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Problem 1

Part A

Let $U_1 ... U_n$ be *i.i.d* and Uniformly distributed on (0,b) where b > 0.
Set $Z_i = -log(U_i)$; where the probability density function p(z) = $\frac{e^{-z}}{b}$
This is found using the change of density formula.

If Y = $\sum_{i=1}^{n} Z_i$, find p(y;b).
It is helpful to think of the series $S_1 = Z_1, S_2 = S_1 + Z_2 .... S_n = S_{n-1} + Z_n$

We can then compute S_n starting from S_2
p(s_2) = P[$s_1 * z_2$] (s) = $\int_{-\infty}^{\infty} P_{s_1}(s-y)P_{z_2}(y)dy$

The normal bounds for p(z) is $\epsilon(-log(b), \infty)$, because we change our function to be $P(s-y)$ we get a new upper bound: $-log(b) < s - y < \infty$ and therefore $y < s + log(b)$

$$p(s_2) = \int_{-log(b)}^{s+log(b)} P_{s_1}(s-y)P_{z_2}(y)dy = \int_{-log(b)}^{s+log(b)} \frac{e^{-s+y}}{b} * \frac{e^{-y}}{b} dy = \frac{e^{-s}}{b^2}[2log(b) + s] \tag{1}$$

Continuing on to S_3 we get

$$\begin{aligned} p(s_3) = \int_{-log(b)}^{s+2log(b)} P_{s_2}(s-y)P_{z_3}(y)dy = \int_{-log(b)}^{s+2log(b)} \frac{e^{-s+y}}{b}[2log(b) + s + y] * \frac{e^{-y}}{b} dy \\ = \frac{e^{-s}}{b^3}[\frac{1}{2}(3log(b) + s)^2] \end{aligned} \tag{2}$$

Continuing like this we see a pattern.

$$\begin{aligned} p(y;b) = \frac{e^{-y}}{b^n}[\frac{1}{(n-1)!}(nlog(b) + y)^{n-1}] \\ \epsilon(-log(b), y + nlog(b)) \end{aligned} \tag{3}$$

Part B

From the pdf computed in part A, find the third moment of Y.

To do this we look at the third moment equation.

$E[X^3] = \int_{-\infty}^{\infty} X^3 F_x dx$

Plugging in for our function we get:

$$\int_{-log(b)}^{y+nlog(b)} y^3 \frac{e^{-y}}{b^n}[\frac{1}{(n-1)!}(nlog(b) + y)^{n-1}] \tag{4}$$

Here I left the bounds according to the pdf of Y. I believe this integral solved would give the correct answer however I have not been able to successfully complete this step.

Part C

To determine the maximum likeihood estimation ob be bases on the two independent observations we need to take the derivative of L such that
$L = argmax_b p(y_1, y_2|b) = argmax_b p(y_1, |b)p(y_2, |b)$
Since $P(y; b) = p(b)p(y|b) and p(b) = 1$ we can plug in our equation from part A to solve this equation.

Problem 2

Part A

The parameters that must be estimated are as follows:
1.) DK 2.) 3DK 3.) 2DK 4.) DK

Part B

Problem 3