CS395: Selected CS1 (Introduction to Machine Learning)

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References:

https://www.coursera.org/learn/machine-learning (Andrew Ng)

Machine learning A to Z: **Kirill Eremenko ©superdatascience**

Linear Algebra review: Matrices and vectors

Vector: Special case of the matrix

n x 1 matrix

$\mathbb{R}^4$

$$y = \begin{bmatrix} 4 \\ 50 \\ 3 \\ 25 \end{bmatrix} \quad 4 - dimentional\ vector\ n = 4$$

$y_i = i^{\text{th}}$ element.

$y_2 = 50$

1-indexed vs 0-indexed:

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \qquad y = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Matrix Addition

$$\begin{bmatrix} 1 & 5 & 9 \\ 2 & 6 & 11 \\ 3 & 7 & 12 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 5 \\ 2 & 4 & 3 \\ 5 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 11 & 5 & 14 \\ 4 & 10 & 14 \\ 8 & 9 & 14 \end{bmatrix}$$

3 x 3   3 x 3   3 x 3

We can add only two matrices that are of **the same dimensions.**

So this example is a 3 x 3 matrix, the result in same dimension 3x3.

Addition and subtraction are **element-wise**, so you simply add or subtract each corresponding element

Scalar Multiplication

$$3 \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \mathrm{x}3$$

2 x 2

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} / 4 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \ \mathrm{x}\ \tfrac{1}{4} = \begin{bmatrix} 1/4 & 1/2 \\ 3/4 & 1 \end{bmatrix}$$

Combination of Operands

Scaler Multiplication

$$3 \times \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} / 3 =$$

Scaler Multiplication

$$\begin{bmatrix} 3 \\ 12 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ \frac{2}{3} \end{bmatrix} =$$

$$\begin{bmatrix} 2 \\ 12 \\ 10\frac{1}{3} \end{bmatrix}$$

Matrix Vector Multiplication

$$\begin{bmatrix} 5 & 1 \\ 2 & 6 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 15 \\ 34 \\ 26 \end{bmatrix}$$

$$A \quad \text{x} \quad x \quad = \quad y$$

$$\begin{bmatrix} 5 & 1 \\ 2 & 6 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 15 \\ 34 \\ 26 \end{bmatrix}$$

3 × 2   2 × **1**   **3** × **1 matrix → 3 dim vector**

To get $y_i, multiply\ A's\ i^{th}\ row\ with\ elements\ of\ vector\ x, and\ add\ them\ up.$

$5 \times 2 + 1 \times 5 = 15$

$2 \times 2 + 6 \times 5 = 34$

$3 \times 2 + 4 \times 5 = 26$

An **m x n matrix** multiplied by an **n x 1 vector** results in an **m x 1 vector**.

Example: If we have the house sizes and the hypotheses (h_θ), and you need to predict the prices of these houses

$house\ sizes$:

2100
3150
4152
1108

$$h_\theta(x) = -40 + 0.25\,x$$

Prediction **4x1 matrix**:

Matrix **4x2**:

$$\begin{bmatrix} 1 & 2100 \\ 1 & 3150 \\ 1 & 4152 \\ 1 & 1108 \end{bmatrix} \times \begin{bmatrix} -40 \\ 0.25 \end{bmatrix} = \begin{bmatrix} -40 \times 1 + 0.25 \times 2100 \\ -40 \times 1 + 0.25 \times 3150 \\ -40 \times 1 + 0.25 \times 4152 \\ -40 \times 1 + 0.25 \times 1108 \end{bmatrix}$$

$h_\theta(2100)$

2x1

Matrix Multiplication

$$\overset{\substack{\text{A}\\ \text{2x3}}}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}} \times \overset{\substack{\text{B}\\ \text{3x2}}}{\begin{bmatrix} 1 & 2 \\ 0 & 5 \\ 2 & 1 \end{bmatrix}} = \overset{\substack{\text{C}\\ \text{2x2}}}{\begin{bmatrix} 7 & 15 \\ 16 & 39 \end{bmatrix}}$$

C_{11}=1+0+6 = 7

C_{21} =4 + 0+ 12 = 16

C_{12} = 2+10+3= 15

C_{22} =8+25+6= 39

An **m x n matrix** multiplied by an **n x o matrix** results in an **m x o** matrix. In the above example, a 2 x 3 matrix times a 3 x 2 matrix resulted in a 2 x 2 matrix.

Example

$house\ sizes:$

2100
3150
4152
1108

$$h_\theta(x) = \boxed{-40 + 0.25\,x}$$
$$h_\theta(x) = \boxed{200 + 0.1\,x}$$
$$h_\theta(x) = \boxed{-150 + 0.4\,x}$$

m x n matrix

Matrix **4x2**:

$$\begin{bmatrix} 1 & 2100 \\ 1 & 3150 \\ 1 & 4152 \\ 1 & 1108 \end{bmatrix} \times \begin{bmatrix} -40 & 200 & -150 \\ 0.25 & 0.1 & 0.4 \end{bmatrix} =$$

n x o matrix

2x3

m x o matrix

Prediction **4x3 matrix**:

$$\begin{bmatrix} 485 & 410 & 690 \\ 748 & 515 & 1110 \\ 998 & 615 & 1511 \\ 237 & 311 & 293 \end{bmatrix}$$

Matrix Multiplication Properties

- Matrices are **not commutative**: $A \times B \ \neq B \times A$
- Matrices are **associative**: $(A \times B) \times C = A \times (B \times C)$
- Identity Matrix $I, or\ I_{n \times n}$ for any A, $A.I = I.A = A$
- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Inverse and Transpose

- The **inverse** of a matrix A $(m \times m)$ is denotedA^{-1} . Multiplying by the inverse results in the identity matrix. $A \times A^{-1} = 1$
- A non square matrix does not have an inverse matrix.

$$\begin{bmatrix} 3 & 4 \\ 2 & 16 \end{bmatrix} \times \begin{bmatrix} 0.4 & -0.1 \\ -0.05 & 0.075 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Transpose

- The **transposition** $B_{ij} = A_{ji}$ of a matrix is like rotating the matrix 90° in clockwise direction and then reversing it.
- $A_{ij} = A_{ij}^{T}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

m x n matrix

$$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

nx m matrix

Thanks