Student Research Group Report

Monte-Carlo Methods for the Heston Model

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Introduction

One of the first diffusion-based models in mathematical finance was introduced in 1973 in the paper by Fisher Black and Myron Sholes [BS73]. However, the model was not very realistic, as it did not take into account the variability of the volatility process, which was proven not to be a constant in the real stock market. The implied volatility of the stock options was not the same for different maturities and strikes.

Later, the class of so-called local volatility models was developed (Dupire et. al.). They fixed the problem of the spot implied volatility: now we could get a perfect fin into the spot prices of the options. However, the local volatility models give us the wrong dynamics, which is crucial to valuate the price of different derivatives.

In 1993, Steven Heston introduced a new diffusion-based model [Hes93], but he made a vital assumption: the variance process is not a constant, not a determenistic function of time and stock price, but follows a diffusion process, called the Cox-Ingersol-Ross (CIR) process. The stochastic volatility models cannot be perfectly calibrated to fit the volatility smile, but they give us a realistic dynamics of the implied volatility surface.

In this paper we revise the Heston model and its most popular simulation methods. We remind the reader of some basic facts abou the Monte-Carlo methods in finance. We also study the empirical speed of convergence of the simulation methods and the accuracy of the option greeks. Futhermore, we implement a multi-threaded versions of the desired simulation teqniques and optimize them for the best possible performance in **Python**.

We provide the reader with the code for the simulation methods and the greeks computation for the results to be reproductible.

Part I

Monte-Carlo Methods for the Heston Model: A Theoretical Review

Chapter 1

A review of the original Heston model

1.1 Basic facts

We shall use the following resources in this chapter: [Hes93] and [Gat12]. Assume that the spot asset's price S at time t follows the diffusion (1.1) – (1.2):

$$dS(t) = \mu S(t)dt + \sqrt{v(t)}S(t)dZ_1(t), \tag{1.1}$$
$$dv(t) = \left(\delta^2 - 2\beta v(t)\right) dt + 2\delta\sqrt{v(t)}dZ_2(t), \tag{1.2}$$

where Z_1, Z_2 are the correlated Wiener processes with $dZ_1 dZ_2 = \rho dt$.

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[Hes93] Steven L. Heston. “A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options”. In: *Review of Financial Studies* 6.2 (1993), pp. 327–343.

[Gat12] Jim Gatheral. *The Volatility Surface*. John Wiley & Sons, Ltd, 2012. Chap. 1-3, pp. 1–42. ISBN: 9781119202073. DOI: https://doi.org/10.1002/9781119202073.