# Binary Matroids with Graphic Cocircuits

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#### Abstract

An excluded minor characterization for the class of binary signed-graphic matroids with graphic cocircuits is provided. In this report we present the necessary computations for the case analysis in the proof.

## An excluded minor characterization

The complete list of regular excluded minors for signed-graphic matroids is provided in [2], specifically:

**Theorem 1.** A regular matroid M is signed-graphic if and only if M has no minor isomorphic to  $M^*(G_1), \ldots, M^*(G_{29}), R_{15} or R_{16}$ .

The matroids  $M^*(G_1), \ldots, M^*(G_{29})$  are the cographic matroids of the 29 non-separable forbidden minors for projective planar graphs, while  $R_{15}$  and  $R_{16}$  are two special matroids whose binary compact representation matrices are given in [2] and in the next section of this Technical Report.

Clearly one could easily produce the complete list of binary excluded minors for signed-graphic matroids by adding to the list of the above 31 regular excluded minors the binary excluded minors for regular matroids (i.e.  $F_7$  and  $F_7^*$ ), since any binary signed-graphic matroid is also regular.

**Theorem 2.** A binary matroid M is signed-graphic if and only if M has no minor isomorphic to  $M^*(G_1), \ldots, M^*(G_{29}), R_{15}, R_{16}, F_7$  or  $F_7^*$ .

We define a cocircuit Y of a matroid M to be graphic if the matroid  $M \setminus Y$  is graphic. The main result in this report is the complete list of excluded minors for the class of binary signed-graphic matroids with graphic cocircuits. Of importance for the proof of that result, Theorem 3 here, is the following Lemma.

**Lemma 1.** If N is a minor of the matroid M then for any cocircuit  $C_N \in \mathcal{C}(N^*)$  there exists a cocircuit  $C_M \in \mathcal{C}(M^*)$  such that  $N \setminus C_N$  is a minor of  $M \setminus C_M$ .

*Proof:* If  $N = M \setminus X/Y$  then by duality  $N^* = M/X \setminus Y$ . Therefore by the definitions of contraction and deletion of a set, we have that for any cocircuit  $C_N \in \mathcal{C}(N^*)$  there exists a cocircuit  $C_M \in \mathcal{C}(M^*)$  such that

- (i)  $C_N \subseteq C_M$ ,
- (ii)  $E(N) \cap C_M = C_N$ ,

which in turn imply that  $C_M - C_N \subseteq X$ . So we have

$$M \backslash C_M = M \backslash \{C_M - C_N\} \backslash C_N \succeq N \backslash C_N$$

**Theorem 3.** Let M be a binary matroid such that all its cocircuits are graphic. Then, M is signed-graphic if and only if M has no minor isomorphic to  $M^*(G_{17})$ ,  $M^*(G_{19})$ ,  $F_7$  or  $F_7^*$ .

*Proof:* M must contain a minor isomorphic to some matroid in the set

$$\mathcal{M} = \{M^*(G_1), \dots, M^*(G_{16}), M^*(G_{18}), M^*(G_{20}), \dots, M^*(G_{29}), R_{15}^*, R_{16}^*\}.$$

By case analysis, verified also by the MACEK software [1], it can be shown that for each matroid  $N \in \mathcal{M}$  there exists a cocircuit  $Y_N \in \mathcal{C}(N^*)$  such that the matroid  $N \setminus Y_N$  contains an  $M^*(K_{3,3})$  or an  $M^*(K_5)$  as a minor, which implies that  $N \setminus Y_N$  is not graphic. Therefore, by Lemma 1, there is a cocircuit  $Y_M \in \mathcal{C}(M^*)$  such that  $N \setminus Y_N$  is a minor of  $M \setminus Y_M$ . Thus,  $M \setminus Y_M$  is not graphic which is in contradiction with our assumption that M has graphic cocircuits.

As already mentioned, this Technical Report is mainly devoted to the computations performed using the MACEK software [1] appearing in the proof of Theorem 3. These computations are provided in detail in the next section.

# **MACEK computations**

Each case, i.e. matroid in  $\mathcal{M}$  in the proof of Theorem 3, will be examined separately and specifically:

- for each cographic matroids in  $\mathcal{M}$ , a compact representation matrix of its dual graphic matroid along with the associated graph  $((G_1,\ldots,G_{16},G_{18},G_{20},\ldots,G_{29}))$  are provided. It is clear that due to matroid duality, it is enough to find a circuit C in each  $M \in \{M(G_1),\ldots,M(G_{16}),M(G_{18}),M(G_{20}),\ldots,M(G_{29})\}$  such that M/C contains an  $M(K_{3,3})$  or an  $M(K_5)$ -minor. The advantage of working with the duals of the cographic matroids in  $\mathcal{M}$  is that someone could graphically see that by contracting a cycle (i.e. the one corresponding to C) in the associated graph, a minor isomorphic to  $K_{3,3}$  or  $K_5$  is contained in the resulting graph. Therefore, in that case, the MACEK computations may be seen just as a validation tool.
- for each of the two non-cographic matroid in  $\mathcal{M}$  (i.e.  $R_{15}$  and  $R_{16}$ ), a compact representation matrix is provided along with the cocircuit to be deleted. The MACEK commands and the outputs showing that each of the resulting matroids contains an  $M^*(K_{3,3})$  or an  $M^*(K_5)$  as a minor are given.

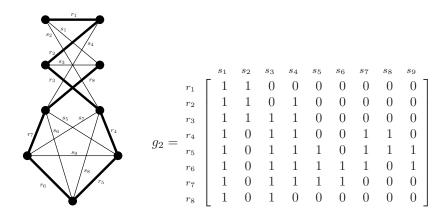
#### The matroid $M(G_1)$ :

$$r_1$$
 $s_3$ 
 $s_4$ 
 $s_2$ 
 $r_2$ 
 $s_5$ 
 $s_6$ 
 $r_7$ 
 $s_{11}$ 
 $s_{10}$ 
 $r_4$ 
 $r_5$ 

## $M(G_1)/\{r_1,s_1,s_3\}$ contains an $M(K_5)$ -minor.

Command: ./macek -pGF2 '!contract 1;!contract -1;!contract -3;!minor' g1 ' $\{grK5, grK33\}$ '
Output: The #1 matroid [g1 $\sim$ c1 $\sim$ c-1 $\sim$ c-3] +HAS+ minor #1 [grK5] in the list  $\{grK5, grK33\}$ .

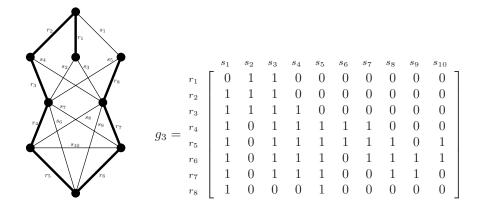
#### The matroid $M(G_2)$ :



$$M(G_2)/\{r_4,r_5,s_9\}$$
 contains an  $M(K_{3,3})$ -minor.

Command: ./macek -pGF2 '!contract 4;!contract 5;!contract -9;!minor' g2 ' $\{grK5, grK33\}$ '
Output: The #1 matroid [g2 $\sim$ c4 $\sim$ c5 $\sim$ c-9] +HAS+ minor #2 [grK33] in the list  $\{grK5, grK33\}$ .

## The matroid $M(G_3)$ :



$$M(G_3)/\{r_1,r_2,r_3,s_2\}$$
 contains an  $M(K_5)$ -minor.

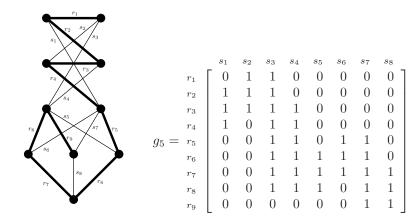
 $\label{local_command: command: 2} \textbf{Command: ./macek -pGF2 '!contract 1;!contract 2;!contract 3;!contract -2;!minor' g3 '{grK5,grK33}' \\ \textbf{Output: The #1 matroid } [g3\sim c1\sim c2\sim c3\sim c-2] \textbf{ +HAS+ minor #1 } [grK5] in the list {grK5 } grK33}.$ 

## The matroid $M(G_4)$ :

$$M(G_4)/\{r_4, r_5, r_6, s_6\}$$
 contains an  $M(K_{3,3})$ -minor.

Command: ./macek -pGF2 '!contract 4;!contract 5;!contract 6;!contract -6;!minor' g4 ' $\{grK5,grK33\}$ '
Output: The #1 matroid [g4 $\sim$ c4 $\sim$ c5 $\sim$ c6 $\sim$ c-6] +HAS+ minor #2 [grK33] in the list  $\{grK5,grK33\}$ .

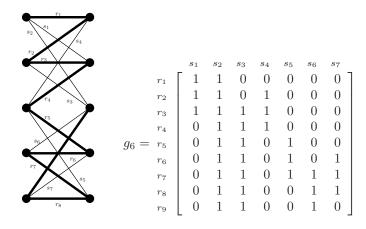
## The matroid $M(G_5)$ :



$$M(G_5)/\{r_7, r_8, r_9, s_8\}$$
 contains an  $M(K_{3,3})$ -minor.

Command:./macek -pGF2 '!contract 7;!contract 8;!contract 9;!contract -8;!minor' g5 '{grK5,grK33}' Output: The #1 matroid [g5 $\sim$ c7 $\sim$ c8 $\sim$ c9 $\sim$ c-8] +HAS+ minor #2 [grK33] in the list {grK5 grK33}.

#### The matroid $M(G_6)$ :



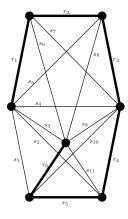
$$M(G_6)/\{r_6, r_7, r_8, s_7\}$$
 contains an  $M(K_{3,3})$ -minor.

Command: ./macek -pGF2 '!contract 6;!contract 7;!contract 8;!contract -7;!minor' g6 ' $\{grK5, grK33\}$ ' Output: Output: The #1 matroid [ $g6\sim c6\sim c7\sim c8\sim c-7$ ] +HAS+ minor #2 [grK33] in the list  $\{grK5, grK33\}$ .

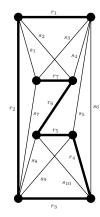
#### The matroid $M(G_7)$ :

$$M(G_7)/\{r_1,r_2,s_5\}$$
 contains an  $M(K_5)$ -minor.

Command: ./macek -pGF2 '!contract 1;!contract 2;!contract -5;!minor' g7 '{grK5,grK33}' Output: The #1 matroid [g7 $\sim$ c1 $\sim$ c2 $\sim$ c-5] +HAS+ minor #1 [grK5] in the list {grK5 grK33}.



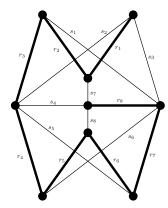
## The matroid $M(G_8)$ :



$$M(G_8)/\{r_4,s_5,s_6\}$$
 contains an  $M(K_5)$ -minor.

Command: ./macek -pGF2 '!contract 4;!contract -5;!contract -6;!minor' g8 ' $\{grK5, grK33\}$ '
Output: The #1 matroid [g8 $\sim$ c4 $\sim$ c-5 $\sim$ c-6] +HAS+ minor #1 [grK5] in the list  $\{grK5, grK33\}$ .

## The matroid $M(G_9)$ :

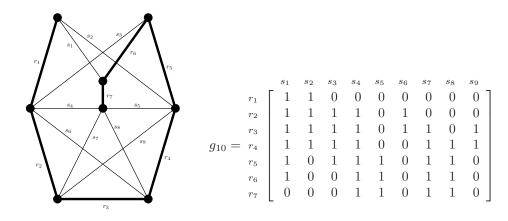


$$g_9 = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ r_6 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ r_7 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ r_8 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$M(G_9)/\{r_1, r_2, r_3, s_2\}$$
 contains an  $M(K_{3,3})$ -minor.

Command: ./macek -pGF2 '!contract 1;!contract 2;!contract 3;!contract -2;!minor' g9 '{grK5,grK33}' Output: The #1 matroid [g9 $\sim$ c1 $\sim$ c2 $\sim$ c3 $\sim$ c-2] +HAS+ minor #2 [grK33] in the list {grK5 grK33}.

## The matroid $M(G_{10})$ :

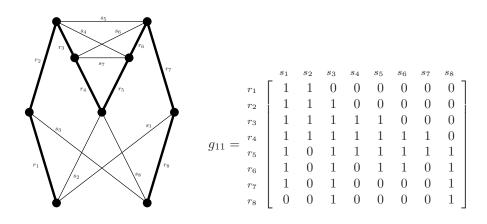


$$M(G_{10})/\{r_1, r_6, s_1, s_3\}$$
 contains an  $M(K_5)$ -minor.

Command: ./macek -pGF2 '!contract 1;!contract 6;!contract -1;!contract -3;!minor' g10 '{grK5,grK33}'

Output: The #1 matroid [g10 $\sim$ c1 $\sim$ c6 $\sim$ c-1 $\sim$ c-3] +HAS+ minor #1 [grK5] in the list {grK5 grK33}.

#### The matroid $M(G_{11})$ :



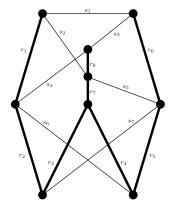
$$M(G_{11})/\{r_3, r_6, s_5, s_7\}$$
 contains an  $M(K_{3,3})$ -minor.

Command: ./macek -pGF2 '!contract 3;!contract 6;!contract -5;!contract -7;!minor' g11 '{grK5,grK33}' Output: The #1 matroid [g11 $\sim$ c3 $\sim$ c6 $\sim$ c-7] +HAS+ minor #2 [grK33] in the list {grK5 grK33}.

## The matroid $M(G_{12})$ :

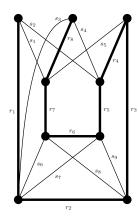
$$M(G_{12})/\{r_8, s_1, s_2, s_3\}$$
 contains an  $M(K_{3,3})$ -minor.

Command: ./macek -pGF2 '!contract 8;!contract -1;!contract -2;!contract -3;!minor' g12 ' $\{grK5, grK33\}$ '
Output: The #1 matroid [g12 $\sim$ c8 $\sim$ c-1 $\sim$ c-2 $\sim$ c-3] +HAS+ minor #2 [grK33] in the list  $\{grK5, grK33\}$ .



$$g_{12} = \begin{bmatrix} r_1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ r_2 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ r_5 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ r_6 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ r_7 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ r_8 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

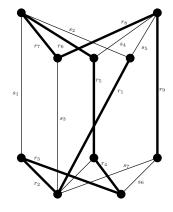
## The matroid $M(G_{13})$ :



$$M(G_{13})/\{r_2, r_6, s_6, s_9\}$$
 contains an  $M(K_{3,3})$ -minor.

Command: ./macek -pGF2 '!contract 2;!contract 6;!contract -6;!contract -9;!minor' g13 '{grK5,grK33}' Output: The #1 matroid [g13 $\sim$ c2 $\sim$ c6 $\sim$ c-9] +HAS+ minor #2 [grK33] in the list {grK5 grK33}.

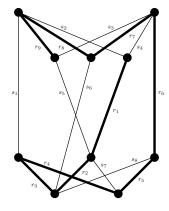
## The matroid $M(G_{14})$ :



 $M(G_{14})/\{r_7,r_8,s_2,s_5\}$  contains an  $M(K_{3,3})$ -minor.

Command: ./macek -pGF2 '!contract 7;!contract 8;!contract -2;!contract -5;!minor' g14 ' $\{grK5, grK33\}$ '
Output: The #1 matroid [g14 $\sim$ c7 $\sim$ c8 $\sim$ c-2 $\sim$ c-5] +HAS+ minor #2 [grK33] in the list  $\{grK5, grK33\}$ .

## The matroid $M(G_{15})$ :

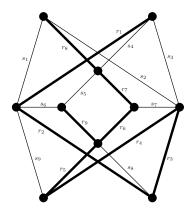


$$g_{15} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ r_6 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ r_7 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ r_8 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ r_9 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$M(G_{15})/\{r_7, r_8, r_9, s_3\}$$
 contains an  $M(K_{3,3})$ -minor.

Command: ./macek -pGF2 '!contract 7;!contract 8;!contract 9;!contract -3;!minor' g15 '{grK5,grK33}' Output: The #1 matroid [g15 $\sim$ c7 $\sim$ c8 $\sim$ c9 $\sim$ c-3] +HAS+ minor #2 [grK33] in the list {grK5 grK33}.

#### The matroid $M(G_{16})$ :



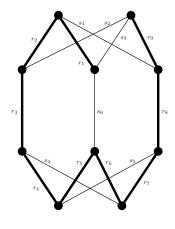
$$M(G_{16})/\{r_6, r_7, r_9, s_5\}$$
 contains an  $M(K_{3,3})$ -minor.

Command: ./macek -pGF2 '!contract 6;!contract 7;!contract 9;!contract -5;!minor' g16 'grK5, grK33'
Output: The #1 matroid [g16 $\sim$ c6 $\sim$ c7 $\sim$ c9 $\sim$ c-5] +HAS+ minor #2 [grK33] in the list grK5 grK33}.

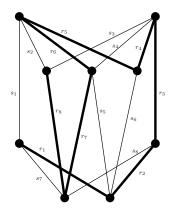
## The matroid $M(G_{18})$ :

$$M(G_{18})/\{r_4, r_7, s_4, s_5\}$$
 contains an  $M(K_{3,3})$ -minor.

Command: ./macek -pGF2 '!contract 4;!contract 7;!contract -4;!contract -5;!minor' g18 ' $\{grK5, grK33\}$ '
Output: The #1 matroid [g18 $\sim$ c4 $\sim$ c7 $\sim$ c-4 $\sim$ c-5] +HAS+ minor #2 [grK33] in the list  $\{grK5, grK33\}$ .



## The matroid $M(G_{20})$ :

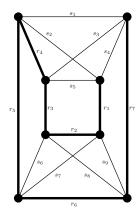


$$g_{20} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ r_1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ r_5 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ r_6 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ r_7 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ r_8 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M(G_{20})/\{r_1, r_2, s_7, s_8\}$$
 contains an  $M(K_{3,3})$ -minor.

Command: ./macek -pGF2 '!contract 1;!contract 2;!contract -7;!contract -8;!minor' g20 '{grK5,grK33}' Output: The #1 matroid [g20 $\sim$ c1 $\sim$ c2 $\sim$ c-7 $\sim$ c-8] +HAS+ minor #2 [grK33] in the list {grK5 grK33}.

## The matroid $M(G_{21})$ :

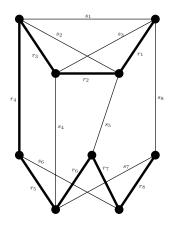


$$g_{21} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ r_6 & & & & & & & & & & & & & & & \\ r_7 & & & & & & & & & & & & & & & \end{bmatrix}$$

# $M(G_{21})/\{r_4, s_2, s_5\}$ contains an $M(K_5)$ -minor.

Command: ./macek -pGF2 '!contract 4;!contract -2;!contract -5;!minor' g21 '{grK5,grK33}' Output: The #1 matroid [g21 $\sim$ c4 $\sim$ c-2 $\sim$ c-5] +HAS+ minor #1 [grK5] in the list {grK5 grK33}.

## The matroid $M(G_{22})$ :

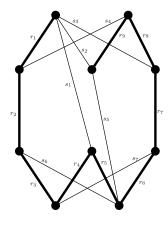


$$g_{22} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ r_5 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ r_6 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ r_7 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ r_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$M(G_{22})/\{r_5, r_6, r_7, s_6\}$$
 contains an  $M(K_5)$ -minor.

Command: ./macek -pGF2 '!contract 5;!contract 6;!contract 7;!contract -6;!minor' g22 'grK5, grK33'
Output: The #1 matroid [g22 $\sim$ c5 $\sim$ c6 $\sim$ c7 $\sim$ c-6] +HAS+ minor #1 [grK5] in the list grK5 grK33.

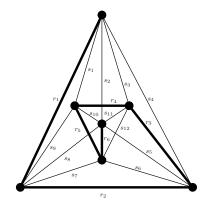
#### The matroid $M(G_{23})$ :



$$g_{23} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ r_7 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ r_8 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ r_9 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

 $M(G_{23})/\{r_1, r_8, s_3, s_4\}$  contains an  $M(K_{3,3})$ -minor.

Command: ./macek -pGF2 '!contract 1;!contract 8;!contract -3;!contract -4;!minor' g23 ' $\{grK5, grK33\}$ '
Output: The #1 matroid [g23 $\sim$ c1 $\sim$ c8 $\sim$ c-4] +HAS+ minor #2 [grK33] in the list  $\{grK5, grK33\}$ .

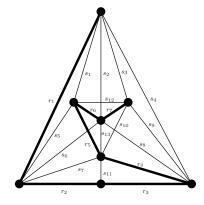


# The matroid $M(G_{24})$ :

 $M(G_{24})/\{r_1, s_1, s_9\}$  contains an  $M(K_5)$ -minor.

Command: ./macek -pGF2 '!contract 1;!contract -1;!contract -9;!minor' g24 '{grK5,grK33}' Output: The #1 matroid [g24 $\sim$ c1 $\sim$ c-9] +HAS+ minor #1 [grK5] in the list {grK5 grK33}.

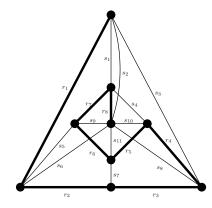
## The matroid $M(G_{25})$ :



$$M(G_{25})/\{r_5, s_{10}, s_{12}\}\$$
contains an  $M(K_5)$ -minor.

Command: ./macek -pGF2 '!contract 5;!contract -10;!contract -12;!minor' g25 ' $\{grK5, grK33\}$ ' Output: The #1 matroid [g25 $\sim$ c5 $\sim$ c-10 $\sim$ c-12] +HAS+ minor #1 [grK5] in the list  $\{grK5, grK33\}$ .

#### The matroid $M(G_{26})$ :



$$M(G_{26})/\{r_5, r_6, r_7, s_4\}$$
 contains an  $M(K_5)$ -minor.

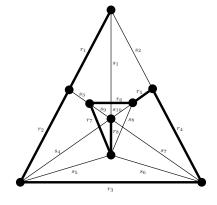
Command: ./macek -pGF2 '!contract 5;!contract 6;!contract 7;!contract -4;!minor' g26 '{grK5,grK33}' Output: The #1 matroid [g26 $\sim$ c5 $\sim$ c6 $\sim$ c7 $\sim$ c-4] +HAS+ minor #1 [grK5] in the list {grK5 grK33}.

## The matroid $M(G_{27})$ :

$$M(G_{27})/\{r_1, r_2, r_3, r_4, s_2\}$$
 contains an  $M(K_5)$ -minor.

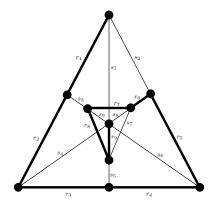
Command: ./macek -pGF2 '!contract 1;!contract 2;!contract 3;!contract 4;!contract -2;!minor' g27
'{grK5,grK33}'

Output: The #1 matroid  $[g27\sim c1\sim c2\sim c3\sim c4\sim c-2]$  +HAS+ minor #1 [grK5] in the list  $\{grK5\ grK33\}$ .



$$g_{27} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 & s_{10} \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ r_6 & & & & & & & & & & \\ r_7 & & & & & & & & & & & \\ r_8 & & & & & & & & & & & & \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

# The matroid $M(G_{28})$ :



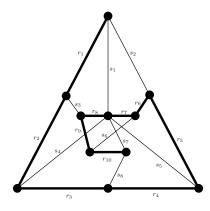
$$g_{28} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ r_7 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ r_8 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

 $M(G_{28})/\{r_1,r_2,r_3,r_4,r_5,s_2\}$  contains an  $M(K_5)$ -minor.

Command: ./macek -pGF2 '!contract 1;!contract 2;!contract 3;!contract 4;!contract 5;!contract -2;!minor' g28 '{grK5,grK33}'

 $\textbf{Output:} \quad \text{The \#1 matroid } [g28 \sim c1 \sim c2 \sim c3 \sim c4 \sim c5 \sim c] \quad \text{+HAS+ minor \#1 } [grK5] \text{ in the list } \{grK5 \ grK33\}.$ 

## The matroid $M(G_{29})$ :



$$M(G_{29})/\{r_1, r_2, r_3, r_4, r_5, s_2\}$$
 contains an  $M(K_{3,3})$ -minor.

Command:./macek -pGF2 '!contract 1;!contract 2;!contract 3;!contract 4;!contract 5;!contract -2;!minor' g29 '{grK5,grK33}'

Output: The #1 matroid [ $g5\sim c7\sim c8\sim c9\sim c-8$ ] +HAS+ minor #2 [grK33] in the list {grK5 grK33}.

#### The matroid $R_{15}$ :

$$r_{15} = \begin{bmatrix} r_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ r_2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ r_6 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ r_7 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

 $R_{15}\setminus\{r_6,r_7,s_8\}$  contains an  $M(K_{3,3})$ -minor.

Command:./macek -pGF2 '!delete 6; !delete 7; !delete -8; !minor' r15 '{"grK5; !dual", "grK33; !dual"}'
Output: The #1 matroid [r15 $\sim$ d6 $\sim$ d7 $\sim$ d-8] +HAS+ minor #2 [grK33#] in the list {grK5# grK33#}.

#### The matroid $R_{16}$ :

$$r_{16} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ r_6 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ r_7 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ r_8 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

 $M(R_{16})\setminus\{r_8,s_1,s_3,s_8\}$  contains an  $M(K_{3,3})$ -minor.

```
Command: ./macek -pGF2 '!delete 8;!delete -1;!delete -3;!delete -8;!minor' r16
'{"grK5;!dual","grK33;!dual"}'
Output: The #1 matroid [r16~d8~d-1~d-3~d-8] +HAS+ minor #2 [grK33#] in the list {grK5# grK33#}.
```

# Acknowledgements

This research has been funded by the European Union (European Social Fund - ESF) and Greek national funds through the Operational Program Education and Lifelong Learning of the National Strategic Reference Framework (NSRF) - Research Funding Program: Thalis. Investing in knowledge society through the European Social Fund.

# References

- [1] P. Hlileny. MACEK 1.2+ MAtroids Computed Efficiently Kit, 2007. http://www.fi.muni.cz/~hlineny/MACEK/.
- [2] H. Qin and D. Slilaty and X. Zhou. The regular excluded minors for signed-graphic matroids. *Combinatorics, Probability and Computing*, 18:953–978, 2009.

