Baum-Welch Algorithm for Hidden Markov Models Using Algebraic Decision Diagrams

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Abstract—This is a placeholder abstract. The whole template is used in semester projects at Aalborg University (AAU).

1 Introduction

In this section we present some introductory ways to use the tools within LaTeX in general, and this template in particular. For example, this is a citation [1], while this is a multi-citation[1, 2].

The column width of the IEEE template is 3.5 inches, so if you generate your plots with this width or less, the output will be the best. For example, Listing 1 contains the code to generate the image in Figure 1 using Python with matplotlib, and exported as pgf (TeX).

```python
import matplotlib.pyplot as plt

plt.rcParams.update({
    "pgf.texsystem": "pdflatex",
    "font.family": "serif",  # use serif/main font
    ↪ for text elements
    "pgf.preamble": "\n".join([
         r"\usepackage[utf8x]{inputenc}",
         r"\usepackage[T1]{fontenc}",
    ]),
})

fig, ax = plt.subplots(figsize=(3.5, 3.5))

ax.plot(range(5))
ax.text(0.5, 3., "serif")
ax.text(0.5, 2., "monospace")
ax.text(2.5, 2., "sans-serif")
ax.set_xlabel(r"µ is not $\mu$")

fig.tight_layout(pad=.5)
fig.savefig("graph.pgf")
```

Listing 1. Code to generate the graph.pgf

1.1 Tables and Figures

1.2 Algorithms, Theorems, and Proofs

There are a few different things outside the normal figure and table floats that are very relevant when writing a scientific paper or article. For example, you may wish to typeset theorems as in Theorem 1.

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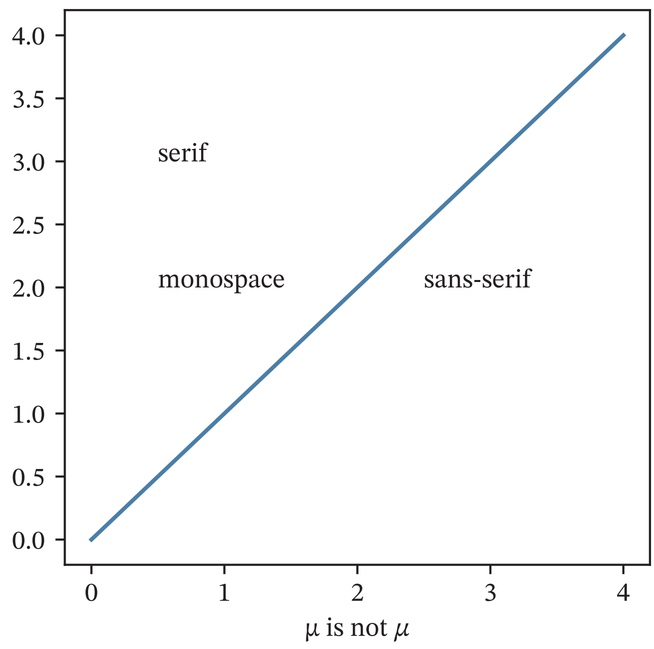

Fig. 1. An example graph drawn using Python's matplotlib library.

Theorem 1 (Pythagorean theorem)**.** *This is a theorem about right triangles and can be summarized in the next equation*

$$x^2 + y^2 = z^2$$

Or ref like Theorem 1 Similarly, for proofs:

Proof. To prove it by contradiction try and assume that the statement is false, proceed from there and at some point you will arrive to a contradiction. □

Note that proofs are not a numbered environment, and as such can't be referenced by default.

2 Preliminaries

This section provides an overview of the theoretical background necessary to understand the rest of the report. We begin by defining the key concepts of a Hidden Markov Model (HMM) and a Markov Decision Process (MDP), which are the two main models used in this report.

TABLE 1
Example of a pretty, twocolumn table.

	Klasser				
Hændelser	Reservation	Gæst	Borgerforening	Kalender	Betaling
Anmodet	✓	✓	✓		
Godkendt	✓		✓		
Afvist	✓		✓		
Redigeret	✓	✓	✓		
Annulleret	✓	✓	✓		✓
Betalt					✓
Refunderet					✓
Kvitteret		✓	✓		
Registreret	✓			✓	
Påmindet		✓	✓		

```
INSERTION-SORT(A, n)
for i ← 2 to n
    key ← A[i]
    // Insert A[i] into the sorted subarray A[1 : i − 1].
    j ← i − 1
    while j > 0 and A[j] > key
        A[j + 1] ← A[j]
        j ← j − 1
    A[j + 1] ← key
```

Algorithm 1. Test

2.1 Hidden Markov Model

HMMs were introduced by Baum and Petrie in 1966 [**NOTFOUND**] and have since been widely used in various fields, such as speech recognition [**NOTFOUND**], bioinformatics [**NOTFOUND**], and finance [**NOTFOUND**].

Definition 1 (Hidden Markov Model)**.** *A Hidden Markov Model (HMM) is a tuple $\mathcal{M} = (S, \mathcal{L}, \ell, \tau, \pi)$, where:*

- *S is a finite set of states.*
- *$\mathcal{L}$ is a finite set of labels.*
- *$\ell : S \to D(\mathcal{L})$ is the emission function.*
- *$\tau : S \to D(S)$ is the transition function.*
- *$\pi \in D(S)$ is the initial distribution.*

$D(X)$ denotes the set of probability distributions over a finite set X. The emission function ℓ describes the probability of emitting a label given a state. The transition function τ describes the probability of transitioning from one state to another. The initial distribution π describes the probability of starting in a given state. An HMM is a statistical model that describes a system that evolves over time. The system is assumed to hold the Markov property, meaning that the future state of the system only depends on the current state and not on the past states. The system is also assumed to be unobservable, meaning that the states are hidden and cannot be directly observed. Instead, the system emits observations, which are used to infer the hidden states.

An example of an HMM is a weather model where the hidden state represents the actual weather (sunny, rainy, or cloudy), but we only observe indirect signals, such as whether someone is carrying an umbrella.

2.2 Continuous Time Hidden Markov Model

In the above definition, we have defined a discrete-time HMM, meaning that the system evolves in discrete time steps. In this report, we are interested in continuous-time systems, where the system evolves in continuous time. To model continuous-time systems, we use a Continuous Time Hidden Markov Model (CTHMM), which is an extension of the HMM to continuous time.

Definition 2 (Continuous Time Hidden Markov Model)**.** *A Continuous Time Hidden Markov Model (CTHMM) is a tuple $\mathcal{M} = (S, \mathcal{L}, \ell, \tau, \pi, \lambda)$, where S, $\mathcal{L}$, ℓ, τ, and π are as defined in the HMM definition.*

- *$\lambda : S \times S \to \mathbb{R}_{\geq 0}$ is the rate function.*

The rate function λ determines the transition rate between states, meaning that the time spent in a state before transitioning follows an exponential distribution.

2.3 Markov Decision Process

MDPs were introduced by Bellman in 1957 [**NOTFOUND**] and have since been widely used in various fields, such as robotics [**NOTFOUND**], finance [**NOTFOUND**], and healthcare [**NOTFOUND**].

Definition 3 (Markov Decision Process)**.** *A Markov Decision Process (MDP) is a tuple $\mathcal{M} = (S, A, \mathcal{L}, \ell, \tau, R, \gamma)$, where:*

- *S is a finite set of states.*
- *A is a finite set of actions.*
- *$\mathcal{L}$ is a finite set of labels.*
- *$\tau : S \times A \to D(S)$ is the transition function.*
- *$R : S \times A \to \mathbb{R}$ is the reward function.*
- *$\pi \in D(S)$ is the initial distribution.*

A MDP works by an agent interacting with an environment. Unlike an HMM, the agent takes actions in the environment and receives rewards. An agent is the decision-maker that interacts with the environment by taking actions. The environment is modeled as a MDP, which consists of a set of states S, a set of actions A, a set of labels $\mathcal{L}$. Each state is directly observable, meaning the agent always knows which state it is in. When the agent takes an action in a state, it transitions to a new state according to the transition function τ.

The reward function R describes the reward received when taking an action in a state. The goal of an MDP is to find a policy $\theta : S \to D(A)$ that maximizes the expected cumulative reward. The policy θ describes the probability of taking an action given a state.

Definition 4 (Policy)**.** *A policy θ is a function that maps states to actions, i.e., $\theta : S \to D(A)$.*

An example of an MDP is a robot navigating a grid, where it can choose actions (move up, down, left, or right) and receives rewards based on reaching certain goal locations

Acronyms

AAU Aalborg University. 1

CTHMM Continuous Time Hidden Markov Model. 2

HMM Hidden Markov Model. 1, 2

MDP Markov Decision Process. 1, 2

References

[1] M. Goossens, F. Mittelbach, and A. Samarin, *The LaTeX Companion*. Reading, Massachusetts: Addison-Wesley, 1993.

[2] G. D. Greenwade, "The Comprehensive Tex Archive Network (CTAN)," *TUGBoat*, vol. 14, no. 3, pp. 342–351, 1993.

Appendix A
Cheatsheet

If something is represented with a greek letter, it is something we calculate.

Symbol	**Meaning**
$\mathbb{R}$	Real numbers
$\mathbb{Q}$	Rational numbers
$\mathbb{N}$	Natural numbers
$s \in S$	States
$l \in L$	Labels
$a \in A$	Actions
$\mathcal{M}$	Markov Model
$\mathcal{H}$	Hypothesis
$o \in O \in \mathcal{O}$	Observations
π	Initial distribution
τ	Transition function
ι or ω	Emission function
α	Forward probabilities
β	Backward probabilities
γ	State probabilities
ξ	Transition probabilities
$\lambda = (\pi, \tau, \omega)$	Model Parameters
ϕ or ψ	Scheduler
μ	Mean
σ	Standard deviation
$\theta = (\mu, \sigma^2)$	Parameters of a distribution
$P(\mathcal{O}; \lambda)$	Probability of $\mathcal{O}$ given λ
$\ell(\lambda; \mathcal{O})$	Log likelihood of λ under $\mathcal{O}$