Sol^n

1st observe that number of possible binary trees with n nodes i.e. structure wise diff only then this number is equal to number of Binary trees having level order traversal $= K_0 K_1 \ldots K_{n-1}$.

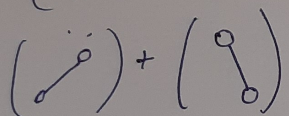
Because for each ~~arrangement~~ structure we would get a <u>unique</u> arrangement of keys.

So we need to find number of all possible structures of binary trees with n nodes.

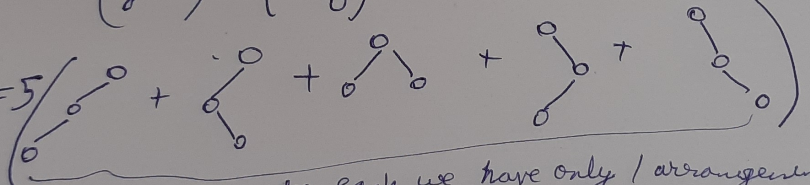
$C_0 = 1$ (no node)

$C_1 = 1$ (o)

$C_2 = 2$



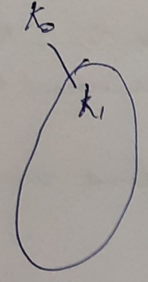
$C_3 = 5$



for each we have only 1 arrangement of K_i's.

Now, K_0 must be a <u>root</u> in all cases.

K_0 has left and right subtrees.

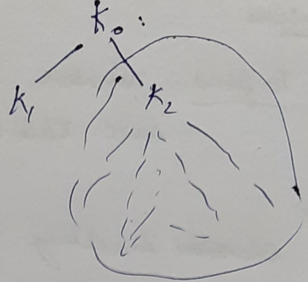


nodes in left subtree $= 0$

nodes in right $= n-1$

no. of such trees $= C_0 \cdot C_{n-1}$

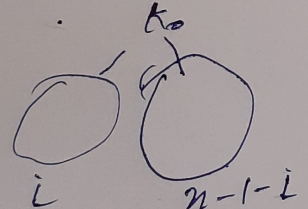
as now K_1 is root node w.r.t that. and ~~left~~ have $(n-1)$ total nodes $\therefore C_{n-1}$



left $= 1$

right $= n-2$

no. of such B.Ts $= C_1 C_{n-1}$



$\Rightarrow$ such B.T.s $= C_i \cdot C_{n-1-i}$

By this we can say

$$C_n = C_0 C_{n-1} + C_1 C_{n-2} + \dots + C_{n-2} C_1 + C_{n-1} C_0$$

$$= \sum_{i=0}^{n-1} C_i C_{n-1-i} = \sum_{i=1}^{n} C_{i-1} C_{n-i}$$

$\Rightarrow C_n$ is <u>Catalan Number</u> $\Rightarrow$ $\boxed{C_n = \dfrac{\binom{2n}{n}}{n+1}}$