Kinematic Modeling of Wheeled Mobile Robots

Robot kinematics deals with the configuration of robots in their work-space, the relations between their geometric parameters, and the constraints imposed in their trajectories. The kinematic equations depend on the geometrical structure of the robot. For example, a fixed robot can have a Cartesian, cylindrical, spherical, or articulated structure, and a mobile robot may have one two, three, or more wheels with or without constraints in their motion. The study of kinematics is a fundamental prerequisite for the study of dynamics, the stability features, and the control of the robot.

- **Internal kinematics** explains the relation between system internal variables (e.g., wheel rotation and robot motion).
- **External kinematics** describes robot position and orientation according to some reference coordinate frame.
- **Forward kinematics** describes robot states as a function of its inputs (wheel speeds, joints motion, wheel steering, etc.).
- **Inverse kinematics** describes the robot inputs that can be calculated for a desired robot state sequence.
- **Motion constraints** appear when a system has less input variables than degrees of freedom (DOF). *Holonomic constraints* prohibit certain robot poses while a *nonholonomic constraints* prohibit certain robot velocities (e.g., the robot can drive only in the direction of the wheels' rotation).
- **Motion on 2D plane:**

  Consider a mobile node on a two-dimensional plane with linear velocity $v(t)$ and angular velocity $\omega(t)$. If the node is making an instantaneous angle $\phi(t)$ measured from x-axis, the horizontal and vertical components of the linear velocity are

  $$v_x(t) = v(t)\cos\phi(t) = \frac{dx}{dt} \quad \text{and} \quad v_y(t) = v(t)\sin\phi(t) = \frac{dy}{dt},$$

  respectively. We may show that:

  $$v(t) = v_x(t)\cos\phi(t) + v_y(t)\sin\phi(t) = \sqrt{v_x(t)^2 + v_y(t)^2} \tag{1}$$

  The tangent angle of each point on the path is defined as

  $$\phi(t) = \texttt{arctan2}\big(v_y(t), v_x(t)\big) \tag{2}$$

where `arctan2` is the four-quadrant inverse tangent function. By calculating the time derivative of $\phi(t)$ the node's angular velocity $\omega(t) = v(t)/R(t)$ is obtained:

$$\omega(t) = \frac{v_x(t)\,\dot{v}_y(t) - v_y(t)\,\dot{v}_x(t)}{v_x(t)^2 + v_y(t)^2} \tag{3}$$

where $\dot{v}_x(t)$ and $\dot{v}_y(t)$ represent acceleration along x-axis and y-axis, respectively; $R(t)$ represents the radius of curvature.

Task 1 (External Kinematics): A mobile robot has to traverse a trajectory on a two-dimensional plane $(x(t), y(t))$ given by the following expressions:

$$x(t) = 8\sin(t)^3 \tag{4}$$
$$y(t) = 8\sin(2t)^3 \tag{5}$$

where time assumes $t \in [-\pi, \pi)$.

1. Find velocities $v_x(t)$ and $v_y(t)$ as time derivatives of $x(t)$ and $y(t)$, respectively.
2. Find accelerations $a_x(t)$ and $a_y(t)$ as time derivatives of $v_x(t)$ and $v_y(t)$, respectively.
3. Obtain linear and angular velocities $v(t)$ and $\omega(t)$ using the expressions (1) and (3), respectively.
4. Simulate this system on MATLAB.
   Time may be generated with N steps
   ```
   N = 500;
   t = linspace(-pi,pi,N);
   ```
   Obtain the plots of $v(t)$ and $\omega(t)$.
   Obtain an animation of mobile node moving on the trajectory $(x(t), y(t))$.

Task 2 (Internal Kinematics): Assume that the robot is a differential-drive with wheel radius $r = 1/4$ and width of the platform $L = 2a = 1/2$. Obtain, simulate and plot the angular velocities of right and left wheel for the velocities computed in **Task 1**.

Note: Submit the report in LaTeX and also put your MATLAB code. Google to know to how to put MATLAB code in LaTeX in a professional manner. You may use symbolic toolbox of MATLAB to obtain derivatives.