COL215 Assignment 3

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Our Algorithm:

We initialized a list (l) to which we will add the final terms of the expansion.

Step1:

We start with the list (s) of minterms in the union of func_TRUE and func_DC. Using the MarkRegion() function, we pair up the possible terms in s and add the pairs into a list (m).
To make pairs, we create a set with all the elements in the list. A set is implemented using hash tables. So, we can add, remove or search for elements in constant time.
We traversed through the list s and checked if its neighbours are in the set. If the neighbours are in the list, we make a pair and add them to the list m. If none of the element's neighbours are in the set, we add the element to the list l. We set s=m and repeat step 2 until we cover all possible expansions. In this way, our list l contains all the valid maximally expanded regions.

Step2:

We remove the redundant terms in list l. For this, we first count the number of times each minterm appears in all the terms of list l combined. Then we traverse through the list l and remove the terms in which all the minterms involved have a count greater than 1 and reduce the count of those minterms by 1. The list l after removing all the redundancies will be the list required.

Complexity analysis:

Complexity of our algorithm: $O(n^2N^2)$

(Where n is the number of minterms in the union of func_TRUE and func_DC and N is the total number of variables)

Analysis:

Step1: Let m_i be the number of terms in the list *Array* at the starting of ith recursion. Each recursion takes $O(m^2N^2)$ time (as the time taken to find the neighbours of a term is $O(N^2)$); so, the time taken by the MarkRegion() function is:

$$\sum_{i=0}^{N} m_i^2 N^2 \leq N^2 \left(\sum_{i=0}^{N} m_i\right)^2$$

Now, $m_i \approx 2^{N-i}(\frac{n}{2^N})^{2^i}$, that is, the number of possible regions of size i multiplied by the probability of the region being valid. So, the sum becomes:

$$N^2 \left(2^N \sum_{i=0}^{N} \frac{1}{2^i}\left(\frac{n}{2^N}\right)^{2^i}\right)^2 \leq N^2 \left(2^N \left(\frac{n}{2^N}\right)\right)^2 = n^2N^2$$

Thus, complexity of this step is $O(n^2N^2)$

Step2: Here the size of our list is $O(nN)$ and we traverse through all the minterms contained in each term of the list. Hence complexity becomes $O(n^2N)$, which is less than that of the second step. Hence the complexity of step 1 dominates.

Test Cases

S.no.	Test Case	Output
1	func_TRUE = ["a'bc'd'", "abc'd'", "a'b'c'd", "a'bc'd", "a'b'cd"] func_DC = ["abc'd"]	["a'b'd", "bc'"]
2	func_TRUE = ["a'b'c'd", "a'b'cd", "a'bc'd", "abc'd", "abc'd'", "ab'c'd'", "ab'cd"] func_DC = ["a'bc'd'", "a'bcd", "ab'c'd"]	["b'd", "bc'", "ac'"]
3	func_TRUE = ["a'b'c", "a'bc", "a'bc'", "ab'c'"] func_DC = ["abc'"]	["a'c", "ac'", "a'b"]
4	func_TRUE = ["a'b'c'd'e'", "a'bc'd'e'", "abc'd'e'", "ab'c'd'e'", "abc'de'", "abcde'", "a'bcde'", "a'bcd'e'", "abcd'e'", "a'bc'de", "abc'de", "abcde", "a'bcde", "a'bcd'e", "abcd'e", "a'b'cd'e", "ab'cd'e"] func_DC = []	['bde', "c'd'e'", 'abd', "cd'e", 'bc']
5	func_TRUE = ["ab", "ab'", "a'b","a'b'"] func_DC = []	[None]
6	func_TRUE = ["a'b'c'd'", "a'b'cd", "a'bc'd", "a'bcd'", "abc'd'", "abcd", "ab'c'd", "ab'cd'"] func_DC = []	["a'b'c'd'", "abc'd'", "a'bc'd", "a'bcd'", "ab'c'd", "ab'cd'", "a'b'cd", 'abcd']
7	func_TRUE=all 2^{11} combinations of terms func_DC=[]	[None]
8	func_TRUE=["a'b'c'd","a'bcd","abc'd'","a'bc'd","a'bcd'","abc'd","abcd","ab'cd"] func_DC=[]	['acd', "a'bc", "a'c'd", "abc'"]

How do these test cases validate our implementation?

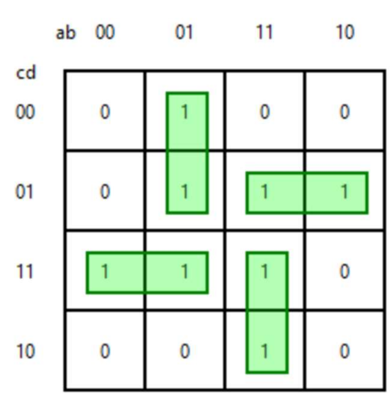
The first 4 test cases are the samples provided in the problem statement.

The fifth one is an edge case when the Boolean function becomes a tautology.

In the sixth case, the function forms a 'checkerboard' on the KMap, i.e., no further optimization is possible.

The seventh one is a big test case in which the function is a tautology for 11 Boolean variables. This case ran in around 5-6 seconds on our devices.

In test case 8 the largest region is not present in the optimal solution:



Test Case 8

After running all these tests, we are convinced that our algorithm is correct and efficient.