COL 106 Lecture 37

Topic : Depth - First Traversal

Depth-first Traversal with Arrival and Departure Time.

arr, dep: global arrays with integer entry for every vertex

count : global integer variable

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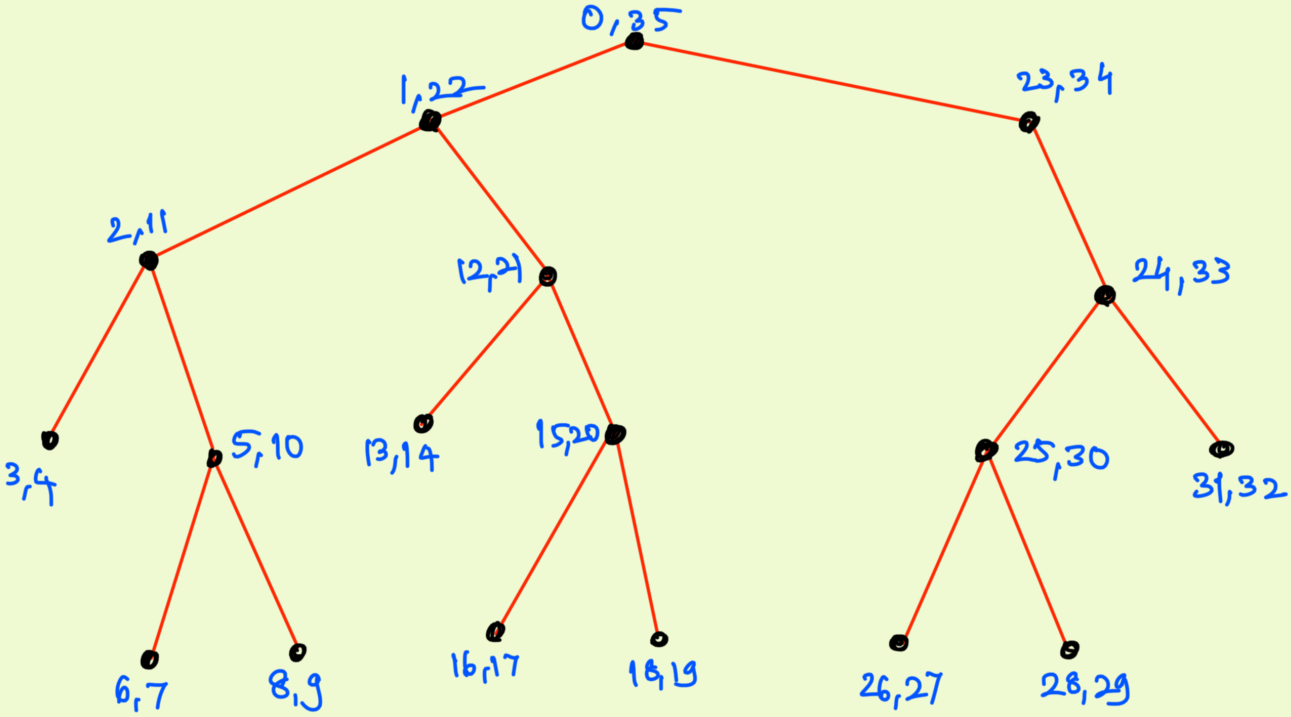
DFT(s):
    Set arr[v] ← -1, dep[v] ← -1 for every vertex v
    count ← 0
    DFT_rec(S)

DFT_rec(v):
    arr[v] ← count
    count ← count + 1.
    For each neighbor u of v:
        If arr[u] = -1:
            prev[u] ← v
            DFT_rec(u)
    dep[v] ← count
    count ← count + 1
```



Assume $G=(V,E)$: connected graph, s: vertex of G.

Run DFT(s) and let $T=(V,E')$, where

$$E' = \{\, \{v, prev[v]\} \mid v \in V \,\}.$$

What kind of graph is T?

Claim: DFT(s) visits every vertex v of G if G is connected

(If G is not connected, DFT(s) visits all vertices in the connected component of s.)

Proof: ?

$|E'| = |V| - 1$

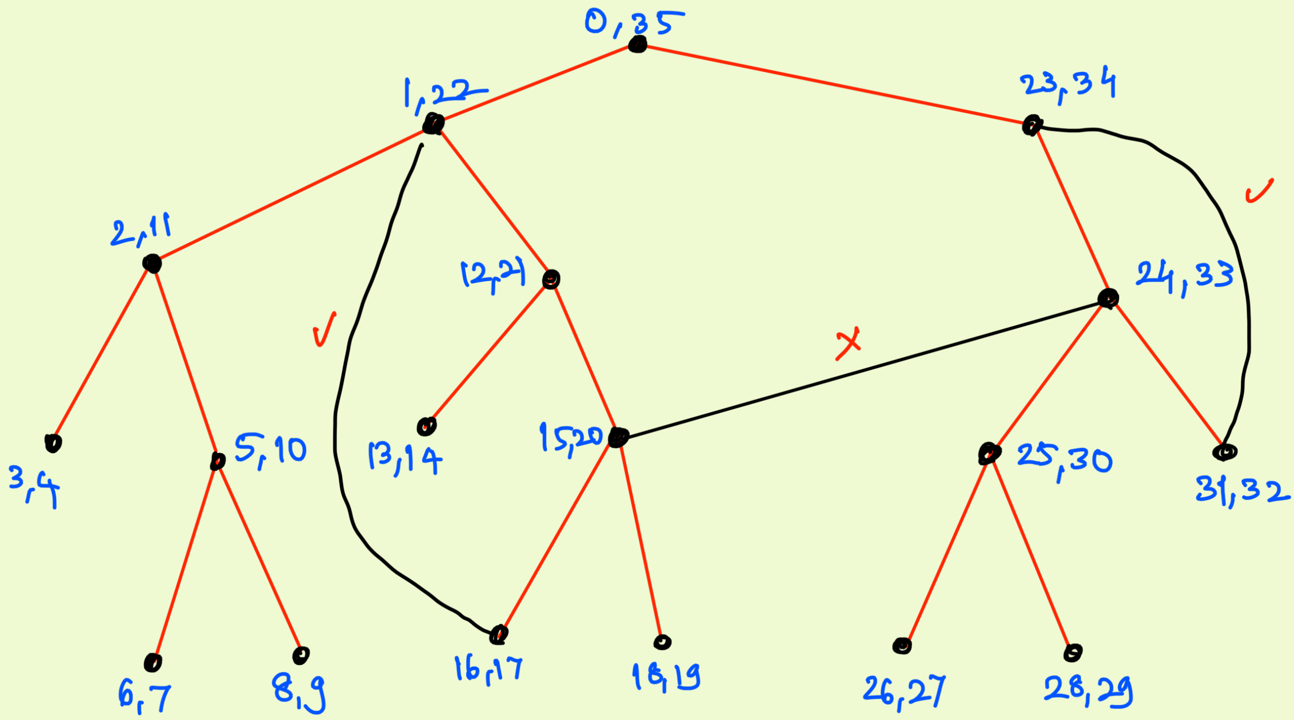
T is connected (because every vertex is connected to s)

$\therefore$ T is a tree.

Edges of T are called "tree edges" of the DFT.

Question: Where are the non-tree edges of the graph with respect to a DFT tree?

(Recall: With respect to a BFT tree, edges are only between vertices in same and adjacent levels.)



Which of the black edges can possibly exist in G?

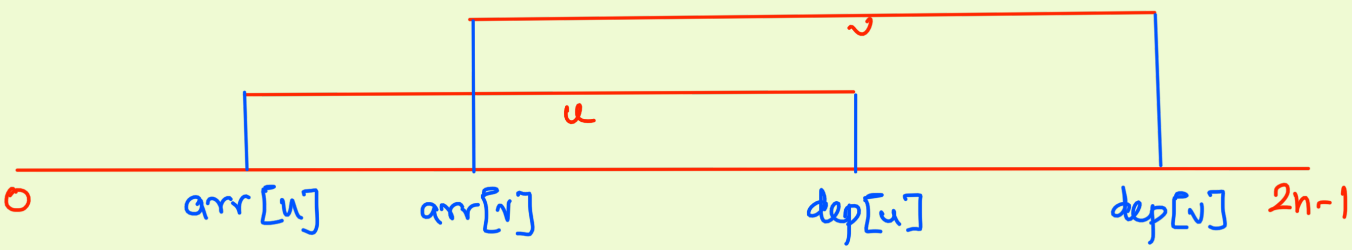
Observation: If $\{u,v\}$ is a non-tree edge, one of u,v is an ancestor of the other in the DFT tree.

The non-tree edges are, therefore, called "back edges".

Laminarity property.

Consider the set $\{[arr[v], dep[v]] \mid v \in V\}$ of intervals.

Is the following scenario possible?



Observation:

1. If u is a descendant of v in a DFT tree, then

$$arr[v] < arr[u] < dep[u] < dep[v]$$

ie $$[arr[u], dep[u]] \subseteq [arr[v], dep[v]]$$

2. If none of u, v is a descendant of the other, then

$$[arr[u], dep[u]] \cap [arr[v], dep[v]] = \emptyset.$$