

Tutorial - 6

Ques. What do you mean by minimum Spanning tree? What is the application of MST.

Ans. A minimum Spanning tree or minimum weight spanning tree is a subset of the edges of a connected, edge-weighted undirected graph that connects all the vertices together, without any cycle and with the minimum possible total edge weight.

Application:

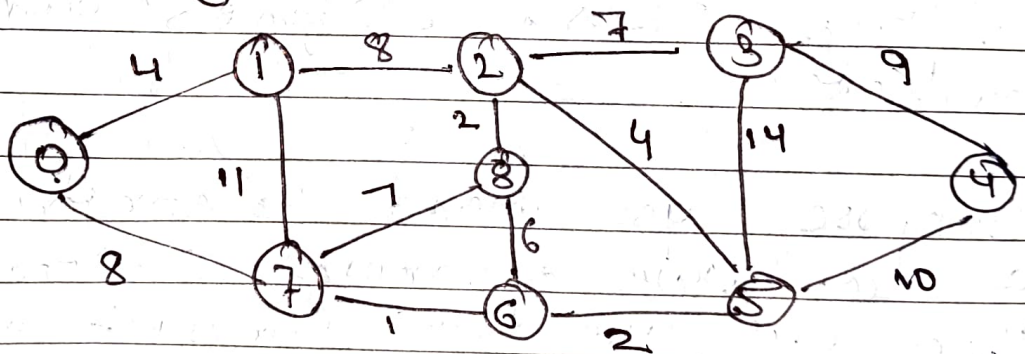
- Designing Local Area Network.
- Laying pipelines connecting offshore drilling sites, refineries and consumer markets.
- Suppose you want to construct highway or railways spanning several cities then we use the concept of MST.
- To reduce cost, you use the concept of MST to connect the houses.

Ques2 Please analyse the time and space complexity of Prim's, Kruskal, Dijkstra and Bellman ford algorithm

Soln

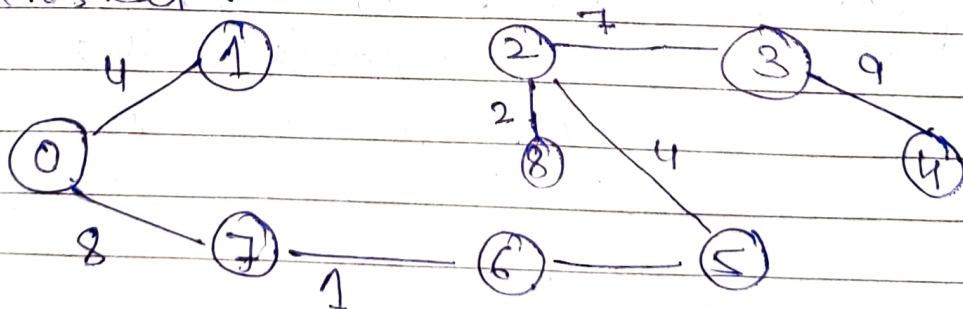
Algorithm	Time Complexity	Space Complexity
Prim's	$O(V^2)$	$O(V+E)$
Kruskal	$O(E \log V)$	$O(\log(E))$
Dijkstra's	$O(V+E)$	$O(V+E)$
Bellman ford	$O(VE)$	$O(V)$

Ques3 Apply Prim's and Kruskal algorithm on the graph to compute MST and its weight



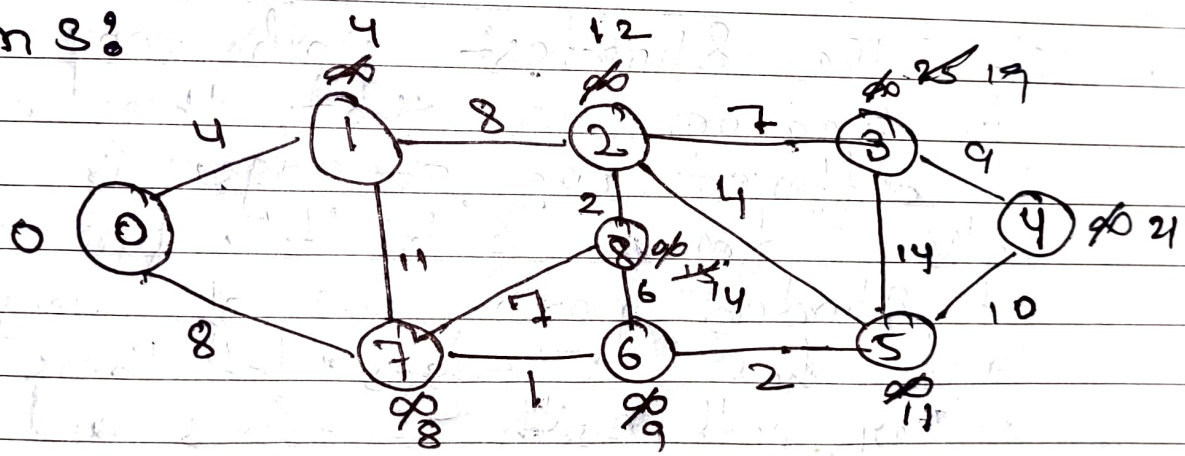
Soln

Kruskal:

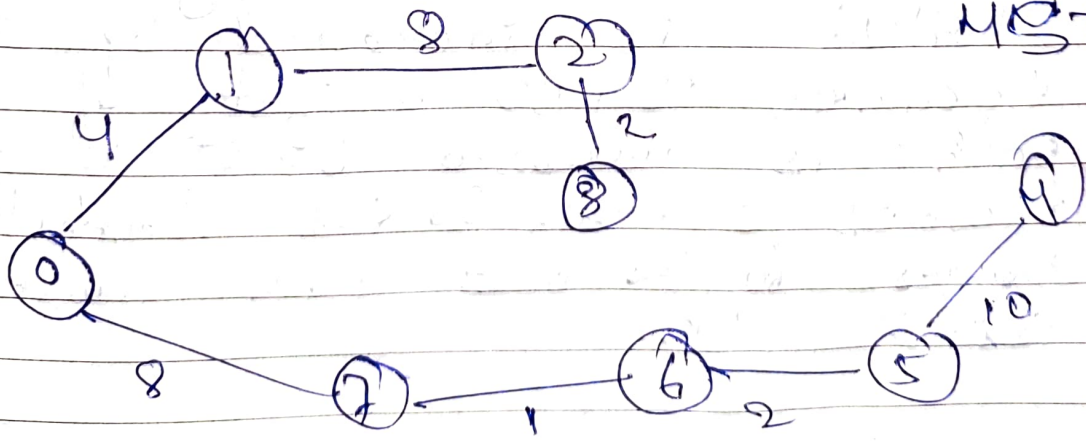


Path	Weight
7 → 6	1
6 → 5	2
2 → 8	2
0 → 1	4
2 → 5	4
8 → 6	6
2 → 3	7
7 → 8	7
1 → 2	8
3 → 4	9
5 → 4	10
1 → 7	11
3 → 5	14

Prims:



MST



Ques 4 Given a weighted graph. You are also given the shortest path from a source vertex 's' to a given destination vertex 't'. Does the shortest path remains same in the modified graph in the following case.

- If weight of every edge is increased by 10 units.
- If weight of every edge is multiplied by 10 units.

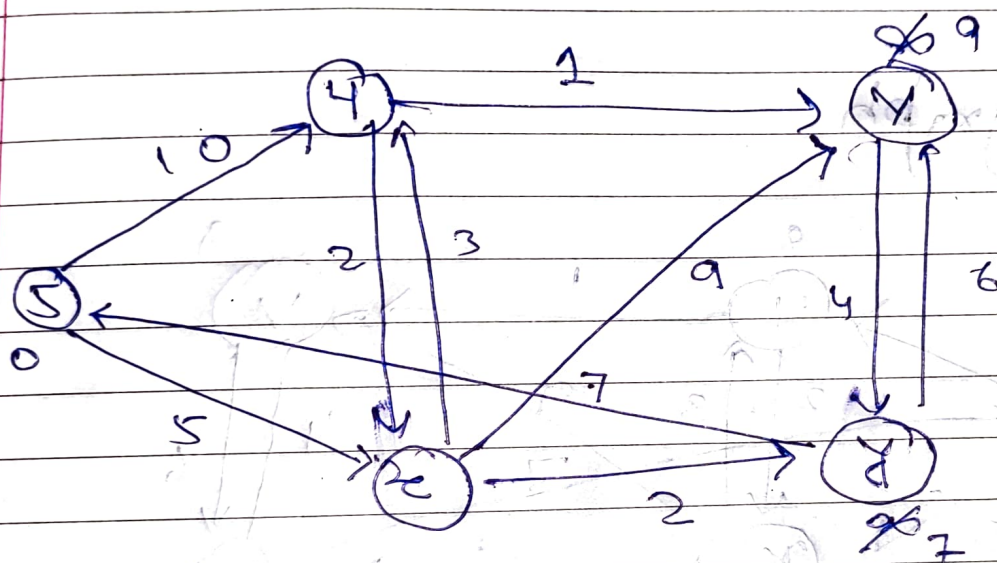
Solⁿ
i)

If The shortest path may change. The reason is that there may be different no. of edges in different paths from 's' to 't'. for example let shortest path be of weight 15 and has 5 edges. let there be another path with 2 edges and total weight is 25. The weight of the shortest is increased by 5th 10 becomes $15 + 50$ weight of other path is increased by 2th 10 it becomes $25 + 20$, so the shortest path changes to the other path whose weight is 45.

- ii) If we multiply all edge weight by 10, the shortest path doesn't change. The reason is simple. Weight of all paths from s to t get multiplied by some amount. The no. of edges on a path doesn't matter.

Ques 5.

Solⁿ Dijkstra's Algorithm



node	Shortest distance from Source node
U	8
X	5
Z	7
Y	9

Date ___/___/___

Saath

Bellman ford algorithm

1 st →	$\begin{matrix} 0 \\ \textcircled{S} \end{matrix}$	$\begin{matrix} 10 \\ \infty \\ \textcircled{4} \end{matrix}$	$\begin{matrix} \infty \\ \textcircled{V} \end{matrix}$	$\begin{matrix} \infty \\ \textcircled{x} \end{matrix}$	$\begin{matrix} \infty \\ \textcircled{y} \end{matrix}$
2 nd →	$\begin{matrix} 0 \\ \textcircled{S} \end{matrix}$	$\begin{matrix} 10 \\ \textcircled{4} \end{matrix}$	$\begin{matrix} \infty \\ \textcircled{V} \end{matrix}$	$\begin{matrix} 5 \\ \textcircled{x} \end{matrix}$	$\begin{matrix} 8 \\ \textcircled{y} \end{matrix}$
3 rd →	$\begin{matrix} 0 \\ \textcircled{S} \end{matrix}$	$\begin{matrix} 10 \\ 8 \\ \textcircled{4} \end{matrix}$	$\begin{matrix} 11 \\ 9 \\ \textcircled{V} \end{matrix}$	$\begin{matrix} 5 \\ \textcircled{x} \end{matrix}$	$\begin{matrix} 7 \\ \textcircled{y} \end{matrix}$
4 th →	$\begin{matrix} 0 \\ \textcircled{S} \end{matrix}$	$\begin{matrix} 8 \\ \textcircled{4} \end{matrix}$	$\begin{matrix} 9 \\ \textcircled{V} \end{matrix}$	$\begin{matrix} 5 \\ \textcircled{x} \end{matrix}$	$\begin{matrix} 7 \\ \textcircled{y} \end{matrix}$

Final graph

