$, given training examples $\{(X_i, Y_i) \in \mathcal{X} \times \mathcal{Y}\}_{i=1}^{N}$

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Question

What are input and output spaces?

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Question

What are input and output spaces?

$\mathcal{X}$: set of (query, item list) pairs $(\mathcal{Q} \times \mathcal{D}^m)^*$
$\mathcal{Y}$: set of all permutations on m items $\mathcal{S}_m$

*: $m = n(q)$ in general; assume fixed for simplicity

How to collect train data?

Desired: given query q and the corresponding set of retrieved items $\mathcal{D}_q$, the ideal ranking $\sigma^*(\mathcal{D}_q)$

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- For a (q, d) with $d \in \mathcal{D}_q$ get relevance labels from human judgment

$$\mathrm{rel}(q, d) \in \mathcal{O},$$

  Where $\mathcal{O}$ can be binary or multi-label(discrete)

- For a $(q, \mathcal{D}_q)$, use search logs

  skips, clicks, add2cart, sale, $\cdots$

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OK! Another simplifying assumption: ranking as sorting scores

$$f(d_i^q) > f(d_j^q) \iff \sigma_f(d_i^q) < \sigma_f(d_j^q)$$

Learning to Rank: Supervised (Batch) Learning

- Risk Minimization

$$R(f) = \mathbb{E}[\ell(f(X), Y)]$$

where the expectation is over the underlying randomness of $X = (q, \mathcal{D}_q)$ and relevance labels Y

Learning to Rank: Supervised (Batch) Learning

- Risk Minimization

$$R(f) = \mathbb{E}[\ell(f(X), Y)]$$

Let $f^* \in \arg\min_f R(f)$. Choose function class $\mathcal{F}_n$ to approximate f^*

$$f^*_{\mathcal{F}_n} \in \arg\min_{f \in \mathcal{F}_n} R(f)$$

Learning to Rank: Supervised (Batch) Learning

- Risk Minimization

$$R(f) = \mathbb{E}[\ell(f(X), Y)]$$

- Empirical Risk Minimization

$$\hat{R}(f) = \sum_{i=1}^{N} \ell(f(X_i), Y_i)$$

Estimate $f^*_{\mathcal{F}_n} = \arg\min_{f \in \mathcal{F}} R(f)$ by

$$\hat{f}_n \in \arg\min_{f \in \mathcal{F}_n} \hat{R}(f)$$

Theoretical Problems

Quantity of interest: Generalization error

$$R(\hat{f}_n) = \mathbb{E}[\ell(\hat{f}_n(X), Y)]$$

where $\hat{f}_n \in \arg\min_{f \in \mathcal{F}} \hat{R}(f)$.

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- bound Excess Risk

$$R(\hat{f}_n) - R(f^*) = \underbrace{R(\hat{f}_n) - R(f^*_{\mathcal{F}_n})}_{\textit{estimation error}} + \underbrace{R(f^*_{\mathcal{F}_n}) - R(f^*)}_{\textit{approximation error}}$$

- show consistency $R(\hat{f}_n) \to_{n \to \infty} R(f^*)$

Ranking vs Classification Error

Assuming binary labels: $rel(q,d) \in \{0,1\}$ for $d \in \mathcal{D}_q = \mathcal{D}_q^+ \cup \mathcal{D}_q^-$

$$
\begin{aligned}
\ell^c(f(X),Y) &= \ell^c\left(f(q;\mathcal{D}_q), rel(q,\mathcal{D}_q)\right) \\
&= \frac{1}{|\mathcal{D}_q|} \sum_{d \in \mathcal{D}_q} \ell\left(f(q,d), rel(q,d)\right)
\end{aligned}
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\ell^r(f(X),Y) &= \ell^r\left(f(q;\mathcal{D}_q), rel(q,\mathcal{D}_q)\right) \\
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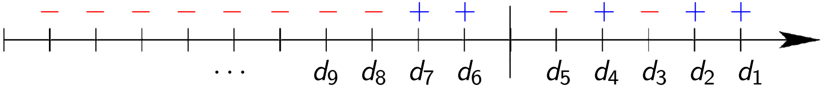

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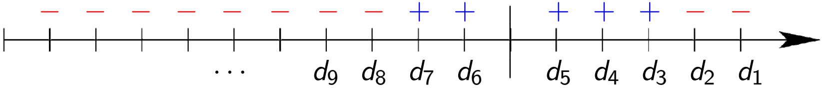


Pairwise Ranking

Convexification Recipe

Use a convex surrogate $\ell(\cdot)$ to $\mathbb{1}(\cdot)$ and perform regularized empirical risk minimization in your favorite function class

$$\operatorname*{arg\,min}_{f \in \mathcal{F}} \quad \sum_{q \in \mathcal{Q}} \frac{1}{|\mathcal{D}_q^+||\mathcal{D}_q^-|} \sum_{\substack{d \in \mathcal{D}_q^+ \\ d' \in \mathcal{D}_q^-}} \ell\left(f(q,d) < f(q,d')\right) + \lambda \rho(f)$$

Pairwise Ranking

suppose that you have pairwise judgments $rel(q, d_i^q) > rel(q, d_j^q)$

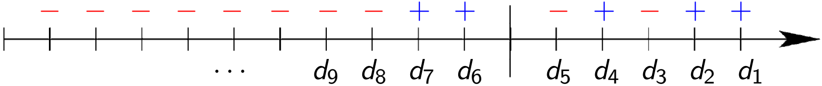
$$\arg\min_{f \in \mathcal{F}} \sum_{q \in \mathcal{Q}} \sum_{\substack{1 \leq i < j \leq |\mathcal{D}_q| \\ rel(q, d_i^q) < rel(q, d_j^q)}} \log\Big(1 + exp(f(d_i^q) - f(d_j^q)\Big)$$

Information Retrieval Measures of Loss

Normalized Discounted Cumulative Gain

$$\ell(f(q,\mathcal{D}_q)), rel(q,\mathcal{D}_q))) = \sum_{d^q \in \mathcal{D}_q} g(rel(q,d^q))h(rank(d^q))$$

usually $g(x) = 2^x - 1$ and $h(x) = \frac{1}{log(1+x)}$



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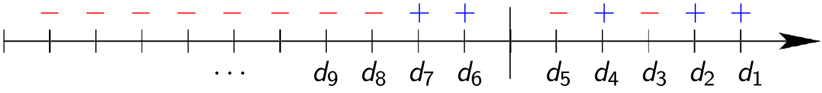

$$rank(d^q) = 1 + \sum_{d' \in \mathcal{D}_q} \mathbb{1}\left(f(q,d^q) < f(q,d')\right)$$

Relevance Judgments are Expensive!

For $(q, \mathcal{D}_q)$, use search logs

skips, clicks, add2cart, sale, $\cdots$

Adhoc Solution

Define relevance labels based on the importance in your ranking objective

$$
\begin{aligned}
rel(q,d) = 1 \qquad & \text{if} \qquad d \in \mathcal{D}_q^{\text{click}} \\
& \cdots \\
rel(q,d) = 5 \qquad & \text{if} \qquad d \in \mathcal{D}_q^{\text{sale}}
\end{aligned}
$$

Train a ranking model using 1-NDCG as loss, gradient boosted trees as function class

State of the Art: LambdaMART

$$\arg\min_{f \in \mathcal{F}} \sum_{q \in \mathcal{Q}} \sum_{\substack{1 \leq i < j \leq |\mathcal{D}_q| \\ rel(q, d_i^q) < rel(q, d_j^q)}} |\Delta L(d_i^q, d_j^q)| \log\Big(1 + exp(f(d_i^q) - f(d_j^q))\Big)$$

$\Delta L(d_i^q, d_j^q)$: difference in the desired, yet hard to optimize measure, ranking loss by swapping positions of (d_i^q, d_j^q)

$\mathcal{F}$: Multiple Additive Regression Trees $F_M(x) = \sum_{i=1}^{M} \Psi(x, \theta_i)$

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Maximum likelihood $\arg\min_f \mathcal{L}(f)$, where

$$\begin{aligned} \mathcal{L}(f) &= \prod_{q \in \mathcal{Q}} \prod_{d_i^q, d_j^q \in \mathcal{D}_q} \mathbb{P}_f(rel(q, d_i^q) > rel(q, d_j^q))^{|\Delta L(d_i^q, d_j^q)|} \\ &= \prod_{q \in \mathcal{Q}} \prod_{d_i^q, d_j^q \in \mathcal{D}_q} \left(\frac{1}{1 + exp\left(\alpha(f(d_i^q) - f(d_j^q))\right)} \right)^{|\Delta L(d_i^q, d_j^q)|} \end{aligned}$$

Unbiased Learning to Rank: Click Models

How does a click imply relevance?

Understanding the click event

C: Click on the item

E: Examining the item

K: position of the item in the ranked list

R: actual Relevance of the item

$\hat{R}$: perceived Relevance of the item

$$
\begin{aligned}
\mathbb{P}(C = 1 | q, d^q, k) &= \mathbb{P}(E = 1, \hat{R} = 1 | q, d^q, k) \\
&= \mathbb{P}(E = 1, R = 1 | q, d^q, k) \\
&= \mathbb{P}(E = 1 | q, d^q, k)\mathbb{P}(R = 1 | E = 1, q, d^q, k) \\
&= \mathbb{P}(E = 1 | k)\mathbb{P}(R = 1 | q, d^q) \\
&= \theta_k \gamma_{q,d^q}
\end{aligned}
$$

Unbiased Learning to Rank: estimating the parameters

How to estimate the parameters $\{\theta_k, \gamma_{q,d^q}\}$?

Data Logs $\mathcal{L} = (q, d, k, c)$ in all search sessions

$$\log \mathbb{P}(\mathcal{L}) = \sum_{(q,d,k,c)\in\mathcal{L}} c \log(\theta_k \gamma_{q,d^q}) + (1-c)\log(1-\theta_k \gamma_{q,d^q})$$

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use Expectation Maximization(EM) to maximize the likelihood

- estimate posterior distribution of hidden variables given observed data

$$\mathbb{P}(E, R | q, d, k, c)$$

- maximize the likelihood given data and posterior probabilities

$$\theta_k = \frac{\sum_{(q,d,k',c)\in\mathcal{L}} \mathbf{1}\{k'=k\}(c + (1-c)\mathbb{P}(E=0|q,d,k,C=0)}{\sum_{(q,d,k',c)\in\mathcal{L}} \mathbf{1}\{k'=k\}}$$

Unbiased Learning to Rank: position debiasing

Biased ranking loss(DCG:rank-Discounted Cumulative Gain)

$$\sum_{(q,d,k,c)\in\mathcal{L}} c\frac{1}{\log(1+rank(d))}$$

Unbiased DCG: $L_{\mathcal{U}} = \sum_{(q,d,k,c)\in\mathcal{L}} \frac{c}{\theta_k}\frac{1}{\log(1+rank(d))}$

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Unbiased DCG: $L_{\mathcal{U}} = \sum_{(q,d,k,c)\in\mathcal{L}} \frac{c}{\theta_k}\frac{1}{\log(1+rank(d))}$

$$\begin{aligned}
\mathbb{E}[L_{\mathcal{U}}] &= \sum_{(q,d,k,c)\in\mathcal{L}} \frac{\mathbb{E}[C|q,d,k]}{\theta_k}\frac{1}{\log(1+rank(d))} \\
&= \sum_{(q,d,k,c)\in\mathcal{L}} \frac{\mathbb{P}(C=1|q,d,k)}{\theta_k}\frac{1}{\log(1+rank(d))} \\
&= \sum_{(q,d,k,c)\in\mathcal{L}} \underbrace{\mathbb{P}(R=1|q,d)}_{rel(q,d)}\frac{1}{\log(1+rank(d))}
\end{aligned}$$

Ranking for Sale!

Gross Merchandise Volume

$$\sum_{q \in \mathcal{Q}, d^q \in \mathcal{D}_q} \mathit{price}(d^q) \mathbf{1}\{d^q \in \mathcal{S}\}$$

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$$\sum_{q \in \mathcal{Q}, d^q \in \mathcal{D}_q} price(d^q) \mathbf{1}\{d^q \in \mathcal{S}\}$$

$$\begin{aligned}
\mathbb{E}[GMV] &= \sum_{q \in \mathcal{Q}, d^q \in \mathcal{D}_q} price(d^q) \mathbb{P}(d^q \in \mathcal{S}) \\
&= \sum_{(q,d,c,s,k) \in \mathcal{L}} \underbrace{price(d^q) \mathbb{P}(R = 1 | q, d^q) \mathbb{P}(S = 1 | q, d^q, R = 1)}_{rel(q,d)}
\end{aligned}$$

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Optimize for (unbiased) NDCG: $\sum_{(q,d,k,c) \in \mathcal{L}} \frac{rel(q,d)}{\log(1 + rank(d))}$

Online Learning to Rank

- $(q, \mathcal{D}_q)$ is presented to ranker
- ranker scores the documents $f_t(d)$ for $d \in \mathcal{D}_q$
- relevance signals are revealed(user interacts with search results)
- ranker suffers $\ell(f_t(q, \mathcal{D}_q)), rel(q, \mathcal{D}_q)))$

minimize regret

$$\sum_{t=1}^{n} \ell(f_t(q, \mathcal{D}_q)), rel(q, \mathcal{D}_q))) - \min_{f \in \mathcal{F}} \sum_{t=1}^{n} \ell(f(q, \mathcal{D}_q)), rel(q, \mathcal{D}_q))),$$

Reinforcement Learning

Define a search session Markov decision process for $(q, \mathcal{D}_q)$

- action space $\mathcal{A}$: all possible ranking functions
- state space $\mathcal{S}$: all possible landing pages; whether a continuation states or termination(purchase/abandon) state

$$\mathcal{S} = \mathcal{S}_C \cup \mathcal{S}_T$$

- reward function $\mathcal{R} : \mathcal{S}_C \times \mathcal{A} \times \mathcal{S} \to \mathbb{R}$; expected sale price of the page if leads to a purchase state otherwise 0
- policy $\pi : \mathcal{S} \to \mathcal{A}$ with a parameter set
- trajectory τ is a series of state, action, rewards
- objective $\mathcal{J}(\theta)$; expected return in a large horizon under trajectory distribution defined by π

$$\arg\max_{\theta} \mathbb{E}_{\tau \sim f_{\pi(\theta)}}[R(\tau)]$$

Conclusion

Ranking is fun!