## Anomalous spin Hall and inverse spin Hall effects in magnetic systems

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Anomalous spin-Hall effects (SHE) and anomalous inverse spin-Hall effects (ISHE) are predicted when the order parameters are involved in the charge-spin interconversion. A spin current whose propagation and polarization are collinear and transverse to an applied charge current can be generated when the charge current is parallel to the magnetization M in a ferromagnet or to the Neel order  $\vec{n}$  in an anti-ferromagnet. When the applied charge current is perpendicular to the order parameter, two spin currents can be generated. One is the spin current polarized along the order parameter and propagating along the charge current direction. The other is the one polarized in the charge current direction and propagating along the order parameter direction. Charge currents proportional to the order parameter are generated by applied spin currents in the anomalous ISHE. When a spin current whose propagation and polarization are collinear is applied, the charge current is along the perpendicular (to the spin current) component of order parameter, and no current for order parameter along the spin current. For applied spin current with propagation and polarization mutually orthogonal to each other, a charge current along the spin current propagation direction is generated if the order parameter is collinear with the polarization of the spin current. A charge current along the polarization of spin current is generated if the order parameter is collinear with the propagation direction of the spin current. Experimental verifications are also proposed. In terms of applications, one of the great advantages of anomalous SHE is that one can control the generated spin or charge current by controlling the magnetization or the Neel order in the magnetic materials.

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Spin current generation, manipulation, and detection are essential themes of spintronics. Similar to electric-currents in electronics that perform all kinds of tasks, spin currents can carry and process information in spintronics, as well as control spin and magnetization direction of memory cells [1, 2] and magnetic domain wall motion [3–6]. Different from an electric current that is the flow of charges, a spin current is the flow of a vector quantity of angular momenta. In the Cartesian coordinates, spin currents are tensor of rank two with nine components, instead of a vector for electric currents. A spin current can exert a torque on spins or on magnetic moments when passing through them, and this torque is component sensitive and useful as a control knob for manipulating spin states of nano-devices [3–6].

Spin currents can be generated in many systems and in many different ways [3-6], but one very attractive method is through the spin Hall effect (SHE) originally proposed by Dyakonov and Perel [12]. Recent escalating research activities on SHE and the terminology is largely due to the rediscovery of the effect by Hirsch [13] and due to the subsequently experimental verifications in semiconductor devices [14, 15]. This well-known SHE says that a charge current  $\vec{J}$  in a material with spin-orbit interaction can generate a spin current  $j_{ij}^s$  polarizing in direction  $\hat{j}$  and flowing in direction  $\hat{i}$  with  $\hat{i}$ ,  $\hat{j}$ , and  $\vec{J}$  mutually perpendicular with each other. Thus, in the Cartesian coordinate this spin current is  $j_{ij}^s = \theta_0 \hbar/(2e)\epsilon_{ijk} J_k$ , where  $\epsilon_{ijk}$  is the Levi-Civita symbol and the Einstein summation convention is applied.  $\theta_0$  is a material parameter called spin Hall angle whose value is an issue of recent debates. Like action-reaction principle in nature, the inverse effect of SHE is called inverse spin Hall effect (ISHE) [16–18] that says a spin current can generate a charge current of  $J_k = \theta_0'(2e)/(\hbar)\epsilon_{ijk}j_{ij}^s$ . Again the Einstein summation convention is applied and  $\theta'_0$  is called inverse spin Hall angle, and interestingly,  $\theta'_0 = \theta_0$ . This conventional SHE was confirmed capable of reversing spins of spin-orbital torque (SOT) memory [1] while the conventional ISHE becomes a standard technique for spin current detection [16–18]. From application point of view, one of the shortcomings of the spin current generated from the conventional SHE is that spin-current polarization must be perpendicular to the spin-current propagation direction. It is highly desirable to generate spin currents whose polarization can be in any possible directions and controllable by other external means so that resulted torques are tunable. This is the subject of the present study.

In this letter I predict the existence of new types of charge-spin interconversion in magnetic materials. Differ from the existing paradigm [12, 13, 16, 17], I show, from the general requirement of a genuine physical quantity being a tensor, that there are, in principle, three new SHEs and ISHEs in magnetic materials when they involve the order parameters of magnetization  $\vec{M}$  in ferromagnets and the Neel order  $\vec{n}$  in anti-ferromagnets. I term them anomalous SHE and anomalous ISHE because they are linear in the order parameter. Specifically, spin currents  $j_{ii}^s$  are converted from a charge current collinear with the order parameter where  $\hat{i}$  is perpendicular to the charge current. When a charge current flows perpendic-

ularly to the order parameter, two new spin currents  $j^s_{ij}$  are generated, where directions  $\hat{i}$  and  $\hat{j}$  are either respectively along electric current and order parameter or respectively along the order parameter and electric current. In the anomalous ISHE, a charge current along the order parameter is generated when a spin current  $j^s_{ii}$  with its polarization and propagation collinear flow perpendicularly to the order parameter. A charge current along either the spin current flow direction or polarization direction is generated by a spin current  $j^s_{ij}$  when  $\hat{i}$  and  $\hat{j}$  are mutually perpendicular to each other, and either  $\hat{i}$  or  $\hat{j}$  is along the order parameter.

Consider a piece of ferromagnetic or anti-ferromagnetic metal with an applied charge current density  $\vec{J}$ . I will take a ferromagnet as an example below. A spin current  $j^s_{ij}$  is generated by  $\vec{J}$ . Since spin current is a tensor of rank 2 and  $\vec{J}$  a tensor of rank 1, the most general relationship between  $j^s_{ij}$  and  $\vec{J}$  [19–21], in the linear response region, is

$$j_{ij}^{s} = \frac{\hbar}{2e} \theta_{ijk}^{\text{SH}} J_k, \tag{1}$$

where  $\theta_{ijk}^{\rm SH}$  is the spin-Hall angle tensor of rank 3 that does not depend on current  $\vec{J}$ . In three dimension, i,j,k=1,2,3 stand for respectively the x,y, and z directions. In the absence of magnetic field, the only available nonzero rank tensors (other than the electric current) are order parameter  $\vec{M}$  or  $\vec{n}$  for a ferromagnet or an antiferromagnet, as well as the Levi-Civita symbol of  $\epsilon_{ijk}=1$  for  $(i,j,k)=(1,2,3),\ (2,3,1),\ (3,1,2),\ \epsilon_{ijk}=-1$  for  $(i,j,k)=(1,3,2),\ (2,1,3,),\ (3,2,1),\$ and  $\epsilon_{ijk}=0$  for any other choices of (i,jk). If the order parameter can also participate in the SHE, and if we restrict ourselves to the linear anomalous SHE and ISHE in  $\vec{M}$  or  $\vec{n}$  (in the case of anti-ferromagnet), then the most general  $\theta_{ijk}^{\rm SH}$  in the case of ferromagnet is

$$\theta_{ijk}^{\rm SH} = \theta_0 \epsilon_{ijk} + \theta_1 M_l \epsilon_{iln} \epsilon_{jnk} + \theta_2 M_l \epsilon_{ink} \epsilon_{jln}, \tag{2}$$

 $\theta_0$  is the usual spin Hall angle that does not interact with  $\vec{M}$ ,  $\theta_{\alpha}$  ( $\alpha=1,2$ ) are anomalous SHE coefficients. It is straight forward to see that the last two terms can be recast as

$$[(\theta_1 + \theta_2)\delta_{ij}\delta_{kl\neq i} + \theta_1\delta_{ik}\delta_{il\neq i} + \theta_2\delta_{il}\delta_{jk\neq i}]M_l.$$
 (3)

In order to see what these anomalous SHEs are, we consider two cases: (1) The charge current is along  $\vec{M}$ . (2) The charge current is perpendicular to  $\vec{M}$ . Without losing generality, let  $\vec{J}$  and  $\vec{M}$  be along the  $\hat{x}$ -axis in case (1). Substituting Eqs. (2) and (3) into Eq. (1), the general SHE in magnetic materials becomes

$$j_{ij}^{s} = \frac{\hbar}{2e} [\theta_0 \epsilon_{ij1} j + (\theta_1 + \theta_2 \delta_{ij \neq 1}) M J]. \tag{4}$$

In case (1), spin current  $j_{22}^s$  and  $j_{33}^s$  proportional to MJ are generated when  $\vec{J}$  and  $\vec{M}$  are collinear and along the

 $\hat{x}$ -axis. Fig. 1(a) is the schematic diagram of the anomalous SHE where the thicker and thinner red arrowed lines denote respectively the charge current source and magnetization directions. The black arrowed line denote two two spin currents whose polarization are indicated by decorated smaller arrows. Clearly, the conventional SHE cannot generate such a spin current.

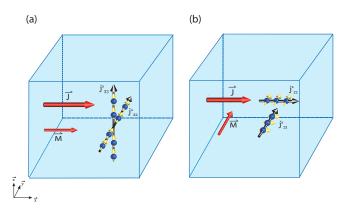


FIG. 1: Schematics of anomalous SHE: (a) The charge current  $\vec{J}$  (the thicker red arrowed line) and magnetization  $\vec{M}$  (the thinner red arrowed line) are along the  $\hat{x}$ -direction. Two anomalous spin current  $j_{22}^s$  and  $j_{33}^s$  (the black arrowed lines) proportional to M and j are generated. (b) The charge current  $\vec{J}$  (the thicker red arrowed line) and magnetization  $\vec{M}$  (the thinner red arrowed line) are respectively along the  $\hat{x}$ -and the  $\hat{y}$ -directions. Two anomalous spin current  $j_{12}^s$  and  $j_{21}^s$  (the black arrowed lines with polarization decorated by the bead-arrow on the lines) proportional to MJ are generated

Without losing generality, we let  $\vec{J}$  and  $\vec{M}$  respectively along the  $\hat{x}$ - and the  $\hat{y}$ -directions in case (2). From Eqs. (1)-(3), we have

$$j_{ij}^s = \frac{\hbar}{2e} (\theta_0 \epsilon_{ij1} + \theta_1 \delta_{i1} \delta_{j2} M + \theta_2 \delta_{i2} \delta_{j1} M) J.$$
 (5)

Eq. (5) says that spin current  $j_{12}^s$  and  $j_{21}^s$ , proportional to the magnetization M and propagating respectively along the  $\hat{x}$ - and the  $\hat{y}$ -directions, are generated. This is different from the conventional SHE that does not allow a spin current polarized or propagating along the charge current direction. The schematic of the new anomalous SHE is shown in Fig. 1(b).

Similarly, an external spin current  $j_{ij}^s$  passing through a ferromagnet can in principle generate a charge current  $\vec{J}$ . By the same arguments, the most general expression for  $\vec{J}$  is

$$J_k = \frac{2e}{\hbar} \theta_{ijk}^{\text{ISH}} j_{ij}^s \tag{6}$$

where  $\theta^{\mathrm{ISH}}_{ijk}$  is the inverse spin-Hall angle tensor of rank 3 that does not depend on spin current  $j^s_{ij}$ . For a ferromagnet whose magnetisation can also participant in the generalized ISHE, the most general  $\theta^{\mathrm{ISH}}_{ijk}$ , up to the linear

term in  $\vec{M}$ , is

$$\theta_{ijk}^{\text{ISH}} = \theta_0' \epsilon_{ijk} + \theta_1' M_l \epsilon_{iln} \epsilon_{jnk} + \theta_2' M_l \epsilon_{ink} \epsilon_{jln}, \tag{7}$$

 $\theta_0'$  is the usual inverse spin Hall angle that does not interact with  $\vec{M}$ ,  $\theta_\alpha'$  ( $\alpha=1,2$ ) are coefficients that characterize the anomalous ISHEs linear in  $\vec{M}$ . If the applied spin current is  $j_{33}^s$ , the conventional ISHE says no electric current can be generated. However, it is very different here, and one has, from Eqs. (6) and (7),

$$J_k = \frac{\hbar}{2e} (\theta_1' + \theta_2') \delta_{ki \neq 3} M_i j_{33}^s. \tag{8}$$

In this case, the theory predicts that a charge current of  $\frac{\hbar}{2e}(\theta_1'+\theta_2')j_{33}^s\vec{M}_\perp$  is generated, where  $\vec{M}_\perp$  is projection of magnetisation vector in the xy-plane. The charge current should be zero if the magnetisation is parallel to the spin current propagation direction. The schematic of this anomalous ISHE is shown is Fig. 2(a). If the applied spin current is  $j_{12}^s$ , then charge current from the ISHE of Eqs. (6) and (7) is

$$J_k = \frac{\hbar}{2e} j_{12}^s [\theta_0' \delta_{k3} + \theta_1' M_2 \delta_{k1} + \theta_2' M_1 \delta_{k2}]. \tag{9}$$

Interestingly, beside of charge current  $\frac{\hbar}{2e}j_{12}^s\theta_0'\hat{z}$  along the  $\hat{z}$ -direction from conventional ISHE, there are two new charge currents  $\frac{\hbar}{2e}j_{12}^s\theta_1'M_2\hat{x}$  and  $\frac{\hbar}{2e}j_{12}^s\theta_2'M_1\hat{y}$  along the  $\hat{x}$ - and  $\hat{y}$ -directions respectively. This two new anomalous ISHEs are illustrated in Figs. 2(c) and 2(d). No anomalous charge current is generated when  $\vec{M}$  is along the z-direction. Of course, the conventional ISHE can generate a charge current along the z direction, and this charge current does not depend on  $\vec{M}$ .

The anomalous SHEs and ISHEs described above work also for the interconversion of charge-spin in an antiferromagnet involving the Neel order parameter  $\vec{n}$ . One needs simply to replace the magnetization  $\vec{M}$  by  $\vec{n}$ . In fact, the theory can even be applied to fictitious order parameters such as topological insulators or 2D materials whose surface direction involve in many physical phenomenon. Indeed, there are already many evidences for the existence of charge-spin conversion beyond current paradigm of SHE. For example, the existence of anomalous SHE may have already observed in ferromagnet [22], in anti-ferromagnets [23–25] and in two-dimensional (2D) Weyl semimetals (a case of anomalous SHE involved fictitious order parameters of 2D materials) [26, 27]. Of course, it shall be interesting to test the predictions made here quantatively.

Here we would like to propose one possible experimental set-up for experimental verification of predictions described above. Figure 3 is the illustration of the set-up. A FM1/NM/FM2 multilayer system lays in the xy-plane, and a charge current  $\vec{J}$  is applied to the bottom ferromagnetic layer FM2 whose magnetization magnetization  $\vec{M}$  is collinear with  $\vec{J}$ . NM is non-magnetic-spacing layer that

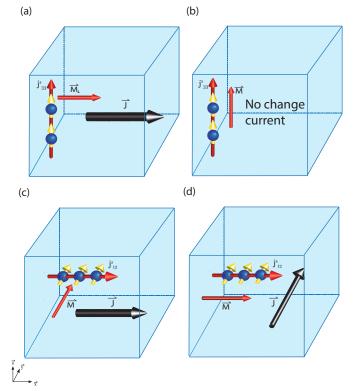


FIG. 2: Schematics of anomalous ISHE: (a) The flow of a spin current  $j_{33}^s$  (the thinner red arrowed line) generates a charge current along  $\vec{M}_{\perp}$  (the thicker red arrowed line), the projection of  $\vec{M}$  in the xy-plane. The charge current  $\vec{J}$  (the black arrowed line) is proportional to  $M_{\perp}$ . (b) No charge current can be generated when  $\vec{M}$  is along the  $\hat{z}$ . (c-d) Charge currents (the black arrowed lines) proportional to the y (c) and x (d) components of the magnetization (the thicker red arrowed lines) are generated by the flow of a spin current  $j_{12}^s$  (the red thinner lines).

can be metal or thin insulating film. The charge current direction is chosen as the x-direction. The charge current generate two spin currents  $j_{33}^s = (\theta_1 + \theta_2) \frac{\hbar}{2e} MJ$ and  $j_{32}^s = \theta_0 \frac{\hbar}{2e} J$  (the black arrowed lines) propagating perpendicularly to the layers. One of them  $j_{32}^s$  is the charge current from the conventional SHE. The other  $j_{33}^s$ is from the anomalous SHE. The two spin currents pass thought the spacing layer and enter into the top ferromagnetic metal layer FM1. If magnetization  $\vec{M}_1$  in FM1 is along the z-direction,  $j_{33}^s$  generates no charge current from anomalous ISHE illustrated in Fig. 2b, as well as no current from the conventional ISHE because the propagation and polarisation of the current are collinear. Spin current  $j_{32}^s$  can generate two charge current in FM2. One of them  $J_1 = \theta_0' \frac{2e}{\hbar} j_{32}^s$  is from the conventional ISHE, and is flow in the x-direction. The other  $\vec{J_2} = \theta_2' \frac{2e}{\hbar} M_1 j_{32}^s$  proportional to  $M_1$  is from the anomalous ISHE that is explained in Fig. 2d. For an open circuit in the y-direction on the top layer (FM1), it shall generate a voltage drop cross the y-direction as denoted by  $V_y$  in Fig. 3.  $V_y$  shall

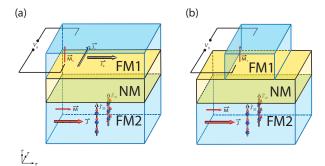


FIG. 3: (Color online) Illustration of an experimental set-up of FM1/NM/FM2 multilayer system lying in the xy-plane. FM2 is a ferromagnetic metal layer of magnetization  $\vec{M}$  (the thinner red arrowed line) for spin current generation. Applied charge current  $\vec{J}$  (the thicker red arrowed line) is collinear with  $\vec{M}$  and along the x-direction. NM is non-magneticspacing layer that can be either a metallic or a thin insulating film. FM1 is another ferromagnetic metal for spin current detection whose magnetization  $\dot{M}_1$  is along the z-direction. Spin currents  $j_{33}^s$  and  $j_{32}^s$  (the black arrowed lines decorated with smaller arrows) are generated by the anomalous  $(j_{33}^s)$  and conventional SHE. The two spin currents passing thought the spacing layer and enter into the top ferromagnetic metal layer FM1.  $\vec{J}_1$  and  $\vec{J}_2$  are two charge currents generated from the anomalous ISHE. A voltage drop  $V_y$  across the y direction in FM1 is generated. (a) A set-up for NM being a thin insulating spacing. (b) For NM being a metal, FM1 is replaced by a Hall bar along the y direction.

change sign when  $\vec{M}_1$  reverses its direction. Both the existence of a non-zero  $V_y$  and its sign change are the features of the anomalous ISHE. Ideally, one would like to eliminate closed circuit in the x-direction on the top layer in order to eliminate possible contribution of usual anomalous Hall contribution to  $V_y$ . This can be achieved either by using a thin insulating spacing or by using a Hall bar as denoted by Fig. 3b. Obviously, the charge current along the y-direction is a uniquely feature of the anomalous ISHE predicted in this theory. Because all existing effects do not result in a  $\vec{M}_2$  dependent  $V_y$ .

According to the anomalous SHE and ISHE described above, a charge current along the x-directions in a ferromagnetic or anti-ferromagnetic metal, whose order parameter is collinear with the current, can generate  $j_{yy}^s$  and  $j_{zz}^s$ . Since the y and z direction is open and no sustainable spin current exist, a spin accumulation of  $\langle s_z \rangle$  and  $\langle s_y \rangle$  will occur on the two surface respectively in the z and y directions. Thus this direction-dependent spin accumulation is another fingerprints of the present theory. Of course, this spin accumulation is hidden in the main order parameter pointing to the x-direction, and how to observe it may be an issue. It may be easier to observe the spin accumulation in an anti-ferromagnet because of zero net magnetization everywhere in the absence of a current.

Clearly, all three anomalous SHEs and ISHEs involve

the coupling between order parameters such as the magnetization of ferromagnets or the Neel order of antiferromagnets. In terms of applications, this nice property allow one to manipulate the generated spin or charge currents by controlling the order parameter. So far, the predictions are based on the general principle of tensor transformation, instead of deriving these terms from a microscopic Hamiltonian. In some sense, similar to energy spectrum analysis in group theory, our theory does not provide the actual possible strengths of the anomalous SHEs and ISHEs,  $\theta_1$ ,  $\theta'_1$ ,  $\theta_2$ , and  $\theta'_2$ . To find these strengths, one needs to compute all possible spin (charge) current conversion from a given microscopic model when an external charge (spin) current is applied. Such a microscopic theory is surely important and necessary although it is foreseeable difficult because of necessity of including electron-electron interactions.

In conclusion, I predict the existence of anomalous SHEs and ISHEs in magnetic materials when the chargespin interconversion involve the order parameters such as the magnetization in ferromagnetic materials and the Neel order in anti-ferromagnetic materials. In particular, spin current  $j_{ii}^s$  can be generated by a charge current collinear with the order parameter and propagating perpendicularly to i. Inversely, a charge current can be generated by a spin current of  $j_{ii}^s$  along the projection of order parameter perpendicular to  $\hat{i}$ . Two anomalous spin currents proportional to the magnitude of order parameter can be generated by an applied charge current when it is perpendicular to the order parameter. One of them propagates along the order parameter and is polarized along the charge current direction. The other propagates along the charge current direction and is polarized along the order parameter. Its inverse effect is the generation of two charge currents by a spin current of  $j_{ij\neq i}^s$ . One current proportional to the i'th component of the order parameter flow along the  $\hat{j}$ -direction, and the other proportional to the j'th component of the order parameter flow along the  $\hat{i}$ -direction.

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<sup>[1]</sup> Luqiao Liu, Chi-Feng Pai, Y. Li, H. W. Tseng, D. C. Ralph, R. A. Buhrman, "Spin-Torque Switching with the Giant Spin Hall Effect of Tantalum", *Science* **336** (6081), 555-558 (2012).

<sup>[2]</sup> Y. Zhang, H. Y. Yuan, X. S. Wang, and X. R. Wang, "Breaking the current density threshold in spin-orbittorque magnetic random access memory", *Phys. Rev. B* 97, 144416 (2018).

- [3] P. Yan, X. S. Wang, and X. R. Wang, "All-magnonic spin-transfer torque and domain wall propagation", Phys. Rev. Lett. 107, 177207 (2011).
- [4] X.S. Wang, P. Yan, Y. H. Shen, G.E.W. Bauer, and X.R. Wang, "Domain wall propagation through spin wave emission", *Phys. Rev. Lett.* 109, 167209 (2012).
- [5] B. Hu and X. R. Wang, "Instability of Walker Propagating Domain Wall in Magnetic Nanowires", Phys. Rev. Lett. 111, 027205 (2013).
- [6] Jiahao Han, Pengxiang Zhang, Justin T. Hou, Saima A. Siddiqui, and Luqiao Liu, "Mutual control of coherent spin waves and magnetic domain walls in a magnonic device", *Science* 366 (6469), 1121-1125 (2019).
- [7] D. K. C. MacDonald and K. Sarginson, "Galvanomagnetic effects in conductors", Repts. Progr. in Phys. 15, 249 (1952).
- [8] E. M. Conwell, "Galvanomagnetic effects in semiconductors at high electric fields, *Phys. Rev.* 123, 454 (1961).
- [9] S. D. Ganichev, E. L. Ivchenko, V. V. Bel'kov, S. A. Tarasenko, M. Sollinger, D. Weiss, W. Wegscheider and W. Prettl, "Spin-galvanic effect", *Nature* 417, 153 (2002).
- [10] The Quantum Hall Effect, edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1990).
- [11] Emerson M. Pugh and Norman Rostoker, "Hall effect in ferromagnetic materials", Rev. Mod. Phys. 25, 151 (1953).
- [12] M. I. Dyakonov and V. I. Perel, "Possibility of orientating electron spins with current". Sov. Phys. JETP Lett. 13, 467 (1971).
- [13] J. E. Hirsch, "Spin Hall effect", Phys. Rev. Lett. 83, 1834 (1999).
- [14] Y. Kato; R. C. Myers; A. C. Gossard; D. D. Awschalom, "Observation of the Spin Hall Effect in Semiconductors". Science 306(5703), 1910-1913 (2004).
- [15] J. Wunderlich; B. Kaestner; J. Sinova; T. Jungwirth, "Experimental Observation of the Spin-Hall Effect in a Two-Dimensional Spin-Orbit Coupled Semiconductor System", Phys. Rev. Lett. 94(4), 047204 (2005).
- [16] E. Saitoh, M. Ueda, H. Miyajima and G. Tatara, "Conversion of spin current into charge current at room temperature: inverse spin-Hall effect", Appl. Phys. Lett. 88, 182509 (2006).
- [17] T. Kimura, Y. Otani, T. Sato, S. Takahashi, and S. Maekawa, "Room-temperature reversible spin Hall effect", Phys. Rev. Lett. 98, 156601 (2007).

- [18] Y. Kajiwara, K. Harii, S. Takahashi, J. Ohe, K. Uchida, M. Mizuguchi, H. Umezawa, H. Kawai, K. Ando, K. Takanashi, S. Maekawa and E. Saitoh, "Transmission of electrical signals by spin-wave interconversion in a magnetic insulator", Nature 464, 262-267 (2010).
- [19] Y. Zhang, H. W. Zhang, and X. R. Wang, "Extraordinary galvanomagnetic effects in polycrystalline magnetic films", Europhys. Lett. 113, 47003 (2016).
- [20] Y. Zhang, X. S. Wang, H. Y. Yuan, S. S. Kang, H. W. Zhang, and X. R. Wang, "Dynamic magnetic susceptibility and electrical detection of ferromagnetic resonance", J. Phys.: Condens. Matter 29, 095806 (2017).
- [21] Y. Zhang, Q. Liu, B. F. Miao, H. F. Ding, and X. R. Wang, "Anatomy of electrical signals and dc-voltage line shape in spin-torque ferromagnetic resonance", *Phys. Rev. B.* 99, 064424 (2019).
- [22] H. Kurebayashi, J. Sinova, D. Fang, A. C. Irvine, T. D. Skinner, J. Wunderlich, V. Novak, R. P. Campion, B. L. Gallagher, E. K. Vehstedt, L. P. Zarbo, K. Vyborny, A. J. Ferguson, and T. Jungwirth, "An antidamping spin-orbit torque originating from the Berry curvature", Nat. Nanotech. 9, 211217 (2014).
- [23] M. Kimata, H. Chen, K. Kondou, S. Sugimoto, P. K. Muduli, M. Ikhlas, Y. Omori, T. Tomita, A. H. MacDonald, S. Nakatsuji, and Y. Otani, "Magnetic and magnetic inverse spin Hall effects in a non-collinear antiferromagnet", *Nature* 565, 627 (2019).
- [24] Y. Liu, Y. Liu, M. Chen, S. Srivastava, P. He, K. L. Teo, T. Phung, S-H. Yang, and H. Yang "Current-induced out-of-plane spin accumulation on the (001) surface of the IrMn3 antiferromagnet", Phys. Rev. Appl. 12, 064046 (2019).
- [25] X. Chen, X. Zhou, R. Cheng, C. Song, J. Zhang, Y. Wu, Y. Ba, H. Li, Y. Sun, Y. You, Y. Zhao, and F. Pan, "Electric field control of Nel spin-orbit torque in an antiferromagnet", *Nat. Mater.* 18, 931 (2019).
- [26] D. MacNeill, G. M. Stiehl, M. H. D. Guimaraes, R. A. Buhrman, J. Park and D. C. Ralph, "Control of spin-orbit torques through crystal symmetry in WTe2/ferromagnet bilayers", Nat. Phys. 13, 300 (2017).
- [27] M. H. Guimaraes, G. M. Stiehl, D. MacNeill, N. D. Reynolds, and D. C. Ralph, "Spin-orbit torques in NbSe2/permalloy bilayers", Nano Lett. 18, 13111316 (2018).