Roll No.

B.E / B.Tech (Full Time) ARREAR END SEMESTER EXAMINATIONS, APRIL / MAY 2014

COMMON TO ALL BRANCHES
Semester II

GE 9151 & ENGINEERING MECHANICS
(Regulation 2008)

Time: 3 Hours   Answer ALL Questions   Max. Marks 100

PART-A (10 x 2 = 20 Marks)

1. The figure represents the **principle of transmissibility**.
The *principle of transmissibility* states that the conditions of equilibrium or motion of a rigid body will remain unchanged if a force **F** acting at a given point of the rigid body is replaced by a force **F'** of the same magnitude and same direction, but acting at a different point, *provided that the two forces have the same line of action*

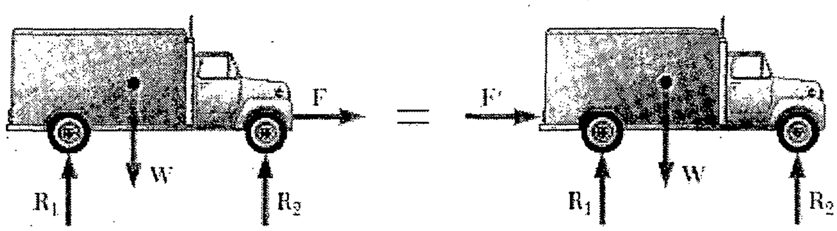


fig 1

2. *Moment of the 250-N force on the handle of the monkey wrench about the center of the* bolt = - 250 cos 15^0 * 200 + 250 sin 15^0 * 30 = **- 46.355 N.m ➔ ANS**

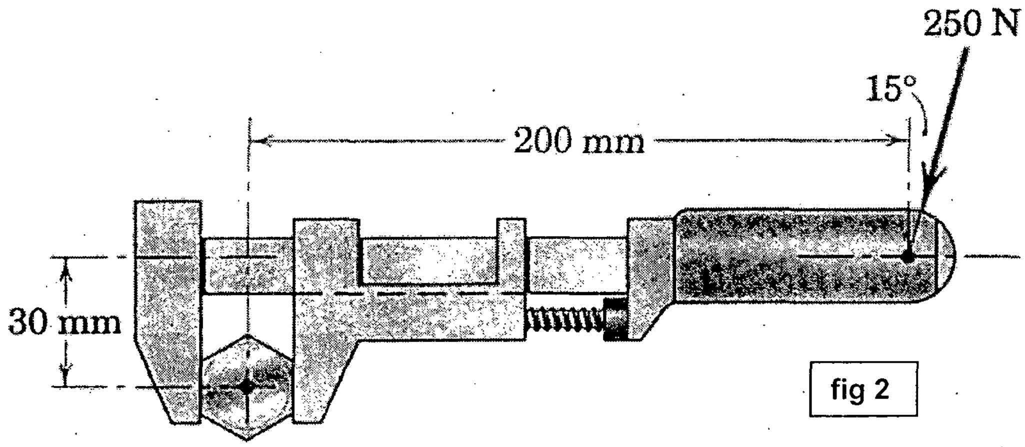


fig 2

3. The balanced teeter-totter (seesaw) signifies that a body is in equilibrium when the moment of all the forces acting on the body is zero. The moment arm of the heavier person is shorter than the moment arm of the lighter person. However the moment of these forces about the point of support is equal and opposite thus resulting in a balanced teeter-totter.

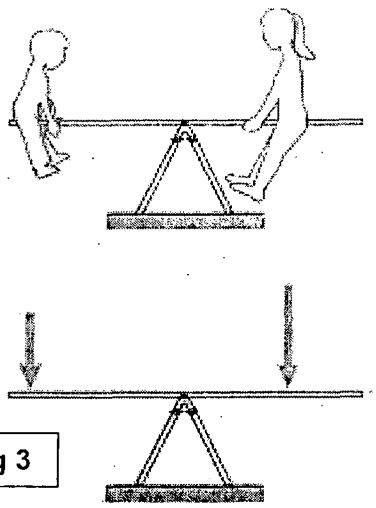


fig 3

(not a free body diagram)

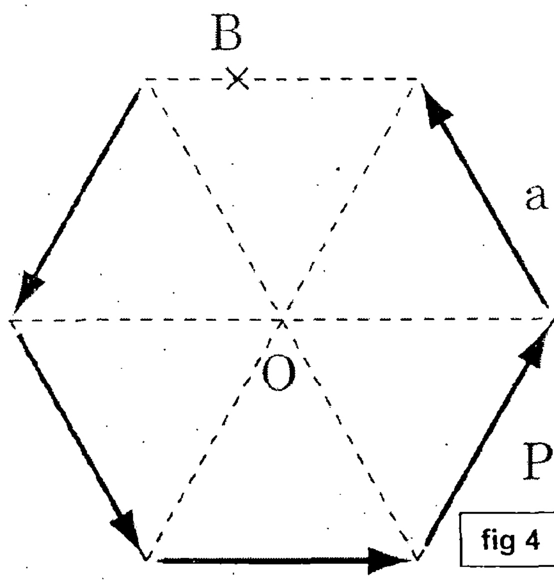


fig 4

4. The equivalent force system of these forces at point B as shown in the figure.
Answers:
(a) Force P (towards right),
Moment √3/2 Pa (clockwise)
(b) Force 4√3 P (upward),
Moment 3√3 Pa (anticlockwise)
(c) **Force P (towards right),**
Moment 3√3 Pa (anticlockwise) → **ANS**
(d) Force P (towards left),
Moment 3√3/2 Pa (clockwise)

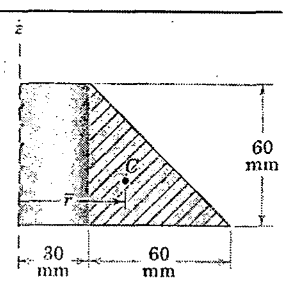
5. Parallel axis theorem used for finding the moment of inertia of an area.

$$I = \bar{I} + Ad^2$$

This formula expresses that the moment of inertia I of an area with respect to any given axis AA' is equal to the moment of inertia $\bar{I}$ of the area with respect to a centroidal axis BB' parallel to AA' *plus* the product of the area A and the square of the distance d between the two axes. This theorem is known as the *parallel-axis*

6. Volume V of the solid generated by revolving the 60-mm right triangular area through 180^0 about the z-axis.

$$V = \theta \bar{r} A = \pi[30 + \tfrac{1}{3}(60)][\tfrac{1}{2}(60)(60)] = 2.83(10^5)\ \text{mm}^3$$ → **ANS**



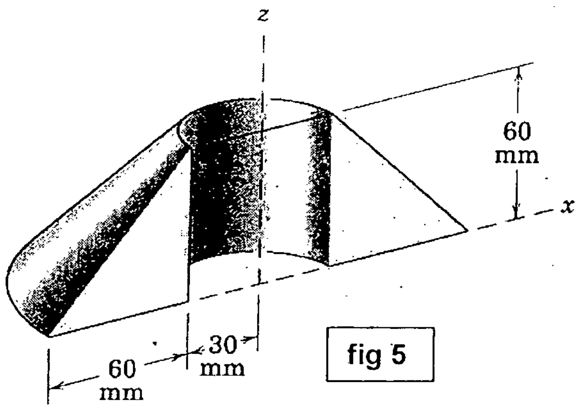


fig 5

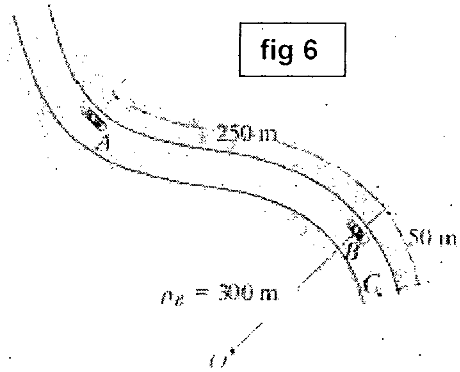


fig 6

7. If the car decelerates uniformly along the curved road from 25 m/s at A to 15 m/s at C, determine the acceleration of the car at B.

Tangential acceleration, $a_t = dv/dt = (dv/ds)\,(ds/dt) = v\,(dv/ds)$

Since car decelerates uniformly, a_t is constant.

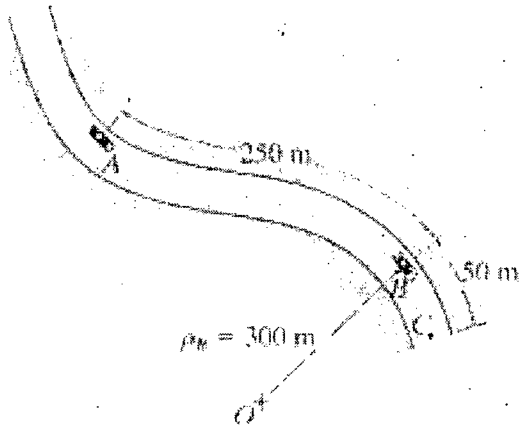
$a_t\,ds = v\,dv$

Integrating between the points A and C, $a_t\,(s_C - s_A) = \frac{1}{2}\,(v_C^2 - v_A^2)$

$$v_C^2 = v_A^2 + 2a_t(s_C - s_A)$$
$$(15\ \text{m/s})^2 = (25\ \text{m/s})^2 + 2a_t(300\ \text{m} - 0)$$
$$a_t = -0.6667\ \text{m/s}^2$$
$$v_B^2 = v_A^2 + 2a_t(s_B - s_A)$$
$$v_B^2 = (25\ \text{m/s})^2 + 2(-0.6667\ \text{m/s}^2)(250\ \text{m} - 0)$$
$$v_B = 17.08\ \text{m/s}$$
$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{(17.08\ \text{m/s})^2}{300\ \text{m}} = 0.9722\ \text{m/s}^2$$
$$a_B = \sqrt{(a_B)_t^2 + (a_B)_n^2}$$
$$= \sqrt{(-0.6667\ \text{m/s}^2)^2 + (0.9722\ \text{m/s}^2)^2}$$
$$= 1.18\ \text{m/s}^2$$

Ans. → **ANS**

Express your answer with the appropriate units.



ANSWER:

$$a = \sqrt{\left(\frac{\frac{v_1^2}{6} + \frac{v_2^2 \cdot 5}{6}}{300}\right)^2 + \left(\frac{v_1^2 - v_2^2}{2 \cdot 300}\right)^2}\ \frac{m}{s^2}$$

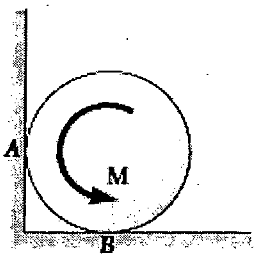
8. **Conditions under which the motion of a projectile is parabolic:**

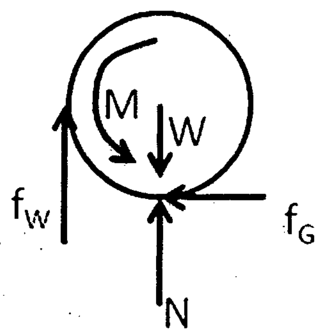
   (i) The air resistance is neglected

   (ii) The acceleration due to gravity is constant and does not vary with altitude.

9. The cylinder shown is of weight W and radius r, and the coefficient of static friction μ_s is the same at A and B. If the cylinder is in equilibrium, draw the free body diagram of the cylinder.

fig 7





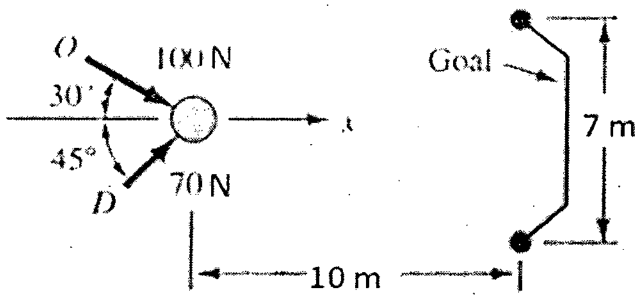
10. **Angle of repose**

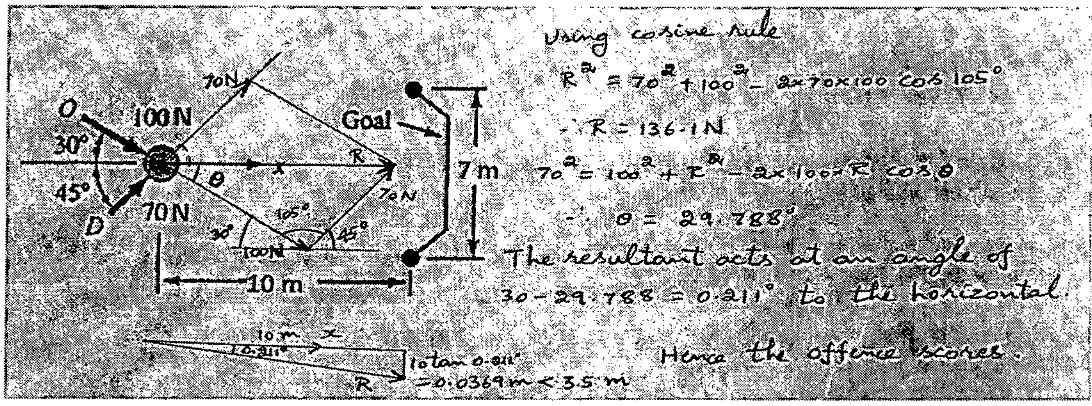
    The value of the angle of inclination corresponding to impending motion is called the *angle of repose*. Clearly, the angle of repose is equal to the angle of static friction φ_s.

<u>Part – B (5 x 16 = 80 marks)</u>

11. Two soccer players approach a stationary ball 10 m away from the goal. Simultaneously, a player on team O (offense) kicks the ball with force 100 N for a split second while a player on team D (defense) kicks with force 70 N during the same time interval. Does the offense score (assuming the goalie is outside the range of the goal).

fig 8





→ **ANS**

12. a) The special purpose milling cutter is subjected to a force of 1200 N and a couple of 240 Nm as shown in fig. 9. Determine the moment of this force couple system about point O. **(10)**

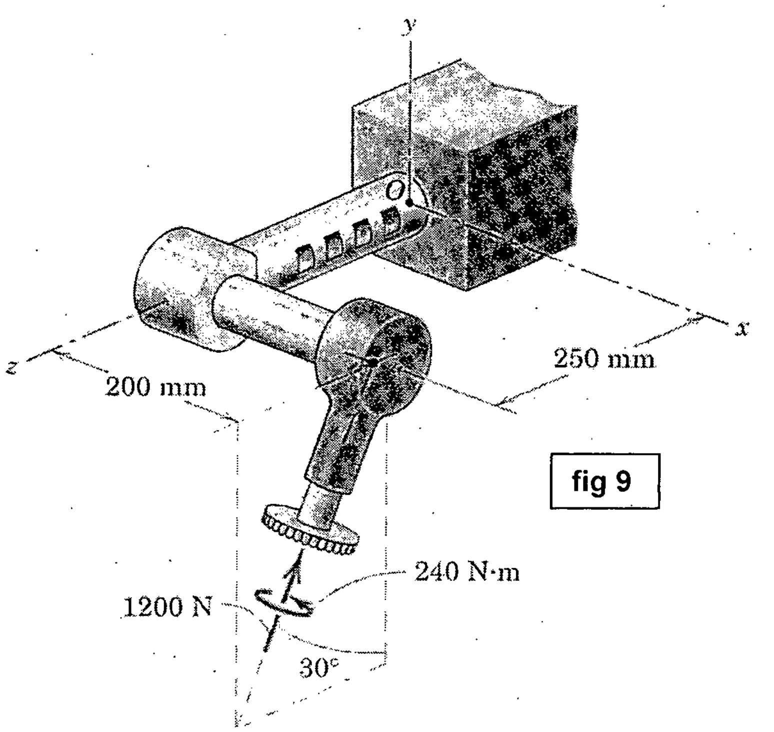


fig 9

R = 1200 sin 30° **j** – 1200 cos 30° **k** = 600 **j** – 1039.23 **k** → **Ans**

Mo = - 600 * 0.25 **i** + 600 * 0.2 **k** + 1039.23 * 0.2 **j** + **M**

$= - 150\ \mathbf{i} + 120\ \mathbf{k} + 207.846\ \mathbf{j} + 240 \sin 30^0\ \mathbf{j} - 240 \cos 30^0\ \mathbf{k}$

$= - 150\ \mathbf{i} + 327.846\ \mathbf{j} - 87.846\ \mathbf{k}$ → **Ans**

[OR]

b)

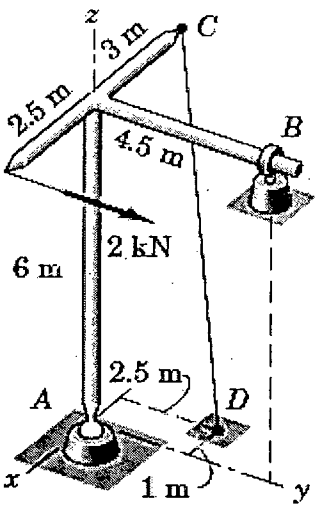
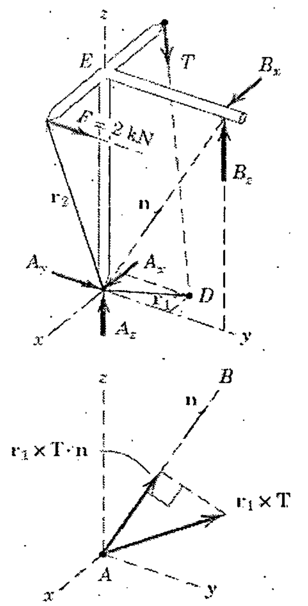


fig 10



Solution. The system is clearly three-dimensional with no lines or planes of symmetry, and therefore the problem must be analyzed as a general space system of forces. The free-body diagram is drawn, where the ring reaction is shown in terms of its two components. All unknowns except T may be eliminated by a moment sum about the line AB. The direction of AB is specified by the unit vector $\mathbf{n} = \frac{1}{\sqrt{6^2 + 4.5^2}}(4.5\mathbf{j} + 6\mathbf{k}) = \frac{1}{5}(3\mathbf{j} + 4\mathbf{k})$. The moment of T about AB is the component in the direction of AB of the vector moment about the point A and equals $\mathbf{r}_1 \times \mathbf{T} \cdot \mathbf{n}$. Similarly the moment of the applied load F about AB is $\mathbf{r}_2 \times \mathbf{F} \cdot \mathbf{n}$. With $\overline{CD} = \sqrt{46.2}$ m, the vector expressions for T, F, $\mathbf{r}_1$, and $\mathbf{r}_2$ are

$$\mathbf{T} = \frac{T}{\sqrt{46.2}}(2\mathbf{i} + 2.5\mathbf{j} - 6\mathbf{k}) \qquad \mathbf{F} = 2\mathbf{j} \text{ kN}$$

$$\mathbf{r}_1 = -\mathbf{i} + 2.5\mathbf{j} \text{ m} \qquad \mathbf{r}_2 = 2.5\mathbf{i} + 6\mathbf{k} \text{ m}$$

The moment equation now becomes

$$[\Sigma M_{AB} = 0] \quad (-\mathbf{i} + 2.5\mathbf{j}) \times \frac{T}{\sqrt{46.2}}(2\mathbf{i} + 2.5\mathbf{j} - 6\mathbf{k}) \cdot \frac{1}{5}(3\mathbf{j} + 4\mathbf{k})$$

$$+ (2.5\mathbf{i} + 6\mathbf{k}) \times (2\mathbf{j}) \cdot \frac{1}{5}(3\mathbf{j} + 4\mathbf{k}) = 0$$

Completion of the vector operations gives

$$-\frac{48T}{\sqrt{46.2}} + 20 = 0 \qquad T = 2.83 \text{ kN} \qquad \textit{Ans.}$$

and the components of T become

$$T_x = 0.833 \text{ kN} \qquad T_y = 1.042 \text{ kN} \qquad T_z = -2.50 \text{ kN}$$

We may find the remaining unknowns by moment and force summations as follows:

$$[\Sigma M_z = 0] \quad 2(2.5) - 4.5B_x - 1.042(3) = 0 \qquad B_x = 0.417 \text{ kN} \qquad \textit{Ans.}$$

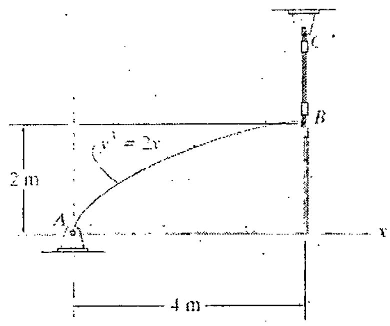
$$[\Sigma M_x = 0] \quad 4.5B_z - 2(6) - 1.042(6) = 0 \qquad B_z = 4.06 \text{ kN} \qquad \textit{Ans.}$$

$$[\Sigma F_x = 0] \quad A_x + 0.417 + 0.833 = 0 \qquad A_x = -1.250 \text{ kN} \qquad \textit{Ans.}$$

$$[\Sigma F_y = 0] \quad A_y + 2 + 1.042 = 0 \qquad A_y = -3.04 \text{ kN} \qquad \textit{Ans.}$$

$$[\Sigma F_z = 0] \quad A_z + 4.06 - 2.50 = 0 \qquad A_z = -1.556 \text{ kN} \qquad \textit{Ans.}$$

9–18. The plate is made of steel having a density of 7850 kg/m^3. If the thickness of the plate is 10 mm, determine the horizontal and vertical components of reaction at the pin A and the tension in cable BC.



Differential Element: The element parallel to the y axis shown shaded in Fig. a will be considered. The area of this element is given by

$$dA = y\,dx = 1.2599x^{1/3}\,dx$$

Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = y/2$.

Area: Integrating,

$$A = \int_A dA = \int_0^{4\text{ m}} 1.2599x^{1/3}\,dx = 0.9449x^{4/3}\Big|_0^{4\text{ m}} = 6\text{ m}^2$$

Thus, the mass of the plate can be obtained from

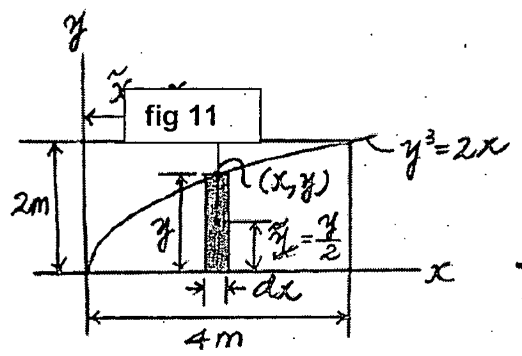
$$m = \rho A t = 7850(6)(0.01) = 471\text{ kg}$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ m}} x\left(1.2599x^{1/3}\,dx\right)}{6} = \frac{\int_0^{4\text{ m}} 1.2599x^{4/3}\,dx}{6} = \frac{0.5399x^{7/3}\Big|_0^{4\text{ m}}}{6} = 2.2857\text{ m}$$

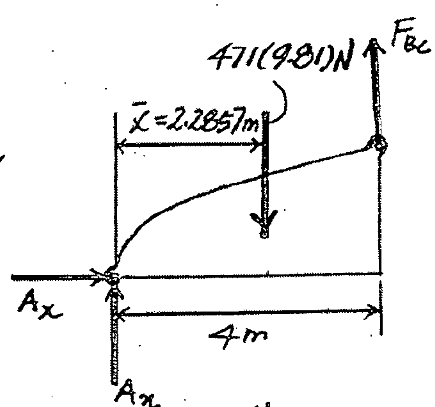
Since the plate has a uniform thickness, its center of gravity coincides with its centroid.

Equations of Equilibrium: By referring to the free body diagram shown in Fig. b.

$\circlearrowleft + \Sigma M_A = 0;$	$F_{BC}(4) - 471(9.81)(2.2857) = 0$	
	$F_{BC} = 2640.27\text{ N} = 2.64\text{ kN}$	**Ans.**
$\overset{+}{\rightarrow} \Sigma F_x = 0;$	$A_x = 0$	**Ans.**
$+\uparrow \Sigma F_y = 0;$	$A_y + 2640.27 - 471(9.81) = 0$	
	$A_y = 1980.24\text{ N} = 1.98\text{ kN}$	**Ans.**



(a)



(b)

[OR]

b) Determine the moment of inertia of the beam's cross-sectional area about the x and y axes.

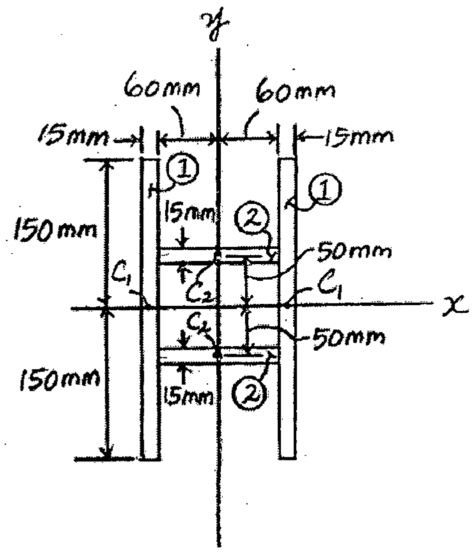
Composite Parts: The composite cross - sectional area of the beam can be subdivided into segments as shown in Fig. *a*. The perpendicular distance measured from the centroid of each segment to the x axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the x axis can be determined using the parallel - axis theorem. Thus,

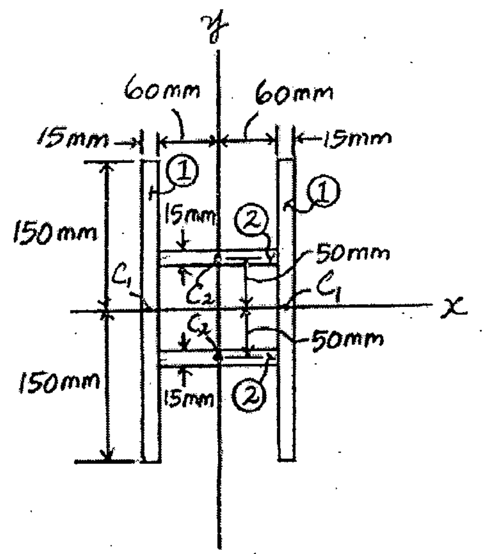
$$I_x = \bar{I}_{x'} + A(d_y)^2$$

$$= \left[2\left(\frac{1}{12}(15)(300^3)\right) + 2(15)(300)(0)^2\right] + \left[2\left(\frac{1}{12}(120)(15^3)\right) + 2(120)(15)(50)^2\right]$$

$$= 67.5(10^6) + 9.0675(10^6) = 76.6(10^6)\ \text{mm}^4$$ **Ans.**



(a)



(a)

Composite Parts: The composite cross - sectional area of the beam can be subdivided into segments as shown in Fig. *a*. perpendicular distance measured from the centroid of each segment to the y axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the x axis can be determined using the parallel - axis t
Thus,

$$I_y = \bar{I}_{y'} + A(d_x)^2$$

$$= \left[2\left(\frac{1}{12}(300)(15^3)\right) + 2(300)(15)(67.5)^2\right] + \left[2\left(\frac{1}{12}(15)(120^3)\right) + 2(120)(15)(0)^2\right]$$

$$= 41.175(10^6) + 4.32(10^6) = 45.5(10^6)\ \text{mm}^4$$ **Ans.**

14. a) (i) The two systems shown start from rest. On the left, two 200 N weights are connected by an inextensible cord, and on the right, a constant 200 N force pulls on the cord. Neglecting all frictional forces which of the following statements is true?

a) Blocks A and C will have the same acceleration.

b) **Block C will have a larger acceleration than block A. → ANS**

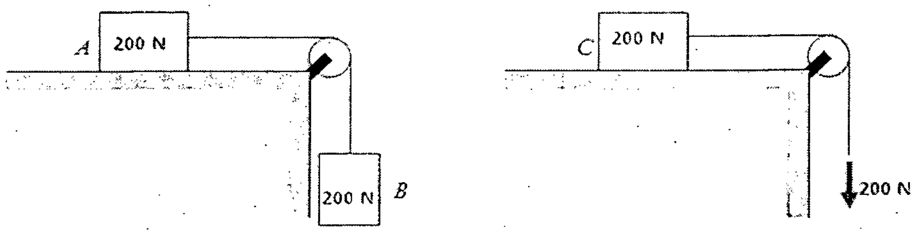
c) Block A will have a larger acceleration than block C.

d) Block A and C will not have any acceleration.

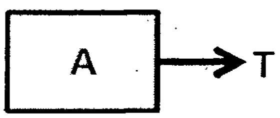
e) None of the above

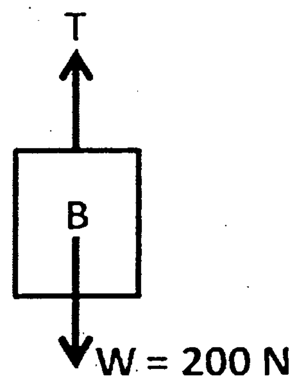
Justify your answer. (8)

fig 13

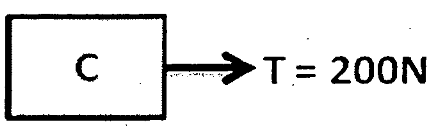


Let a_A be the acceleration of block A.
Let a_B be the acceleration of block B.
When block A moves to the right by x_A, block B moves down by y_B.
$x_A = y_B$
Differentiating w.r.t. time
$v_A = v_B$
Differentiating w.r.t. time
$a_A = a_B \rightarrow (1)$
Let T be the tension in the cable





$\Sigma Fx = m.a_A$
$T = (200/9.81).a_A \rightarrow (2)$
$\Sigma Fy = m.a_B$
$W - T = (200/9.81)\ a_B$
Substituting from eqn. (1), $200 - T = (200/9.81)\ a_A \rightarrow (3)$
Substituting from eqn. (2), $200 - (200/9.81).a_A = (200/9.81)\ a_A$
$9.81 = 2a_A \rightarrow \mathbf{a_A = 4.905\ m/s^2}$



Let a_C be the acceleration of block C.
$\Sigma Fx = m.a_C$
$T = (200/9.81).a_C$
$200 = (200/9.81).a_C \rightarrow \mathbf{a_C = 9.81\ m/s^2}$
Thus Block C will have a larger acceleration than block A. → ANS

(ii)

Solution. The force registered by the scale and the velocity both depend on the acceleration of the elevator, which is constant during the interval for which the forces are constant. From the free-body diagram of the elevator, scale, and man taken together, the acceleration is found to be

$$[\Sigma F_y = ma_y] \qquad 8300 - 7360 = 750a_y \qquad a_y = 1.257 \text{ m/s}^2$$

The scale reads the downward force exerted on it by the man's feet. The equal and opposite reaction R to this action is shown on the free-body diagram of the man alone together with his weight, and the equation of motion for him gives

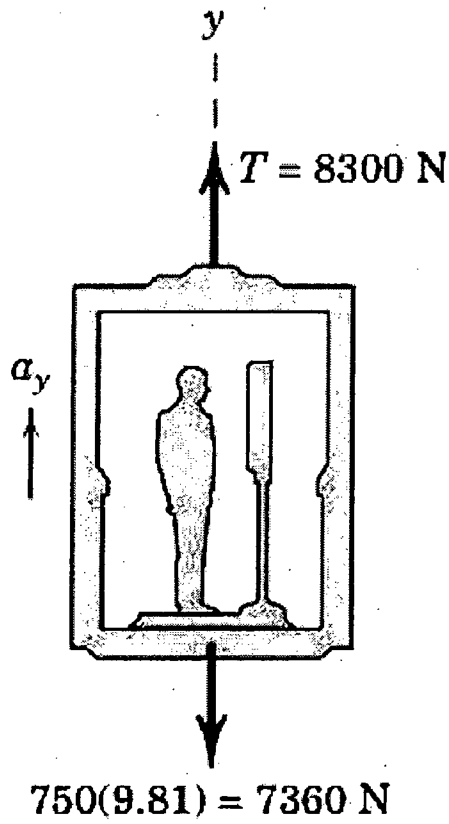
① $$[\Sigma F_y = ma_y] \qquad R - 736 = 75(1.257) \qquad R = 830 \text{ N} \qquad \textit{Ans.}$$

The velocity reached at the end of the 3 seconds is

$$\left[\Delta v = \int a\, dt\right] \qquad v - 0 = \int_0^3 1.257\, dt \qquad v = 3.77 \text{ m/s} \qquad \textit{Ans.}$$

(8)

fig 14



[OR]

b)

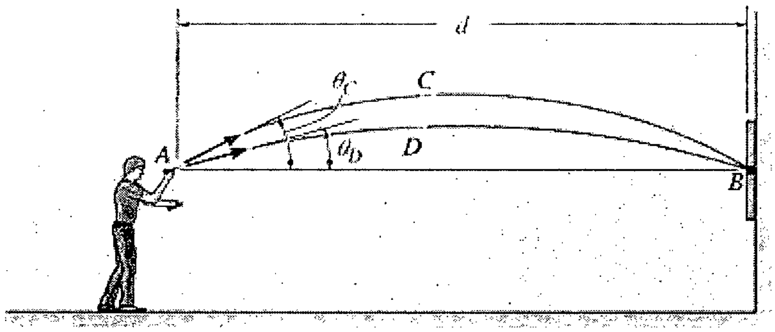
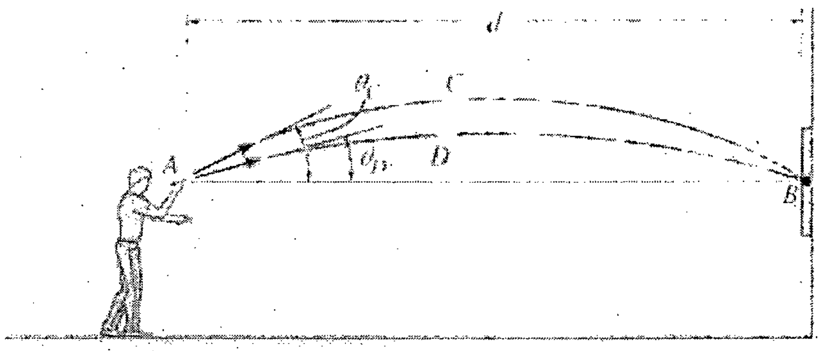


fig 15

Given:



$v_0 = 10\ \frac{\text{m}}{\text{s}}$

$d = 5\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

Guesses $\quad \theta_C = 70\ \text{deg} \quad \theta_D = 15\ \text{deg} \quad \Delta t = 2\ \text{s} \quad t = 1\ \text{s}$

Given $\quad d = v_0 \cos(\theta_C) t \qquad 0 = \frac{-g}{2} t^2 + v_0 \sin(\theta_C) t$

$d = v_0 \cos(\theta_D)(t - \Delta t) \qquad 0 = \frac{-g}{2}(t - \Delta t)^2 + v_0 \sin(\theta_D)(t - \Delta t)$

$$\begin{pmatrix} \theta_C \\ \theta_D \\ t \\ \Delta t \end{pmatrix} = \text{Find}(\theta_C, \theta_D, t, \Delta t) \qquad t = 1.972\ \text{s} \quad \Delta t = 1.455\ \text{s} \qquad \begin{pmatrix} \theta_C \\ \theta_D \end{pmatrix} = \begin{pmatrix} 75.313 \\ 14.687 \end{pmatrix} \text{deg}$$

15. a)

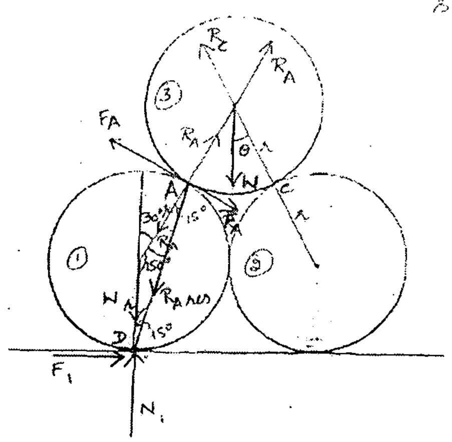


fig 16

$\sin\theta = \frac{r}{2r} = \frac{1}{2}$

$\therefore \theta = 30°$

$F_A = \mu_s R_A \qquad F_C = \mu_s R_C$

$\Sigma F_y = 0 \Rightarrow R_A\cos\theta + R_C\cos\theta + F_C\sin\theta + F_A\sin\theta = W \rightarrow (1)$

$R_A = R_C = R$

$\therefore (1) \Rightarrow 2R\cos\theta + 2\mu R\sin\theta = W$

$2R[\cos\theta + \mu\sin\theta] = W$

$\theta = 30°$

$\therefore R = \frac{W}{\mu + \sqrt{3}}$

Roller 1 is a 3 force member.

It will be in equilibrium, if the 3 forces are concurrent.

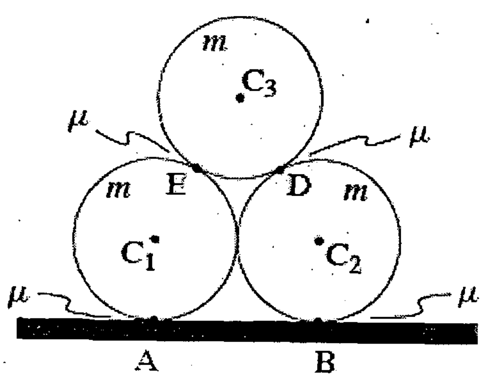
The weight, W and the reaction from the ground pass through pt. D.

$\therefore$ Resultant reaction at pt. A of the ~~no~~normal rxn. R_A and the friction force F_A should also pass through pt. D.

The resultant $R_{A\,res}$ makes an angle 15° with the normal reaction.

$\therefore$ ~~tan 15°~~ $\mu_s = \tan\phi = \tan 15°$

$\therefore \mu_s = 0.268$ **→ ANS**



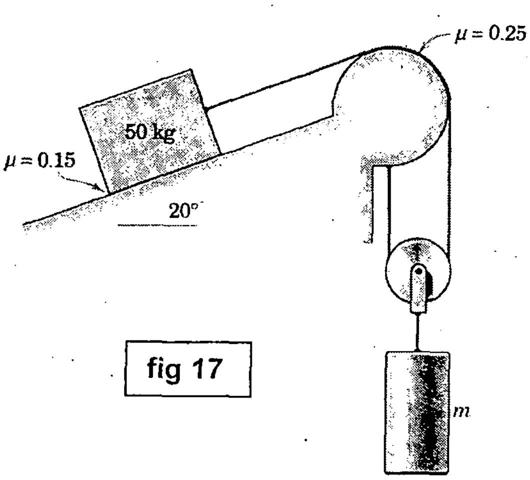
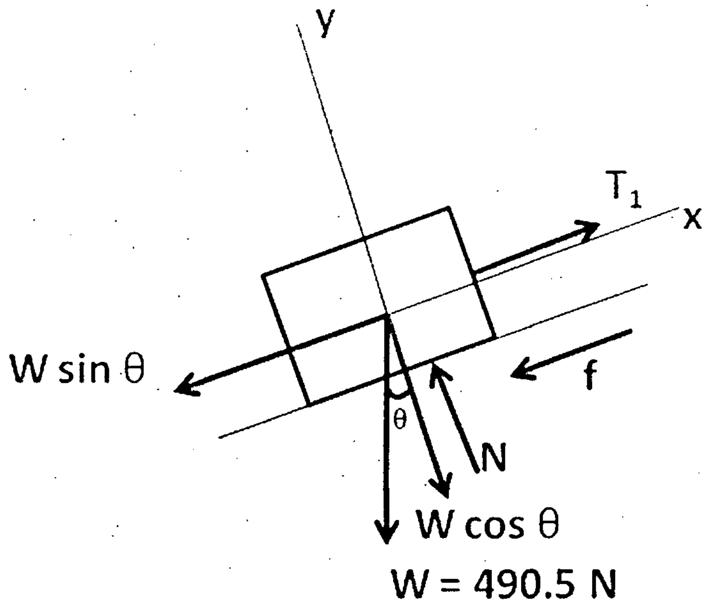


fig 17

[OR]

b) Determine the range of cylinder mass m for which the system is in equilibrium. The coefficient of friction between the 50-kg block and the incline is 0.15 and that between the cord and cylindrical support is 0.25.



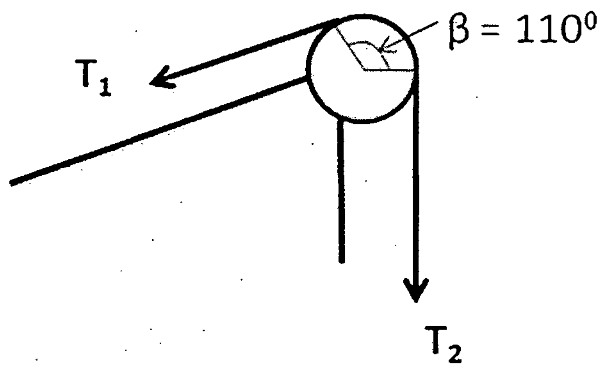
Let T_1 be the tension in the cable.
$\Sigma Fy = 0 \Rightarrow N = W \cos\theta = 490.5 * \cos 20^0 = 460.919$ N → (1)

CASE I

When the block is moving up, friction force acts downward.
$\Sigma Fx = 0 \Rightarrow T_1 - W \sin\theta - f = 0 \Rightarrow T_1 - 490.5 * \sin 20^0 - \mu N = 0$
$T_1 - 167.76 - 0.15 * 460.919 = 0 \rightarrow$ $\mathbf{T_1 = 236.9}$ **N**

CASE II

When the block is moving down, friction force acts upward.
$\Sigma Fx = 0 \Rightarrow T_1 - W \sin\theta + f = 0 \Rightarrow$ $\mathbf{T_1 = 98.622}$ **N**



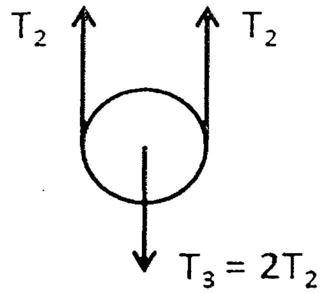
CASE I: When The block moves up the inclined plane, T_1 is the slack side, T_2 is the tight side.

$\left[\frac{T_2}{T_1} = e^{\mu_s \beta}\right]$ ➔ $T_2 = T_1\, e^{\mu\beta} = 236.9 * e^{0.25*(110/180*\pi)} =$ **382.83 N**

CASE II: When The block moves down the inclined plane, T_2 is the slack side, T_1 is the tight side

$\left[\frac{T_1}{T_2} = e^{\mu_s \beta}\right]$ ➔ $T_2 = T_1 / e^{\mu\beta} = 98.622 / e^{0.25*(110/180*\pi)} =$ **61.027 N**

$\Sigma Fy = 0 \rightarrow W = T_3 = 2T_2 \rightarrow 9.81m = 2T_2$



CASE I:

$9.81\ m = 2 * 382.83 \rightarrow$ **$m = 78.05$ kg**

CASE II:

$9.81\ m = 2 * 61.027 \rightarrow$ **$m = 12.44$ kg**

$12.44\ kg \leq m \leq 78.05\ kg$ $\rightarrow$ ANS