

Name: _____

Entry number: _____

COL106

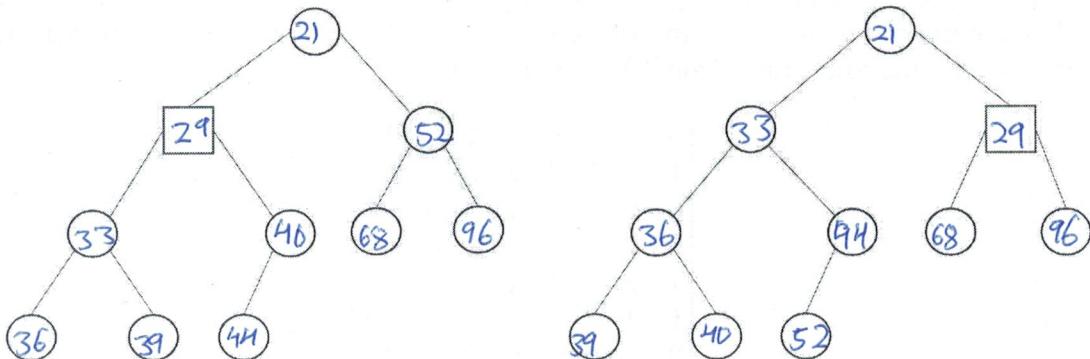
Quiz A

Duration: 30 minutes

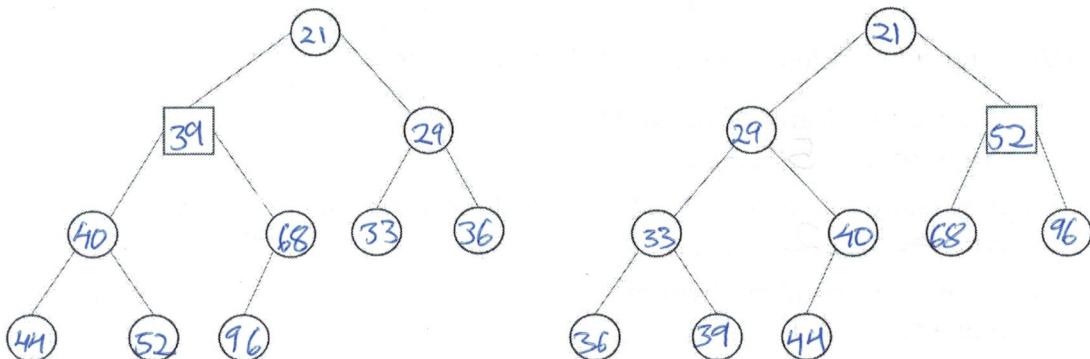
Read the following instructions before you begin writing.

1. Keep a pen, your identity card, and optionally a water bottle with you. Keep everything else away from you, at the place specified by the invigilators.
2. Write your entry number and name on every page. (Sheets will be separated prior to grading.)
3. Answer only in the designated space. Think before you use this space. No additional space will be provided for writing answers.
4. No clarifications will be given during the exams. If something is unclear or ambiguous, make reasonable assumptions and state them clearly. The instructors reserve the right to decide whether your assumptions were indeed reasonable.

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1. The set of keys in a min-heap is the set $\{21, 29, 33, 36, 39, 40, 44, 52, 68, 96\}$. Complete the diagrams by filling keys in the circles/square to create a valid min-heap so that
 - (a) (1 point) the square node has the **smallest** possible key



- (b) (2 points) the square node has the **largest** possible key



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Quiz A

Duration: 30 minutes

2. (4 points) A k -AVL tree is a binary search tree in which the heights of the left and right subtrees of every node differ by at most k (the usual AVL trees discussed in class are 1-AVL trees). Determine whether each of the following statements is true or false.

1. The worst-case time for searching a key in a k -AVL tree decreases as k increases.

Answer: False

2. The worst-case time for finding the predecessor of a node of an k -AVL tree decreases as k increases.

Answer: False

3. The largest possible height of a k -AVL tree containing a fixed number n of nodes increases as k increases.

Answer: True

4. The smallest possible number of nodes in a k -AVL tree of a fixed height h increases as k increases.

Answer: False

3. (3 points) G is an unknown graph over the vertex set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and A is the unknown adjacency matrix of G , whose i 'th row and column represent the vertex i (numbering starts from 0). We know that

$$A^2 = \begin{bmatrix} 5 & 1 & 0 & 0 & 1 & 1 & 2 & 2 & 2 & 1 \\ 1 & 5 & 1 & 3 & 2 & 2 & 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 1 & 3 & 1 & 2 & 0 & 1 & 0 & 1 \\ 1 & 2 & 1 & 1 & 2 & 0 & 0 & 1 & 0 & 1 \\ 1 & 2 & 0 & 2 & 0 & 4 & 2 & 2 & 1 & 1 \\ 2 & 1 & 0 & 0 & 0 & 2 & 4 & 2 & 3 & 1 \\ 2 & 1 & 1 & 1 & 1 & 2 & 2 & 4 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 & 3 & 1 & 3 & 1 \\ 1 & 3 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 3 \end{bmatrix}$$

Using this information, answer the following questions.

1. What is the degree of the vertex 0?

Answer: 5

2. How many common adjacent vertices do vertices 1 and 5 have?

Answer: 2

3. What is the number of edges?

Answer: 17

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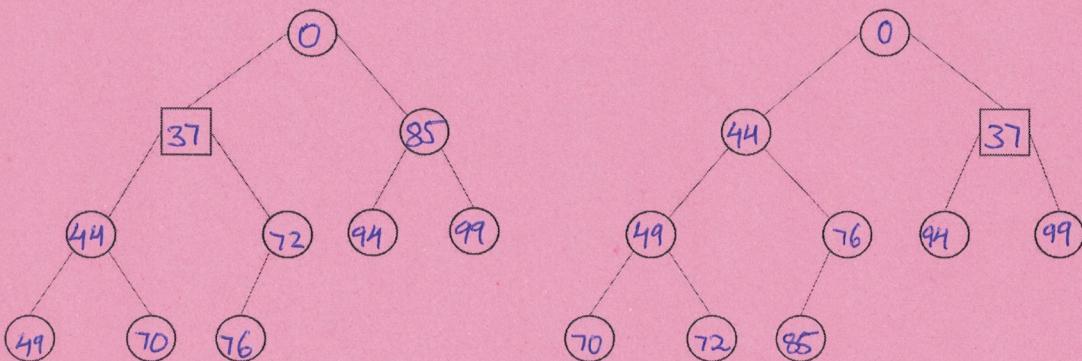
Quiz B

Duration: 30 minutes

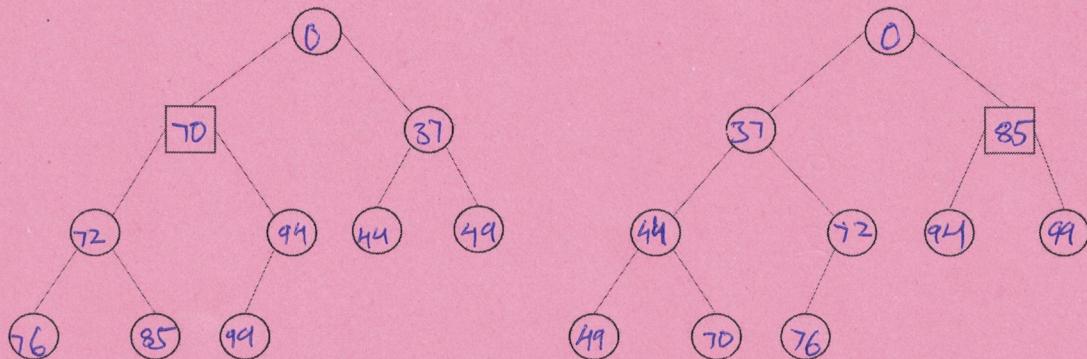
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1. The set of keys in a min-heap is the set $\{0, 37, 44, 49, 70, 72, 76, 85, 94, 99\}$. Complete the diagrams by filling keys in the circles/square to create a valid min-heap so that
 - (a) (1 point) the square node has the **smallest** possible key



- (b) (2 points) the square node has the **largest** possible key



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Entry number: _____

COL106

Quiz B

Duration: 30 minutes

2. (4 points) A k -AVL tree is a binary search tree in which the heights of the left and right subtrees of every node differ by at most k (the AVL trees discussed in class are 1-AVL trees). Determine whether each of the following statements is true or false.

1. The smallest possible number of nodes in a k -AVL tree of a fixed height h increases as k increases.

Answer: False

2. The worst-case time for searching a key in a k -AVL tree increases as k increases.

Answer: True

3. The worst-case time for finding the predecessor of a node of an k -AVL tree decreases as k increases.

Answer: False

4. The largest possible height of a k -AVL tree containing a fixed number n of nodes decreases as k increases.

Answer: True False

3. (3 points) G is an unknown graph over the vertex set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and A is the unknown adjacency matrix of G , whose i 'th row and column represent the vertex i (numbering starts from 0). We know that

$$A^2 = \begin{bmatrix} 7 & 3 & 4 & 1 & 5 & 6 & 4 & 3 & 2 & 4 \\ 3 & 5 & 5 & 1 & 4 & 3 & 2 & 1 & 2 & 3 \\ 4 & 5 & 7 & 1 & 4 & 4 & 3 & 2 & 3 & 4 \\ 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 0 & 2 \\ 5 & 4 & 4 & 1 & 6 & 5 & 3 & 2 & 2 & 3 \\ 6 & 3 & 4 & 1 & 5 & 7 & 4 & 3 & 2 & 4 \\ 4 & 2 & 3 & 1 & 3 & 4 & 5 & 2 & 3 & 5 \\ 3 & 1 & 2 & 1 & 2 & 3 & 2 & 4 & 0 & 2 \\ 2 & 2 & 3 & 0 & 2 & 2 & 3 & 0 & 3 & 3 \\ 4 & 3 & 4 & 2 & 3 & 4 & 5 & 2 & 3 & 6 \end{bmatrix}$$

Using this information, answer the following questions.

1. What is the degree of the vertex 4?

Answer: 6

2. How many common adjacent vertices do vertices 1 and 8 have?

Answer: 2

3. What is the number of edges?

Answer: 26

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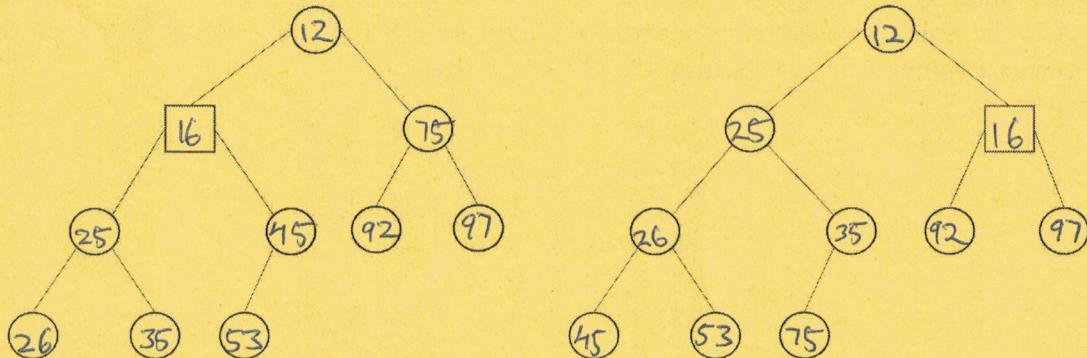
COL106

Quiz C

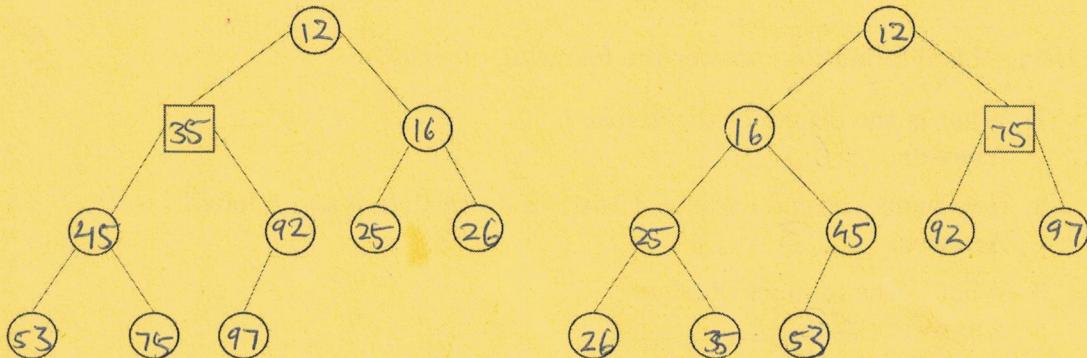
Duration: 30 minutes

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-
1. The set of keys in a min-heap is the set $\{12, 16, 25, 26, 35, 45, 53, 75, 92, 97\}$. Complete the diagrams by filling keys in the circles/square to create a valid min-heap so that
 - (a) (1 point) the square node has the **smallest** possible key



- (b) (2 points) the square node has the **largest** possible key



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Entry number: _____

COL106

Quiz C

Duration: 30 minutes

2. (4 points) A k -AVL tree is a binary search tree in which the heights of the left and right subtrees of every node differ by at most k (the AVL trees discussed in class are 1-AVL trees). Determine whether each of the following statements is true or false.

1. The largest possible height of a k -AVL tree containing a fixed number n of nodes decreases as k increases.

Answer: False

2. The smallest possible number of nodes in a k -AVL tree of a fixed height h decreases as k increases.

Answer: True

3. The worst-case time for searching a key in a k -AVL tree increases as k increases.

Answer: True

4. The worst-case time for finding the predecessor of a node of an k -AVL tree increases as k increases.

Answer: True

3. (3 points) G is an unknown graph over the vertex set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and A is the unknown adjacency matrix of G , whose i 'th row and column represent the vertex i (numbering starts from 0). We know that

$$A^2 = \begin{bmatrix} 6 & 3 & 1 & 4 & 3 & 3 & 1 & 4 & 2 & 2 \\ 3 & 6 & 1 & 5 & 3 & 2 & 2 & 2 & 1 & 5 \\ 1 & 1 & 3 & 0 & 3 & 2 & 1 & 3 & 2 & 2 \\ 4 & 5 & 0 & 6 & 3 & 3 & 1 & 2 & 2 & 4 \\ 3 & 3 & 3 & 3 & 6 & 3 & 2 & 4 & 4 & 4 \\ 3 & 2 & 2 & 3 & 3 & 5 & 0 & 4 & 3 & 3 \\ 1 & 2 & 1 & 1 & 2 & 0 & 2 & 1 & 0 & 2 \\ 4 & 2 & 3 & 2 & 4 & 4 & 1 & 6 & 3 & 3 \\ 2 & 1 & 2 & 2 & 4 & 3 & 0 & 3 & 4 & 2 \\ 2 & 5 & 2 & 4 & 4 & 3 & 2 & 3 & 2 & 6 \end{bmatrix}$$

Using this information, answer the following questions.

1. What is the degree of the vertex 1

Answer: 6

2. How many common adjacent vertices do vertices 0 and 5 have?

Answer: 3

3. What is the number of edges?

Answer: 25

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Entry number: _____

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Quiz D

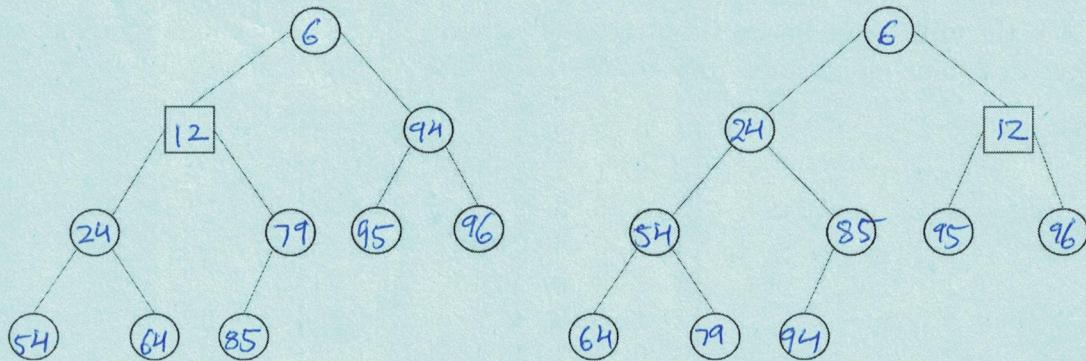
Duration: 30 minutes

Read the following instructions before you begin writing.

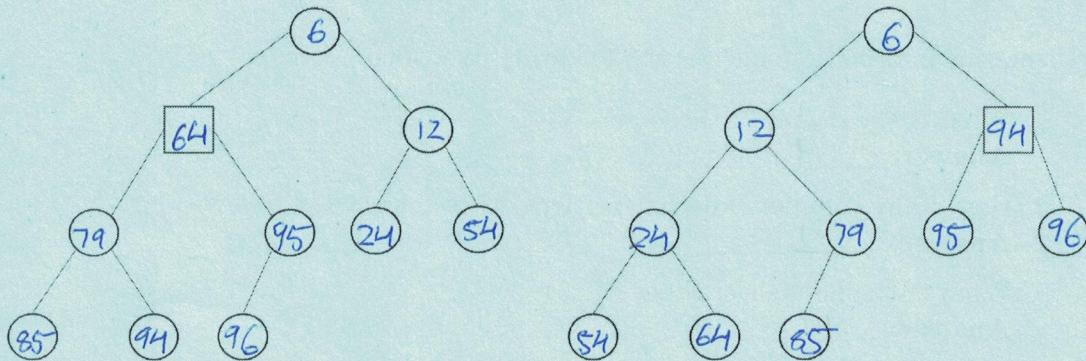
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4. No clarifications will be given during the exams. If something is unclear or ambiguous, make reasonable assumptions and state them clearly. The instructors reserve the right to decide whether your assumptions were indeed reasonable.

-
1. The set of keys in a min-heap is the set $\{6, 12, 24, 54, 64, 79, 85, 94, 95, 96\}$. Complete the diagrams by filling keys in the circles/square to create a valid min-heap so that

- (a) (1 point) the square node has the **smallest** possible key



- (b) (2 points) the square node has the **largest** possible key



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Quiz D

Duration: 30 minutes

2. (4 points) A k -AVL tree is a binary search tree in which the heights of the left and right subtrees of every node differ by at most k (the AVL trees discussed in class are 1-AVL trees). Determine whether each of the following statements is true or false.

1. The worst-case time for finding the predecessor of a node of an k -AVL tree increases as k increases.

Answer: True

2. The largest possible height of a k -AVL tree containing a fixed number n of nodes increases as k increases.

Answer: True

3. The smallest possible number of nodes in a k -AVL tree of a fixed height h decreases as k increases.

Answer: True

4. The worst-case time for searching a key in a k -AVL tree decreases as k increases.

Answer: False

3. (3 points) G is an unknown graph over the vertex set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and A is the unknown adjacency matrix of G , whose i 'th row and column represent the vertex i (numbering starts from 0). We know that

$$A^2 = \begin{bmatrix} 4 & 0 & 2 & 2 & 1 & 1 & 3 & 2 & 0 & 3 \\ 0 & 2 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 2 & 1 & 6 & 3 & 2 & 3 & 1 & 0 & 3 & 2 \\ 2 & 1 & 3 & 4 & 0 & 0 & 1 & 1 & 1 & 2 \\ 1 & 1 & 2 & 0 & 4 & 4 & 1 & 2 & 1 & 1 \\ 1 & 1 & 3 & 0 & 4 & 5 & 1 & 2 & 2 & 1 \\ 3 & 0 & 1 & 1 & 1 & 1 & 3 & 2 & 0 & 2 \\ 2 & 1 & 0 & 1 & 2 & 2 & 2 & 4 & 0 & 1 \\ 0 & 1 & 3 & 1 & 1 & 2 & 0 & 0 & 3 & 0 \\ 3 & 0 & 2 & 2 & 1 & 1 & 2 & 1 & 0 & 3 \end{bmatrix}.$$

Using this information, answer the following questions.

1. What is the degree of the vertex 3?

Answer: 4

2. How many common adjacent vertices do vertices 2 and 6 have?

Answer: 1

3. What is the number of edges?

Answer: 19