Mathematics problems

1 Elementary algebra

Problem 1.1. Simplify
$$\frac{z^{11}}{z^4 \cdot z^5}$$

Problem 1.2. Solve for x:
$$6^2 \cdot 3^x \cdot 2^x = 6^9$$

Problem 1.3. Calculate the missing value. If $x \cdot y$ is 5, then $x^{-3}y^{-3} = \ldots$

Problem 1.4. Calculate
$$\frac{\sqrt{3^{10}}}{\sqrt{9^3}}$$

Problem 1.5. True or False (x and y and z are real numbers):

(a) $x + y = y + x$

(b) $x(y + z) = xy + xz$

(c) $x^{y+z} = x^y + x^z$

(d) $\frac{x^y}{x^z} = x^{y-z}$

Problem 1.6. Find the solution set for the inequality below:
$$\frac{4x - 10}{4} \geq 4$$

2 Functions of one variable

Problem 2.1 (Based on SYD 2.5.6). The relationship between temperatures measured in Celsius and Fahrenheit is linear. $0°$C is equivalent to $32°$F and $100°$C is the same as $212°$F. Which temperature is measured by the same number on both scales?

Problem 2.2. Take the following function $f(x) = 7x + 3$. Find y if $f(y) = 52$.

Problem 2.3. Find all values of x that satisfy:
$$10^{x^2 - 2x + 2} = 100$$

Problem 2.4. Solve the following problem. If the annual GDP growth of a country is 2%, how long does it take the economy to double its GDP?

Problem 2.5. Calculate the following value
$$\ln\left(\frac{1}{e^3}\right)$$

3   Calculus

Problem 3.1. Calculate the following sum

$$\sum_{i=0}^{\infty} \left(\frac{1}{8^i} + 0.5^i \right)$$

Problem 3.2. Find the following limit

$$\lim_{x \to 3} \frac{x-3}{2}$$

Problem 3.3. Find the slope of the function $f(x) = x^2 - 4$ at $(-1, -3)$.

Problem 3.4. Find the following derivative

$$\frac{d}{dx} \frac{x^2 + 3}{x + 2}$$

Problem 3.5. Find the following second derivative

$$\frac{d^2}{dx^2} 4x^3 + 4$$

Problem 3.6. Is the function $f(x) = \frac{1}{x}$ continuous at 0? Why?

Problem 3.7. Consider the following function. Find all of its stationary points and classify them as local minima, local maxima or inflection points. Also decide whether it is convex or concave. If it has one or more inflection points then define where it is locally concave or locally convex.

$$f(x) = 3x^3 - 9x$$

Problem 3.8. Let $f(x, y) = x^3 y^2$. Calculate $f(2, 3)$

Problem 3.9. Consider the following function: $f(x, y) = \ln(2x - y)$. For what combinations of x and y is this function defined?

Problem 3.10. Find the following partial derivative:

$$\frac{\partial^2}{\partial x^2} x^5 + x^2 y^3$$

Problem 3.11. Find the local maxima or minima of the following function:

$$f(x, y) = \sqrt{xy} - 0.25x - 0.25y$$

Problem 3.12. Solve the following constrained optimization problem using Lagrange's method: $\max x^2 y^2$ s.t. $x + y = 5$

4   Linear algebra

Problem 4.1. Take the following matrices:

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

What is $A \cdot B$?

Problem 4.2. Take the following matrices:

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

What is $B \cdot A$?

Problem 4.3. What is the transpose of the following matrix?

$$\begin{bmatrix} 3.3 & 5.1 & 4.7 \\ 2 & 6.1 & 1.23 \\ 4 & 5.76 & 0 \end{bmatrix}$$

Problem 4.4. Calculate the determinant of

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

5 Probability theory

Problem 5.1. You run an experiment where you flip a coin four times. Each time you get either heads (H) or tails (T). What is the sample space of your experiment?

Problem 5.2. Assume that in a certain country 1% of the population uses a certain drug. You have a way to test drug use, which will give you a positive result in 99% of the cases where the individual is indeed a drug user and a negative result in 99.5% of the cases where the individual doesn't use the drug. What is the probability that someone with a positive drug test is indeed a drug user?

Problem 5.3. You run an experiment in which you toss a dice twice and sum up the results. What is the expected value of this sum?