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Math_task.

1 Elementary Algebra

P 1.1. $\frac{z^{11}}{z^4 \cdot z^5} = \frac{z^{11}}{z^9} = z^2$

P 1.2. $6^2 \cdot 3^x \cdot 2^x = 6^9$

$6^2 \cdot 6^x = 6^9$

$6^{2+x} = 6^9$

$x = 7$

P 1.3. $x \cdot y = 5, \quad x^{-3} \cdot y^{-3} = ?$

$x^{-3} \cdot y^{-3} = (x \cdot y)^{-3} = 5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

P 1.4. $\frac{\sqrt{3^{10}}}{\sqrt{9^3}} = \frac{\sqrt{3^{10}}}{\sqrt{3^6}} = \sqrt{3^4} = 3^2 = 9$

P 1.5. (a) $x + y = y + x$ // True

(b) $x(y+z) = xy + xz$ // True

(c) $x^{y+z} = x^y + x^z$ // False

(d) $\frac{x^y}{x^z} = x^{y-z}$ // True

P 1.6. $\frac{4x-10}{4} \geq 4$

$2x - 5 \geq 8$

$2x \geq 13 \qquad x \geq 6.5, \quad x \in [6.5; +\infty)$

2 Functions of one variable

P2.1. The relationship between C and F can be described as following:

$$F = a \cdot C + b$$

$$\begin{cases} 32 = a \cdot 0 + b \\ 212 = a \cdot 100 + b \end{cases} \Rightarrow \begin{cases} b = 32 \\ 212 = a \cdot 100 + 32 \end{cases} \Rightarrow \begin{cases} b = 32 \\ a = \frac{9}{5} = 1.8 \end{cases}$$

$$F = 1.8\,C + 32$$

If $F = C$, then: $C = 1.8\,C + 32$

$C = -40$ // $C = F = -40$

P2.2. $f(x) = 7x + 3$

$f(y) = 52$

$y = ?$

$f(y) = 7y + 3$

$7y + 3 = 52$

$y = 7$

P2.3. $10^{x^2 - 2x + 2} = 100$

$10^{x^2 - 2x + 2} = 10^2$

$x^2 - 2x + 2 = 2$

$x^2 - 2x = 0$

$x(x - 2) = 0$

$x_1 = 0$

$x_2 = 2$

P2.4. Years to double = 72 / Interest rate // The rule of 72

Years to double $= \frac{72}{2} = 36$ years

P2.5 $\ln\left(\frac{1}{e^3}\right) = \ln(1) - \ln(e^3) = 0 - 3 = -3$

3 Calculus

3.1. $\sum_{i=0}^{\infty}\left(\frac{1}{8^i}+0,5^i\right)=\sum_{i=0}^{\infty}\frac{1}{8^i}+\sum_{i=0}^{\infty}\frac{1}{2^i}=$

$=\frac{1}{1-\frac{1}{8}}+\frac{1}{1-\frac{1}{2}}=\frac{8}{7}+2=\frac{8+14}{7}=\frac{22}{7}$

3.2. $\lim_{x\to 3}\frac{x-3}{2}=\frac{3-3}{2}=0$

3.3. $f(x)=x^2-4$

(2) $\left[\frac{dy}{dx}\right]_{(-1,-3)}=2(-1)=-2$

(1) $\frac{dy}{dx}=\frac{d}{dx}(x^2-4)$

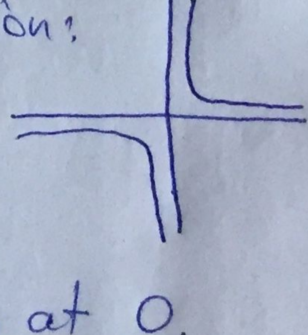
$\frac{dy}{dx}=2x$

3.4. $\frac{d}{dx}\frac{x^2+3}{x+2}=\frac{2x(x+2)-(x^2-3)}{(x+2)^2}=\frac{2x^2+4x-x^2-3}{(x+2)^2}=$

$=\frac{x^2+4x-3}{(x+2)^2}$

3.5. $\frac{d^2}{dx^2}4x^3+4=((4x^3+4)')'=(12x^2)'=24x$

3.6. $f(x)=\frac{1}{x}$ $\Rightarrow$ Graphical representation:



(1) The function has asymptotes;

(2) $\lim_{x\to 0_-}\neq\lim_{x\to 0_+}$;

(1)+(2) = The function is not continuous at 0.

3.8

$f(x,y) = x^3 y^2$

$f(2,3) = 2^3 \cdot 3^2 = 8 \cdot 9 = 72$

3.9. $f(x,y) = \ln(2x - y)$

$2x - y > 0$ // for function to be defined

$2x > y$

$x > \frac{y}{2}$

3.10. $\frac{d^2}{dx^2} x^5 + x^2 y^3 = 5x^4 + 2xy^3$

$(5x^4 + 2xy^3)' = 20x^3 + 2y^3$

5. Probability theory

5.1.

$P(\text{flipping coin 4 times}) = 2^4 = 16 \Rightarrow$

$\Rightarrow \{$ HHHH HHHT HTHT TTHH TTTH
HTHH THHH HTTH THHT TTHT
HHTH HHTT HTTT THTH THTT
TTTT $\}$

5.2.

Country A

Drug Users 1%

99% (+) 1% (−)

Non-Drug users 99%

0,5% (+) 99.5 (−)

Applying Bayes theorem:

$$P(DU|+) = \frac{0{,}99 \cdot 0.01}{0{,}99 \cdot 0{,}01 + 0{,}005 \cdot 0{,}99} = \frac{0{,}0099}{0{,}01485} = 0{,}6666$$

$$P(DU|+) \approx 0{,}67$$

5.3.

$$E(X) = \sum x\,(P(X) = x) = 2 \cdot \frac{1}{36} + 3 \cdot \frac{2}{36} + 4 \cdot \frac{3}{36} + 5 \cdot \frac{4}{36} + 6 \cdot \frac{5}{36} + 7 \cdot \frac{6}{36} + 8 \cdot \frac{5}{36} + 9 \cdot \frac{4}{36} + 10 \cdot \frac{3}{36} + 11 \cdot \frac{2}{36} + 12 \cdot \frac{1}{36} = \frac{2+6+12+20+30+42+40+36+30+22+12}{36} = \frac{252}{36} = 7$$