Exercise Sheet 2

Spatial position and orientation (part 2)

Sven Böttger

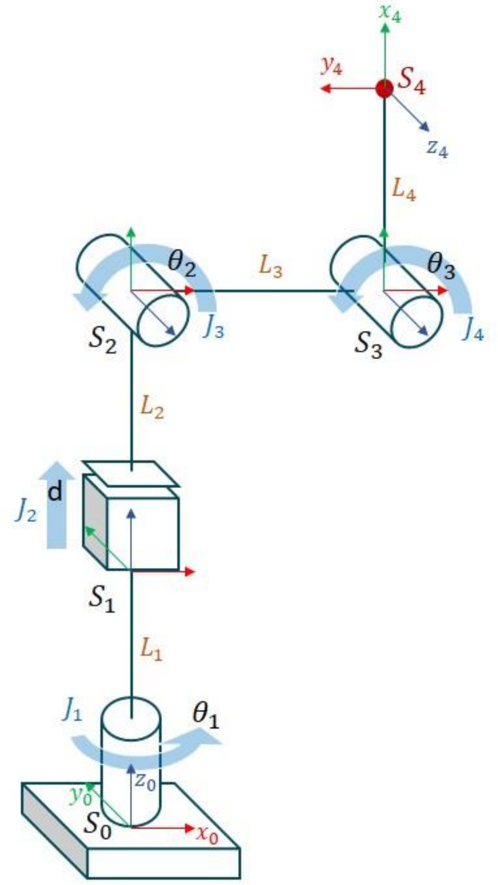
1 Spatial position, orientation and forward kinematics

$$^{0}M_1 = \begin{pmatrix} c_{\theta_1} & -s_{\theta_1} & 0 & 0 \\ s_{\theta_1} & c_{\theta_1} & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^{1}M_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & l_2 + d \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^{2}M_3 = \begin{pmatrix} c_{\theta_2} & -s_{\theta_2} & 0 & l_3 * \cos(\theta_2) \\ s_{\theta_2} & c_{\theta_2} & 0 & l_3 * \sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

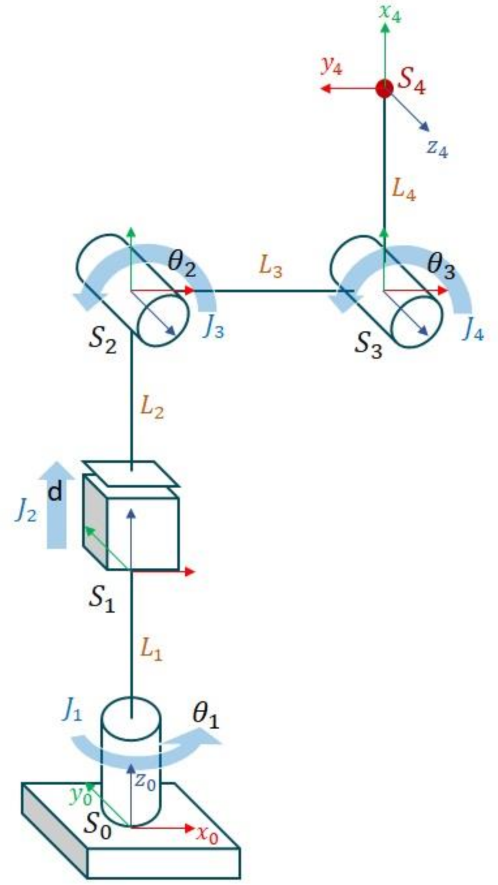
$$^{3}M_4 = \begin{pmatrix} c_{\theta_3} & -s_{\theta_3} & 0 & l_4 * \cos(\theta_3) \\ s_{\theta_3} & c_{\theta_3} & 0 & l_4 * \sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



1.1 Forward kinematics

a. Derive a transformation (one homogeneous matrix) between the base coordinate system S_0 and the robot's end-effector S_4.

$$^{0}M_4 = {}^{0}M_1 \times {}^{1}M_2 \times {}^{2}M_3 \times {}^{3}M_4$$



1.1 Forward kinematics

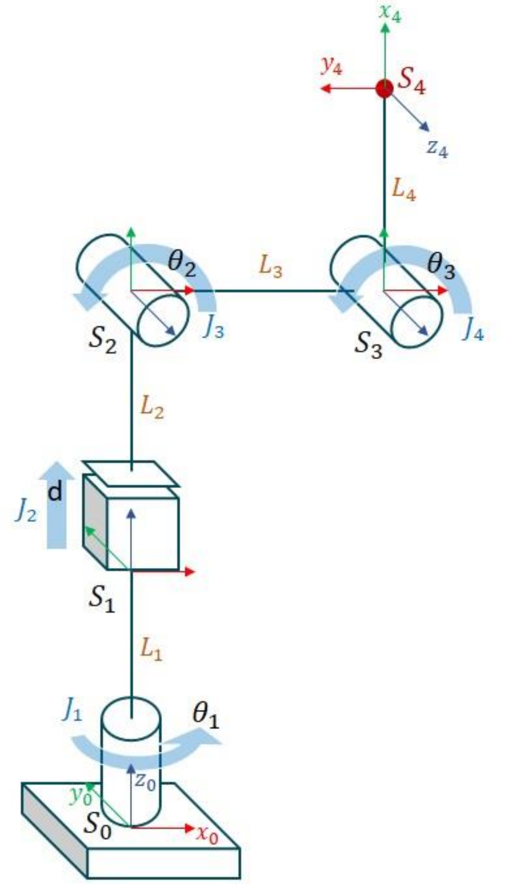
$$^{0}M_4 =$$

$$\begin{pmatrix} c_1c_2c_3 - c_1s_2s_3 & -c_1c_2s_3 - c_1c_3s_2 & s_1 & l_3c_1c_2 + l_4c_1c_2c_3 - l_4c_1s_2s_3 \\ s_1c_2c_3 - s_1s_2s_3 & -s_1c_2s_3 - s_1c_3s_2 & -c_1 & l_3c_2s_1 + l_4c_2c_3s_1 - l_4s_1s_2s_3 \\ s_3c_2 + s_2c_3 & c_3c_2 - s_2s_3 & 0 & d + l_1 + l_2 + l_3s_2 + l_4c_2s_3 + l_4c_3s_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

1.1 Forward kinematics

b. Calculate the end-effector coordinates in respect to the robot's base given:

$$
\begin{aligned}
l_1 &= l_2 = 10\,mm \\
l_3 &= l_4 = 50\,mm \\
\theta_1 &= -60° \\
\theta_2 &= 30° \\
\theta_3 &= 60 \\
d &= 20\,mm
\end{aligned}
$$

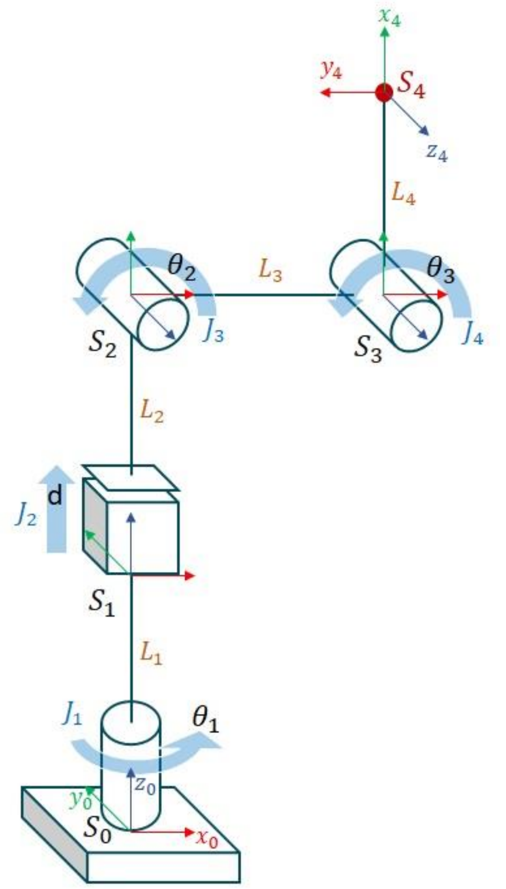


1.1 Forward kinematics

a. Calculate the end-effector coordinates in respect to the robot's base given:

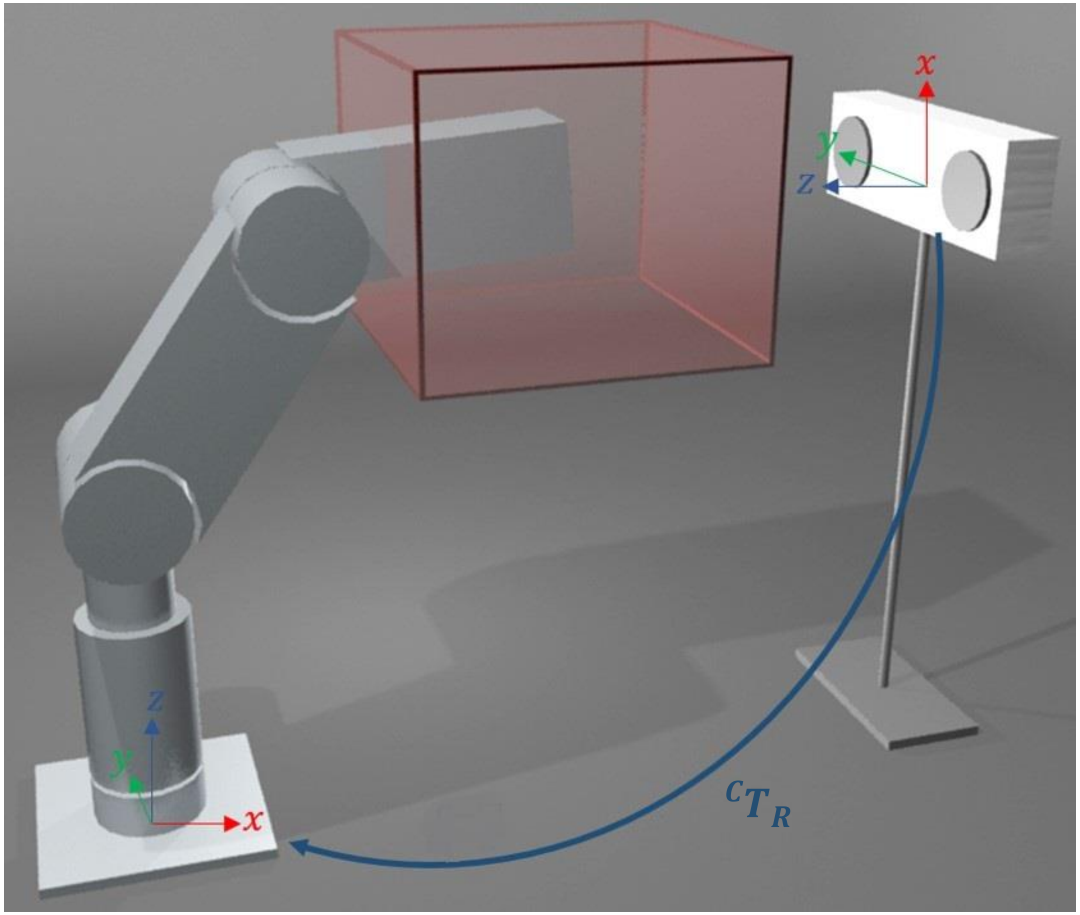
$$ {}^{0}P = {}^{0}M_4 \times {}^{4}P $$

$$ {}^{0}P = \begin{pmatrix} 0 & -0.5 & -0.866 & 21.65 \\ 0 & 0.866 & -0.5 & -37.5 \\ 1 & 0 & 0 & 115 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$



1.2 Transformations between coordinate systems

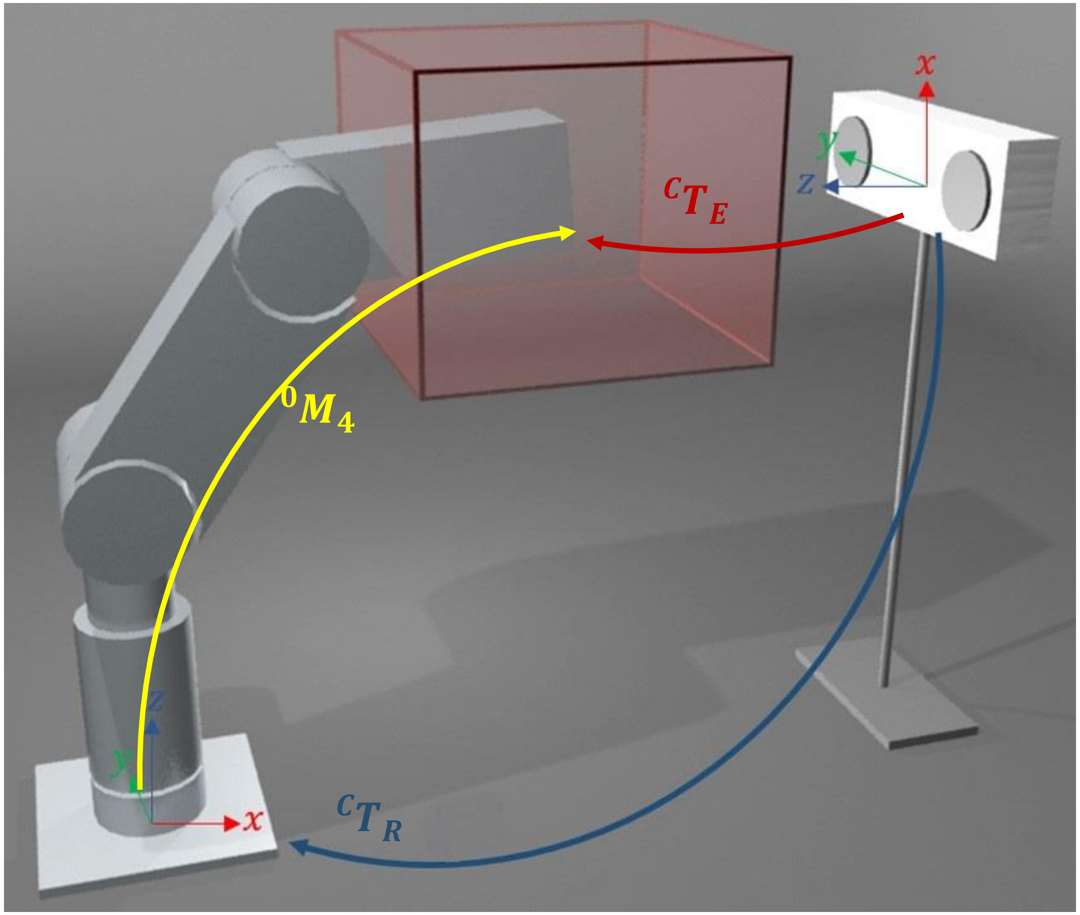
$$^{C}T_{R} = \begin{pmatrix} 0 & 0 & 1 & -100 \\ 0 & 1 & 0 & -20 \\ -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



1.2 Transformations between coordinate systems

a. Derive and sketch a transformation ${}^{C}T_{E}$ between the end-effector and the camera system.

$$ {}^{C}T_{E} = {}^{C}T_{R} \times {}^{0}M_{4} $$



1.2 Transformations between coordinate systems

$$^{C}T_{E} =$$

$$\begin{pmatrix} s_3c_2 + s_2c_3 & c_3c_2 - s_2s_3 & 0 & d + l_1 + l_2 + l_3s_2 + l_4c_2s_3 + l_4c_3s_2 - 100 \\ s_1c_2c_3 - s_1s_2s_3 & -s_1c_2s_3 - s_1c_3s_2 & -c_1 & l_3s_1c_2 + l_4s_1c_2c_3 - l_4s_1s_2s_3 - 20 \\ c_1s_2s_3 - c_1c_2c_3 & c_1c_2s_3 + c_1c_3s_2 & -s_1 & -l_4c_1c_2c_3 + l_4c_1s_2s_3 - l_3c_1\,c_2 + 120 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

1.2 Transformations between coordinate systems

b. Check whether the robot is within a safe pose or not, given:

$$
\begin{aligned}
l_1 &= l_2 = 10\,mm \\
l_3 &= l_4 = 50\,mm \\
\theta_1 &= -60° \\
\theta_2 &= 30° \\
\theta_3 &= 60 \\
d &= 20\,mm
\end{aligned}
$$

1.2 Transformations between coordinate systems

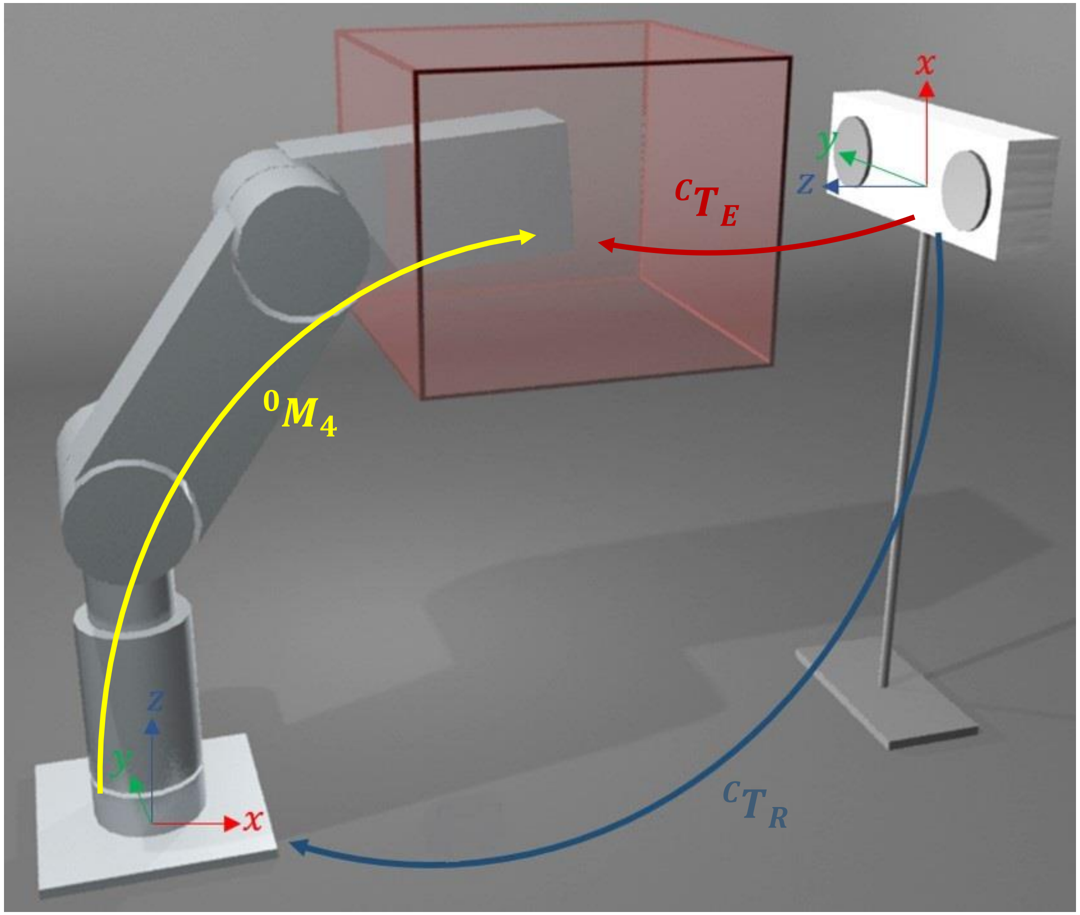
We already have

$$ {}^{0}P = \begin{pmatrix} 0 & -0.5 & -0.866 & 21.65 \\ 0 & 0.866 & -0.5 & -37.5 \\ 1 & 0 & 0 & 115 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

so

$$ {}^{C}P = {}^{C}T_R \times {}^{0}P $$

$$ {}^{C}P = \begin{pmatrix} 1 & 0 & 0 & 15 \\ 0 & 0.866 & -0.5 & -57.5 \\ 0 & 0.5 & 0.866 & 98.35 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$



1.2 Transformations between coordinate systems

c. Derive the end-effector coordinates with respect to the robot base, given its position according to camera:

$$v = \begin{pmatrix} 0 \\ 0 \\ 50\ mm \end{pmatrix}$$

1.2 Transformations between coordinate systems

$$^{R}Q = {}^{R}T_{C} \times {}^{C}Q$$

Whereas

$$^{R}T_{C} = {}^{C}T_{R}^{-1}$$

Therefore

$$^{R}Q = {}^{C}T_{R}^{-1} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^{R}Q = \begin{pmatrix} 0 & 0 & -1 & 70 \\ 0 & 1 & 0 & 20 \\ 1 & 0 & 0 & 100 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$