



Digital Logic and System Design

4. Gate-level Minimisation

COL215, I Semester 2022-2023

Venue: LHC 111

'E' Slot: Tue, Wed, Fri 10:00-11:00

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Optimising a Gate-level Implementation

- Improving a gate-level design
- Objective?
 - Area
 - Delay
 - Power/Energy
 - Temperature
 - Testability/Reliability/Security/...
- Smaller is better
 - Always?

Literal Count

- Optimal circuit: **minimum number of literals**
- Focus on **Area**
 - **#literals** proportional to **#transistors**
- Simplification in using Literals:
 - Ignoring complement operations
 - Ignoring wires
 - Ignoring blank spaces
 - Ignoring transistor dimensions

Circular Design Flow

- Literal Count useful in spite of limitations
 - Fast decisions
 - Don't need to generate full gate-level circuit, physical layout
- Gate-level decision
 - depends on geometry
 - ...which depends on gate-level decision 😊
- **Fast estimates** are critical!

Logic minimisation

- Optimization by applying Boolean Algebra theorems
 - Problem: in what order?
- **Map** method (**Karnaugh Map** or **K-Map**)
 - Pictorial representation of Truth Table
 - **2^n squares/cells** for n-variable function
 - one square for each input combination (truth table row)
 - value = **1** if corresponding minterm included in function
 - otherwise, value = **0**

a	b	c	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Truth Table

0	1	1	0
1	0	0	1

Map of function F

2-variable Karnaugh Map

a	b	F
0	0	0
0	1	1
1	0	1
1	1	0

Truth Table



a\b	0	1
0	0	1
1	1	0

Karnaugh Map of F

	b'	b
a'	00 $a'b'$	01 $a'b$
a	10 ab'	11 ab

a\b	0	1
0	00 m_0	01 m_1
1	10 m_2	11 m_3

a\b	0	1
0	0	1
1	1	0

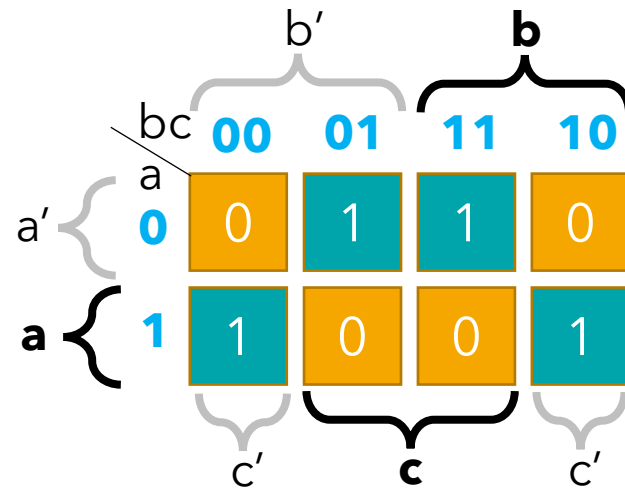
$a'b$ (top-right cell)
 ab' (bottom-left cell)

$$F = a'b + ab'$$

3-variable Karnaugh Map

a	b	c	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Truth Table



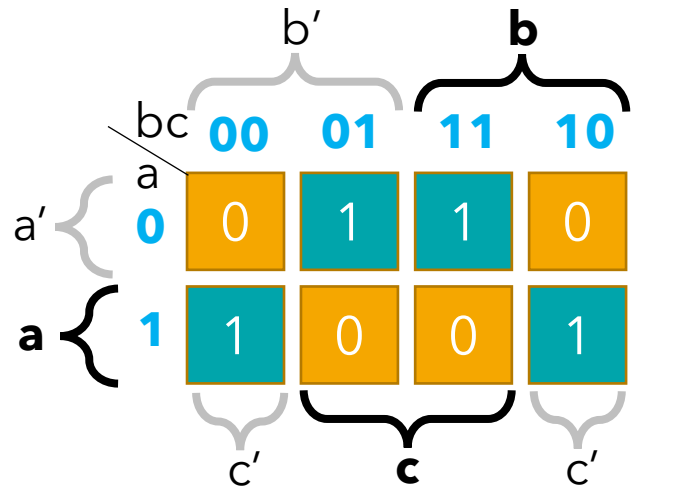
Gray Code sequence
Why?

$$F = a'b'c + a'bc + ab'c' + abc'$$

3-variable Karnaugh Map

a	b	c	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Truth Table



$$F = a'b'c + a'bc + ab'c' + abc'$$

bc	00	01	11	10
a				
0	m ₀	m ₁	m ₃	m ₂
1	m ₄	m ₅	m ₇	m ₆

$$F = m_1 + m_3 + m_4 + m_6 \\ = \sum (1, 3, 4, 6)$$