CS396: Selected CS2

(Deep Learning for visual recognition)

Spring 2022

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Lectures (Course slides) are based on Stanford course :
Convolutional Neural Networks for Visual Recognition (CS231n):
http://cs231n.stanford.edu/index.html

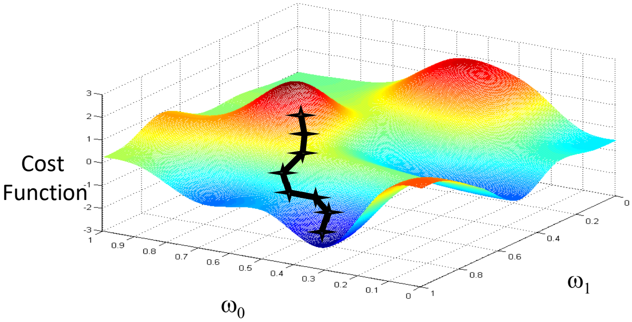
Lecture 5: Training Networks, Part 2

Topics

- Fancier optimization
- Regularization

Following the Gradient

- Compute the best direction along which we should change our parameter (weight) vector that is mathematically guaranteed to be the direction of the **steepest descend.**
- This direction will be related to the **gradient** of the cost function.



Gradient Descent

Recap

- We make steps down the cost function in the direction with the steepest descent, and the size of each step is determined by the parameter **α**, which is called the **learning rate.**
- Type equation here.
- The gradient descent algorithm is:

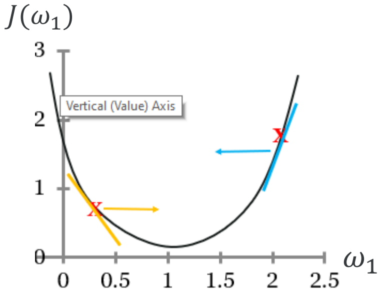
repeat until <u>**convergence**</u> {

$$\omega_j := \omega_j - \alpha \frac{\partial}{\partial \omega_j} J(\omega_0, \omega_1)$$

$$(\text{for } j = 0 \text{ and } j = 1)$$

}

Learning rate (step size)

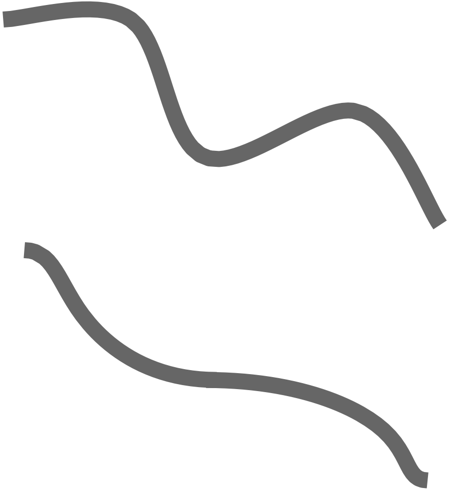


Positive slope (positive number)→ ω_1 will decrease
Negative slope (negative number)→ ω_1 will increase

Andrew Ng

Optimization: Problems with SGD

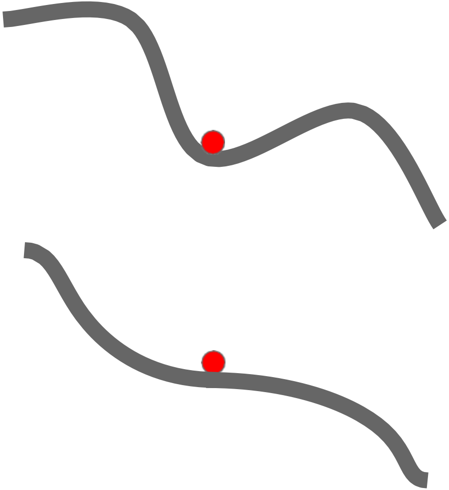
What if the loss function has a **local minima** or **saddle point**?



Optimization: Problems with SGD

What if the loss function has a **local minima** or **saddle point**?

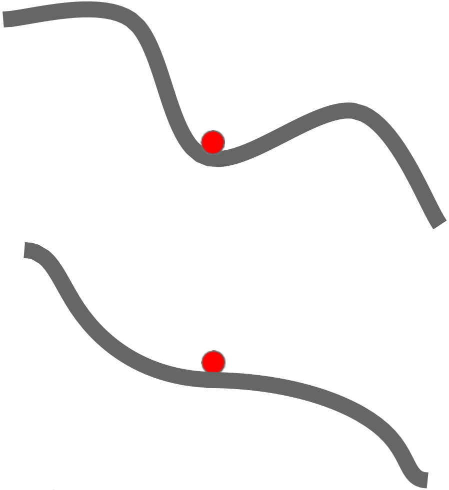
Zero gradient, gradient descent gets stuck



Optimization: Problems with SGD

What if the loss function has a **local minima** or **saddle point**?

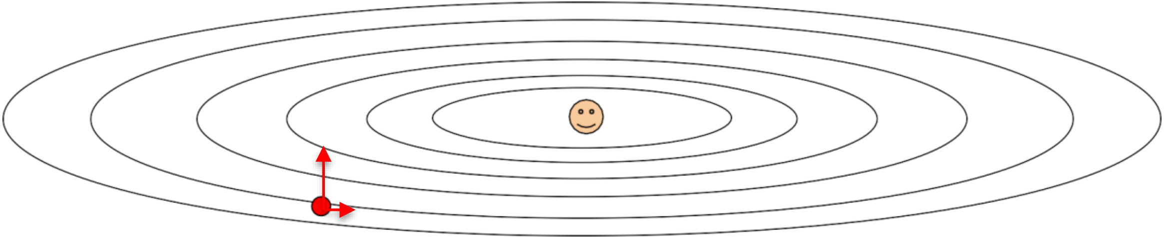
Saddle points much more common in high dimension



Dauphin et al, "Identifying and attacking the saddle point problem in high-dimensional non-convex optimization", NIPS 2014

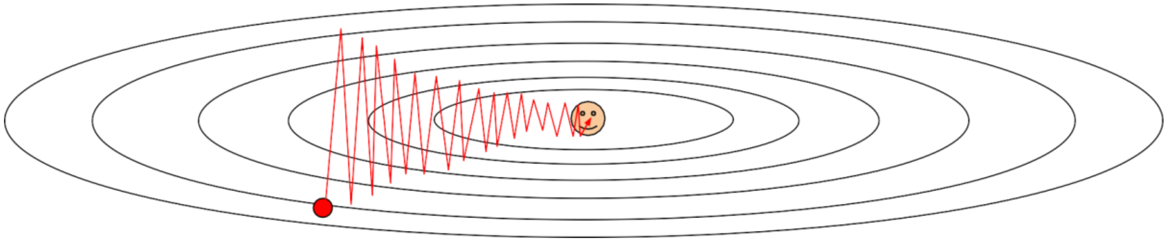
Optimization: Problems with SGD

What if loss changes quickly in one direction and slowly in another?
What does gradient descent do?



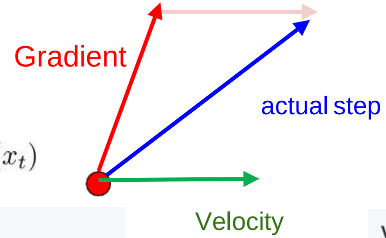
Optimization: Problems with SGD

What if loss changes quickly in one direction and slowly in another?
What does gradient descent do?
Very slow progress along shallow dimension, jitter along steep direction



So, we need to slow down changes on vertical dimension and increase it on the horizontal dimension.

SGD + Momentum



SGD

$$x_{t+1} = x_t - \alpha \nabla f(x_t)$$

```
while True:
  dx = compute_gradient(x)
  x -= learning_rate * dx
```

SGD+Momentum

$$v_{t+1} = \rho v_t + \nabla f(x_t)$$
$$x_{t+1} = x_t - \alpha v_{t+1}$$

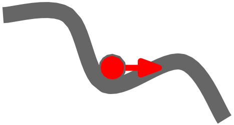
```
vx = 0
while True:
  dx = compute_gradient(x)
  vx = rho * vx + dx
  x -= learning_rate * vx
```

- Build up “velocity” as a running <u>mean</u> of gradients
- Rho gives “friction”; typically rho=0.9 or 0.99

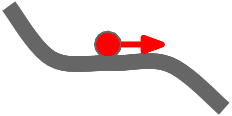
Sutskever et al, “On the importance of initialization and momentum in deep learning”, ICML 2013

SGD + Momentum

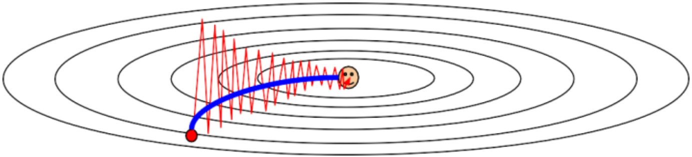
Local Minima



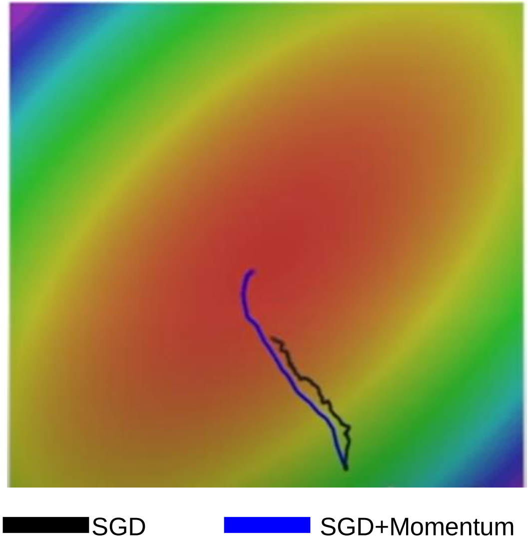
Saddle points



Poor Conditioning

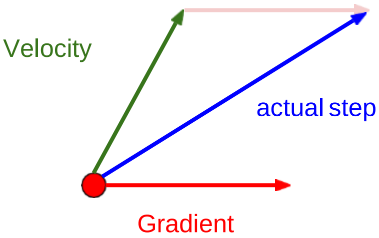


Gradient Noise



SGD+Momentum

Momentum update:

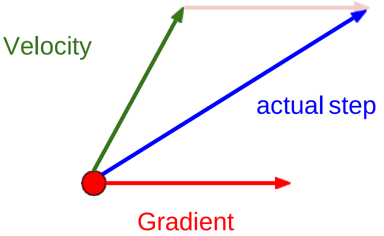


Combine gradient at current point with velocity to get step used to update weights

Nesterov, “A method of solving a convex programming problem with convergence rate O(1/k^2)”, 1983
Nesterov, “Introductory lectures on convex optimization: a basic course”, 2004
Sutskever et al, “On the importance of initialization and momentum in deep learning”, ICML 2013

Nesterov Momentum (Nesterov accelerated gradient)

Momentum update:



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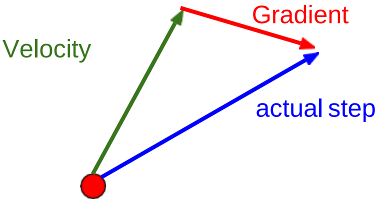
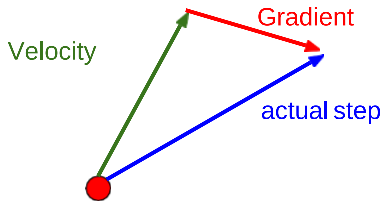
Nesterov Momentum



“Look ahead” to the point where updating using velocity would take us; compute gradient there and mix it with velocity to get actual update direction

Nesterov Momentum

$$v_{t+1} = \rho v_t - \alpha \nabla f(x_t + \rho v_t)$$
$$x_{t+1} = x_t + v_{t+1}$$



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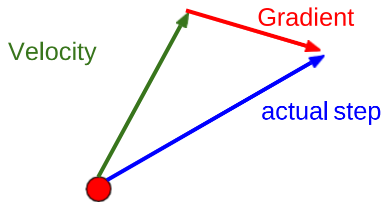
Nesterov Momentum

$$v_{t+1} = \rho v_t - \alpha \nabla f(\boxed{x_t + \rho v_t})$$

$$x_{t+1} = x_t + v_{t+1}$$

Annoying, usually we want update in terms of $x_t, \nabla f(x_t)$



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$$x_{t+1} = x_t + v_{t+1}$$

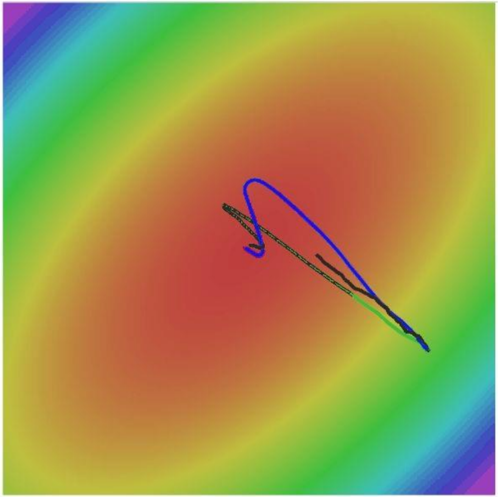
Annoying, usually we want update in terms of $x_t, \nabla f(x_t)$

Change of variables $\tilde{x}_t = x_t + \rho v_t$ and rearrange:

$$v_{t+1} = \rho v_t - \alpha \nabla f(\tilde{x}_t)$$
$$\begin{aligned} \tilde{x}_{t+1} &= \tilde{x}_t - \rho v_t + (1+\rho) v_{t+1} \\ &= \tilde{x}_t + v_{t+1} + \rho(v_{t+1} - v_t) \end{aligned}$$

The **first equation** looks exactly like updating the velocity in the SGD momentum case. At the **second equation**, we have our current point plus our current velocity plus **a weighted difference** between our current velocity and our previous velocity. Here, Nesterov momentum is kind of incorporating some kind of **error-correcting** term between your current velocity and your previous velocity.

Nesterov Momentum



SGD

SGD+Momentum

Nesterov (nag)

AdaGrad

```
grad_squared = 0
while True:
  dx = compute_gradient(x)
  grad_squared += dx * dx
  x -= learning_rate * dx / (np.sqrt(grad_squared) + 1e-7)
```

Added element-wise scaling of the gradient based on the historical sum of squares in each dimension

“Per-parameter learning rates”
or “adaptive learning rates”

Duchi et al, “Adaptive subgradient methods for online learning and stochastic optimization”, JMLR 2011

AdaGrad

```
grad_squared = 0
while True:
  dx = compute_gradient(x)
  grad_squared += dx * dx
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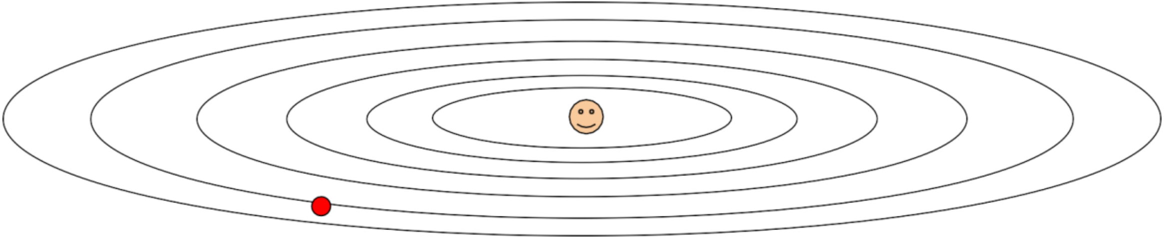
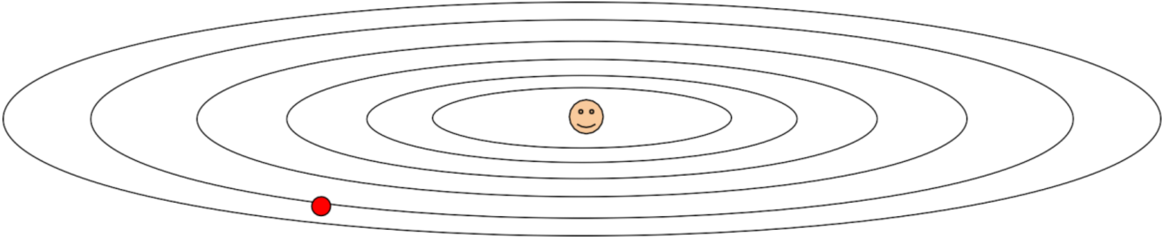


Q: What happens with AdaGrad?

AdaGrad

```
grad_squared = 0
while True:
  dx = compute_gradient(x)
  grad_squared += dx * dx
  x -= learning_rate * dx / (np.sqrt(grad_squared) + 1e-7)
```

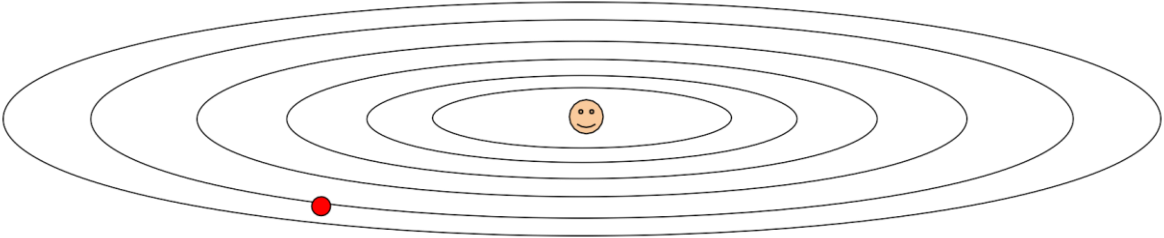


Q: What happens with AdaGrad?

Progress along “steep” directions is damped;
progress along “flat” directions is accelerated

AdaGrad

```
grad_squared = 0
while True:
  dx = compute_gradient(x)
  grad_squared += dx * dx
  x -= learning_rate * dx / (np.sqrt(grad_squared) + 1e-7)
```



Q2: What happens to the step size over long time?

AdaGrad

```
grad_squared = 0
while True:
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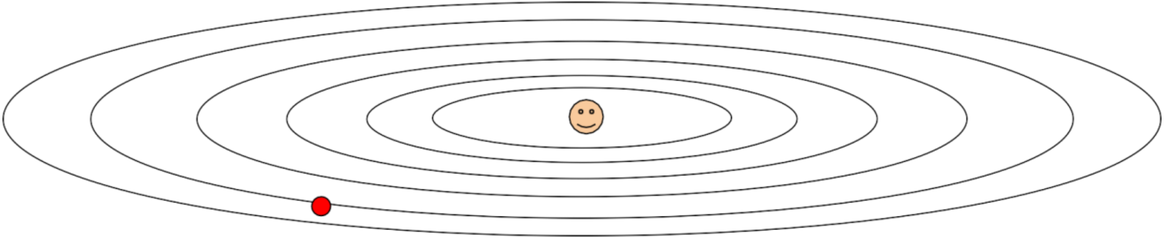
  grad_squared += dx * dx
  x -= learning_rate * dx / (np.sqrt(grad_squared) + 1e-7)
```



Q2: What happens to the step size over long time?

Decays to zero

RMSProp

AdaGrad

```
grad_squared = 0
while True:
  dx = compute_gradient(x)
  grad_squared += dx * dx
  x -= learning_rate * dx / (np.sqrt(grad_squared) + 1e-7)
```

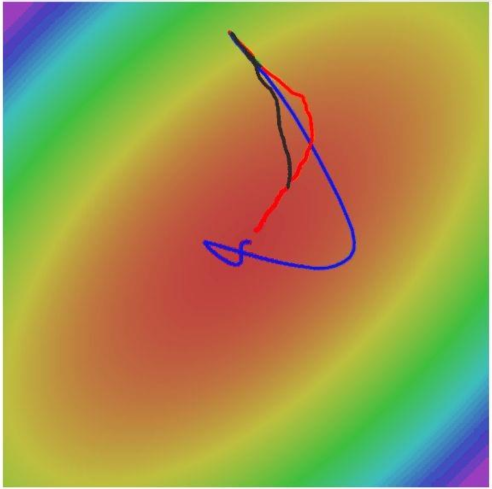
RMSProp

```
grad_squared = 0
while True:
  dx = compute_gradient(x)
  grad_squared = decay_rate * grad_squared + (1 - decay_rate) * dx * dx
  x -= learning_rate * dx / (np.sqrt(grad_squared) + 1e-7)
```

Tieleman and Hinton, 2012

Typically, decay rate = 0.99

RMSProp



Note:
AdaGrad just gets stuck due to the problem of continually decaying learning rates. For that, it is not shown in this comparison.

Adam (almost)

```
first_moment = 0
second_moment = 0
while True:
  dx = compute_gradient(x)
  first_moment = beta1 * first_moment  + (1 - beta1) * dx
  second_moment = beta2 * second_moment + (1 - beta2) * dx * dx
  x -= learning_rate * first_moment / (np.sqrt(second_moment) + 1e-7))
```

Kingma and Ba, “Adam: A method for stochastic optimization”, ICLR 2015

Adam (almost)

```
first_moment = 0
second_moment = 0
while True:
  dx = compute_gradient(x)
  first_moment = beta1 * first_moment  + (1 - beta1) * dx
  second_moment = beta2 * second_moment + (1 - beta2) * dx * dx
  x -= learning_rate * first_moment / (np.sqrt(second_moment) + 1e-7))
```

Momentum

AdaGrad / RMSProp

Sort of like RMSProp with momentum

Q: What happens at first timestep?

Kingma and Ba, "Adam: A method for stochastic optimization", ICLR 2015

Adam (full form)

```
first_moment = 0
second_moment = 0
for t in range(1, num_iterations):
  dx = compute_gradient(x)
  first_moment = beta1 * first_moment  + (1 - beta1) * dx
  second_moment = beta2 * second_moment + (1 - beta2) * dx * dx
  first_unbias = first_moment / (1 - beta1 ** t)
  second_unbias = second_moment / (1 - beta2 ** t)
  x -= learning_rate * first_unbias / (np.sqrt(second_unbias) + 1e-7))
```

Momentum

Bias correction

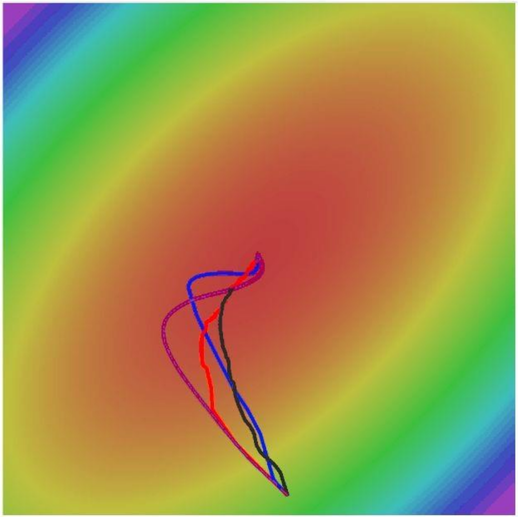
AdaGrad / RMSProp

Bias correction for the fact that first and second moment estimates start at zero

Adam with beta1 = 0.9, beta2 = 0.999, and learning_rate = 1e-3 or 5e-4 is a great starting point for many models!

Kingma and Ba, “Adam: A method for stochastic optimization”, ICLR 2015

Adam



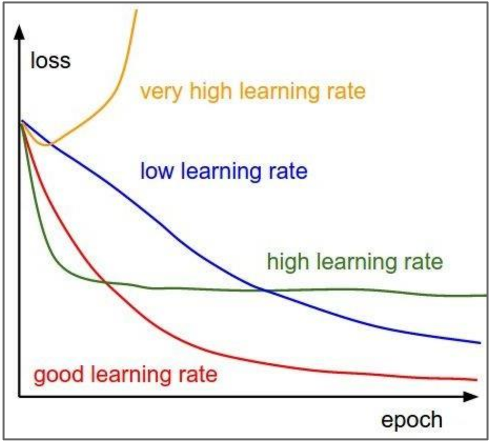
SGD

SGD+Momentum

RMSProp

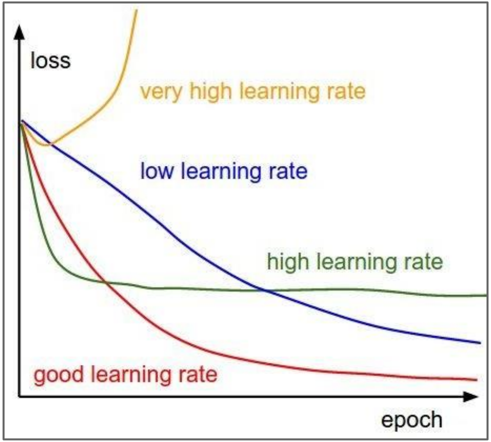
Adam

SGD, SGD+Momentum, Adagrad, RMSProp, Adam all have **learning rate** as a hyperparameter.



Q: Which one of these learning rates is best to use?

SGD, SGD+Momentum, Adagrad, RMSProp, Adam all have **learning rate** as a hyperparameter.



=> Learning rate decay over time!

step decay:
e.g. decay learning rate by half every few epochs.

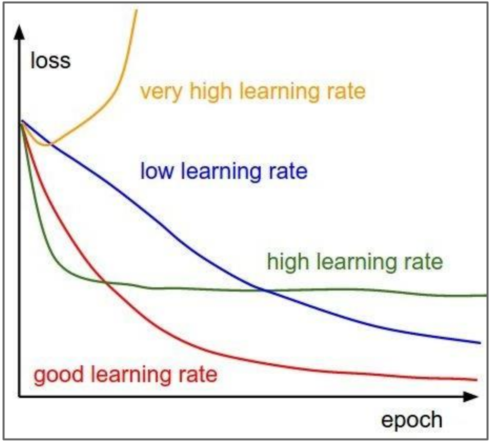
exponential decay:

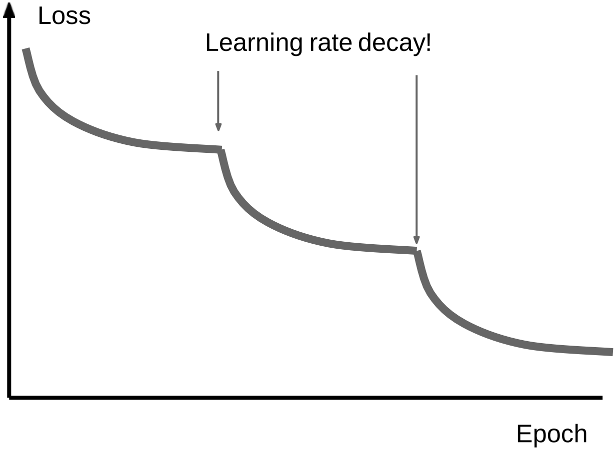
$$\alpha = \alpha_0 e^{-kt}$$

1/t decay:

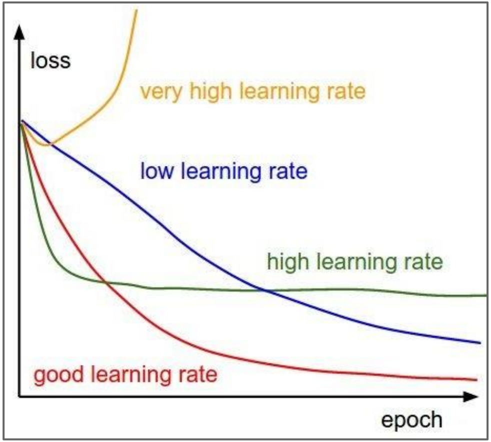
$$\alpha = \alpha_0 / (1 + kt)$$

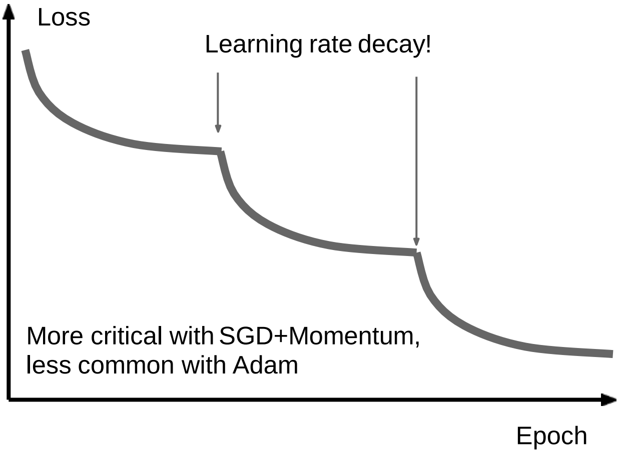
SGD, SGD+Momentum, Adagrad, RMSProp, Adam all have **learning rate** as a hyperparameter.



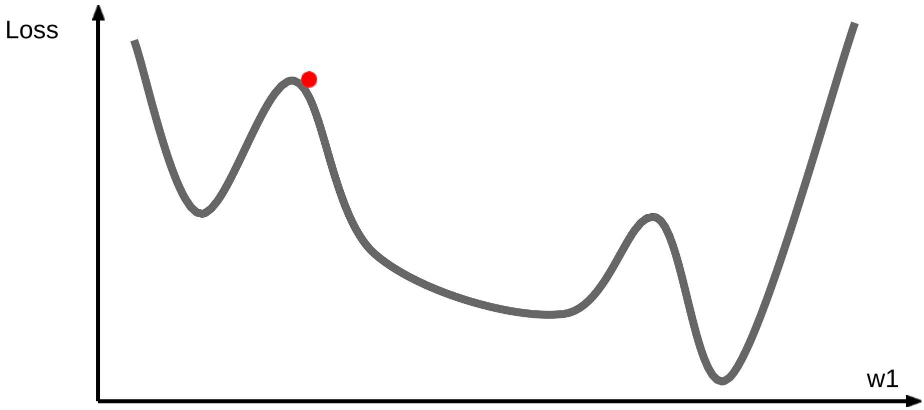


SGD, SGD+Momentum, Adagrad, RMSProp, Adam all have **learning rate** as a hyperparameter.



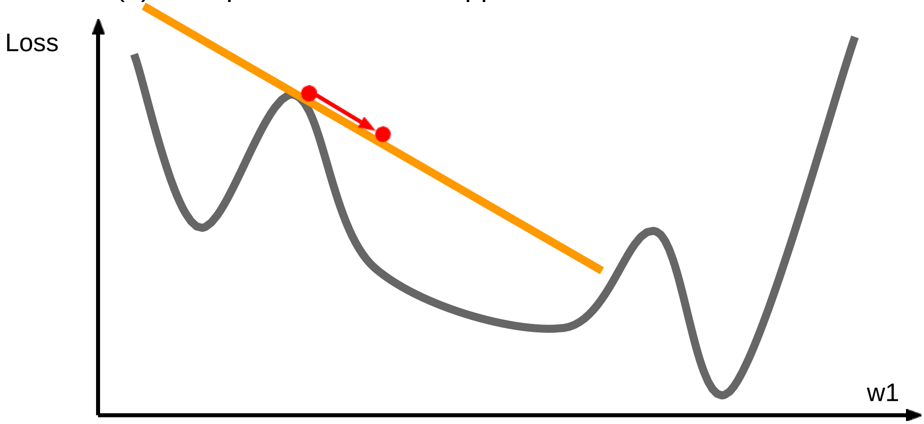


First-Order Optimization



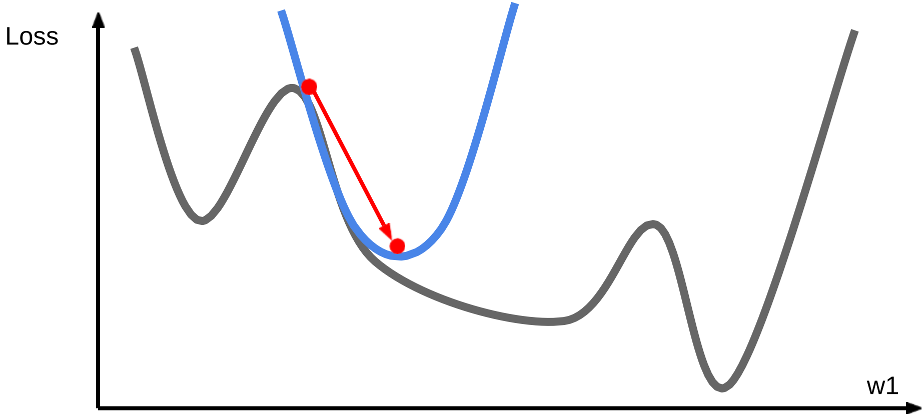
First-Order Optimization

(1) Use gradient form linear approximation
(2) Step to minimize the approximation



Second-Order Optimization

(1) Use gradient **and Hessian** to form **quadratic** approximation
(2) Step to the **minima** of the approximation



Advantage and Disadvantage of Second-Order Optimization

Advantage:
- No hyperparameters
- No learning rate

Disadvantage:
- High complexity which is bad for deep learning

L-BFGS (Limited-memory BFGS)

- **Usually works very well in full batch, deterministic mode** i.e. if you have a single, deterministic f(x) then L-BFGS will probably work very nicely

- **Does not transfer very well to mini-batch setting**. Gives bad results. Adapting second-order methods to large-scale, stochastic setting is an active area of research.

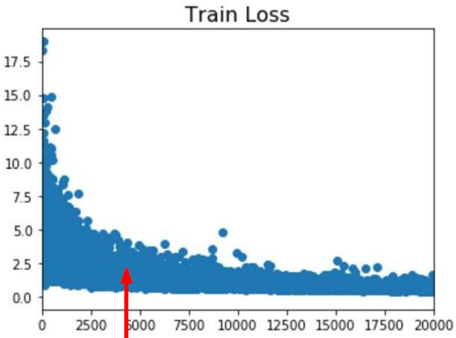
Le et al, "On optimization methods for deep learning, ICML 2011"
Ba et al, "Distributed second-order optimization using Kronecker-factored approximations", ICLR 2017

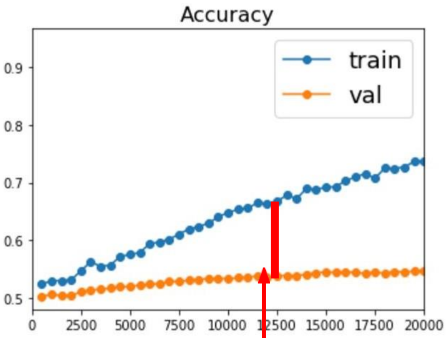
In practice:

- **Adam** is a good default choice in many cases
- **SGD+Momentum** with learning rate decay often outperforms Adam by a bit, but requires more tuning

- If you can afford to do full batch updates then try out **L-BFGS** (and don’t forget to disable all sources of noise)

Beyond Training Error





Better optimization algorithms help reduce training loss

But we really care about error on new data - how to reduce the gap?

Model Ensembles

1. Train multiple independent models or
2. Use multiple snapshots of a single model during training

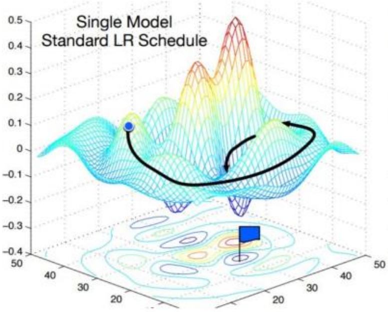
At test time average their results

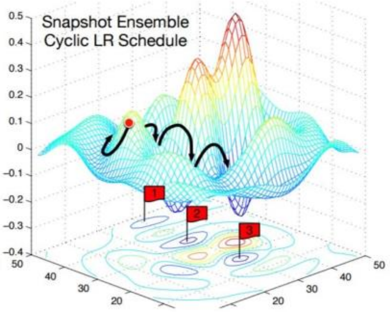
(Take average of predicted probability distributions, then choose argmax)

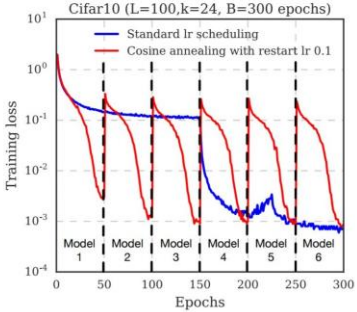
Enjoy 2% extra performance

Model Ensembles: Tips and Tricks

Instead of training independent models, use multiple snapshots of a single model during training!



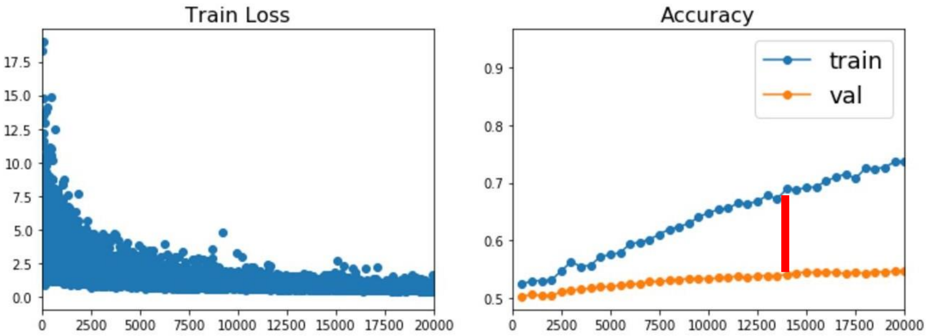




Loshchilov and Hutter, “SGDR: Stochastic gradient descent with restarts”, arXiv 2016
Huang et al, “Snapshot ensembles: train 1, get M for free”, ICLR 2017

Cyclic learning rate schedules can make this work even better!

How to improve single-model performance?

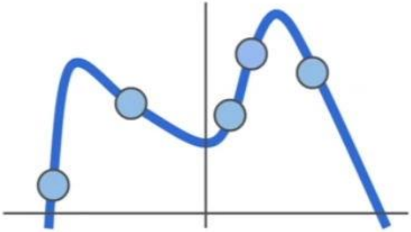


Regularization

What is missing in this Loss function?

$$\underbrace{L(W) = \frac{1}{N}\sum_{i=1}^{N} L_i(f(x_i, W), y_i)}$$

Data loss: Model predictions should match training data (encourage overfitting)



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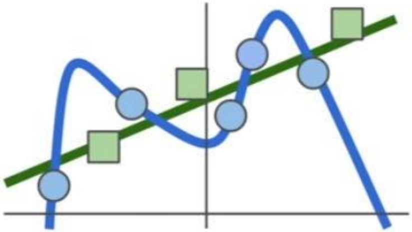
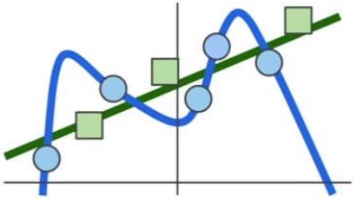


Regularization

$$L(W) = \underbrace{\frac{1}{N}\sum_{i=1}^{N} L_i(f(x_i, W), y_i)}_{\text{Data loss}} + \underbrace{\lambda R(W)}_{\text{Regularization}}$$

Data loss: Model predictions should match training data



Regularization: Technique to discourage the complexity of the model (i.e, express preferences over weights). It does this by penalizing the loss function. This helps to solve the overfitting problem.

Regularization parameter λ

Regularization works on assumption that smaller weights generate simpler model and thus helps avoid **overfitting**.

λ is the **penalty term** or **regularization parameter** which determines how much to penalizes the weights.

- **λ = 0,** then the regularization term becomes zero (back to the original Loss function).

- **λ is large**, the weights become close to zero (i.e. a very simple model have **underfitting**).

λ is a *hyperparameter* between 0 and a large value.

Regularization function *R(W)*

Simple Examples:

L2 regularization: $R(W) = \sum_k \sum_l W_{k,l}^2$

L1 regularization: $R(W) = \sum_k \sum_l |W_{k,l}|$

Elastic net (L1 + L2): $R(W) = \sum_k \sum_l \beta W_{k,l}^2 + |W_{k,l}|$

More complex:

Dropout

Batch normalization

Stochastic depth, fractional pooling, etc

L1 regularization

L1 regularization: $R(W) = \sum_k \sum_l |W_{k,l}|$

Built-in feature selection : L1 regularization does feature selection. It does this by **assigning insignificant input features with zero weight** and useful features with a non zero weight.

$$\boldsymbol{w_1} = [\mathbf{1}, \mathbf{0}, \mathbf{0}, \mathbf{0}]$$

Sparsity : In L1 regularization we shrink the parameters to zero. When input features have weights closer to zero that leads to sparse L1 norm. In Sparse solution majority of the input features have zero weights and very few features have non zero weights.

L2 regularization

$$\underline{\text{L2 regularization:}}\ R(W) = \sum_k \sum_l W_{k,l}^2$$

L2 regularization forces the weights to be small but does not make them zero and does **non-sparse solution**.

$$w_2 = [0.25, 0.25, 0.25, 0.25]$$

L2 regularization likes to "spread out" the weights

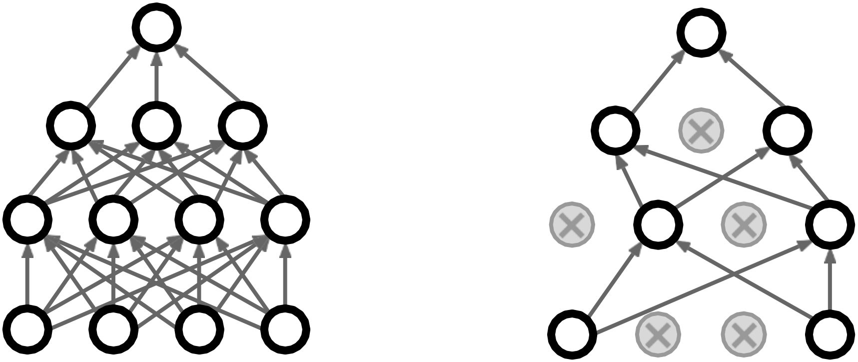
*L2 has **no feature selection,** it gives better prediction when output variable is a function of all input features*

L2 regularization is able to learn complex data patterns

Regularization: Dropout

In each forward pass, randomly set some neurons to zero

Probability of dropping is a hyperparameter; 0.5 is common



Srivastava et al, “Dropout: A simple way to prevent neural networks from overfitting”, JMLR 2014

Regularization: Dropout

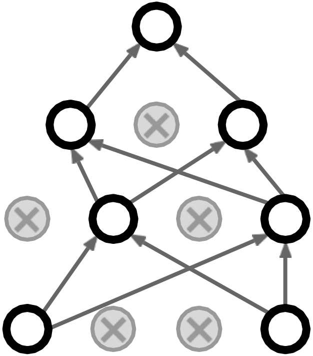
```
p = 0.5 # probability of keeping a unit active. higher = less dropout

def train_step(X):
  """ X contains the data """

  # forward pass for example 3-layer neural network
  H1 = np.maximum(0, np.dot(W1, X) + b1)
  U1 = np.random.rand(*H1.shape) < p # first dropout mask
  H1 *= U1 # drop!
  H2 = np.maximum(0, np.dot(W2, H1) + b2)
  U2 = np.random.rand(*H2.shape) < p # second dropout mask
  H2 *= U2 # drop!
  out = np.dot(W3, H2) + b3

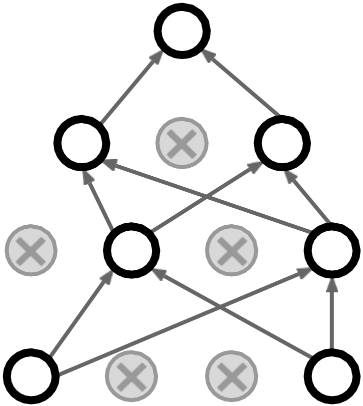
  # backward pass: compute gradients... (not shown)
  # perform parameter update... (not shown)
```

Example forward pass with a 3-layer network using dropout

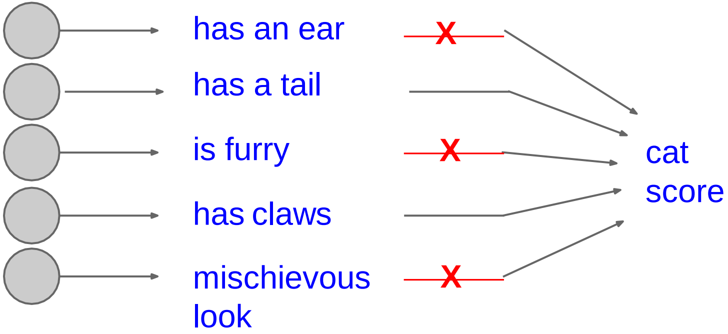


Regularization: Dropout

How can this possibly be a good idea?

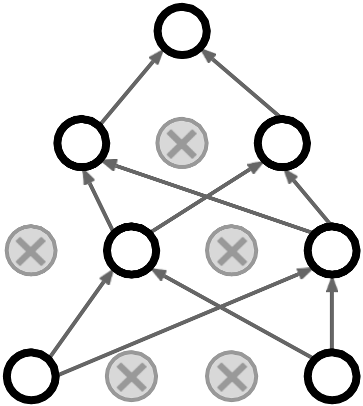


Forces the network to have a redundant representation;
Prevents co-adaptation of features



Regularization: Dropout

How can this possibly be a good idea?



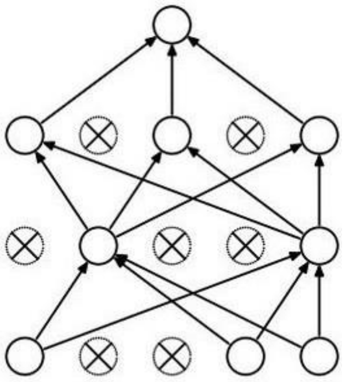
Another interpretation:

Dropout is training a large **ensemble** of models (that share parameters).

Each binary mask is one model

An FC layer with 4096 units has $2^{4096} \sim 10^{1233}$ possible masks!

At test time….



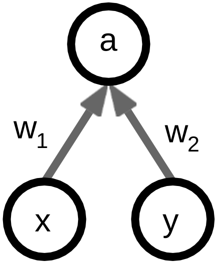
Ideally:
want to integrate out all the noise

Monte Carlo approximation:
do many forward passes with different dropout masks, average all predictions

At test time….

Can in fact do this with a single forward pass! (approximately)

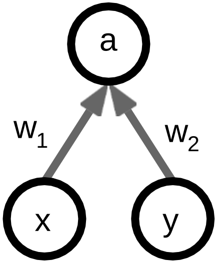
Leave all input neurons turned on (no dropout).



(this can be shown to be an approximation to evaluating the whole ensemble)

Dropout: Test time

Can in fact do this with a single forward pass! (approximately)
Leave all input neurons turned on (no dropout).



At test time we have: $E[a] = w_1 x + w_2 y$

During training we have:

$$E[a] = \frac{1}{4}(w_1 x + w_2 y) + \frac{1}{4}(w_1 x + 0y) + \frac{1}{4}(0x + 0y) + \frac{1}{4}(0x + w_2 y) = \frac{1}{2}(w_1 x + w_2 y)$$

With p=0.5, using all inputs in the forward pass would inflate the activations by 2x from what the network was "used to" during training!

=> Have to compensate by scaling the activations back down by ½

At test time, **multiply** by dropout probability

Dropout: Test time

```
def predict(X):
  # ensembled forward pass
  H1 = np.maximum(0, np.dot(W1, X) + b1) * p # NOTE: scale the activations
  H2 = np.maximum(0, np.dot(W2, H1) + b2) * p # NOTE: scale the activations
  out = np.dot(W3, H2) + b3
```

At test time all neurons are active always

=> We must scale the activations so that for each neuron:

<u>output at test time</u> = <u>expected output at training time</u>

Dropout Summary

```python
""" Vanilla Dropout: Not recommended implementation (see notes below) """

p = 0.5 # probability of keeping a unit active. higher = less dropout

def train_step(X):
  """ X contains the data """

  # forward pass for example 3-layer neural network
  H1 = np.maximum(0, np.dot(W1, X) + b1)
  U1 = np.random.rand(*H1.shape) < p # first dropout mask
  H1 *= U1 # drop!
  H2 = np.maximum(0, np.dot(W2, H1) + b2)
  U2 = np.random.rand(*H2.shape) < p # second dropout mask
  H2 *= U2 # drop!
  out = np.dot(W3, H2) + b3

  # backward pass: compute gradients... (not shown)
  # perform parameter update... (not shown)

def predict(X):
  # ensembled forward pass
  H1 = np.maximum(0, np.dot(W1, X) + b1) * p # NOTE: scale the activations
  H2 = np.maximum(0, np.dot(W2, H1) + b2) * p # NOTE: scale the activations
  out = np.dot(W3, H2) + b3
```

drop in forward pass

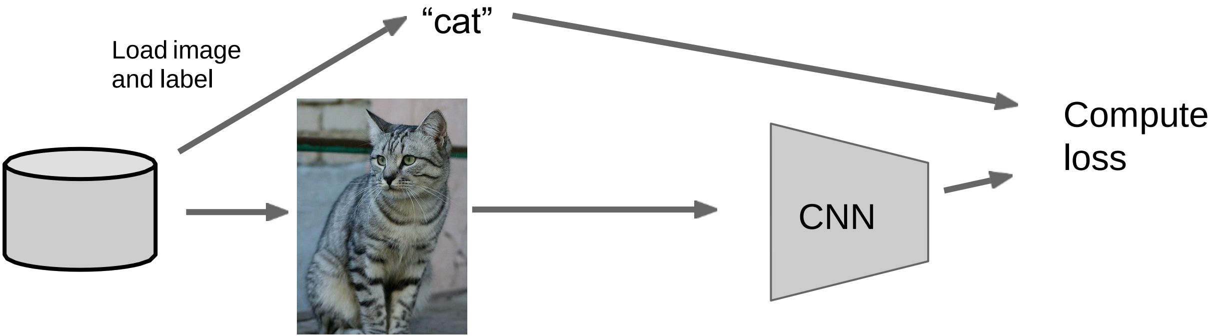
scale at test time

More common: “Inverted dropout”

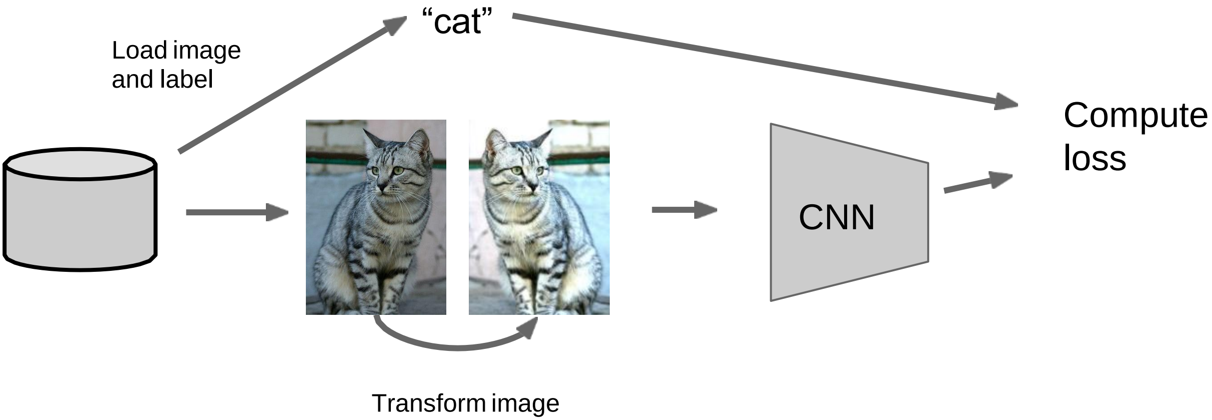
```python
p = 0.5 # probability of keeping a unit active. higher = less dropout

def train_step(X):
  # forward pass for example 3-layer neural network
  H1 = np.maximum(0, np.dot(W1, X) + b1)
  U1 = (np.random.rand(*H1.shape) < p) / p # first dropout mask. Notice /p!
  H1 *= U1 # drop!
  H2 = np.maximum(0, np.dot(W2, H1) + b2)
  U2 = (np.random.rand(*H2.shape) < p) / p # second dropout mask. Notice /p!
  H2 *= U2 # drop!
  out = np.dot(W3, H2) + b3

  # backward pass: compute gradients... (not shown)
  # perform parameter update... (not shown)

def predict(X):
  # ensembled forward pass
  H1 = np.maximum(0, np.dot(W1, X) + b1) # no scaling necessary
  H2 = np.maximum(0, np.dot(W2, H1) + b2)
  out = np.dot(W3, H2) + b3
```

test time is unchanged!

Regularization: Data Augmentation



Regularization: Data Augmentation

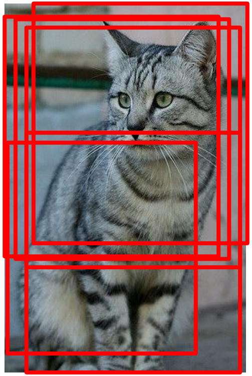


Random crops and scales

Training: sample random crops / scales

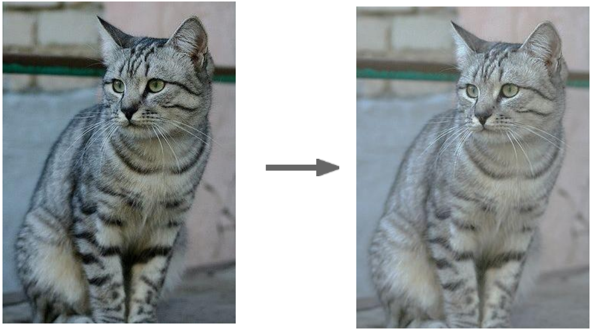
ResNet:

1. Pick random L in range [256, 480]
2. Resize training image, short side = L
3. Sample random 224 x 224 patch



Color Jitter

Simple: Randomize
contrast and brightness



Data Augmentation

Get creative for your problem!

Random mix/combinations of :

- translation
- rotation
- stretching
- shearing,
- …

Regularization: A common pattern

Training: Add random noise

Testing: Marginalize over the noise

Examples:

Dropout

Batch Normalization

Data Augmentation

Regularization: A common pattern

Training: Add random noise

Testing: Marginalize over the noise

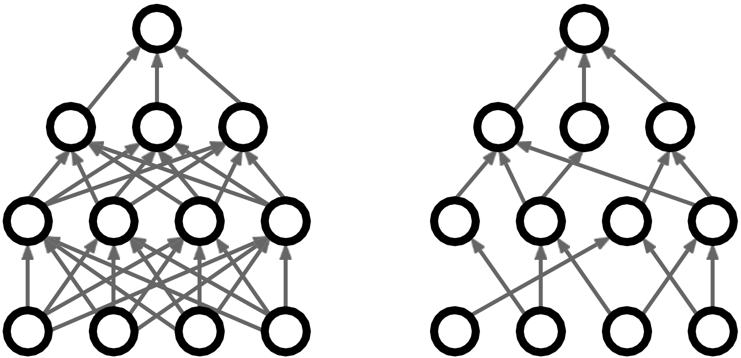
Examples:

Dropout

Batch Normalization

Data Augmentation

DropConnect

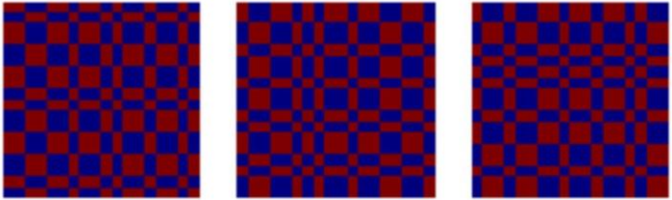


Wan et al, “Regularization of Neural Networks using DropConnect”, ICML 2013

Regularization: A common pattern

Training: Add random noise
Testing: Marginalize over the noise

Examples:
Dropout
Batch Normalization
Data Augmentation
DropConnect
Fractional Max Pooling

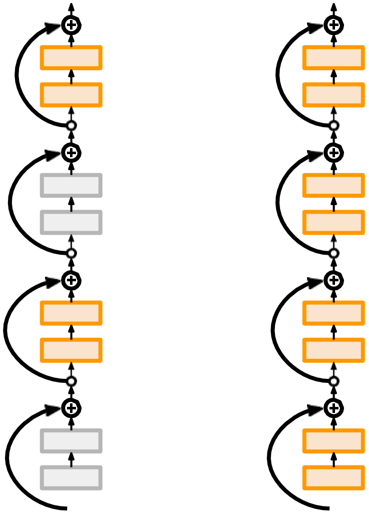


Graham, “Fractional Max Pooling”, arXiv 2014

Regularization: A common pattern

Training: Add random noise
Testing: Marginalize over the noise

Examples:
Dropout
Batch Normalization
Data Augmentation
DropConnect
Fractional Max Pooling
Stochastic Depth



Huang et al, “Deep Networks with Stochastic Depth”, ECCV 2016

Summary

- Optimization
  - Momentum, RMSProp, Adam, etc
- Regularization
  - Dropout, etc