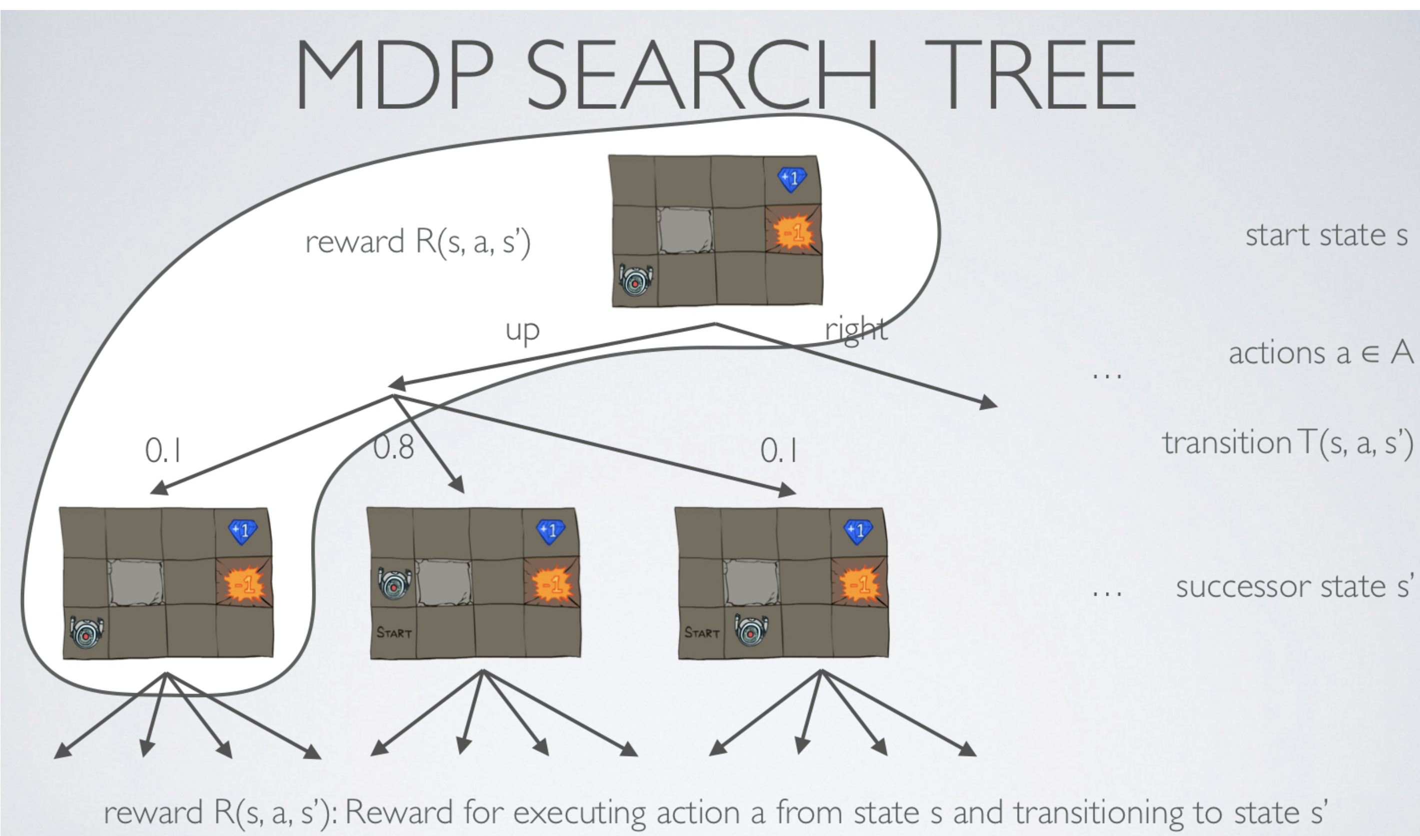
Introduction to AI

Markov Models

Markov Decision Processes (MDP)

- An MDP is defined by
  - A set of states *S*. ie. The coordinates of the robot.
  - A set of actions *A*. ie. Move up / down / left / right
  - A transition function T(s, a, s'): P(s' | s, a), s' means the next state.
  - A reward function R(s, a, s'): Reward of taking action a in s and ending up in s'.
    - Sometimes just R(s, a) / R(s) / R(s')
  - A discount factor γ: how much we care about next states.
  - A start state and maybe a terminal state.

MDP Search Tree



- The expected value starting in s, taking action a: $Q^*(s,a) = \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$

    - ```
      def getQValue(self, state, action):
        # transition: a list of (nextState, prob) pairs   T(s, a, s')
        transition = self.mdp.getTransitionStatesAndProbs(state, action)
        # Q*(s, a)
        qValue = 0.0
        for nextState, prob in transition:
            # reward: R(s, a, s')
            reward = self.mdp.getReward(state, action, nextState)
            qValue += prob * (reward + self.discount * self.getValue(nextState))
        return qValue
      ```

- The optimal value of the state s: $V^*(s) = max_a Q^*(s,a)$
- Optimal action to take at state s: $\pi^*(s) = argmax_a Q^*(s,a)$

    - ```
      def getPolicy(self, state):
        # V*(s) = max_a(Q*(s, a))
        qValueByAction = util.Counter() # use counter.argmax() to get the key with the larg
        for action in self.mdp.getPossibleActions(state):
            qValueByAction[action] = self.getQValue(state, action)
        # Returns None if there is no key.
        return qValueByAction.argMax()
      ```

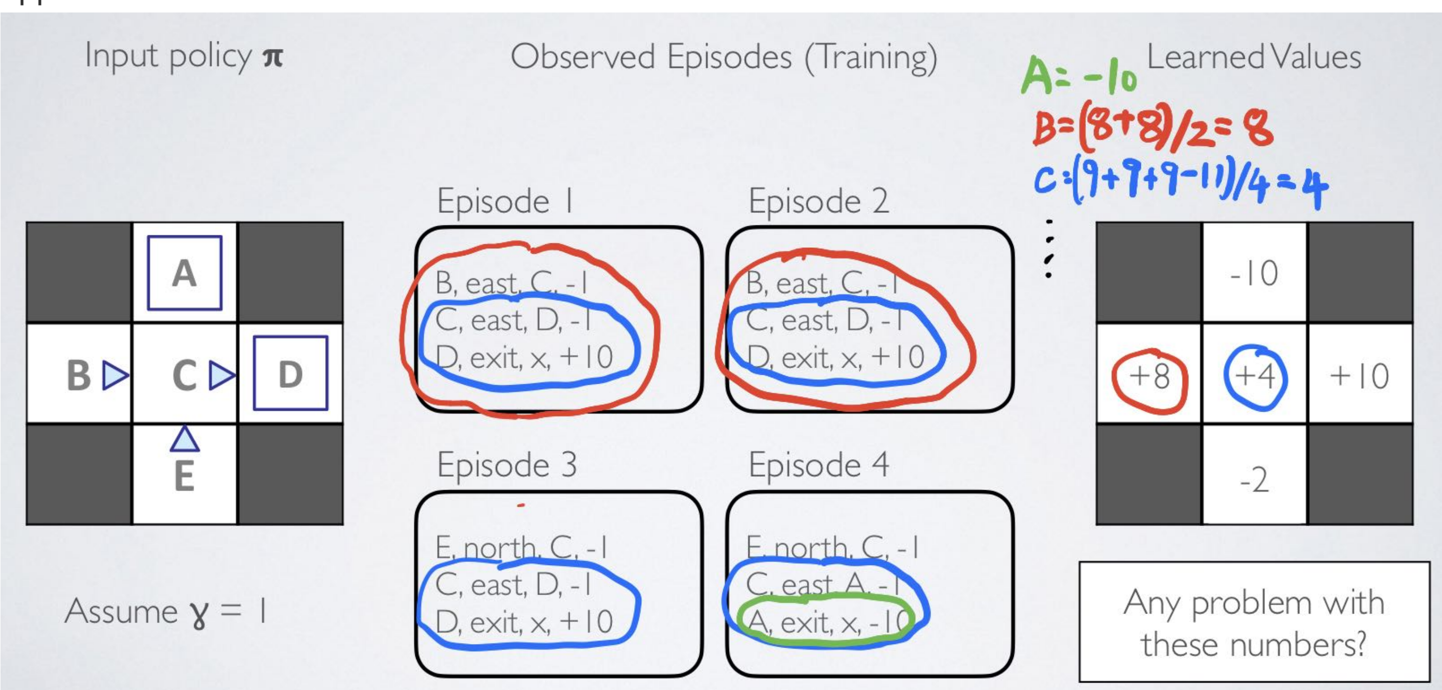
Reinforcement Learning

Model-based Learning

Learn transition T(s, a, s') and reward R(s, a, s) of the MDP model by counting outcomes s' for each s and a, then use value iteration as we did in the MDP.

Model-free Learning

Learn and approximate the value function $V^*(s)$ and $Q^*(s,a)$ directly, then extract policy based on approximated values.



> It wastes information about state connections, and each state must be learned separately. So it takes a long time to learn.

Q-Learning

Learn Q values based on samples after each action.

$$Q(s,a) = Q(s,a) + \alpha * difference \ (\alpha = learning\ rate)$$

$$difference = new\ value - old\ value = [reward + \gamma V^*(s')] - Q(s,a)$$
$$V^*(s') = max_{a'} Q(s', a')$$

```
def update(self, state, action, nextState, reward):
    # Returns self.qValue[(state, action)]
    oldValue = self.getQValue(state, action)
    # getValue(): Returns max_action Q(state,action)
    newValue = reward + self.discount * self.getValue(nextState)
    difference = newValue - oldValue
    # Update Q value
    self.qValue[(state, action)] += self.alpha * difference
```

> Q: When to explore instead of exploit the current policy? (in order to find potential better policies)
> A: Random actions -- For every time step. with probability ϵ, act randomly, with probability $1 - \epsilon$, act on the best policy we got.

Approximate Q-Learning

> Q-Learning is not scalable. It needs a lot of memory to store all Q-values and takes too long to learn good actions for every state.

Goal: Generalize experiences and knowledge learned to new, simular situations.
Idea: Descirbe a state using a vector of features (important properties like $1/(distance\ to\ dot)^2$, distance to closest ghost, ...). Features need to be normalized to [-1, 1] or [0, 1]. (the same idea of machine learning)

$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + ... + w_n f_n(s,a)$. Learn the weights.

note that there are fewer weights than states, because many states might share the same features, so it is faster to learn.

Iteration:
$$w_i = w_i + \alpha * difference * f_i(s,a)$$
$$difference = new\ value - old\ value = [reward + \gamma V^*(s')] - Q(s,a)$$
$$V^*(s') = max_{a'} Q(s', a')$$

```python
def getQValue(self, state, action):
    # dot product of matrices
    return self.weights * self.featExtractor.getFeatures(state, action)

def update(self, state, action, nextState, reward):
    difference = (reward + self.discount * self.getValue(nextState)) - self.getQValue(state, act
    features = self.featExtractor.getFeatures(state, action)
    for feature, value in features.items():
        self.weights[feature] += self.alpha * difference * value
```