Exercise Sheet 1

Applications, spatial position and orientation (part 1)

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1 Medical robotics applications

a. Name three **types of medical robots** and an **example application** for each. Briefly explain the **advantages gained from using robotics** in your examples.

1. **Navigation**
   Ex. Surgical drill
   Precise positioning, motion compensation…

2. **Motion replication**
   Ex. Minimally-invasive surgery
   Motion downscaling, reduce tremor…

3. **Imaging**
   Ex. Robotic ultrasound
   Automation, speed…

4. **Rehabilitation and prosthesis**
   Ex. Exoskeletons
   Replace damaged structures, autonomous rehabilitation…

2 Spatial position and orientation

a. Rotation of α around the y-axis of the base coordinate system ($R(y, \alpha)$), then β around the z-axis of the base coordinate system ($R(z, \beta)$), and finally γ around the x-axis of the base coordinate system ($R(x, \gamma)$):
Calculate the 3×3 matrix R_{YPR}.
Is this an extrinsic or intrinsic rotation?

2 Spatial position and orientation

First, find the individual rotation matrices:

$$R(y,\alpha) = \begin{pmatrix} c_\alpha & 0 & s_\alpha \\ 0 & 1 & 0 \\ -s_\alpha & 0 & c_\alpha \end{pmatrix}$$

$$R(z,\beta) = \begin{pmatrix} c_\beta & -s_\beta & 0 \\ s_\beta & c_\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R(x,\gamma) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\gamma & -s_\gamma \\ 0 & s_\gamma & c_\gamma \end{pmatrix}$$

2 Spatial position and orientation

Then, multiply the matrices to yield the final rotation matrix:

$$R_{YPR} = R(x,\gamma) \cdot R(z,\beta) \cdot R(y,\alpha)$$

Note the multiplication order → Extrinsic rotation

Last to first!

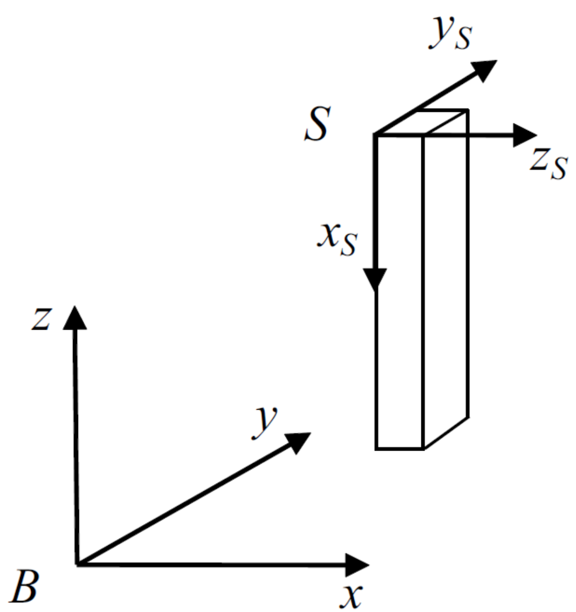
2 Spatial position and orientation

We get

$$R_{YPR} = \begin{pmatrix} c_\alpha . c_\beta & -s_\beta & c_\beta . s_\alpha \\ s_\alpha . s_\gamma + c_\alpha c_\gamma s_\beta & c_\beta . c_\gamma & c_\gamma . s_\alpha . s_\beta - c_\alpha s_\gamma \\ c_\alpha . s_\beta . s_\gamma - c_\gamma . s_\alpha & c_\beta . s_\gamma & c_\alpha . c_\gamma + s_\alpha . s_\beta . s_\gamma \end{pmatrix}$$

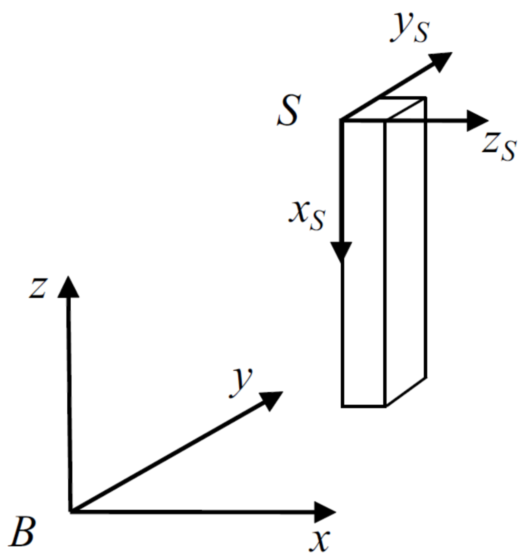
2 Spatial position and orientation

b. If the rotations described in part (a) lead to an object orientation as seen in Figure 1, find the YPR angles α, β and γ and calculate the rotation matrix R_{YPR}.



2 Spatial position and orientation

First, find YPR angles from the figure:



$$\alpha = 90°, \beta = 0\,, \gamma = 0$$

2 Spatial position and orientation

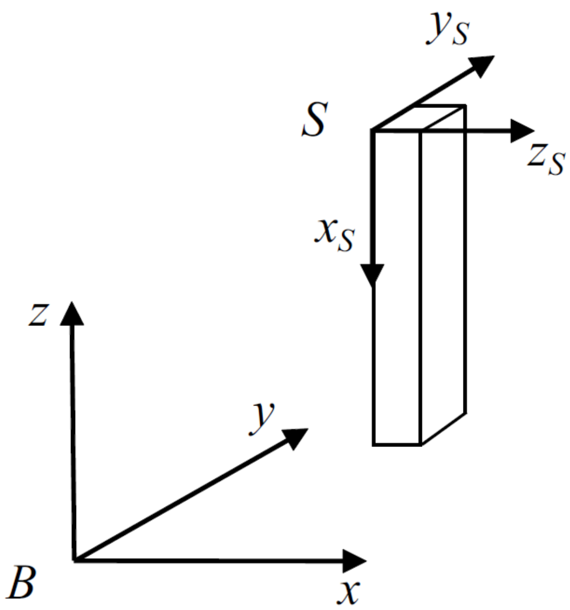
Then implement the values in the rotation matrix R_{YPR}:

$$R(y,\alpha) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \quad R(z,\beta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad R(x,\gamma) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_{YPR}(\alpha,\beta,\gamma) = R(y,\alpha) \cdot R(z,\beta) \cdot R(x,\gamma) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

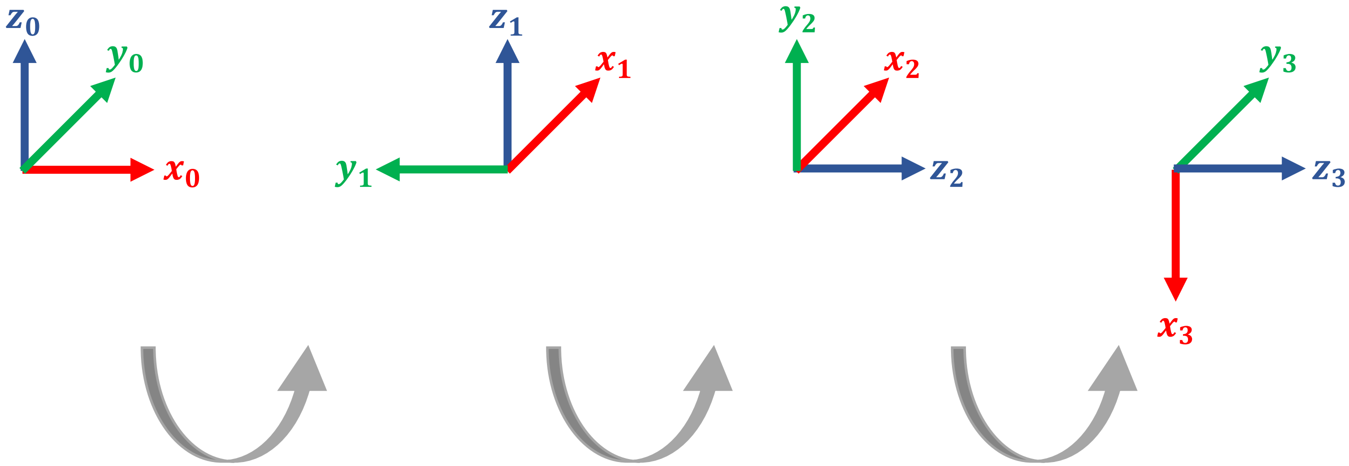
2 Spatial position and orientation

c. Derive the Euler angles θ, δ and φ of the object coordinate system S. Do you get the same angles as YPR?



Assuming that Figure 1 represents the following orientation:
rotation of θ around the z-axis of the **base** coordinate system ($R(z,\theta)$), then δ around the x-axis of the **new** coordinate system ($R(x',\delta)$), and finally φ around the z-axis of the **new** coordinate system ($R(z'',\varphi)$)

2 Spatial position and orientation



$\theta = 90°$ $\delta = 90°$ $\varphi = -90°$

(alternatively -90°, -90°, 90)