Question 1

The root must contain the min key in all the 4 cases.

Minimum possible key in the square node [0.5 mark each]: put the second-minimum key in the square, and then complete the placement of keys in an arbitrary manner respecting the heap condition.

Maximum possible key in the square node [1 mark each]: For the diagram on the left (resp. right), the square node has 6 (resp. 3) descendants including itself, so its content can be at most the 6th-highest (resp. 3rd-highest) key. Moreover, it is possible to complete the placement of keys respecting the heap condition by putting the 6th-highest (resp. 3rd-highest) key in the square. So, a correct answer is one which satisfies the heap condition, and which has the 6th-highest (resp. 3rd-highest) key in the square.

Question 2 [1 mark each]

The following statements are true.

1. The smallest possible number of nodes in a $k$-AVL tree of a fixed height $h$ decreases as $k$ increases.
2. The largest possible height of a $k$-AVL tree containing a fixed number $n$ of nodes increases as $k$ increases.
3. The worst-case time for searching a key in a k-AVL tree increases as k increases.
4. The worst-case time for finding the predecessor of a node of an $k$-AVL tree increases as $k$ increases.

In each question paper, some of these statements have been negated and the statements have been permuted.

Question 3 [1 mark each]

1. The degree of vertex i is the (i,i)th entry of A^2.
2. The number of common neighbors of vertices i and j is the (i,j)th entry of A^2.
3. The number of edges is half the sum of degrees, that is, half the sum of diagonal entries of A^2.