KaHIP v0.53 – Karlsruhe High Quality Partitioning User Guide

Peter Sanders and Christian Schulz
Karlsruhe Institute of Technology (KIT), Karlsruhe, Germany
Email: {sanders, christian.schulz}@kit.edu

Abstract

This paper severs as a user guide to the graph partitioning framework KaHIP (Karlsruhe High Quality Partitioning). We give a rough overview of the techniques used within the framework and describe the user interface as well as the file formats used. Moreover, we provide a short description of the current library functions provided within the framework.

Contents

1 Introduction

Given a graph $G = (V, E)$ and a number $k > 1$, the graph partitioning asks for a division of the graphs vertex set into k disjoint blocks of roughly equal size such that some objective function is minimized. The most common formulation minimizes the number of edges that run between the blocks. An example is given in Figure 1. Nowadays, the graph partitioning problem has many applications in different areas such as parallel scientific computing or graph computations [31, 7, 36, 12, 4, 5, 26], route planning [19, 22, 33, 20, 18, 21, 9, 10], VLSI Design [1, 2, 16, 8] or for solving sparse linear equation systems [13][1]. There has been a vast amount of research on graph partitioning. We refer the reader to [6] for more material on graph partitioning.

KaHIP - Karlsruhe High Quality Partitioning - is a family of graph partitioning programs based on the publications [27, 28, 30, 15, 25, 29, 32]. It includes KaFFPa (Karlsruhe Fast Flow Partitioner) in its variants Strong, Eco and Fast, KaFFPaE (KaFFPaEvolutionary) which is a parallel evolutionary algorithm that uses KaFFPa to provide combine and mutation operations, as well as KaBaPE which extends the evolutionary algorithm. Moreover, we include algorithms to output a vertex separator from a given partition.

The purpose of the manual is to give a very rough overview over the techniques used in the partitioning programs, as well as to serve as a guide and manual on how to use our algorithms. We start with a short overview of the algorithms implemented within our graph partitioning framework. This is followed by a description of the graph format that is used. It is basically the same graph format that is used by Metis [17] and Chaco [14], and that has been used during the 10th DIMACS Implementation Challenge on Graph Clustering and Graph Partitioning [3]. We then give an overview over the user interface of KaFFPa, KaFFPaE and KaBaPE and explain which program to use to derive node separators. We finish this manual with the description of the library functions provided by current release.

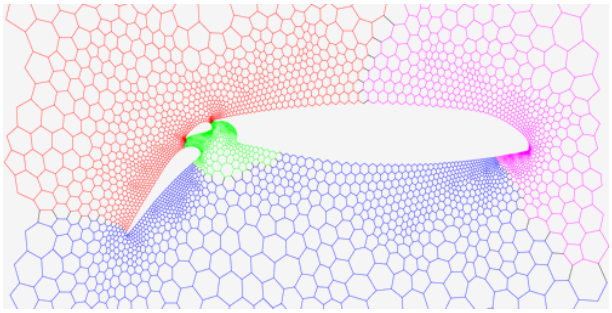


Figure 1: An example mesh that is partitioned into four blocks indicated by the colors.

[1] Let us know if we miss your application here!

2 Graph Partitioning Algorithms within KaHIP

We now give a rough overview over the algorithms implemented in our graph partitioning framework. For details on the algorithms, we refer the interested reader to the corresponding papers. Figure 2 gives an overview over the techniques used with the framework.

2.1 KaFFPa

A successful heuristic for partitioning large graphs is the *multilevel graph partitioning* approach, where the graph is recursively *contracted* to create smaller graphs which should reflect the same basic structure as the input graph. After applying an *initial partitioning* algorithm to the smallest graph, the contraction is undone and, at each level, a *local search* method is used to improve the partitioning induced by the coarser level. KaFFPa is a multilevel graph partitioning framework which contributes a number of improvements to the multilevel scheme which lead to enhanced partitioning quality. This includes flow-based methods, improved local search and repeated runs similar to the approaches used in multigrid solvers.

Local improvement algorithms are usually variants of the FM algorithm [11]. The variant KaFFPa uses is organized in rounds. In each round, a priority queue P is used that is initialized with all vertices that are incident to more than one block, in a random order. The priority is based on the gain $g(i) = \max_P g_P(i)$ where $g_P(i)$ is the decrease in edge cut when moving i to block P. Local search then repeatedly looks for the highest gain node v and moves it to the corresponding block that maximizes the gain. However, in each round a node is moved at most once. After a node is moved, its unmoved neighbors become eligible, i.e. its unmoved neighbors are inserted into the priority queue. When a stopping criterion is reached, all movements after the best-found cut that occurred within the balance constraint are undone. This process is repeated several times until no improvement is found.

Max-Flow Min-Cut Local Improvement. KaFFPa additionally uses more advanced local search algorithms. The first method is based on max-flow min-cut computations between pairs of blocks, in other words, a method to improve a given bipartition. Roughly speaking, this improvement method is applied between all pairs of blocks that share a non-empty boundary. The algorithm basically constructs a flow problem by growing an area around the given boundary vertices of a pair of blocks such that each s-t cut in this area yields a feasible bipartition of the original graph/pair of blocks *within* the balance constraint. One can then apply a max-flow min-cut algorithm to obtain a min-cut in this area and therefore a non-decreased cut between the original pair of blocks. This is improved in multiple ways, for example, by iteratively applying the method, searching in larger areas for feasible cuts, and applying most balanced minimum cut heuristics. For more details we refer the reader to [27].

Multi-try FM. The second method for improving a given partition is called multi-try FM. This local improvement method moves nodes between blocks in order to decrease the cut. Previous k-way methods were initialized with *all* boundary nodes, i.e., all boundary nodes were eligible for movement at the beginning. Roughly speaking, the multi-try FM algorithm is a k-way improvement method that is initialized with a *single* boundary node, thus achieving a more localized search. This is repeated several rounds. The algorithm has a higher chance to escape local optima. For more details about the multi-try FM algorithm we refer the reader to [27, 32].

Iterated Multilevel Algorithms. KaFFPa extends the concept of *iterated multilevel algorithms* which was introduced for graph partitioning by Walshaw et al. [34]. The main idea is to iterate the multilevel-scheme using different random seeds for coarsening and uncoarsening. Once the graph is partitioned, edges that are between two blocks are not contracted so that the given partition can be used as initial partition on the coarsest level. This ensures non-decreased partition quality since the refinement algorithms of KaFFPa guarantee no worsening. KaFFPa uses F-cycles as a potentially stronger iterated multilevel algorithm. The detailed description of F-cycles for the multilevel graph partitioning framework can be found in [27].

2.2 KaFFPaE

KaFFPaE (KaFFPaEvolutionary) is a distributed evolutionary algorithm to tackle the graph partitioning problem. KaFFPaE uses KaFFPa to provide new effective combine and mutation operators. Roughly speaking, to combine two partitions of a population of the algorithm the coarsening phase of KaFFPa is modified such that no cut edge of either of the input partitions is contracted. This ensures that both input partitions can be used as initial partition on the coarsest level and moreover that exchanging good parts of solutions can be exchanged effectively. Intuitively, the combine operator assembles good parts of solutions into a single partition. The combine operation framework is very flexible so that a partition can be combined with an arbitrary domain specific graph clustering. Moreover, the algorithm is parallelized such that each process has its own population of partitions and independently performs combine and mutation operations. This is combined with a scalable communication protocol similar to randomized rumor spreading to distribute high quality partitions among the processes. Overall, the system is able to improve the best known partitioning results for many inputs and also in a short amount of time for graphs of moderate size.

2.3 KaBaPE

KaFFPa and KaFFPaE compute partitions of very high quality when some imbalance $\epsilon > 0$ is allowed. However, they are not very good for small values of ϵ, in particular for the perfectly balanced case $\epsilon = 0$. Hence, we developed new techniques for the graph partitioning problem with strict balance constraints, that work well for small values of ϵ including the perfectly balanced case. The techniques relax the balance constraint for node movements, but globally maintain balance by combining multiple local searches. This is done by reducing the combination problem to finding negative cycles in a directed graph, exploiting the existence of efficient algorithms for this problem. From a meta-heuristic point of view the proposed algorithms increase the neighborhood of a

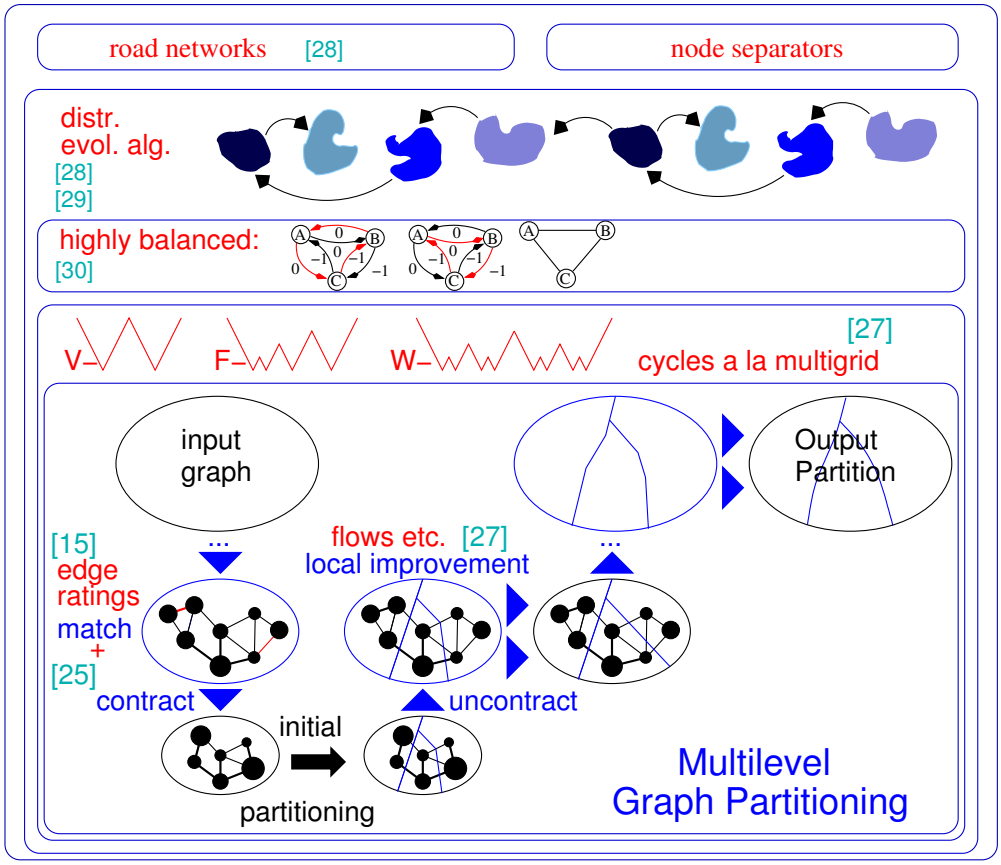


Figure 2: Overview over the techniques used in the KaHIP graph partitioning framework.

strictly balanced solution in which local search is able to find better solutions. Moreover, we provide efficient ways to explore this neighborhood. Experiments indicate that previous algorithms have not been able to find such rather complex movements. KaBaPE also provided balancing variants of these techniques that are able to make infeasible partitions feasible. In contrast to previous algorithms such as Scotch [23], Jostle [35] and Metis [17], KaBaPE is able to *guarantee* that the output partition is feasible.

2.4 Node Separators

The node separator problem asks to partition the node set of a graph into three sets A, B and S such that the removal of S disconnects A and B. A common way to obtain a node separator is the following. First, we compute a partition of the graph into two sets V_1 and V_2. Clearly, the boundary nodes in V_1 would yield a feasible separator and so would the boundary nodes in the opposite block V_2. Since we are interested in a small separator, we could simply use the smaller set of boundary nodes.

We use the method of Pothen et al. [24] which employs the set of cut edges of the partition. As in the original work, we use this method in our framework as a post-processing step to compute a node separator from a set of cut edges. The method computes the smallest node separator that can be found by using a subset of the boundary nodes. The main idea is to compute a subset S of the boundary nodes such that each cut edge is incident to at least one of the nodes in S. Such a set called a *vertex cover*. It is easy to see that S is a node separator since the removal of S eliminates all cut edges.

It is worth mentioning that we also provide an extended method which can be used to obtain a k-way node separator, i.e. k blocks $V_1, \ldots, V_k$ and a set S such that after the removal of the nodes in S there no edge running between the blocks $V_1, \ldots, V_k$. We do this by computing a k-partition using KaFFPa and applying the described flow problem between all pairs of blocks that share a non-empty boundary afterwards. All pair-wise separators together are then be used as a k-way separator.

3 Graph Format

3.1 Input File Format

The graph format used by our partitioning programs is basically the same as used by Metis [17], Chaco [14] and the graph format that has been used during the 10th DIMACS Implementation Challenge on Graph Clustering and Partitioning. Note that self-loops and parallel edges are *not* allowed.

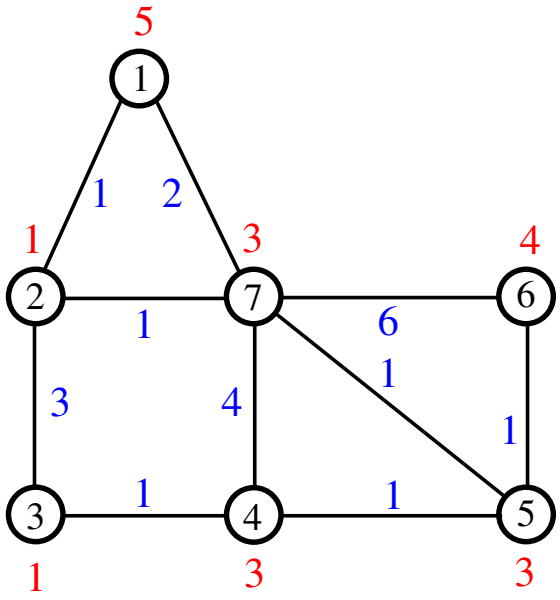
To give a description of the graph format, we follow the description of the Metis 4.0 user guide very closely. A graph $G = (V, E)$ with n vertices and m edges is stored in a plain text file that contains $n + 1$ lines (excluding comment lines). The first line contains information about the size and the type of the graph, while the remaining n lines contain information for each vertex of G. Any line that starts with % is a comment line and is skipped.

The first line in the file contains either two integers, n m, or three integers, n m f. The first two integers are the number of vertices n and the number of undirected edges of the graph, respectively. Note that in determining the number of edges m, an edge between any pair of vertices v and u is counted *only once* and not twice, i.e. we do not count the edge (v, u) from (u, v) separately. The third integer f is used to specify whether or not the graph has weights associated with its vertices, its edges or both. If the graph is unweighted then this parameter can be omitted. It should be set to 1 if the graph has edge weights, 10 if the graph has node weights and 11 if the graph has edge and node weights.

The remaining n lines of the file store information about the actual structure of the graph. In particular, the ith line (again excluding comment lines) contains information about the ith vertex. Depending on the value of f, the information stored in each line is somewhat different. In the most general form (when $f = 11$, i.e. we have node and edge weights) each line has the following structure:

$$c\, v_1\, w_1\, v_2\, w_2 \ldots v_k\, w_k$$

where c is the vertex weight associated with this vertex, $v_1, \ldots, v_k$ are the vertices adjacent to this vertex, and $w_1, \ldots, w_k$ are the weights of the edges. Note that the vertices are numbered starting from 1 (not from 0). Furthermore, the vertex-weights must be integers greater or equal to 0, whereas the edge-weights must be strictly greater than 0.




```
7  10  11
5  2  1  7  2
1  1  1  3  3  7  1
1  2  3  4  1
3  3  1  7  4  5  1
3  4  1  7  1  6  1
4  5  1  7  6
3  1  2  2  1  4  4  5  1  6  6
```

Figure 3: An example graph and its representation in the graph format. The IDs of the vertices are drawn within the cycle, the vertex weight is shown next to the circle (red) and the edge weight is plotted next to the edge (blue).

3.2 Output File Formats

3.2.1 Partition

The output format of a partition is also similar to the output format of a partition provided by Metis. It is basically a text file named *tmppartitionk* where k is the number of blocks given as input to the partitioning program. This file contains n lines. In each line the block ID of the corresponding vertex is given, i.e. line i contains the block ID of the vertex i (here the vertices are numbered from 0 to $n-1$). The block IDs are numbered consecutively from 0 to $k-1$.

3.2.2 Node Separator

If the output is a node separator then the same format as used for a partition is used. However, in this case the nodes of the separator get the block ID k where as the other nodes maintain their original block id.

3.3 Troubleshooting

KaHIP should not crash. If KaHIP crashes it is mostly due to the following reasons: the provided graph contains self-loops or parallel edges, there exists a forward edge but the backward edge is missing or the forward and backward edges have different weights, or the number of vertices or edges specified does not match the number of vertices or edges provided in the file. Please use the *graphcheck* tool provided in our graph partitioning package to verify whether your graph has the right input format. If our graphcheck tool tells you that the graph that you provided has the correct format and KaHIP crashes anyway, please write us an email.

4 User Interface

KaHIP contains the following programs: kaffpa, kaffpaE, partition_to_vertex_separator, graphchecker. To compile these programs you need to have Argtable, g++, OpenMPI and scons installed (we use argtable-2.10, g++-4.8.0, OpenMPI-1.4.1 and scons-1.2). Once you have that you can execute *compile.sh* in the main folder of the release. When the process is finished the binaries can be found in the folder *deploy*. We now explain the parameters of each of the programs briefly.

4.1 KaFFPa

Description: This is the multilevel graph partitioning program.

Usage:
kaffpa file –k=<int> [–help] [–seed=<int>] [–preconfiguration=variant] [–imbalance=<double>] [–time_limit=<double>] [–enforce_balance] [–input_partition=<string>]

Options:

file	Path to graph file that you want to partition.
–k=<int>	Number of blocks to partition the graph into.
–help	Print help.
–seed=<int>	Seed to use for the random number generator.
–preconfiguration=variant	Use a preconfiguration. (Default: eco) [strong|eco|fast]. Strong should be used if quality is paramount, eco if you need a good tradeoff between partition quality and running time, and fast if partitioning speed is in your focus.
–imbalance=<double>	Desired balance. Default: 3 (%).
–time_limit=<double>	Time limit in seconds s. The default value is set to 0s, i.e. one partitioner call will be made. If you set a time limit t, kaffpa will repeatedly call the multilevel method until the time limit is reached and return the best solution found.
–enforce_balance	Use this option only on graphs without vertex weights. If this option is enabled, kaffpa guarantees that the output partition is feasible, i.e. fulfills the specified balance constraint.
–input_partition=<string>	You can specify an input partition. If you do so, KaFFPa will try to improve it.

4.2 KaFFPaE / KaBaPE

Description: This is the distributed evolutionary algorithm to tackle the graph partitioning problem. It includes also the perfectly balance case $\epsilon = 0$.

Usage:

mpirun -n P kaffpaE file –k=<int> [–help] [–seed=<int>] [–preconfiguration=variant] [–imbalance=<double>] [–time_limit=<double>] [–mh_enable_quickstart] [–mh_optimize_communication_volume] [–mh_enable_kabapE] [–mh_enable_tabu_search] [–kabaE_internal_bal=<double>] [–input_partition=<string>]

Options:

P	Number of processes to use.
file	Path to graph file that you want to partition.
–k=<int>	Number of blocks to partition the graph into.
–help	Print help.
–seed=<int>	Seed to use for the random number generator.
–preconfiguration=variant	Use a preconfiguration. (Default: eco) [strong|eco|fast]. Strong should be used if quality is paramount, eco if you need a good trade-off between partition quality and running time, and fast if partitioning speed is in your focus.
–imbalance=<double>	Desired balance. Default: 3 (%).
–time_limit=<double>	Time limit in seconds s. The default value is set to 0s, i.e. one partitioner call will be made by each PE. In order to use combine operations you *have to* set a time limit $t > 0$. kaffpaE will return the best solution after the time limit is reached. A time limit $t = 0$ means that the algorithm will only create the initial population.
–mh_enable_quickstart	Enables the quickstart option. In this case each PE creates a few partitions and these partitions are distributed among the PEs.
–mh_optimize_communication_volume	Modifies the fitness function of the evolutionary algorithm so that communication volume is optimized.
–mh_enable_kabapE	Enables the combine operator of *KaBaPE*.
–mh_enable_tabu_search	Enables our version of combine operation by block matching.
–kabaE_internal_bal=<double>	The internal balance parameter for KaBaPE (Default: 0.01) (1 %)
–input_partition=<string>	You can specify an input partition. If you do so, KaFFPaE will try to improve it.

4.3 Node Separators

Description: This is the program that computes a k-way vertex separator given a k-way partition of the graph.

Usage:
partition_to_vertex_separator file –k=<int> –input_partition=<string> [–help] [–seed=<int>]

Options:

file	Path to the graph file.
–k=<int>	Number of blocks the graph is partitioned in by using the input partition.
–input_partition=<string>	Input partition to compute the vertex separator from.
–help	Print help.
–seed=<int>	Seed to use for the random number generator.

4.4 Graph Format Checker

Description: This program checks if the graph specified in a given file is valid.

Usage:
graphchecker file

Options:

file	Path to the graph file.

5 Library

Some of the programs above can be directly accessed in C/C++ by using the library that we provide, i.e. the graph partitioners KaFFPaFast, KaFFPaEco and KaFFPaStrong (more will follow) can be called using this library. In this section we describe how the library can be used, the methods can be accessed and describe the parameters of these functions. The functions described in this section can be found in the file *interface/kaHIP_interface.h*.

5.1 Graph Data Structure

To make it easy for you to try KaHIP, we use a more or less similar interface to that provided by Metis. Most importantly, we use the same graph data structure. The graph data structure is an adjacency structure of the graph which includes weights for the vertices and edges if there are any. The adjacency structure is basically a compressed sparse row (CSR) format. We closely follow the description of the Metis 4.0 user guide:

The CSR format is a widely used scheme for storing sparse graphs. In this format the adjacency structure of a graph with n vertices and m edges is represented using two arrays *xadj* and *adjncy*. The *xadj* array is of size $n+1$ whereas the *adjncy* array is of size $2m$ (this is because for each edge between vertices v and u we store both a forward edge (v,u) and a backward edge (u,v)).

The adjacency structure of the graph is stored as follows. First of all, the vertex numbering starts from 0. The adjacency list of vertex i is stored in the array *adjncy* starting at index *xadj*[i] and ending at (but not including) index *xadj*[$i+1$] (i.e. *adjncy*[*xadj*[i]] through and including *adjncy*[*xadj*[$i+1$]-1]). That is, for each vertex i, its adjacency list is stored in consecutive locations in the array *adjncy*, and the array *xadj* is used to point to where it begins and where it ends.

The weights of the vertices (if any) are stored in an additional array called *vwgt*. If the graph has n vertices then the array contains n elements, and *vwgt*[i] will store the weight of the ith vertex. The vertex-weights must be integers greater or equal to zero. If all the vertices of the graph have the same weight (e.g. the graph is unweighted), then the array *vwgt* can be set to NULL.

The weights of the edges (if any) are stored in additional array called adjwgt. This array contains $2m$ elements, and the weight of edge *adjncy*[j] is stored at location *adjwgt*[j]. The edge-weights must be integers greater than zero. The weight of a forward edge (u,v) has to be equal to the weight of the backward edge (v,u) and parallel edges as well as self-loops are not allowed. If all the edges of the graph have the same weight (e.g. the graph is unweighted), then the array *adjwgt* can be set to NULL.

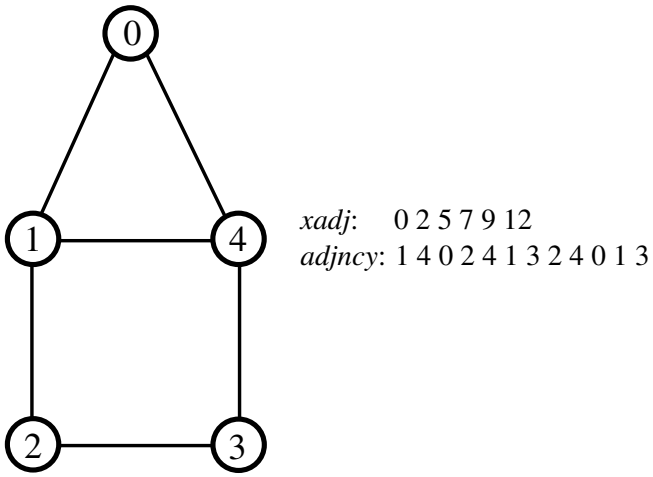


Figure 4: An example unweighted graph with its graph data structure.

5.2 Functions

Currently we provide three functions, e.g. kaffpa_strong, kaffpa_eco, kaffpa_fast, which all have the same declaration. We restrict ourselves to the description of the kaffpa_strong which corresponds to the strong configuration of KaFFPa. kaffpa_eco and kaffpa_fast correspond to the eco and fast configuration of KaFFPa respectively.

Partitioner Call

void kaffpa_strong(int* n, int* *vwgt*, int* *xadj*, int* *adjcwgt*, int* *adjncy*,
int* *nparts*, double* *imbalance*, bool *suppress_output*, int *seed*,
int* *edgecut*, int* *part*);

Parameters

n	Number of vertices of the graph.
vwgt	This is the vertex weight array described in Section 5.1. The array should have size n. If your graph does not have vertex weight you can use a null pointer as an argument.
xadj	This is the xadj array described in Section 5.1 which holds the pointers to the adjacency lists of the vertices. The array should have size $n + 1$.
adjcwgt	This is the adjacency weight array described in Section 5.1 which holds the weights of the edges if they exists. The array should have size $2m$. If your graph does not have edge weights you can use a null pointer as an argument.
adjncy	This is the adjacency array described in Section 5.1 which holds the adjacency lists of the vertices. The array should have size $2m$.
nparts	This parameter specifies the number of blocks you want the graph to be partitioned in.
imbalance	This parameter controls the amount of imbalance that is allowed. For example, setting it to 0.03 specifies an imbalance of 3% which means on unweighted graphs that each block has to fulfill the constraint $\|V_i\| \leq (1 + 0.03)\|V\|/k$.
suppress_output	If this option is enabled then no output of the partitioning library is printed to stdout.
seed	Seed to use for the random number generator.
edgecut	This is an *output* parameter. It represents the edge cut of the computed partition.
part	This is an *output* parameter. It has to be an already allocated array of size n. After the function call this array contains the information of the blocks of the vertices, i.e. the block of the ith vertex is given in *part*[i].

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