



Digital Logic and System Design

7. Sequential Logic

COL215, I Semester 2022-2023

Venue: LHC 111

'E' Slot: Tue, Wed, Fri 10:00-11:00

Instructor: Preeti Ranjan Panda

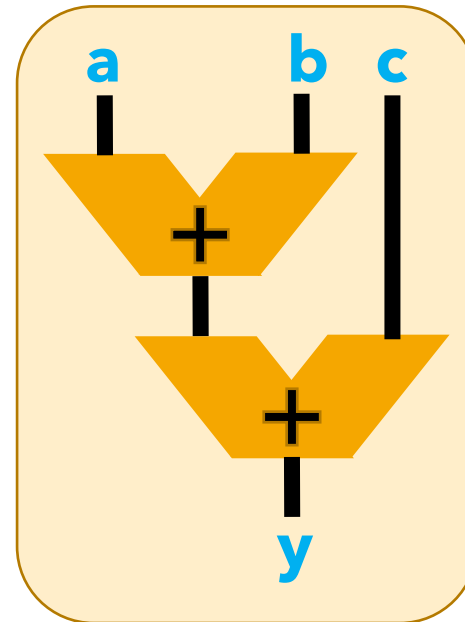
panda@cse.iitd.ac.in

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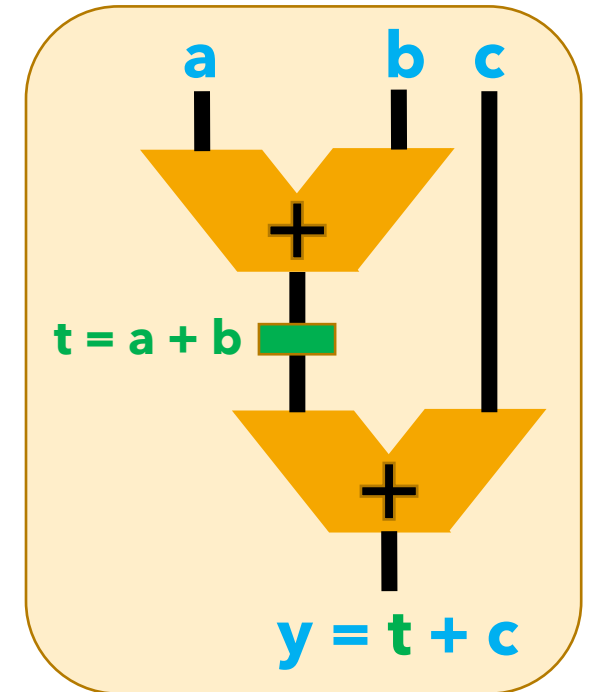
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Sharing Resources

- Implement: $y = a + b + c$
- ...with only **ONE adder**
- **REUSE** the adder
- Need to **STORE** intermediate value
 - $t = a + b$
 - $y = t + c$



2 Adders



Store intermediate value t
Use later for 2nd addition

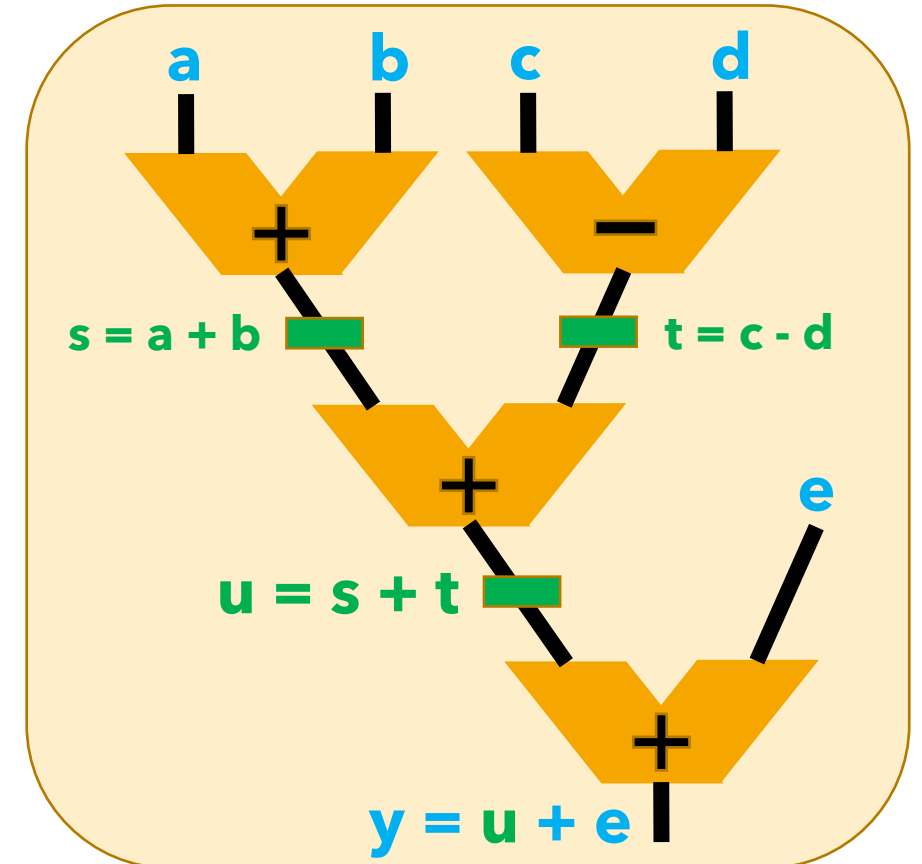
Complex Computation

- Break up complex function/computation
 - intermediate values
 - similar to software
- Efficient resource utilization
 - same adder **reused** for several additions

$$y = a + b + c - d + e$$

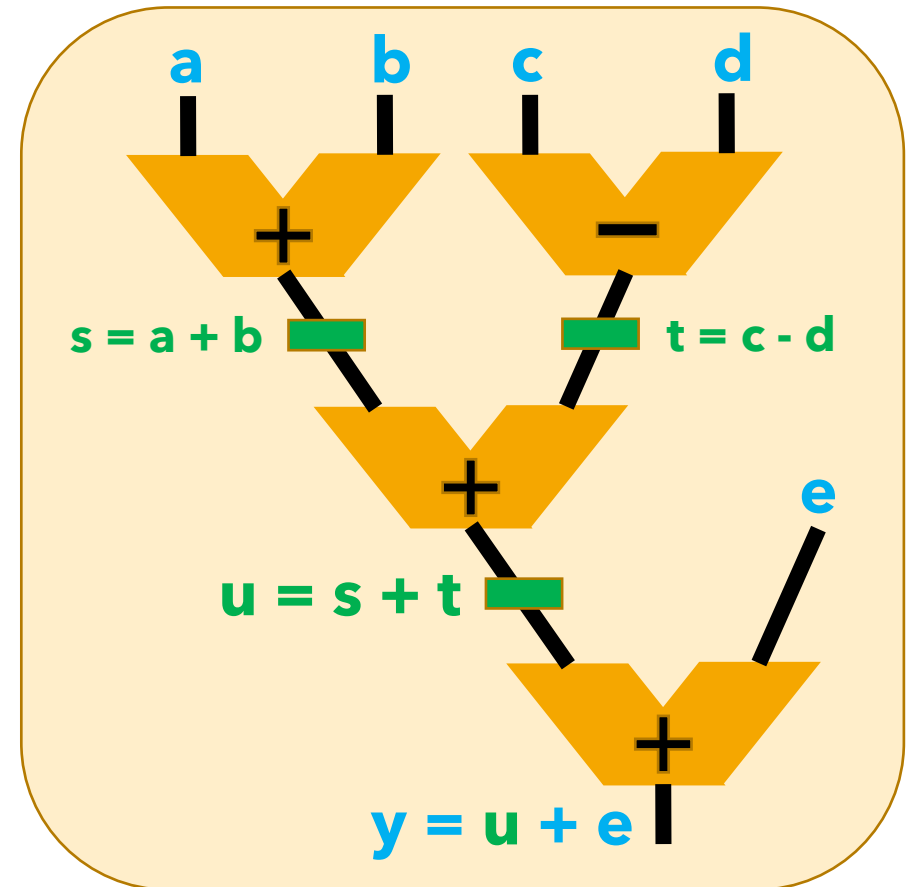


$$\begin{aligned} s &= a + b \\ t &= c - d \\ u &= s + t \\ y &= u + e \end{aligned}$$



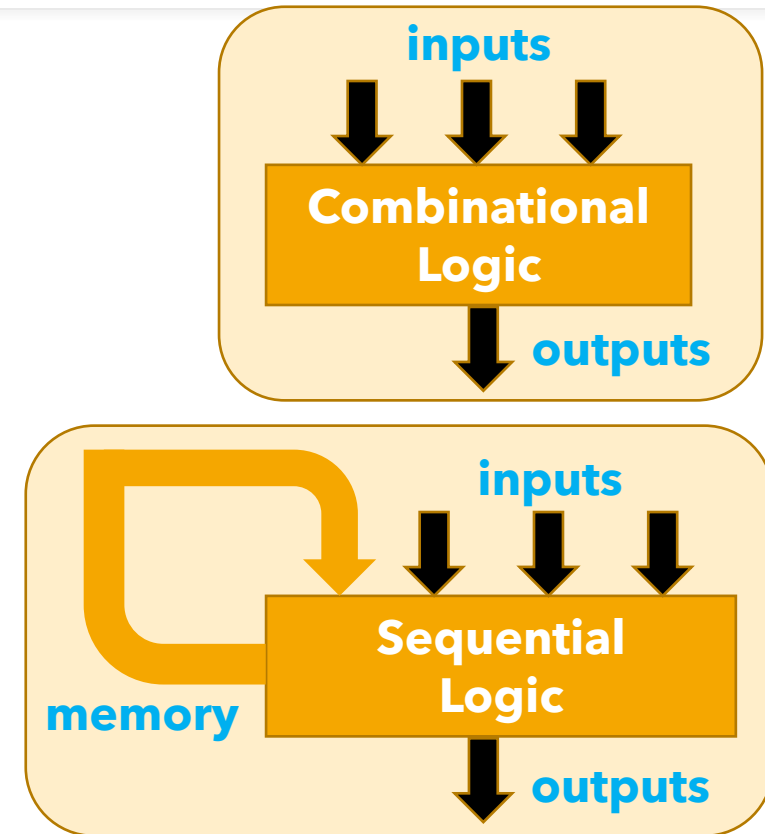
New Hardware Components

- Elementary operation:
 - **Store** a value
 - use it later
- **Control** and **Sequencing**
 - Manage **order** of operations
 - Decide what action is performed by unit (**ADD** vs **SUB**)
- Provide alternative **paths** for operands



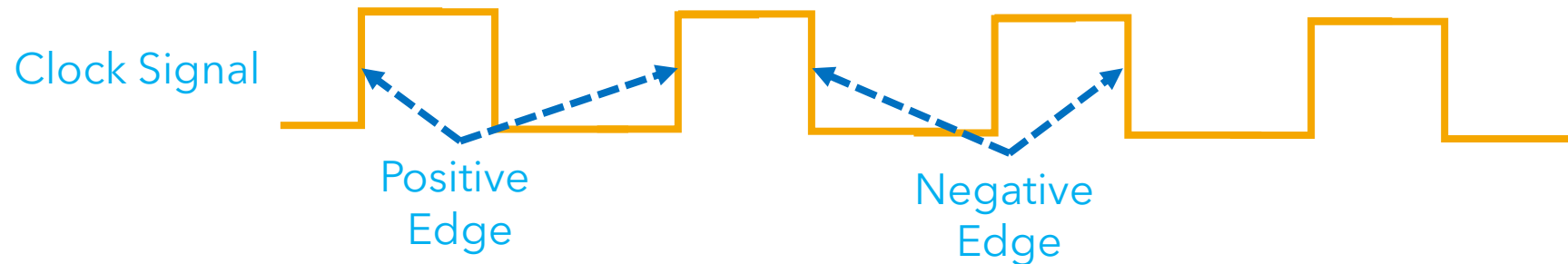
Combinational vs. Sequential Logic

- Combinational:
 - **Output** is a function only of **present inputs**
 - Independent of **earlier** input values
- Sequential:
 - Output is a function of **present input AND earlier inputs**
 - **memory** is stored



Synchronous Design

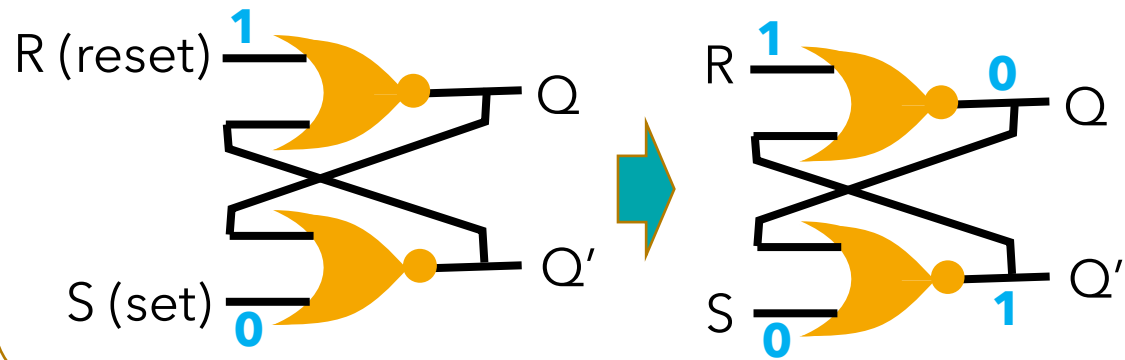
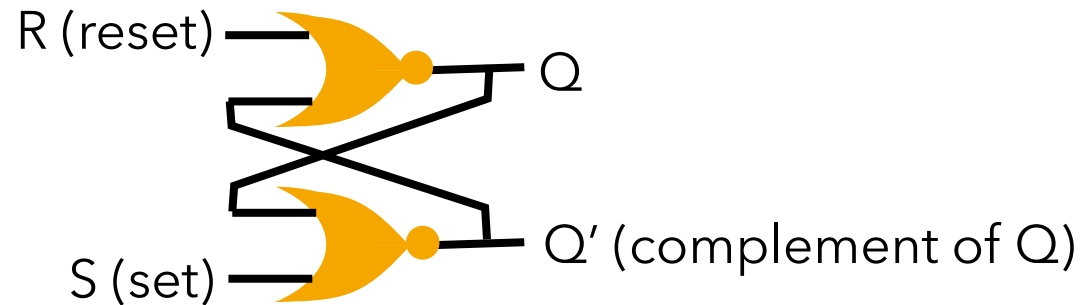
- Computation and Storage are associated with **CLOCKS**
 - Periodic signals
 - Events triggered at **Clock Edges**
 - **positive edge: (0 to 1) transition**
 - **negative edge: (1 to 0) transition**



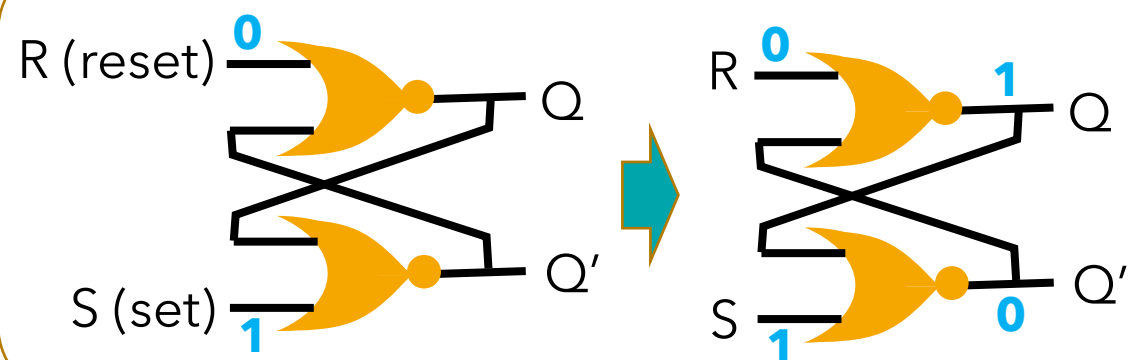
Alternative: Asynchronous Design

- No common clock
- Completion signalled by individual components
 - Contrast: In Synchronous Design completion is checked only at clock edges

Storage Element: SR Latch

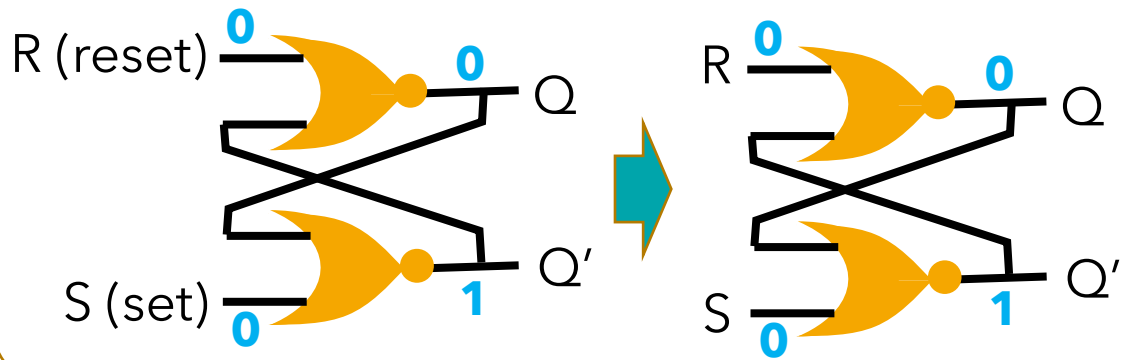
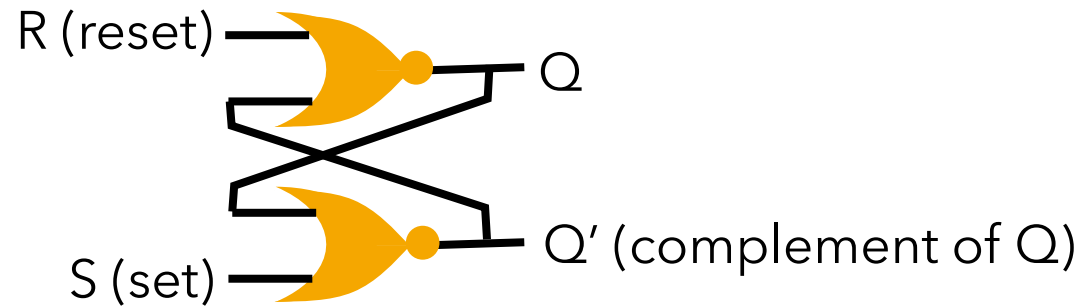


RS = 10: Outputs **QQ' = 01** (Q **reset** to 0)

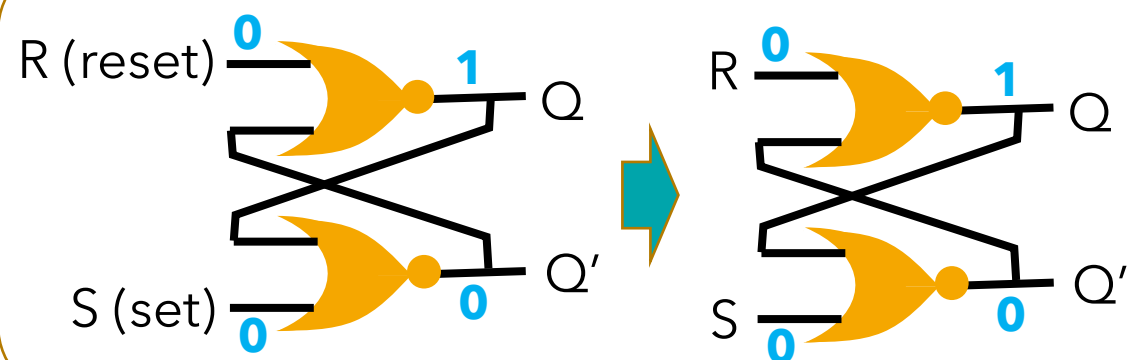


RS = 01: Outputs **QQ' = 10** (Q **set** to 1)

Storing the Q value

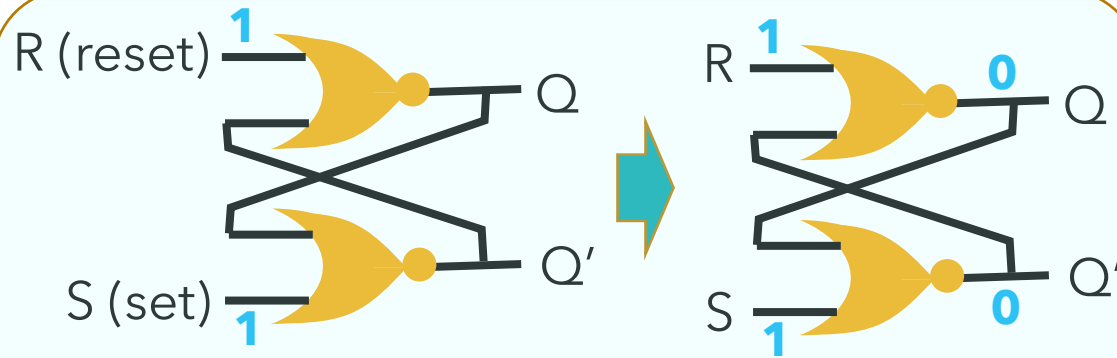


RS = 00: Outputs **QQ' = 01** maintained (no change)

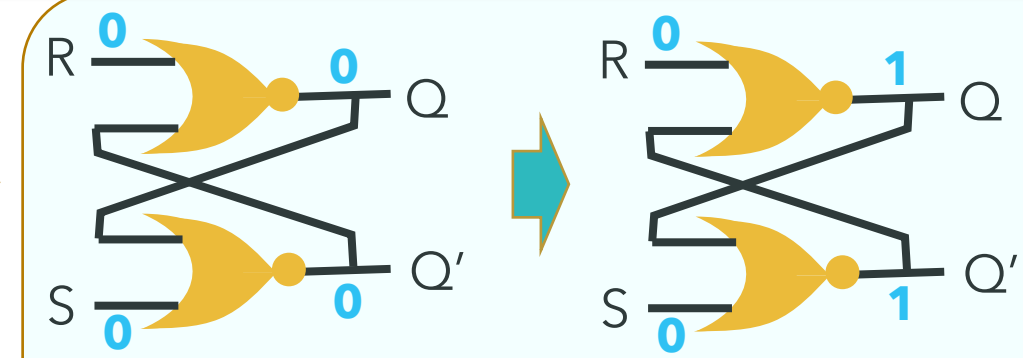


RS = 00: Outputs **QQ' = 10** maintained (no change)

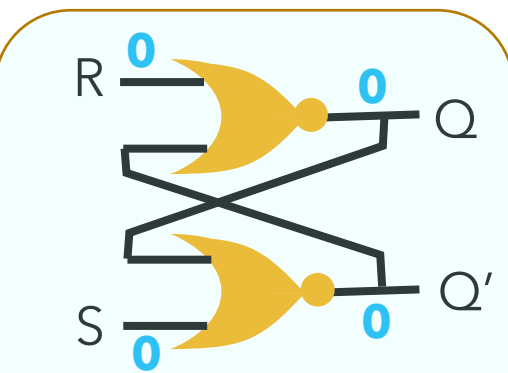
Unstable conditions in SR Latch



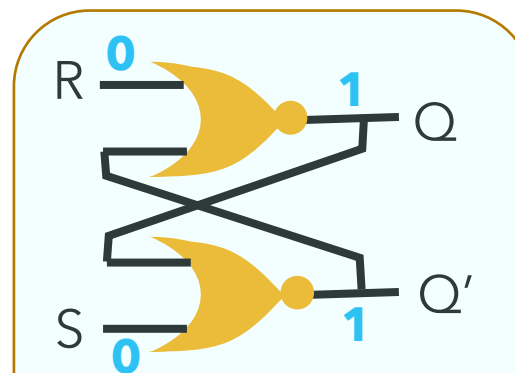
$RS = 11$: Outputs $QQ' = 00$



$RS = 00$: Outputs $QQ' = 11$



Outputs $QQ' = 00$

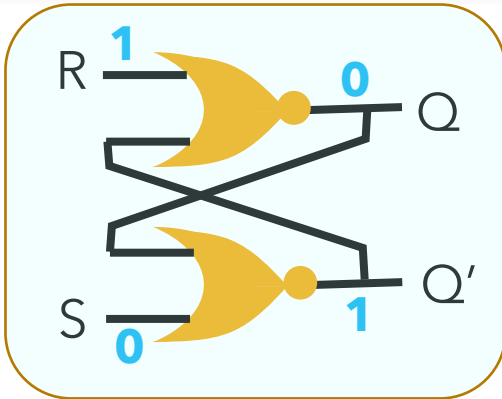


Outputs $QQ' = 11$

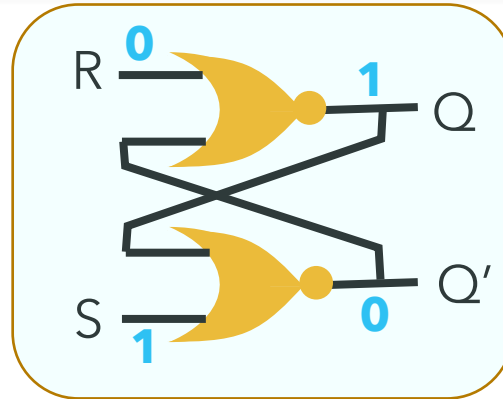
●●● Unstable (Oscillating QQ')

$SR=11$ is NOT ALLOWED!

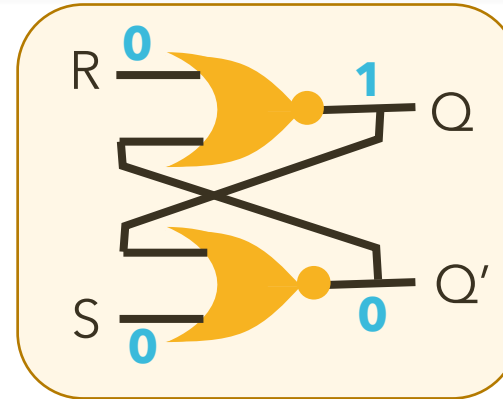
We have a **storage/memory device**!



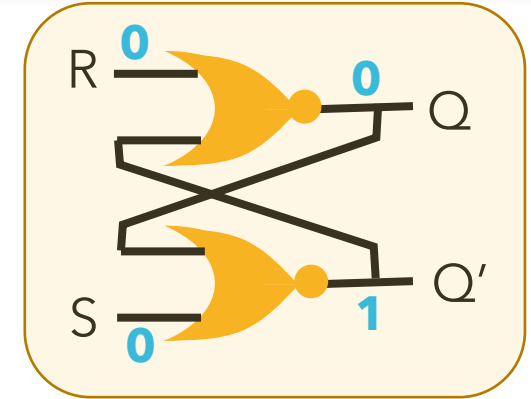
RS = 10: Output **Q=0**



RS = 01: Output **Q=1**



RS = 00: Output **Q unchanged**



Working of the SR Latch

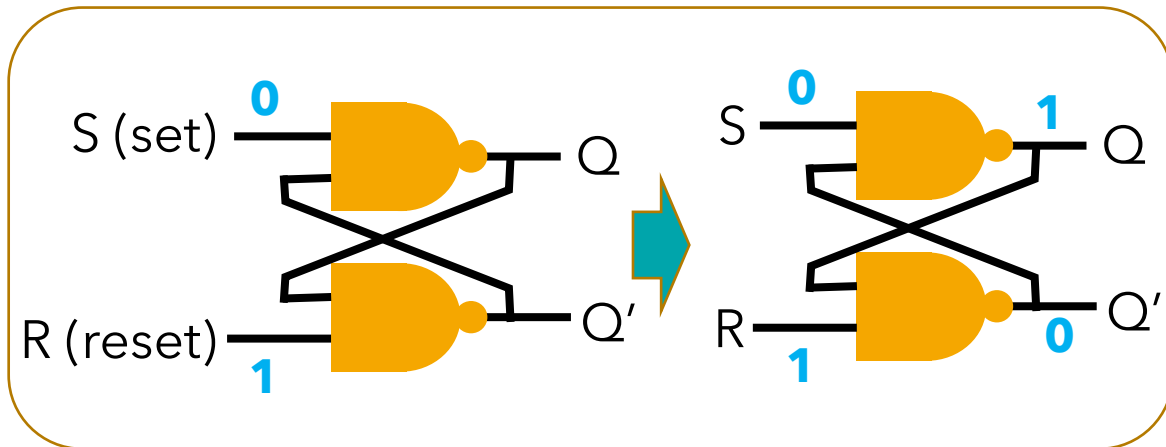
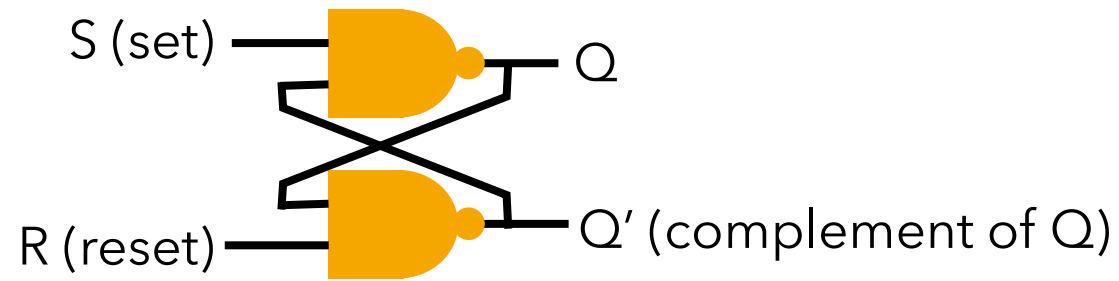
- Normally, $SR=00$
- If $Q=0$ needed, make $SR=01$. Return to $SR=00$.
- If $Q=1$ needed, make $SR=10$. Return to $SR=00$.
- Don't set $SR=11$
- Don't go directly from $SR=01$ to $SR=10$

S	R	Q	Q'
1	0	1	0
0	0	1	0
0	1	0	1
0	0	0	1
1	1	0	0

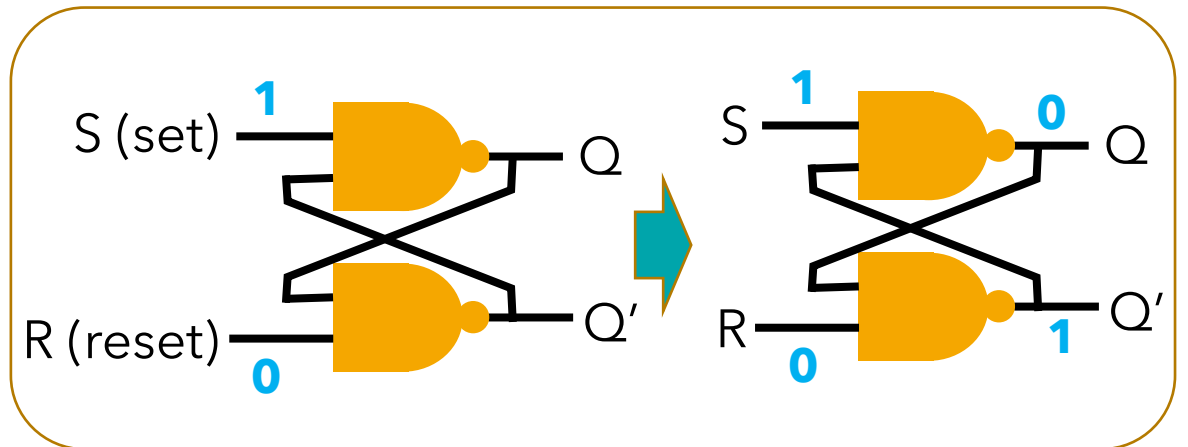
Function Table

Not Allowed

SR Latch with NAND gates

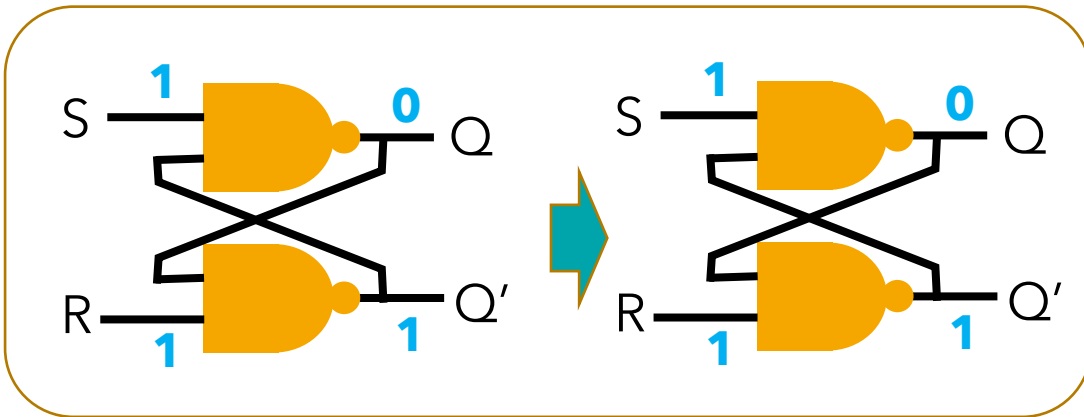
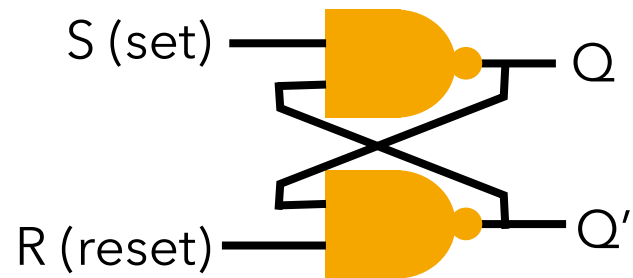


SR = 01: Outputs **QQ' = 10** (Q **set** to **1**)

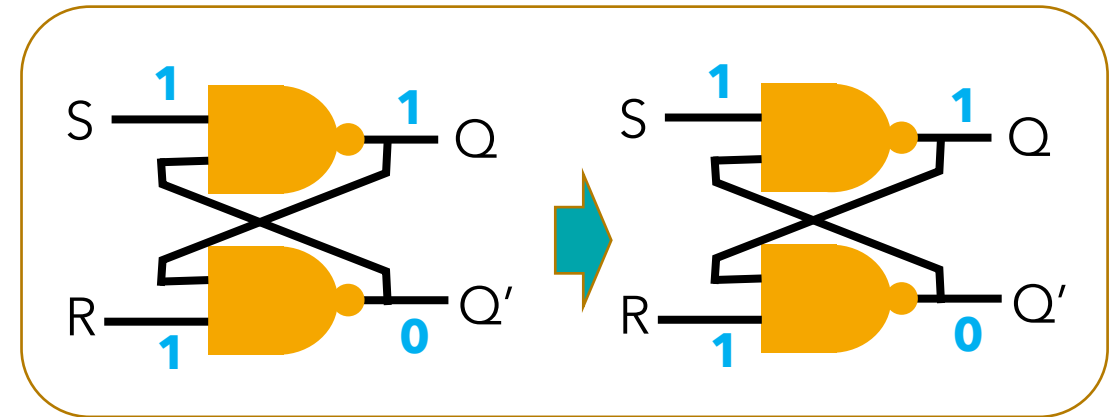


SR = 10: Outputs **QQ' = 01** (Q **reset** to **0**)

Storing the Q value

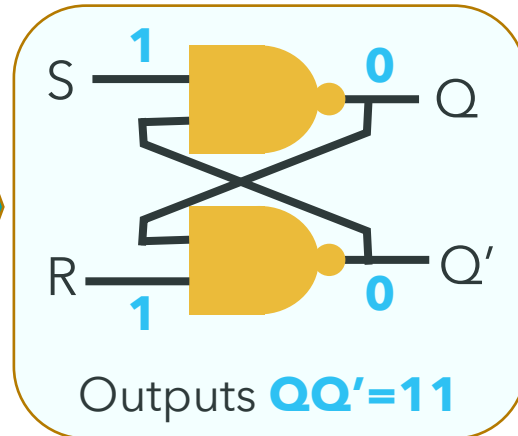
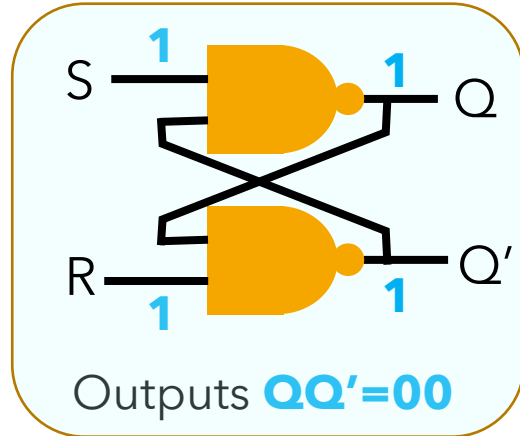
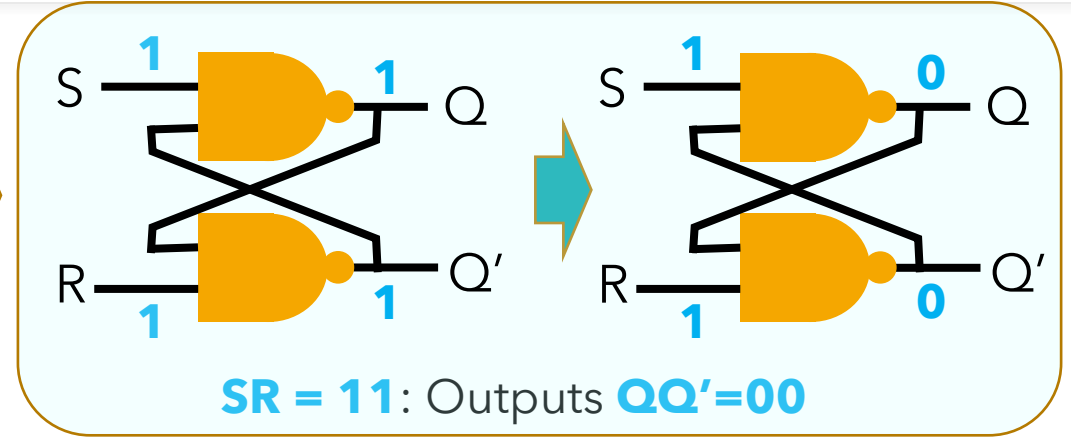
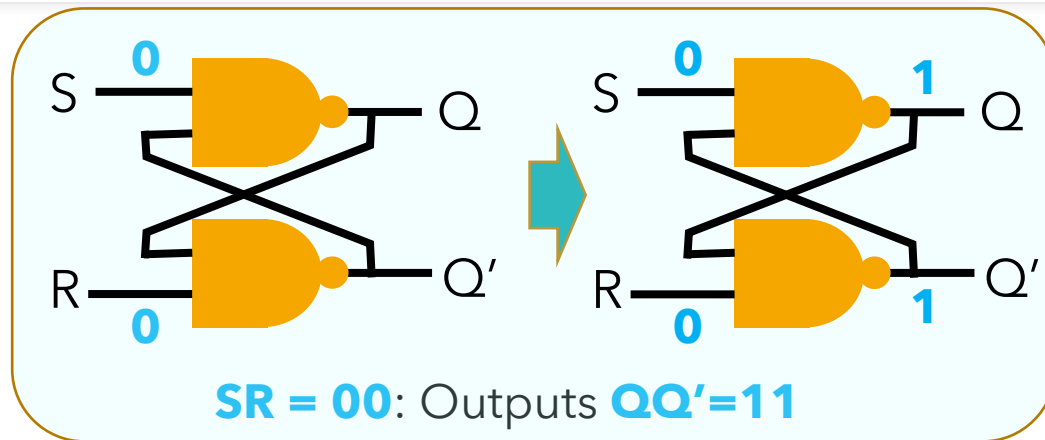


SR = 11: Outputs $QQ'=01$ maintained (no change)



SR = 11: Outputs $QQ'=10$ maintained (no change)

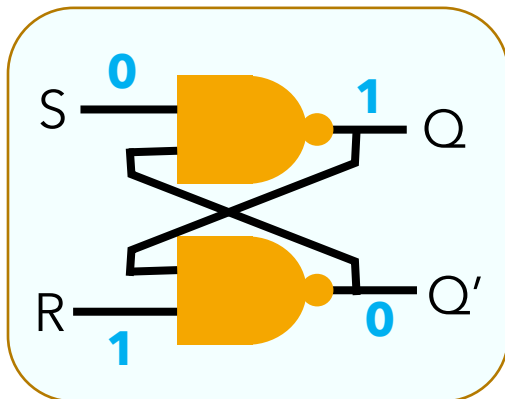
Unstable conditions in NAND Latch



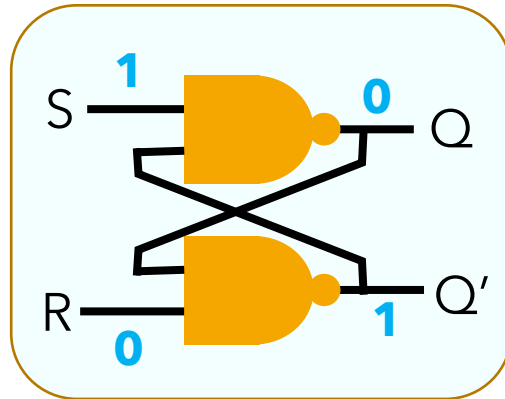
●●● Unstable (Oscillating QQ')

$SR=00$ is NOT ALLOWED!

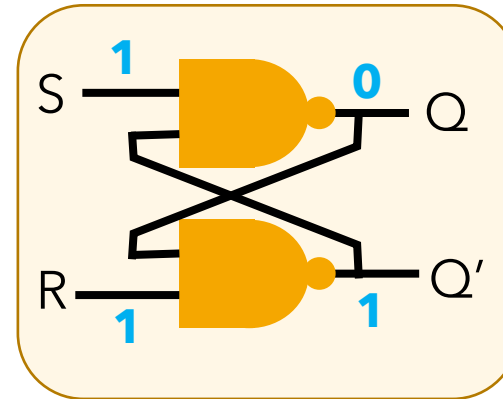
NAND latch is also a **storage/memory device**



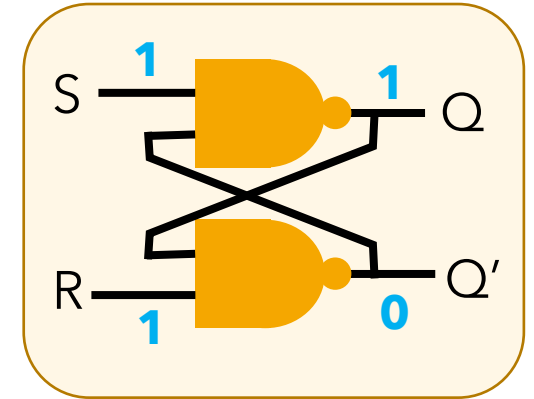
SR = 01: Output Q=1



SR = 10: Output Q=0



SR = 11: Output Q unchanged



Working of the NAND SR Latch

- Normally, SR=11
- If Q=0 needed, make SR=10. Return to SR=11.
- If Q=1 needed, make SR=01. Return to SR=11.
- Don't set SR=00
- Don't go directly from SR=01 to SR=10

S	R	Q	Q'
0	1	1	0
1	1	1	0
1	0	0	1
1	1	0	1
0	0	1	1

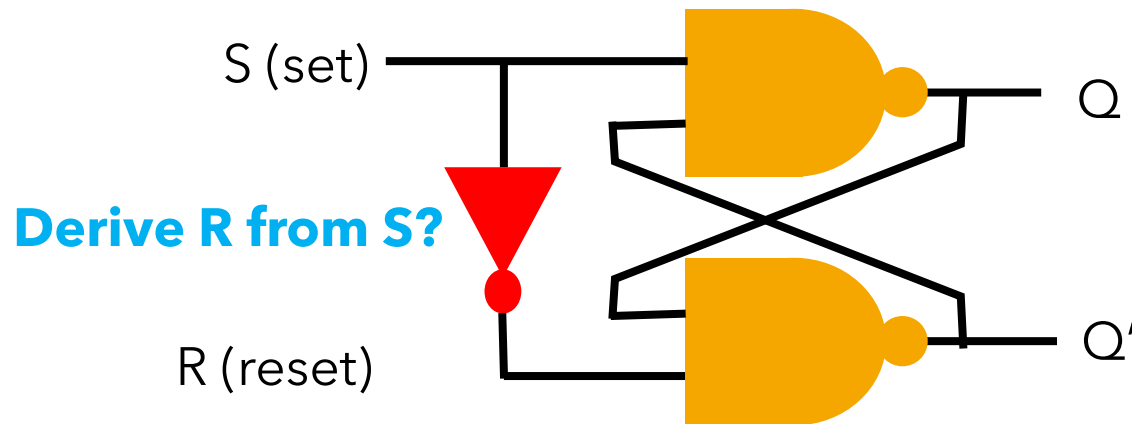
Function Table

Not Allowed

NAND/NOR Latch needs enhancement

- Signal to **enable writing**
- When enabled, **S** and **R** are complements (**01** or **10**)

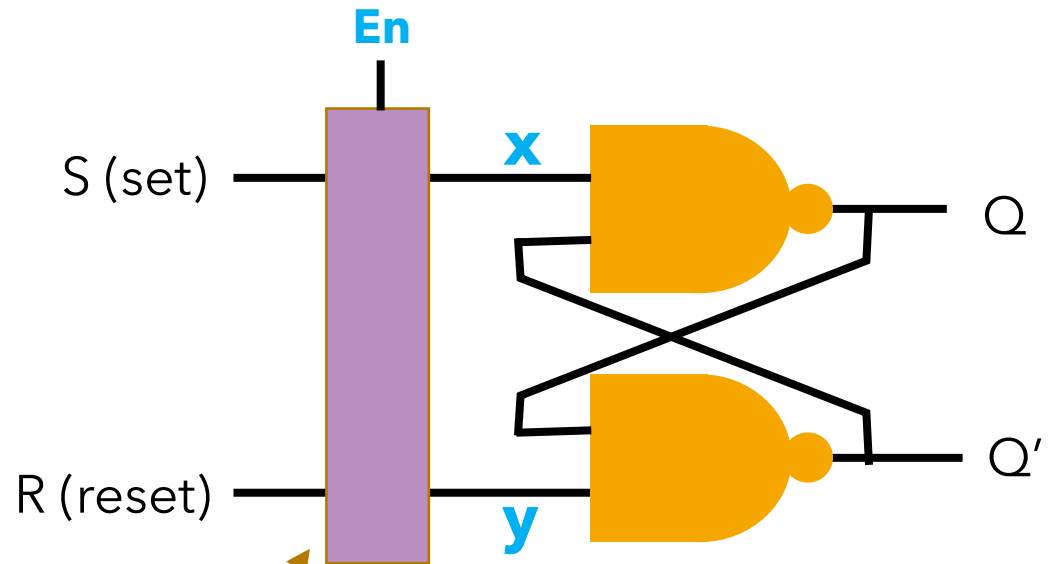
Ensuring complement



Unsafe. May transition through **SR=11**

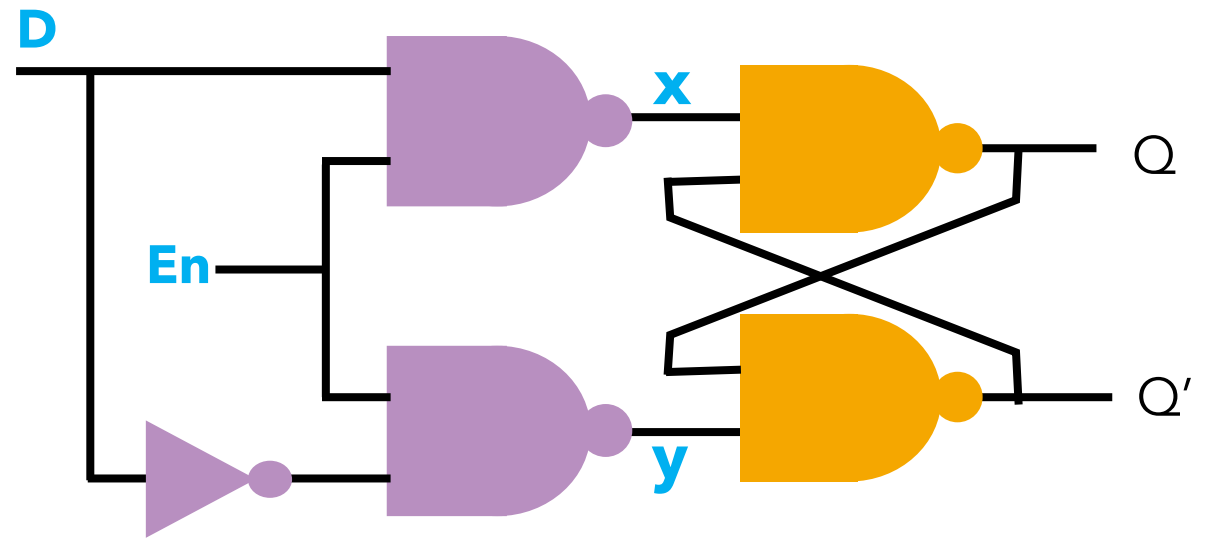
Enabling writing

- **En** (enable) signal
 - When **En=0**: **x = y = 1**
 - save state
 - When **En = 1**:
 - **x = S'**
 - **y = R'**
- What should go here?



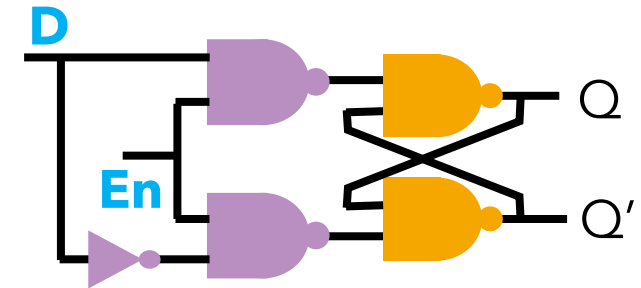
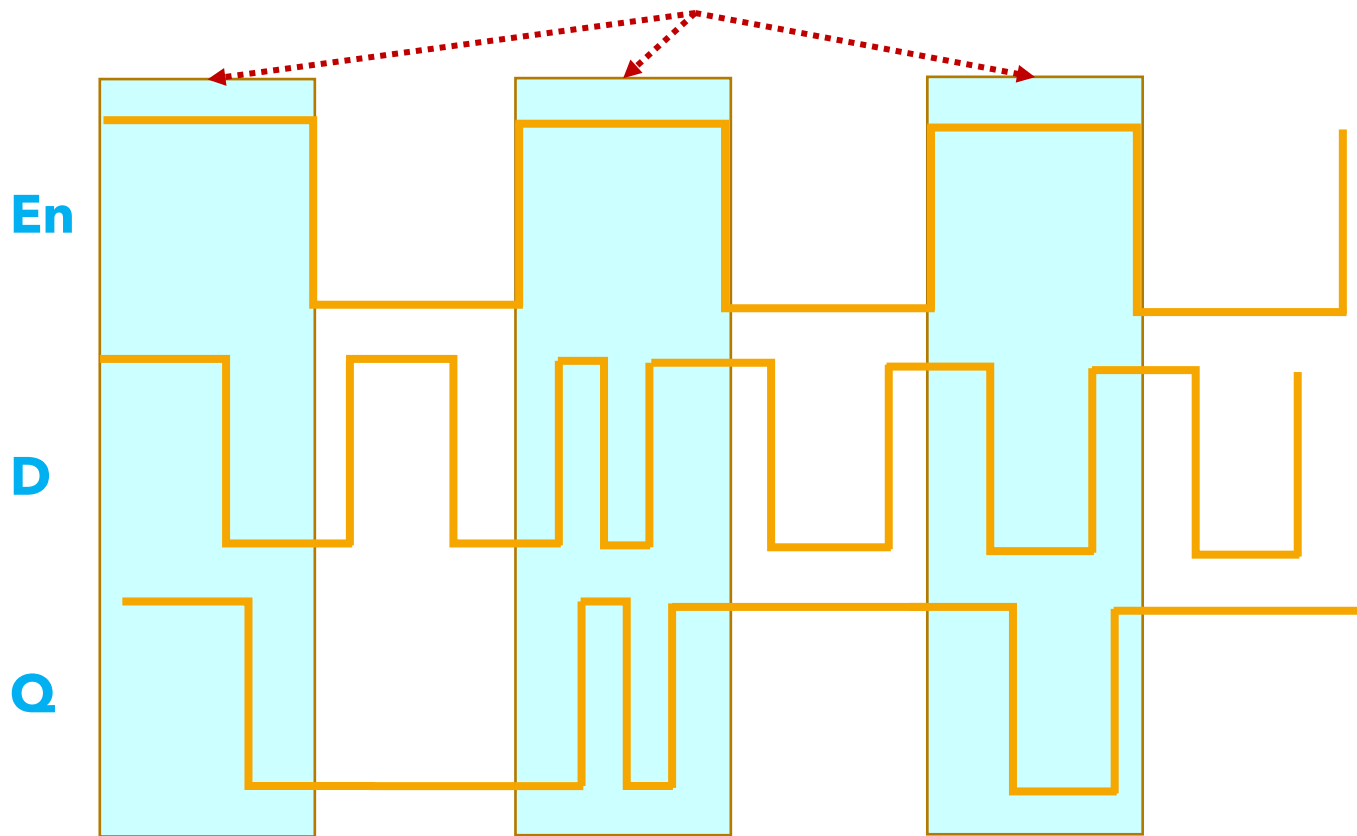
The D (data) Latch

- **En** (enable) signal
 - When **En=0**: **x = y = 1**
 - save state
 - When **En = 1**:
 - **x = D'**
 - **y = D**
- **D** input **ignored** when **En = 0**
- When **En=1**: **Q = D**
- Q follows D when **En enabled**
- Last D value saved when **En disabled**



The D Latch Functionality

Q follows **D** when **En** is ON



previous Q value

En	D	Q
0	0	q
0	1	q
1	0	0
1	1	1

Function Table

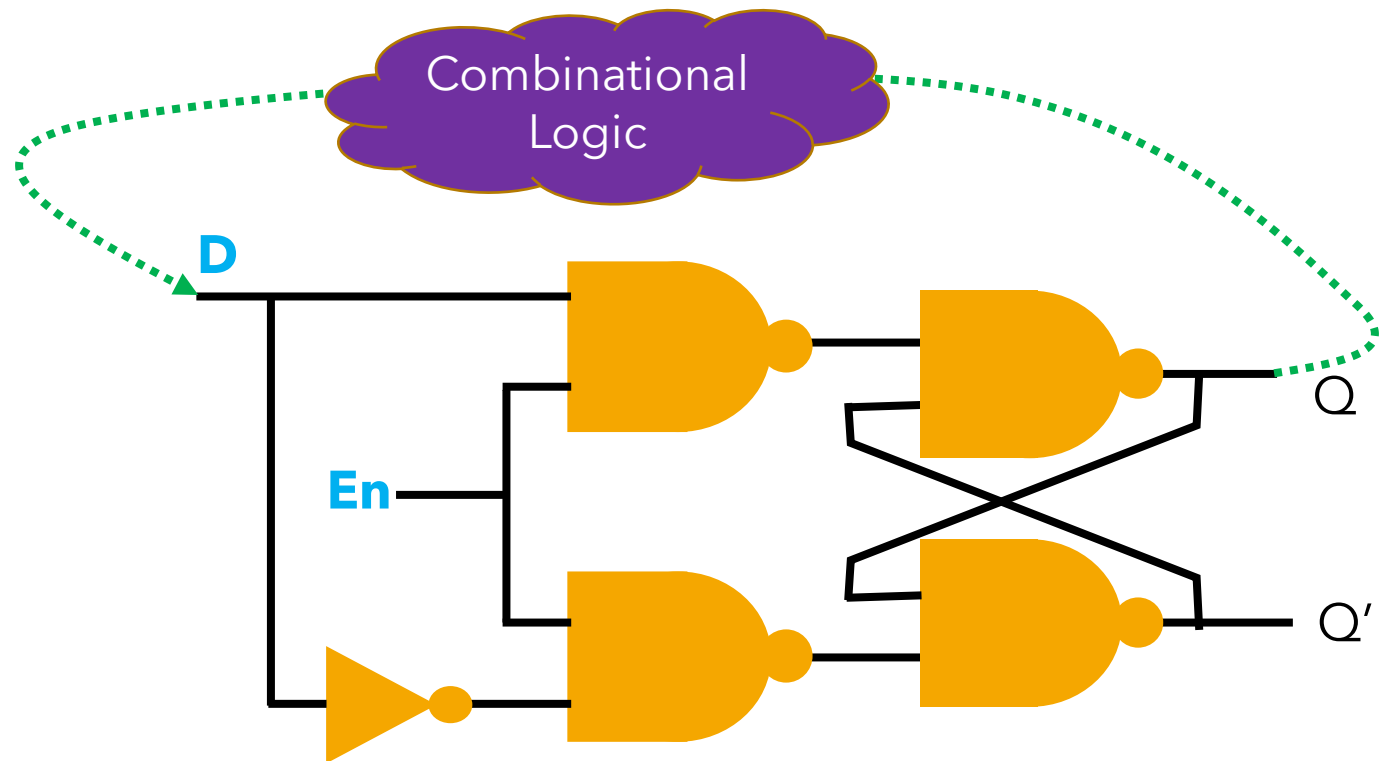
previous Q value

En	D	Q
0	x	q
1	x	D

Function Table

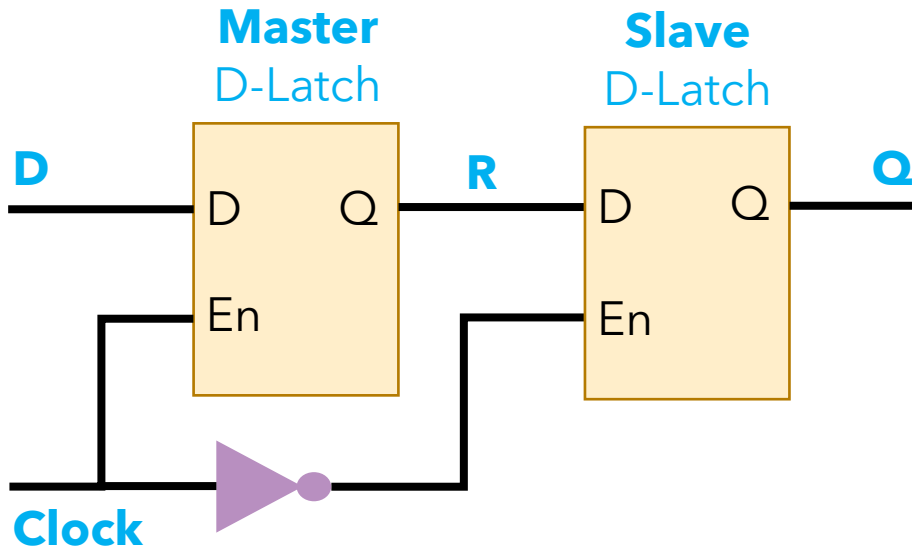
Concerns about Latches

- **D** input might be function of output Q of same latch
 - **loop** in signal path
 - OK in sequential designs
- **En** might be **ON** for entire duration
 - ...causing **oscillations**
- **Safety:** in Latch-based designs, need to be careful of feedback/loop paths
 - e.g., use different **En** signals in different latch stages

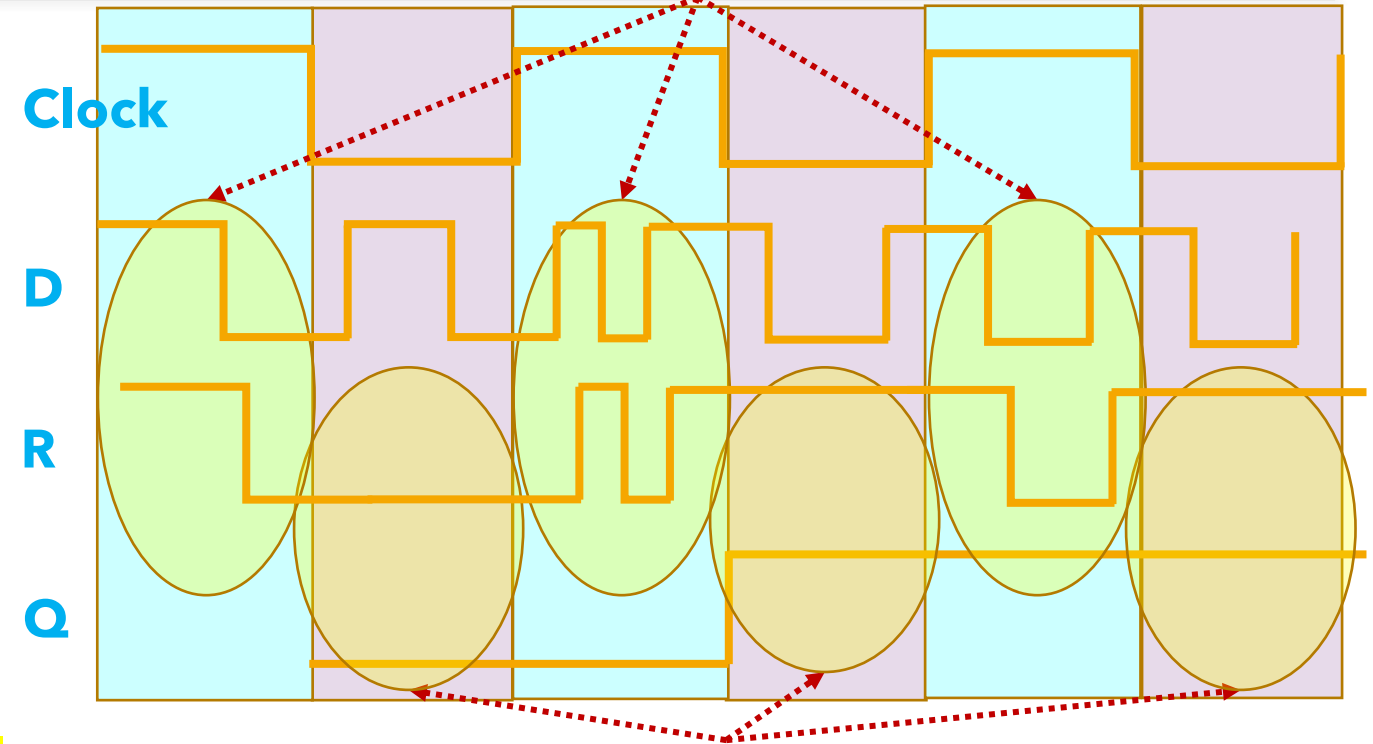


Master-Slave Flip-Flop

R follows **D** when **Clock=1 (Master Enabled)**

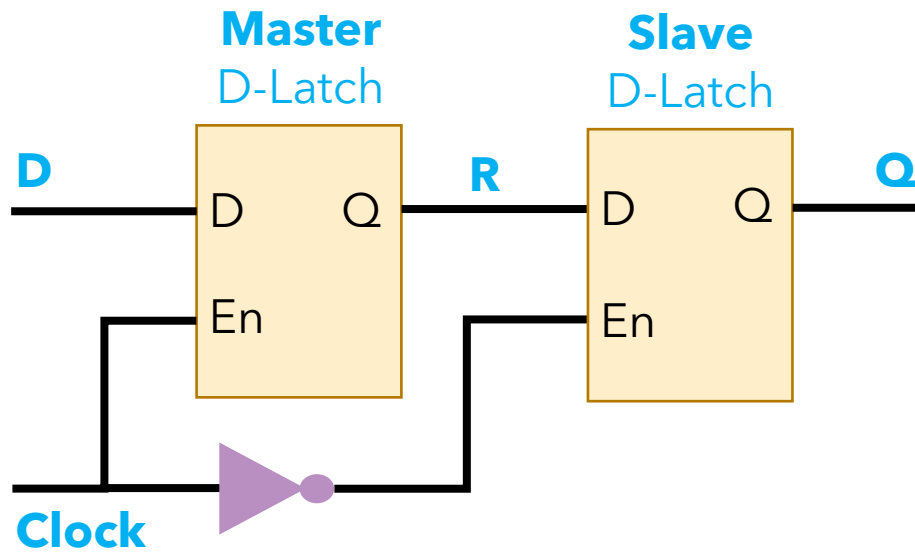


Clock=1: Master Enabled. Slave disabled.
Clock=0: Slave Enabled. Master disabled.

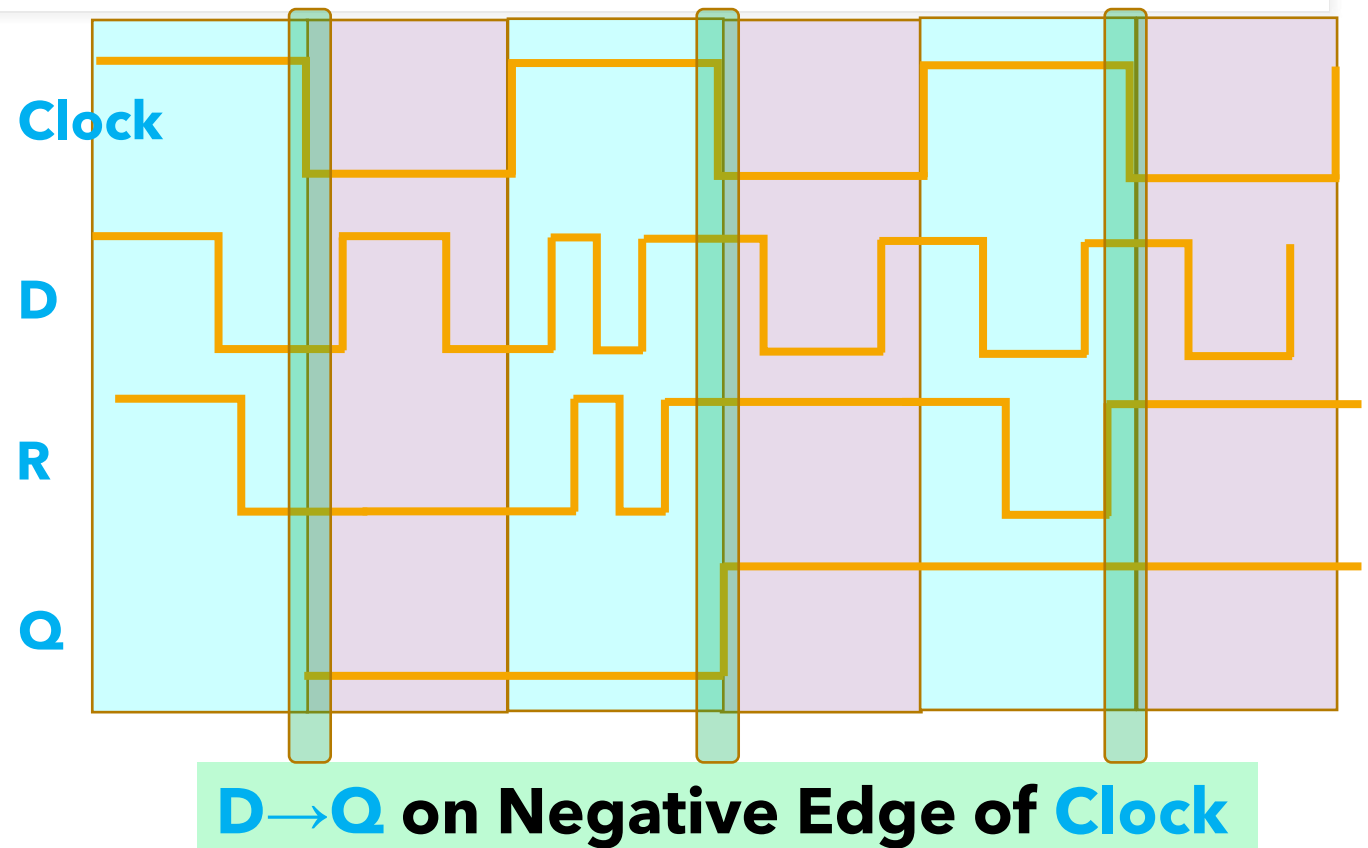


Q follows **R** when **Clock=0 (Slave Enabled)**

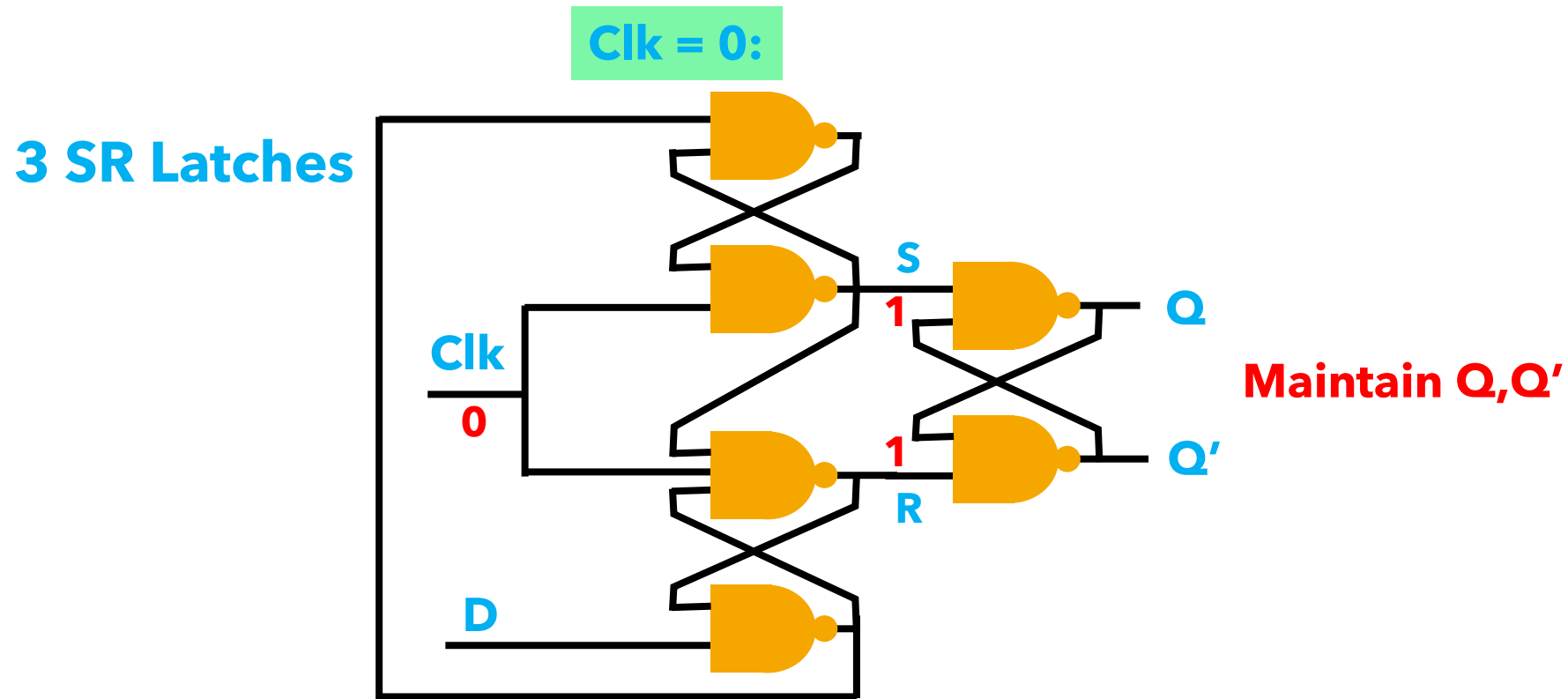
Master-Slave Flip-Flop



Clock=1: Master Enabled. Slave disabled.
Clock=0: Slave Enabled. Master disabled.

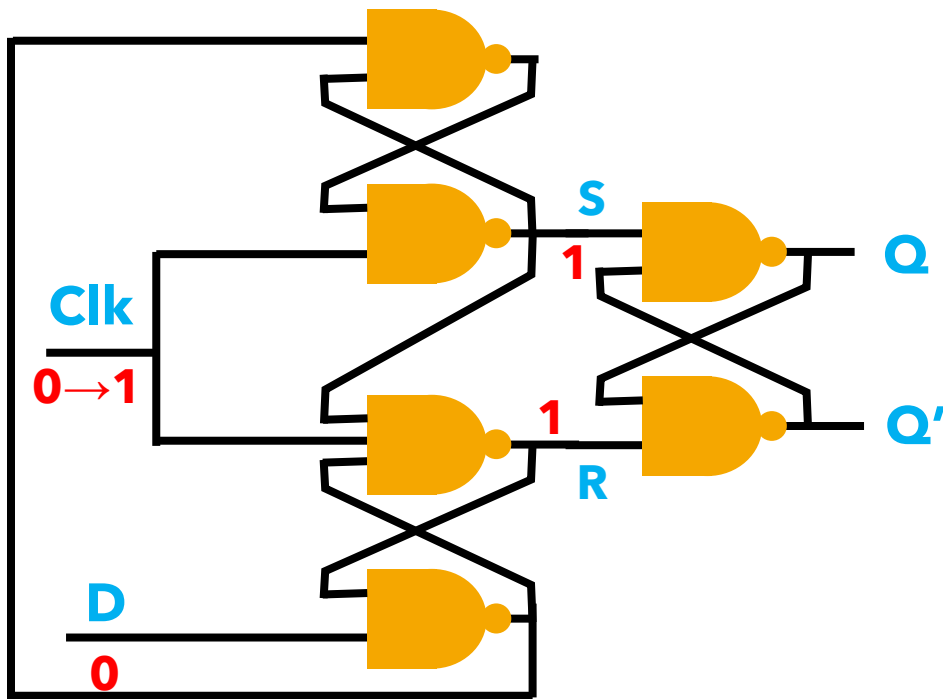


D Flip-flop: Alternative Design



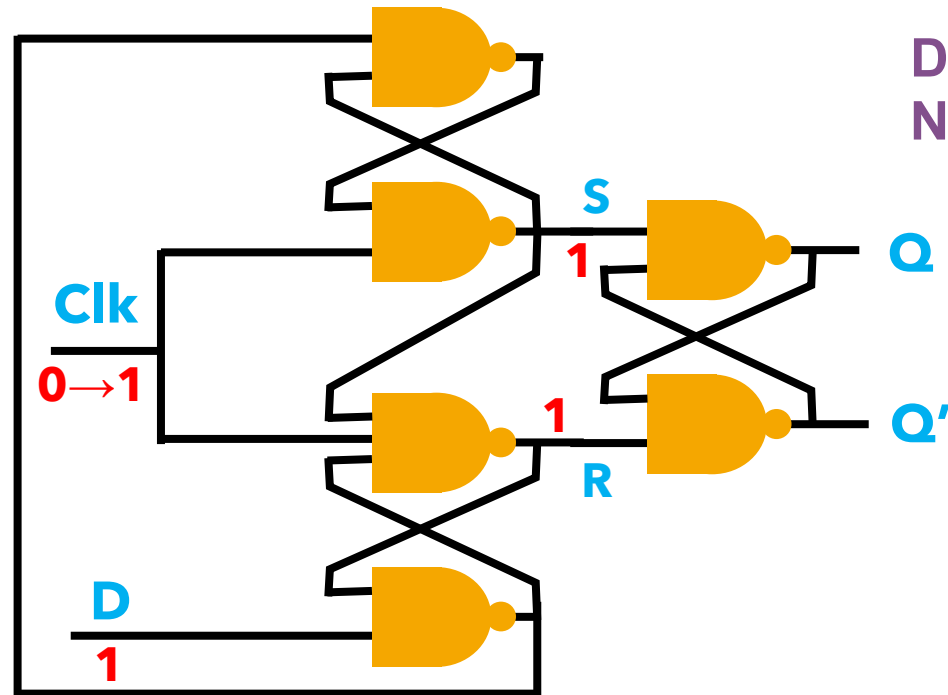
D Flip-flop: Alternative Design

Clk = 0 → 1, D = 0



Clk=0 → 1 causes R=1 → 0, Q=0
Further changes in D?

Clk = 0 → 1, D = 1



D → Q on Clk edge
No changes later

Clk=0 → 1 causes S=1 → 0, Q=1
Further changes in D?

Positive Edge-triggered Flip-flop

- D is transferred to Q at rising Clk edge
- No changes on falling Clk edge
- No changes in rest of Clk period

Flip-flop Timing Considerations

Setup Time

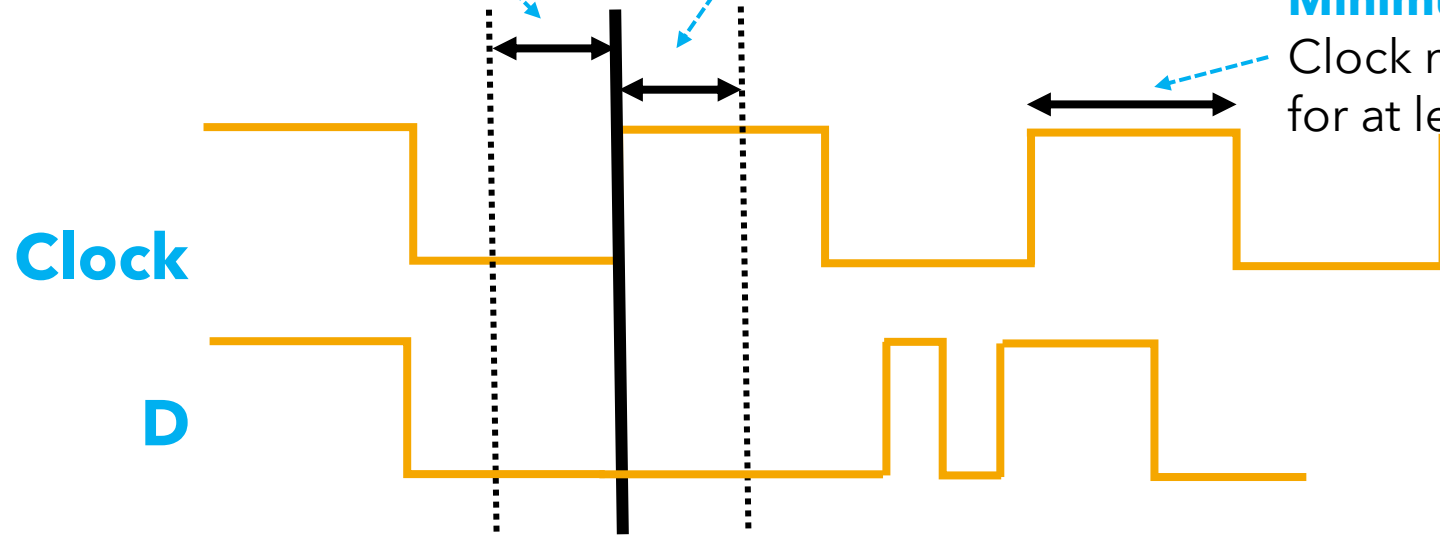
D needs to be constant for this duration **before clock edge**

Hold Time

D needs to be constant for this duration **after clock edge**

Minimum Pulse Width

Clock needs to be constant for at least this duration



Other flip-flop designs

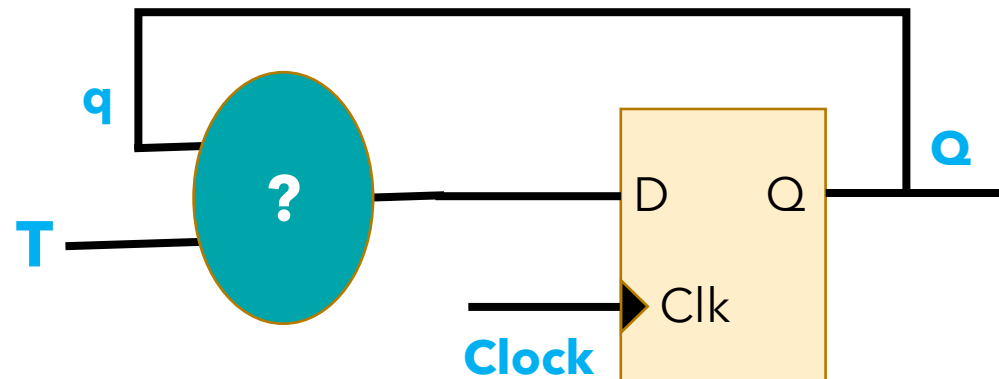
- T flip-flop
 - $T=0$: $Q=q$
 - $T=1$: $Q=\text{NOT } q$
- JK flip-flop
 - $J=1, K=0$: $Q=1$
 - $J=0, K=1$: $Q=0$
 - $J=1, K=1$: $Q=\text{NOT } q$
 - $J=0, K=0$: $Q=q$

T (Toggle) Flip-Flop

T Flip-flop Specification

On clock edge:

- $T=0$: $Q=q$ maintain state
- $T=1$: $Q=\text{NOT } q$ invert



T Flip-Flop implemented using D Flip-flop

T Flip-flop Specification

- $T=0$: $Q=q$
- $T=1$: $Q=\text{NOT } q$

T	q	Q
0	0	0
0	1	1
1	0	1
1	1	0

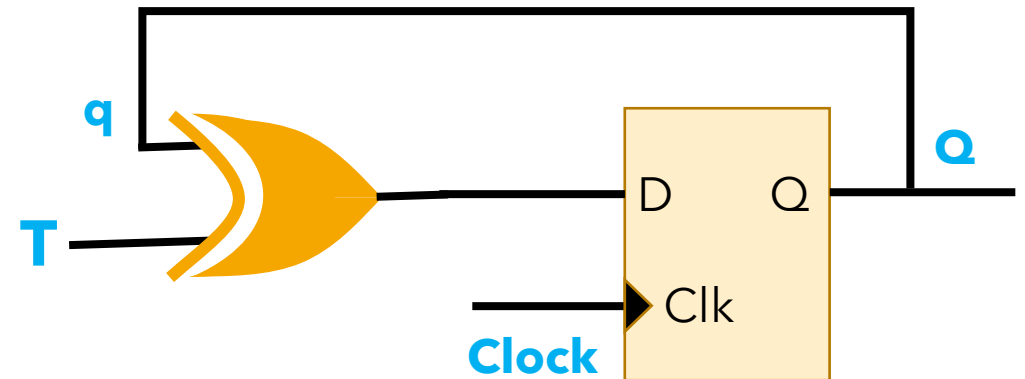
Function Table

$Q=q$

$Q=\text{NOT } q$

$$Q = q \oplus T$$

On clock edge: **Q** takes the value of **$q \oplus T$**
 $(q \oplus T)$ is sent to D input

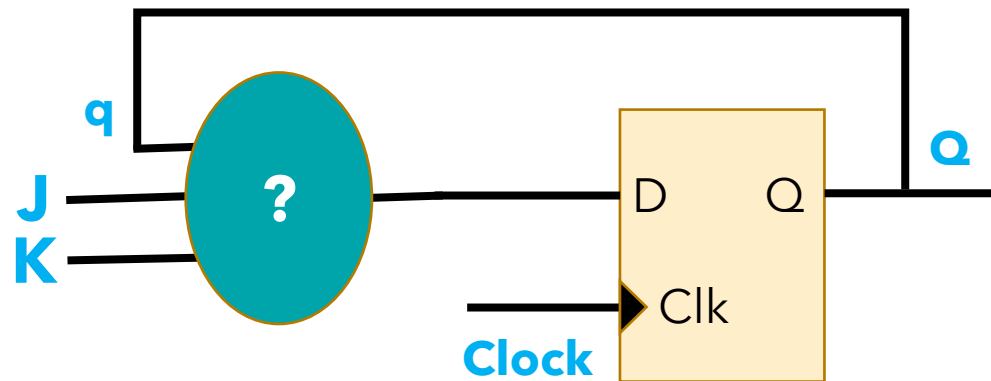


JK Flip-Flop

Specification

On clock edge:

- $J=1, K=0: Q=1$
- $J=0, K=1: Q=0$
- $J=1, K=1: Q=\text{NOT } q$
- $J=0, K=0: Q=q$



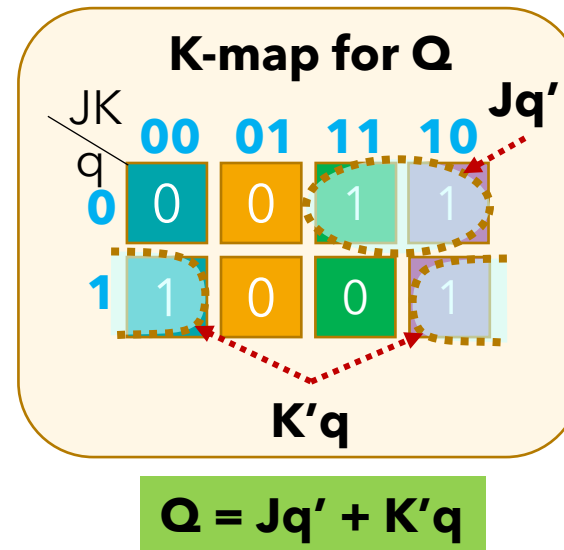
JK Flip-Flop implemented using D Flip-flop

Function Table

J	K	q	Q
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

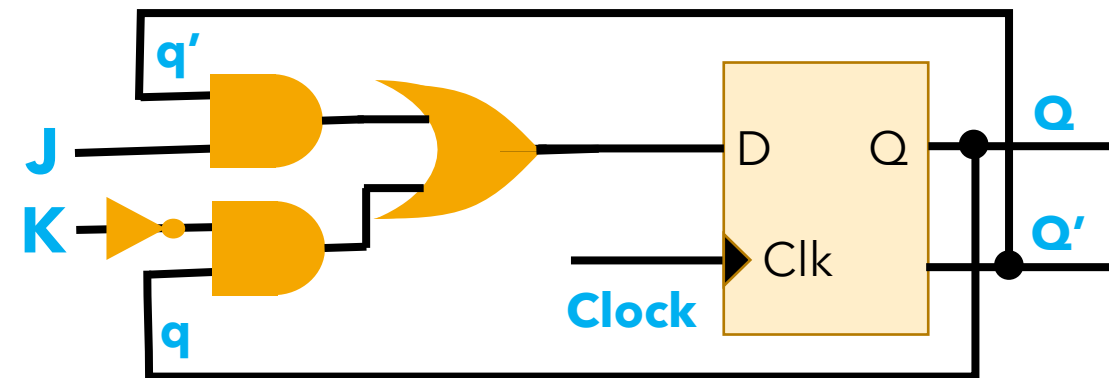
Annotations for Function Table:

- Q=q (for J=0, K=0)
- Q=0 (for J=0, K=1)
- Q=1 (for J=1, K=0)
- Q=NOT q (for J=1, K=1)



Specification

- J=1,K=0: Q=1
- J=0,K=1: Q=0
- J=1,K=1: Q=NOT q
- J=0,K=0: Q=q



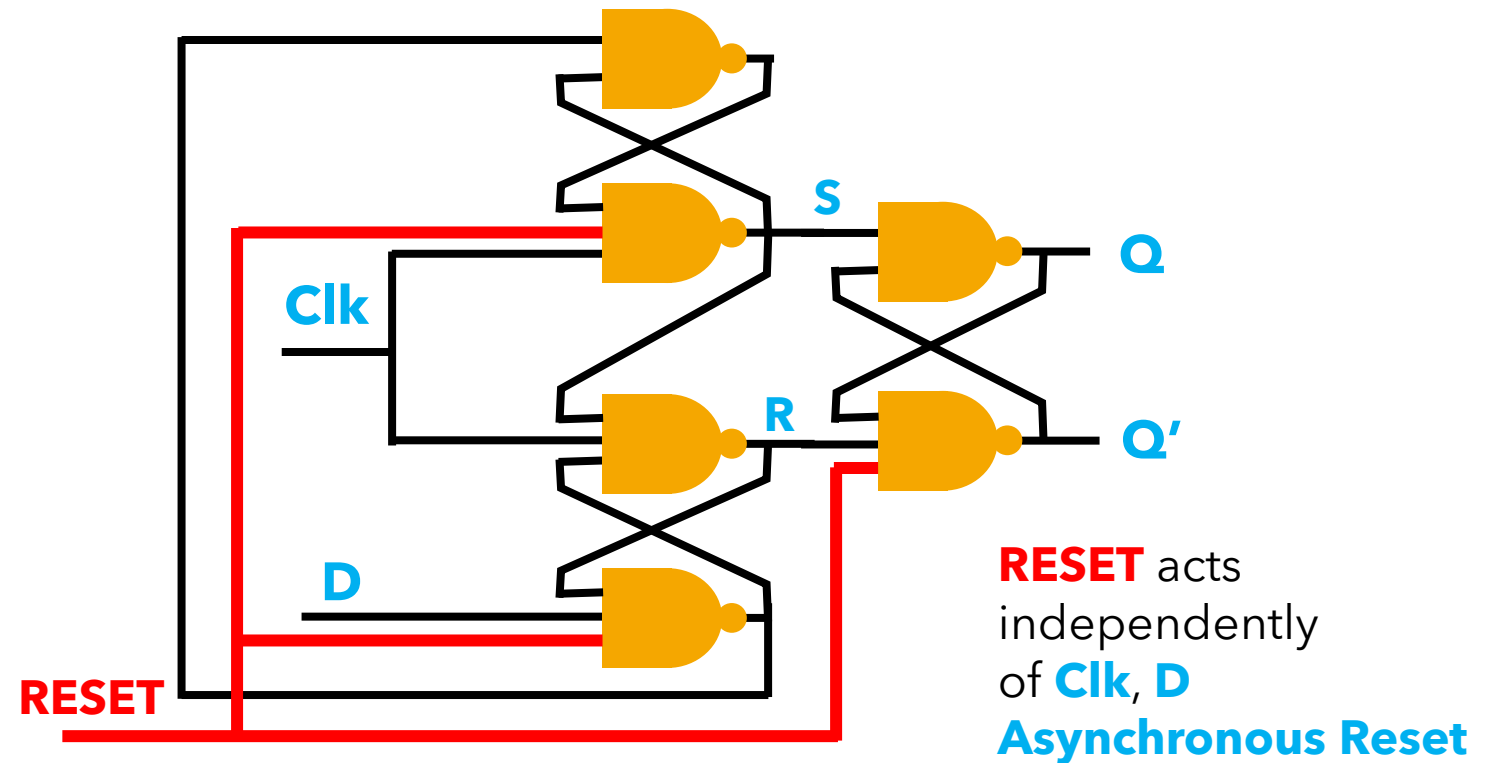
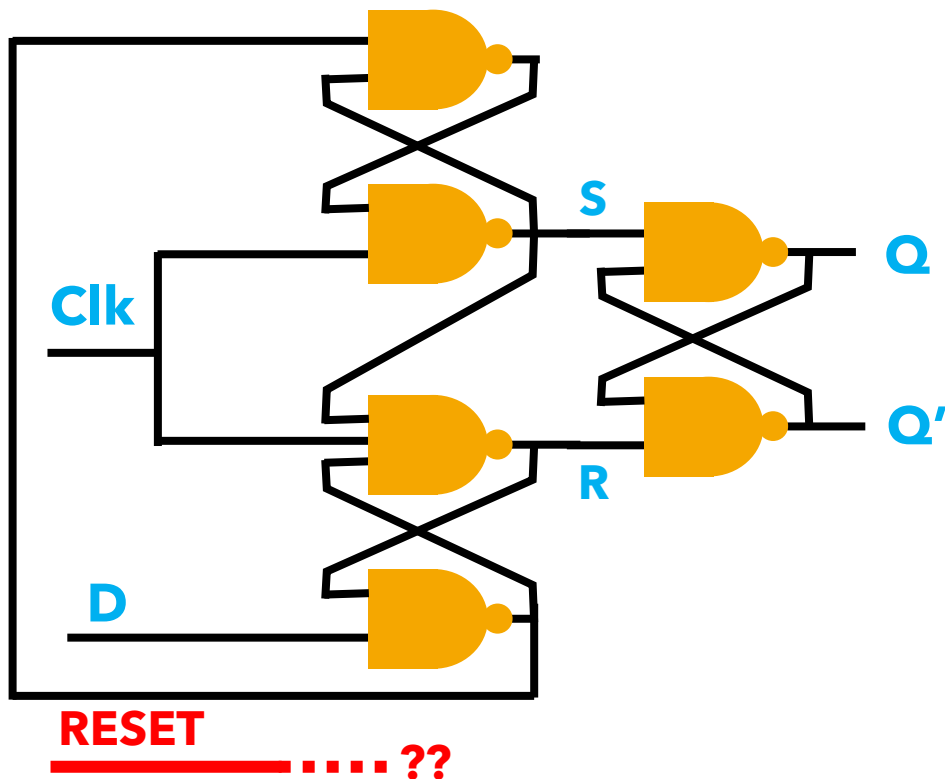
On clock edge: **Q** takes the value of **$Jq' + K'q$**
This is sent to D input

Latches vs. Flip-flops

- Latches are **Level-triggered**
 - Q follows D when **En=1 (level = 1)**
- Flip-flops are **Edge-triggered**
 - D value captured **on clock edge**
 - Feedback/loop is eliminated

D Flip-flop: Asynchronous Inputs

New Input RESET:
RESET=0 causes Q=0



Characteristic Tables: Flip-flop Properties

D	Q (t+1)	
0	0	Reset
1	1	Set

D Flip-flop
Characteristic Table

T	Q (t+1)	
0	Q(t)	No change
1	Q'(t)	Complement

T Flip-flop
Characteristic Table

J	K	Q (t+1)	
0	0	Q(t)	No change
0	1	0	Reset
1	0	1	Set
1	1	Q'(t)	Complement

JK Flip-flop
Characteristic Table

Characteristic Equations

D	Q (t+1)	
0	0	Reset
1	1	Set

D Flip-flop
 $Q(t+1) = D$

T	Q (t+1)	
0	Q(t)	No change
1	Q'(t)	Complement

T Flip-flop
 $Q(t+1) = Q \oplus T$

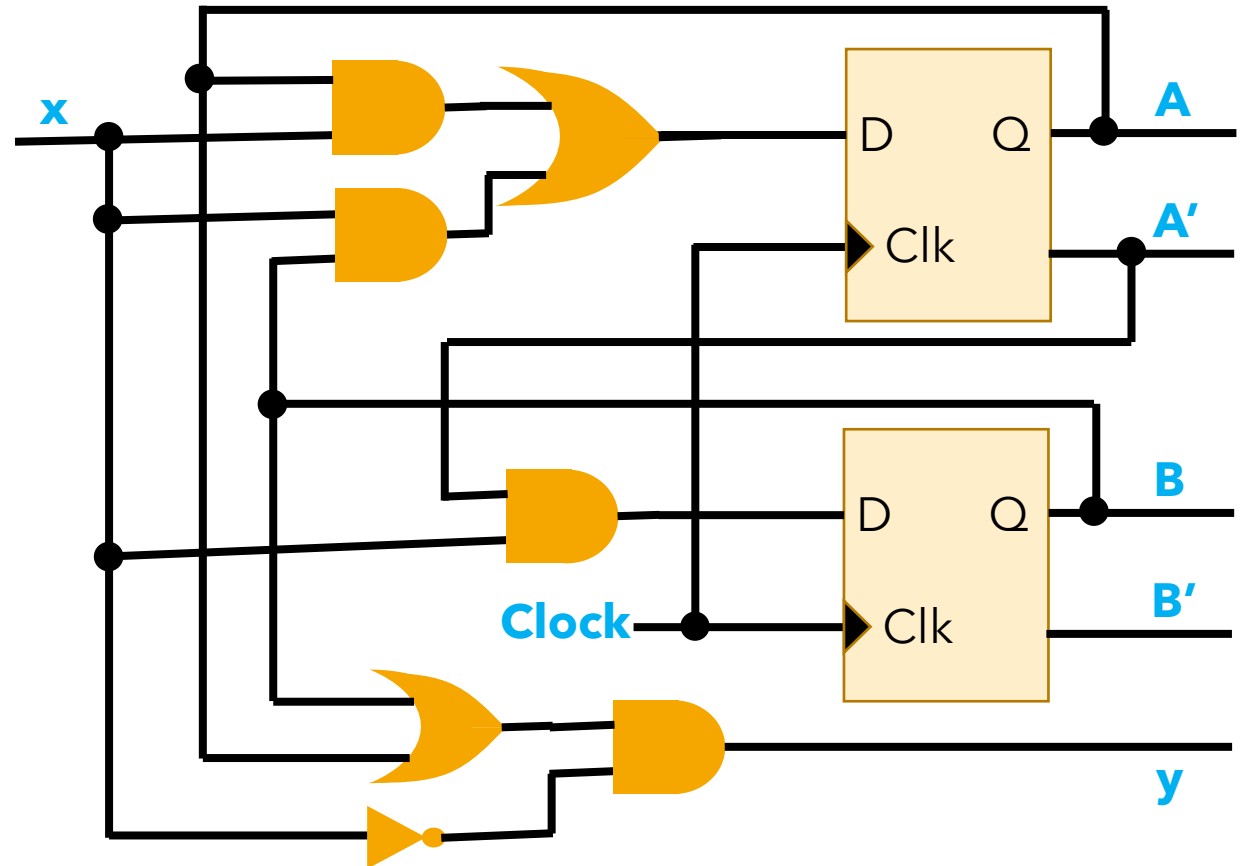
J	K	Q (t+1)	
0	0	Q(t)	No change
0	1	0	Reset
1	0	1	Set
1	1	Q'(t)	Complement

JK Flip-flop
 $Q(t+1) = Q'J + QK'$

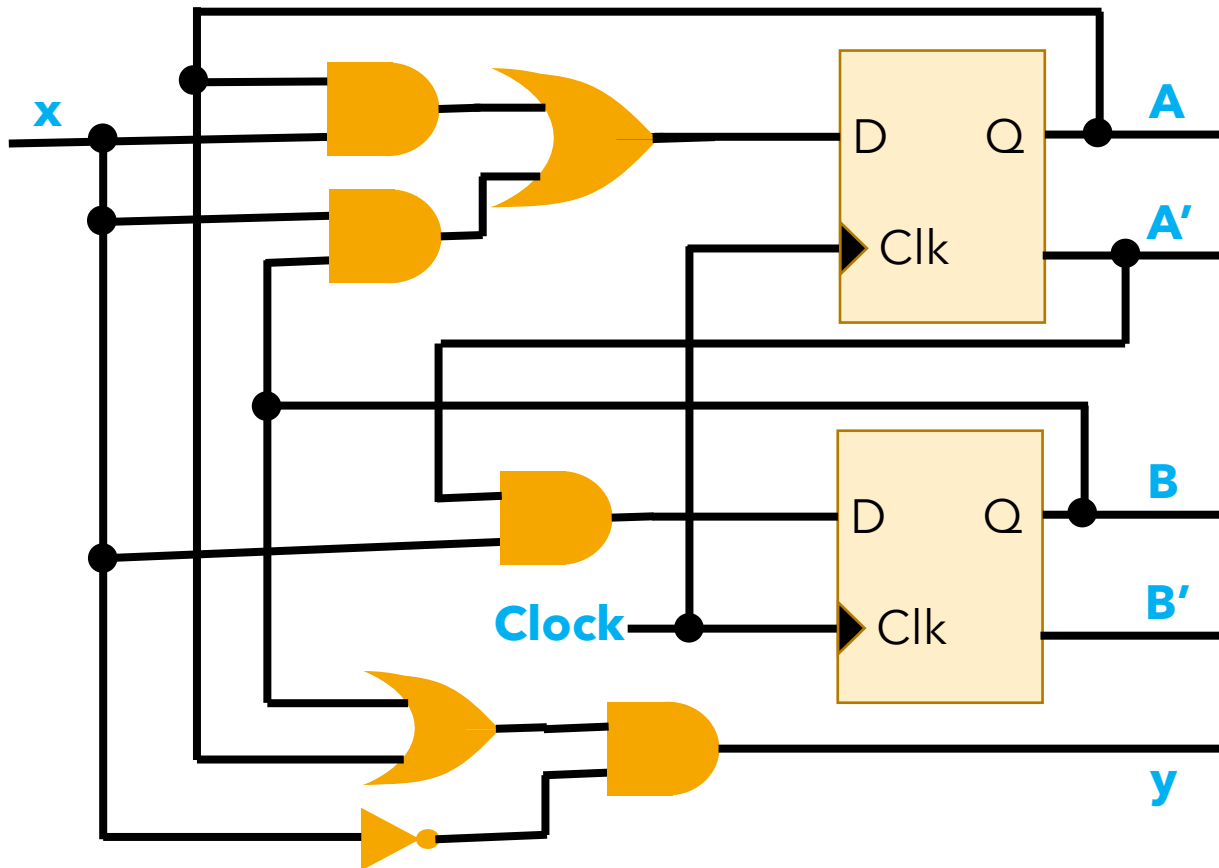
Q (= Q(t)): Value of Q just before clock edge
Clock is implicit (not written in equation)

100%

- **Inputs**
- **Outputs**
- Combinational **logic**
 - **functions of inputs and Qs**
- A set of **flip-flops** (any type)
- A **clock** (common to all flip-flops)



Deriving State Equations



Equations for new values of A, B, y:

$$A(t+1) = A(t) x(t) + B(t) x(t)$$

$$B(t+1) = A'(t) x(t)$$

$$y = x'(t)(A(t) + B(t))$$

Simplify: Drop the (t)

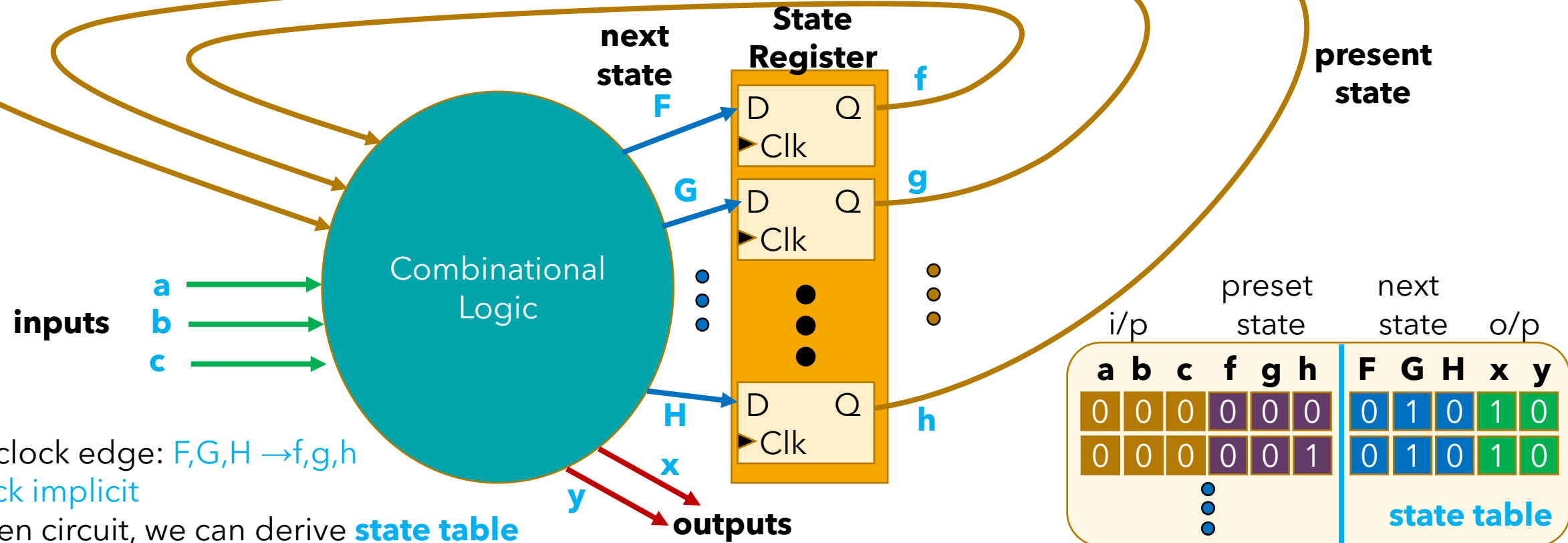
$$A(t+1) = Ax + Bx$$

$$B(t+1) = A'x$$

$$y = x'(A+B)$$

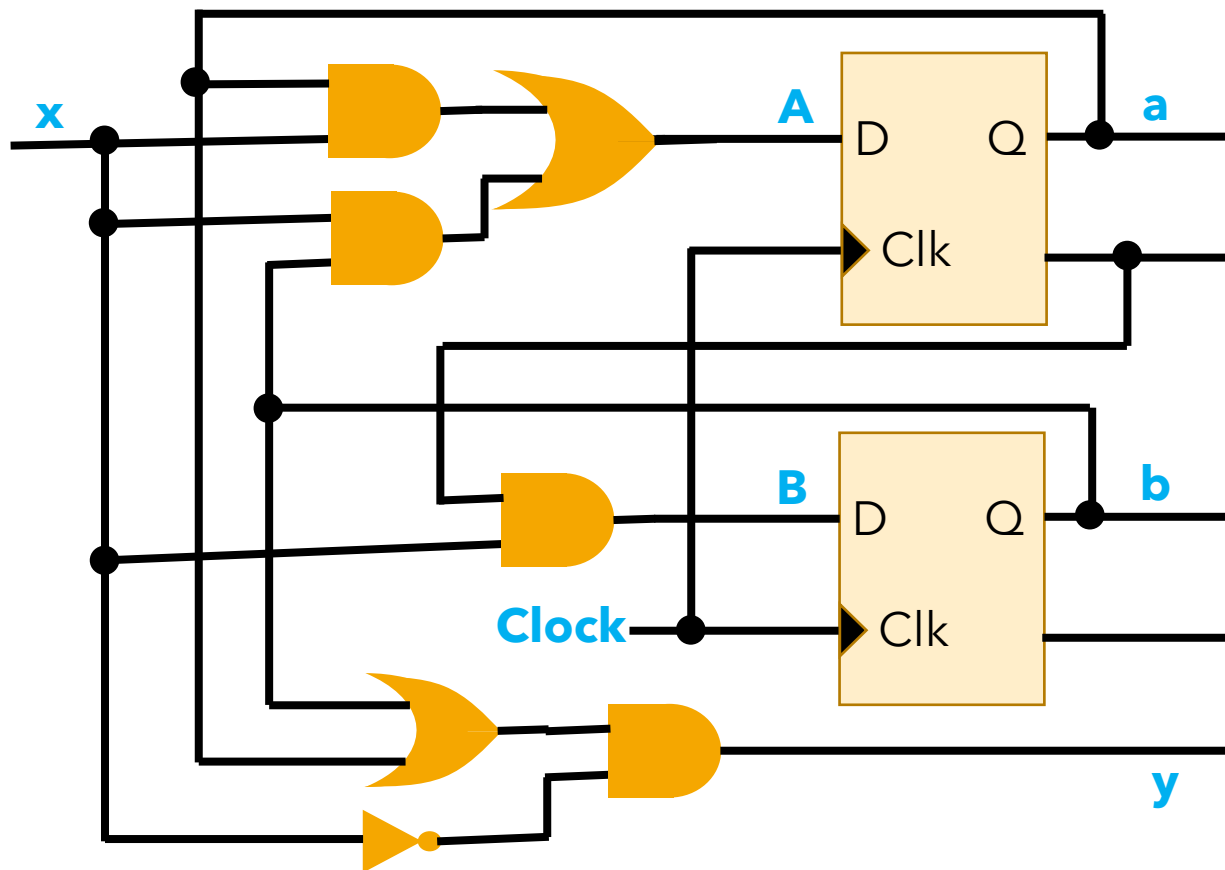
Generates A, B, y values in next clock cycle in terms of values in current clock cycle

Sequential Designs



- on clock edge: $F, G, H \rightarrow f, g, h$
- clock implicit
- Given circuit, we can derive **state table**
- Given state table, can derive **circuit**

Deriving State Table



State Table

a	b	x	A	B	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

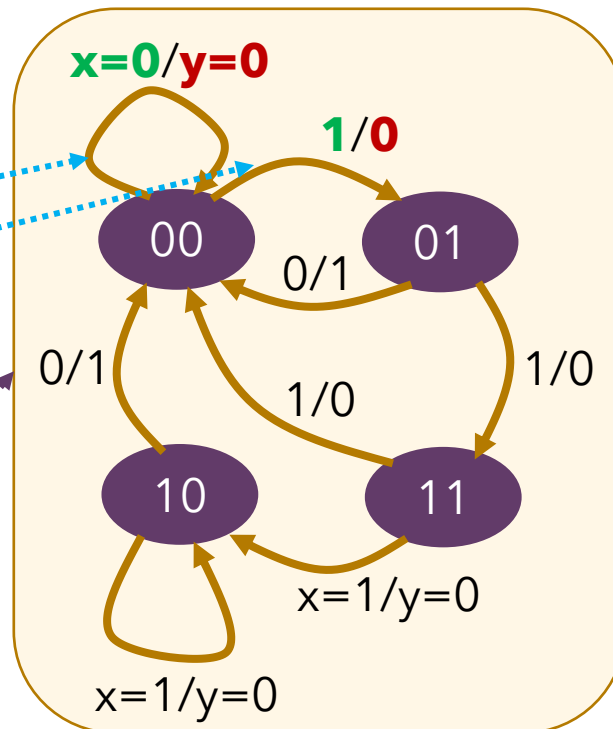
State Diagram (Finite State Machine - FSM)

Q values of Flip-flops together constitute the **State** of the system
Our design has 4 **states**: **00**, **01**, **10**, **11** (**ab/AB** values)

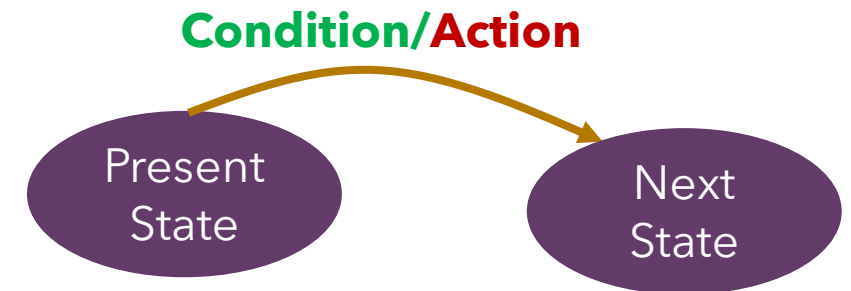
State Table

present state			next state		
a	b	x	A	B	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

State Diagram



State Transition

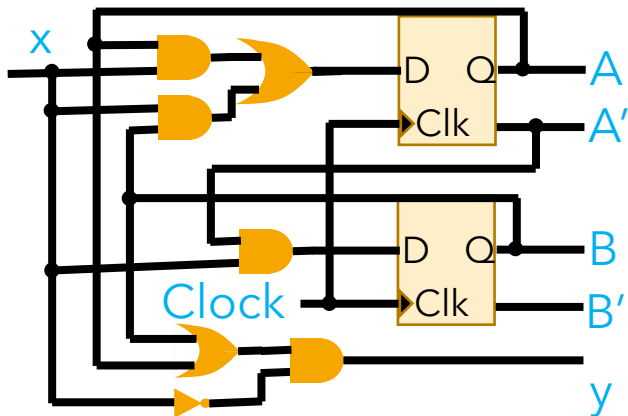


Each **Row** in State Table becomes: one **Edge/State Transition** in FSM

Same
Information

Sequential Analysis

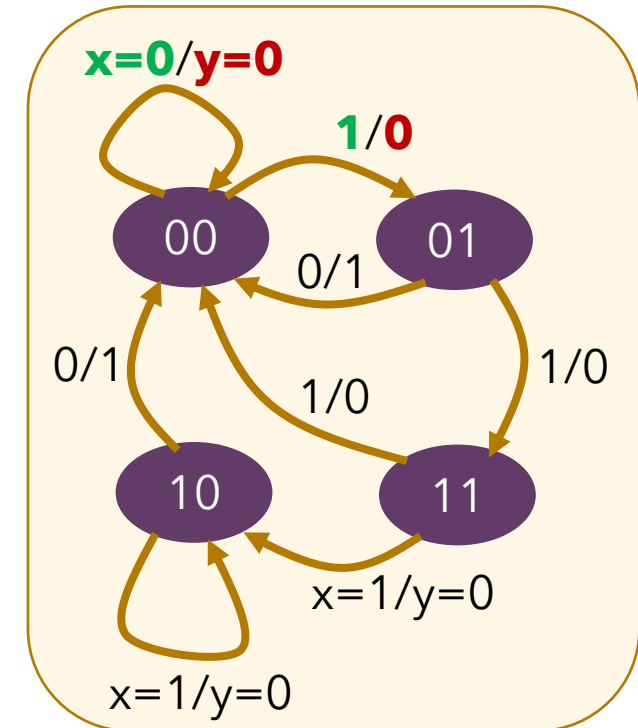
Sequential Design



State Table

present state			next state		
a	b	x	A	B	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

State Diagram



Synthesis

Example Finite State Machine

Specification: Counter

Output $P = 1$ every 3rd clock cycle
Output $P = 0$ in other cycles
If input $R = 0$, reset count to zero
If input $R = 1$, resume counting

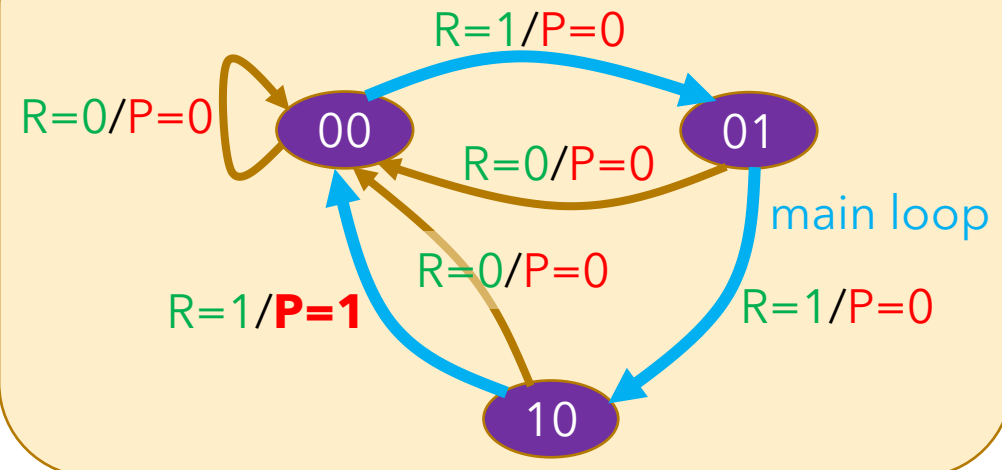
States

00 1st cycle
01 2nd cycle
10 3rd cycle

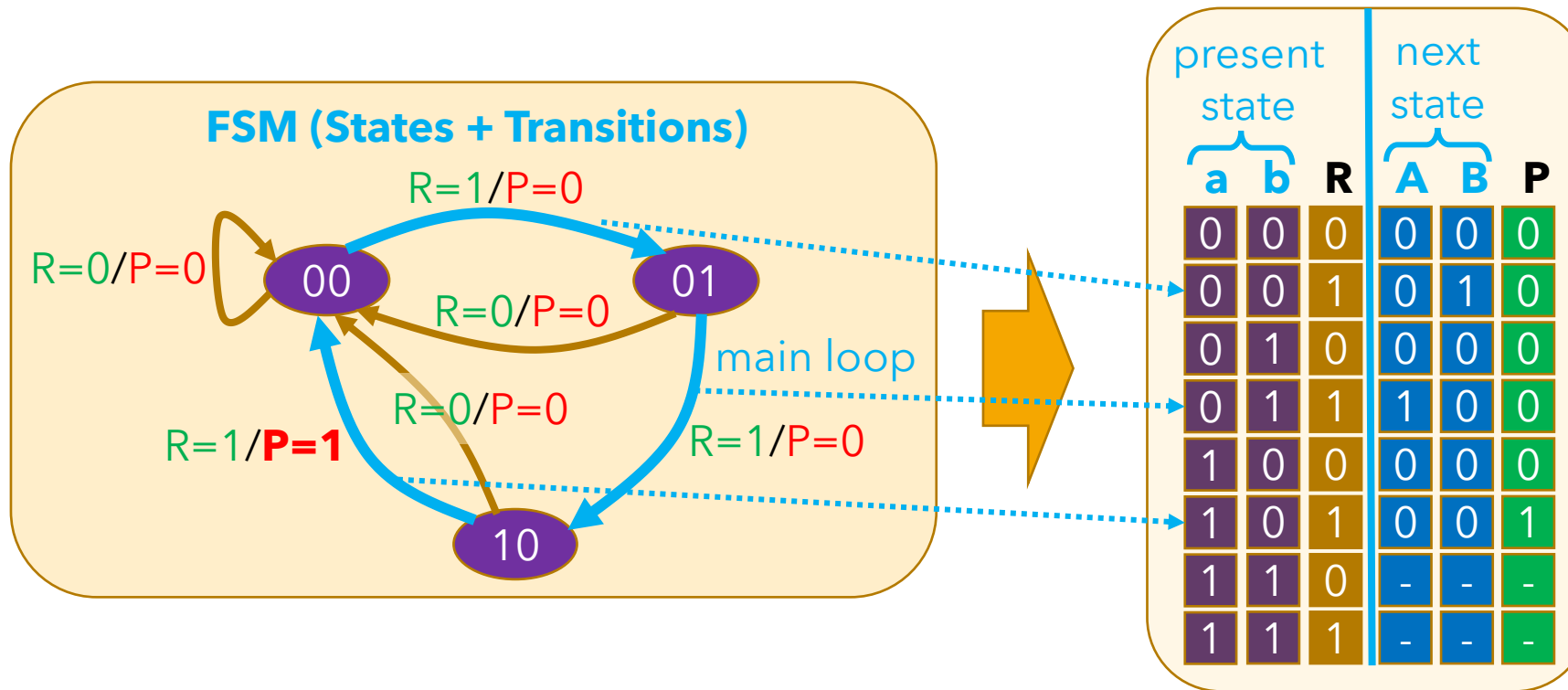
Strategy

- Let us count: 0, 1, 2, 0, 1, 2, 0, 1, 2....
- Every time we hit 0, set $P=1$

FSM (States + Transitions)

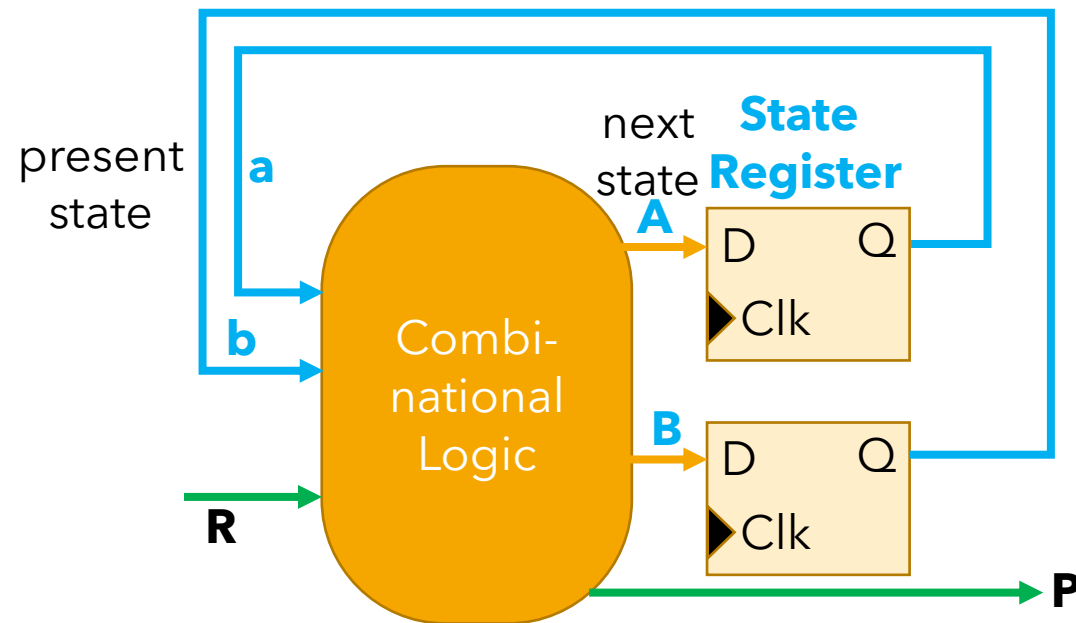


Deriving State Table from FSM



Deriving Sequential Design from State Table

present state			next state		
a	b	R	A	B	P
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	1	0	0	1
1	1	0	-	-	-
1	1	1	-	-	-



Boolean Expressions for Next State, Output

present state			next state		
a	b	R	A	B	P
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	1	0	0	1
1	1	0	-	-	-
1	1	1	-	-	-



K-map for A

ab	00	01	11	10
R				
0	0	0	x	0
1	0	1	x	0

$A = bR$

K-map for B

ab	00	01	11	10
R				
0	0	0	x	0
1	1	0	x	0

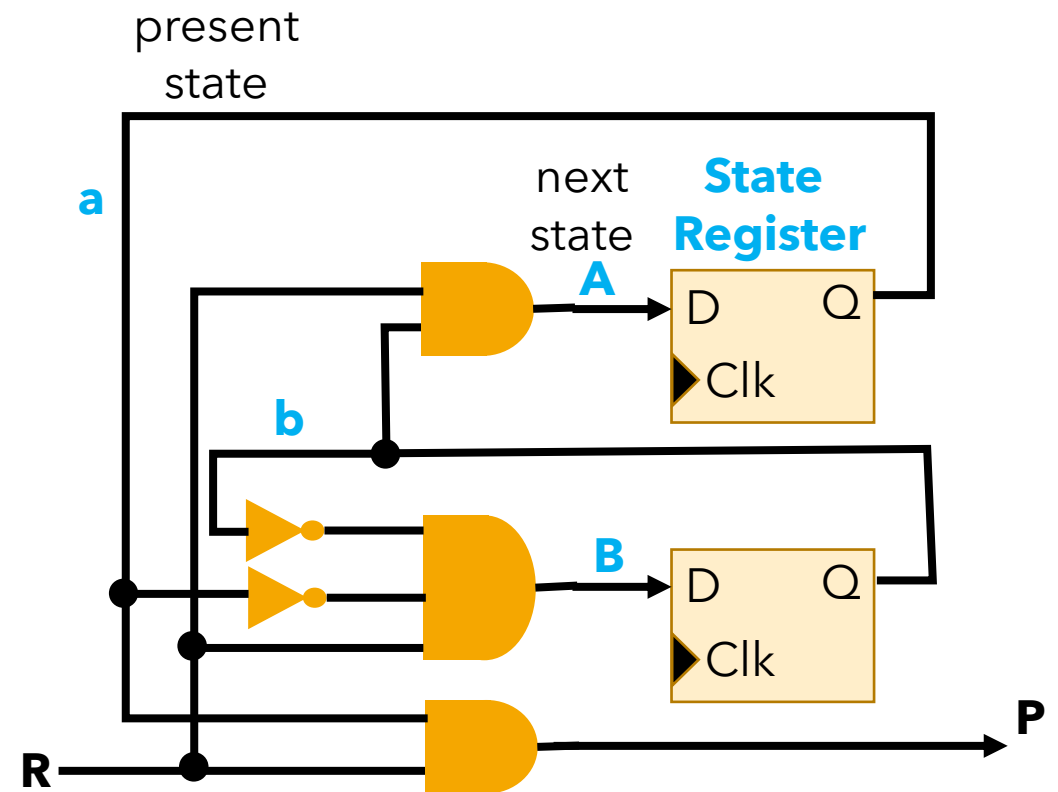
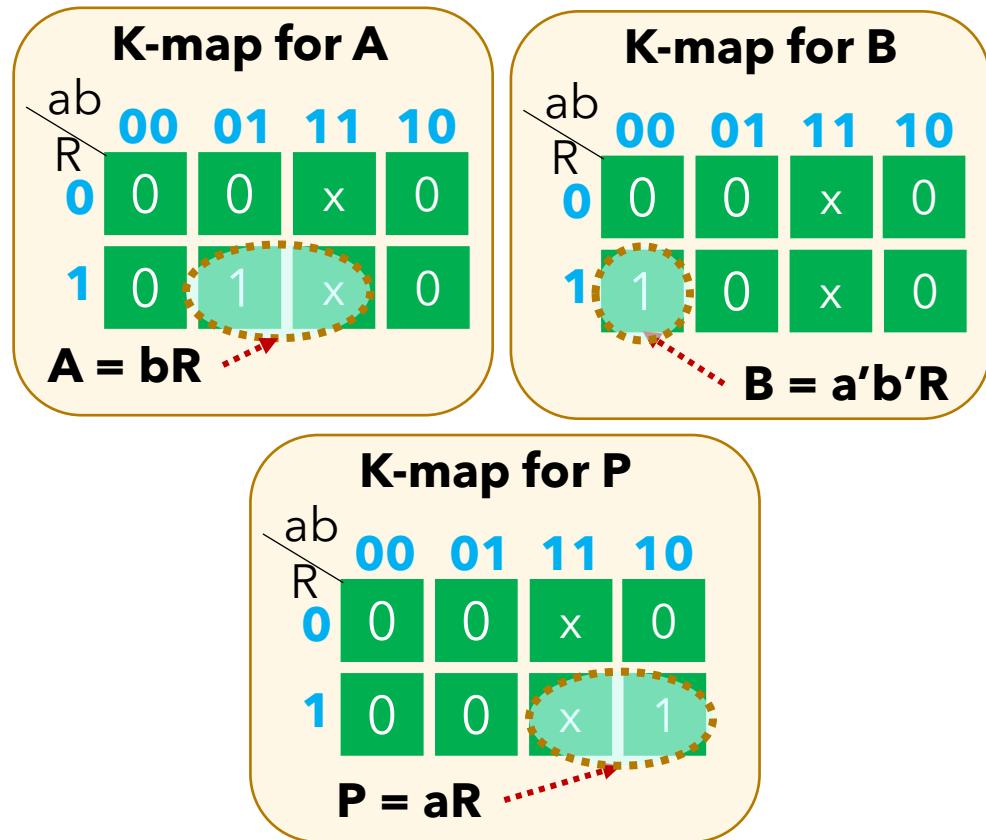
$B = a'b'R$

K-map for P

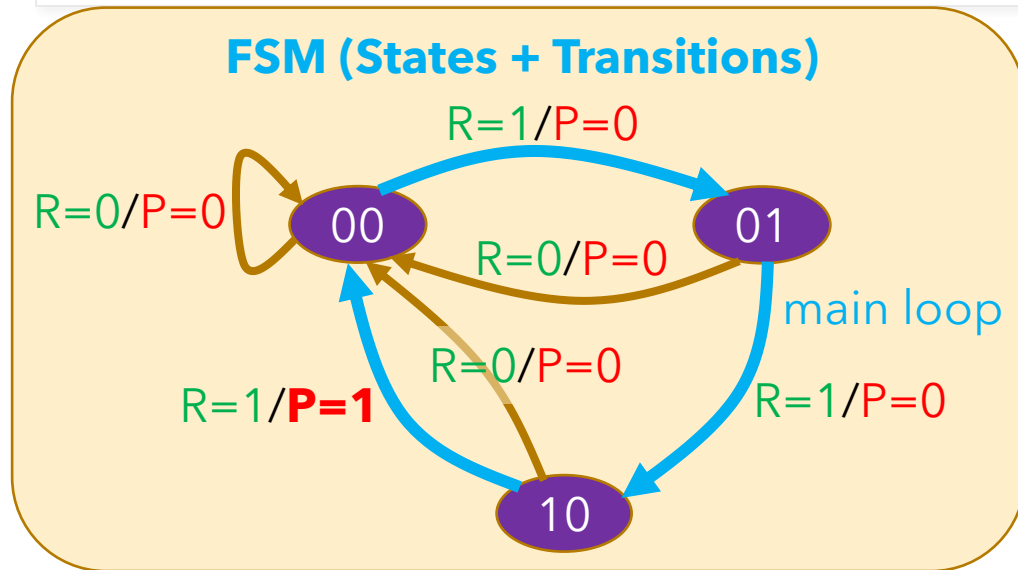
ab	00	01	11	10
R				
0	0	0	x	0
1	0	0	x	1

$P = aR$

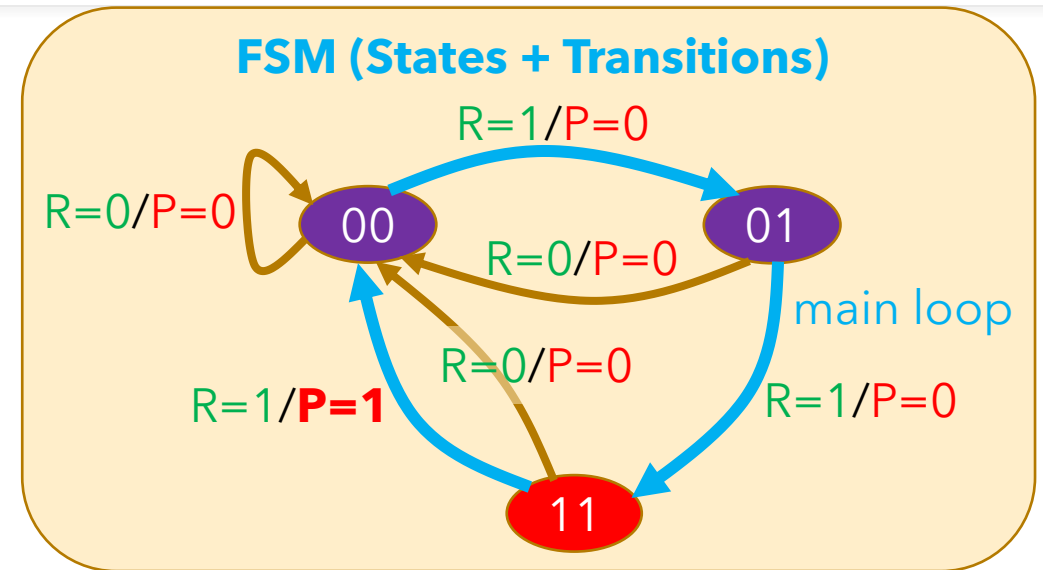
Gate-level Design



What if we used a different sequence?



Replace by:



Specification

Output P = 1 every 3rd clock cycle
Output P = 0 in other cycles
If input R = 0, reset count to zero
If input R = 1, resume counting

Sequence of states: **0,1,3,0,1,3,...**

Output P sequence **unchanged!**

Specification still met.

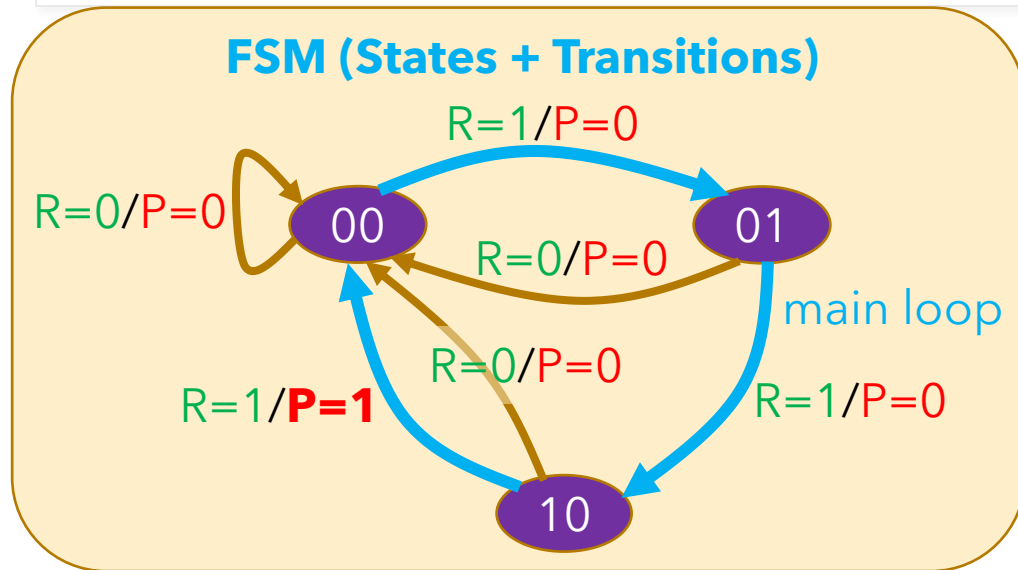
Values **00, 01**, etc., don't matter

What changes?

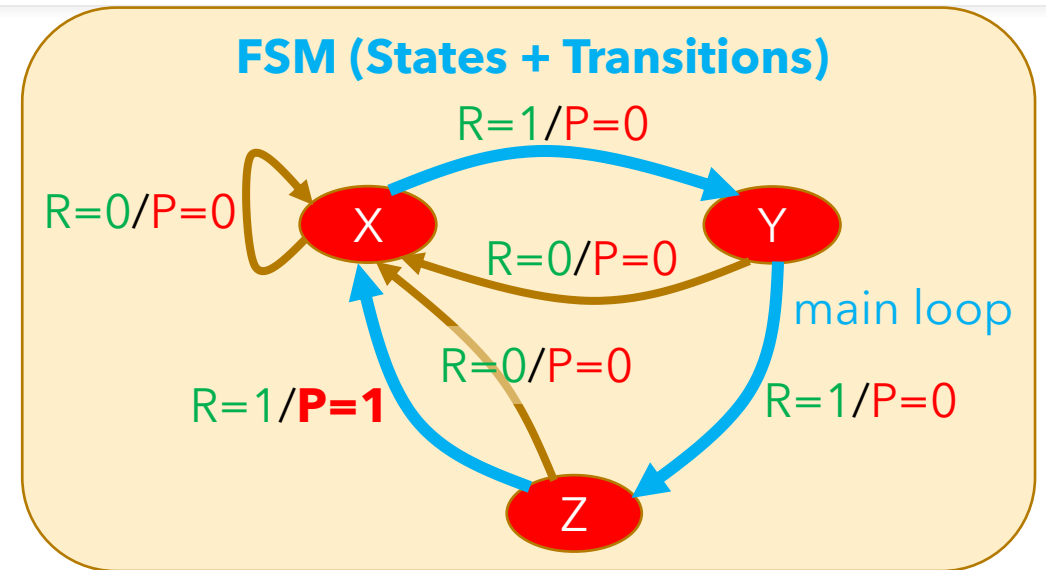
What remains unchanged?

Is the functionality still delivered?

FSM States can be Symbolic



Replace by:



Specification still met.

Values **00**, **01**, etc., don't matter

States can be **Symbolic** (labelled **X**, **Y**, **Z**)

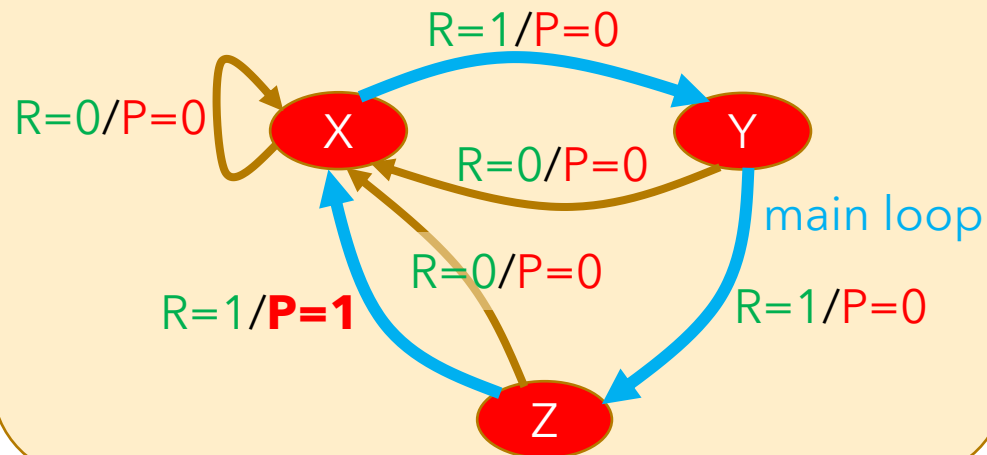
Start with Symbolic FSM

Specification

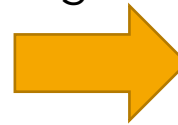
Output $P = 1$ every 3rd clock cycle
Output $P = 0$ in other cycles
If input $R = 0$, reset count to zero
If input $R = 1$, resume counting



FSM (Symbolic States + Transitions)

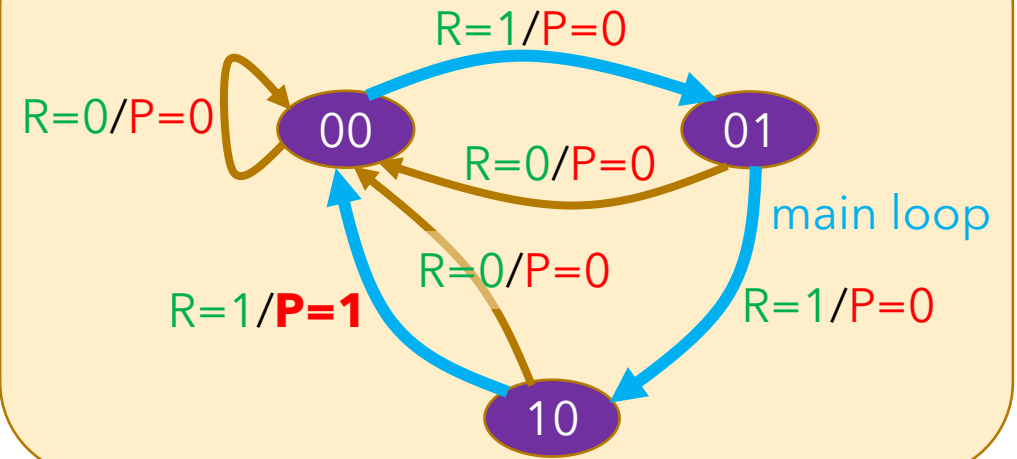


State
Encoding/
State
Assignment



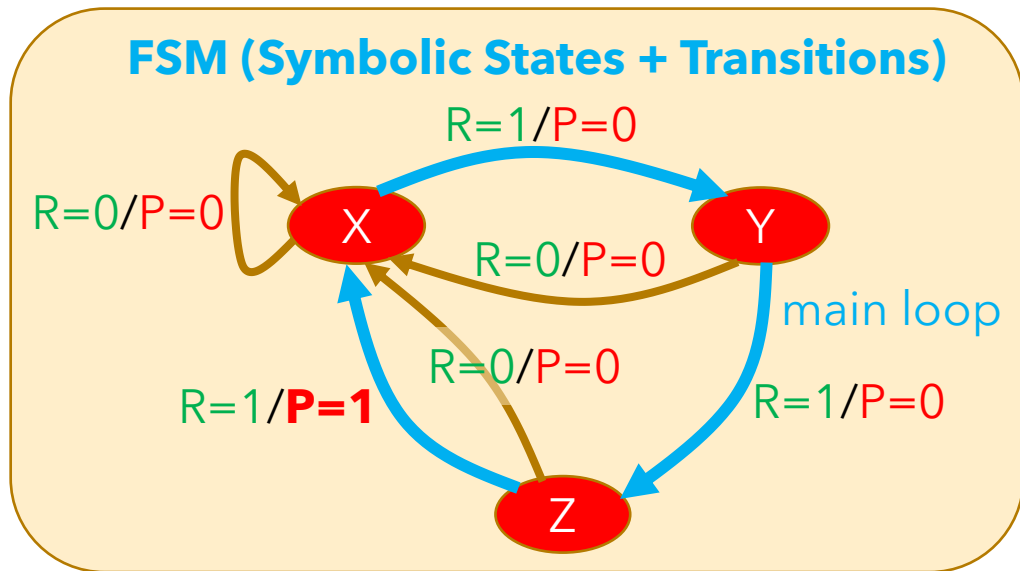
X=00
Y=01
Z=10

Encoded FSM



Other encodings also valid

Symbolic State Table



Symbolic State Table

PS	R	NS	P
X	0	X	0
X	1	Y	0
Y	0	X	0
Y	1	Y	0
Z	0	X	0
Z	1	X	1

PS: present state
NS: next state

**Encoded State Table
(Truth Table)**

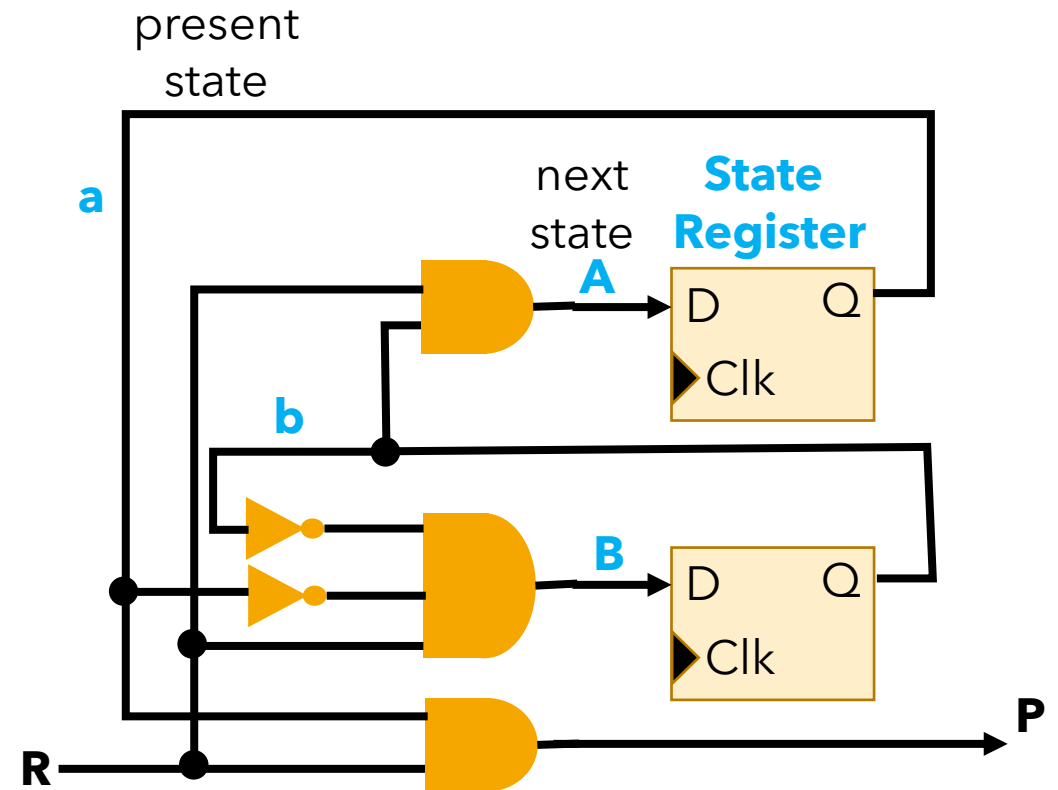
present state			next state		
a	b	R	A	B	P
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	1	0	0	1

No need for
 don't care rows

Finally, Sequential Circuit

Encoded State Table
(Truth Table)

present state			next state		
a	b	R	A	B	P
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	1	0	0	1



Another FSM Example

- Design a Sequence Detector
 - Detect a sequence of 3 or more 1's on an input line

Input: 0 1 1 0 1 1 1 0 1 0 1 1 1 1 0 1...

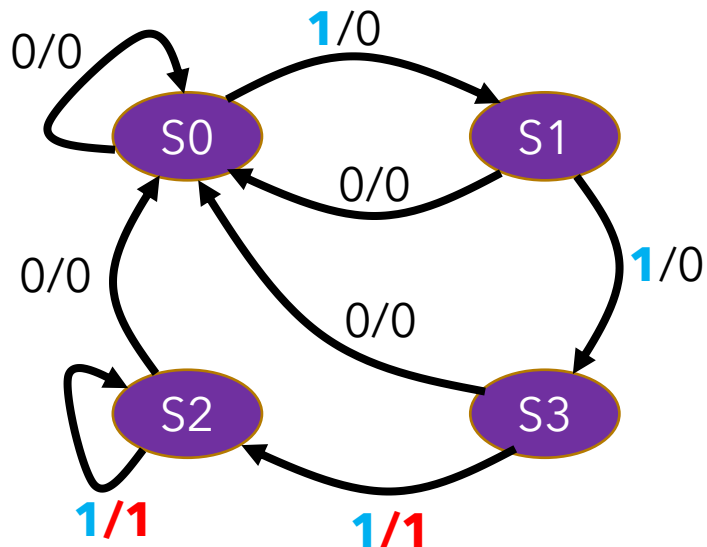
Output: 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1 1 0 0...

- Design FSM
 - Inputs? Outputs? States? Transitions?

FSM Design

Input: 0 1 1 0 1 1 1 0 1 0 1 1 1 1 0 1

Output: 0 0 0 0 0 0 0 1 0 0 0 0 0 1 1 0 0



Style: **Mealy Machine**

Output is a function of both **state** and **input**.
Output is associated with transition.

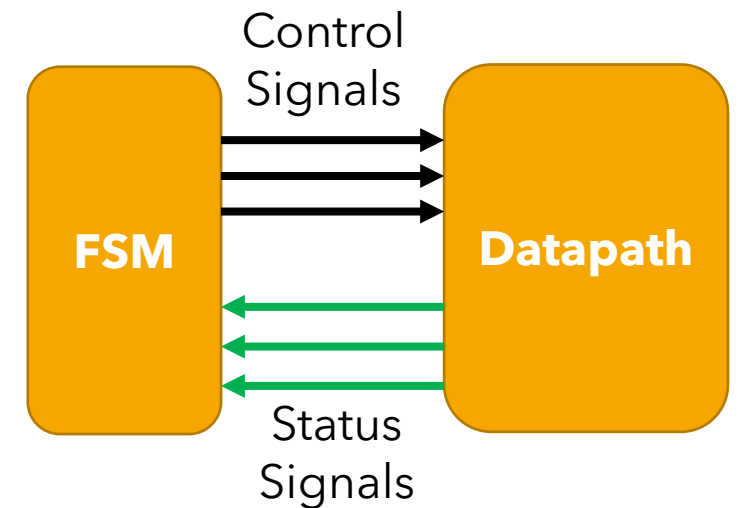
Alternative Style: **Moore Machine**

Output is a function of **state** only
(Ref: Textbook. Figure 5.27)

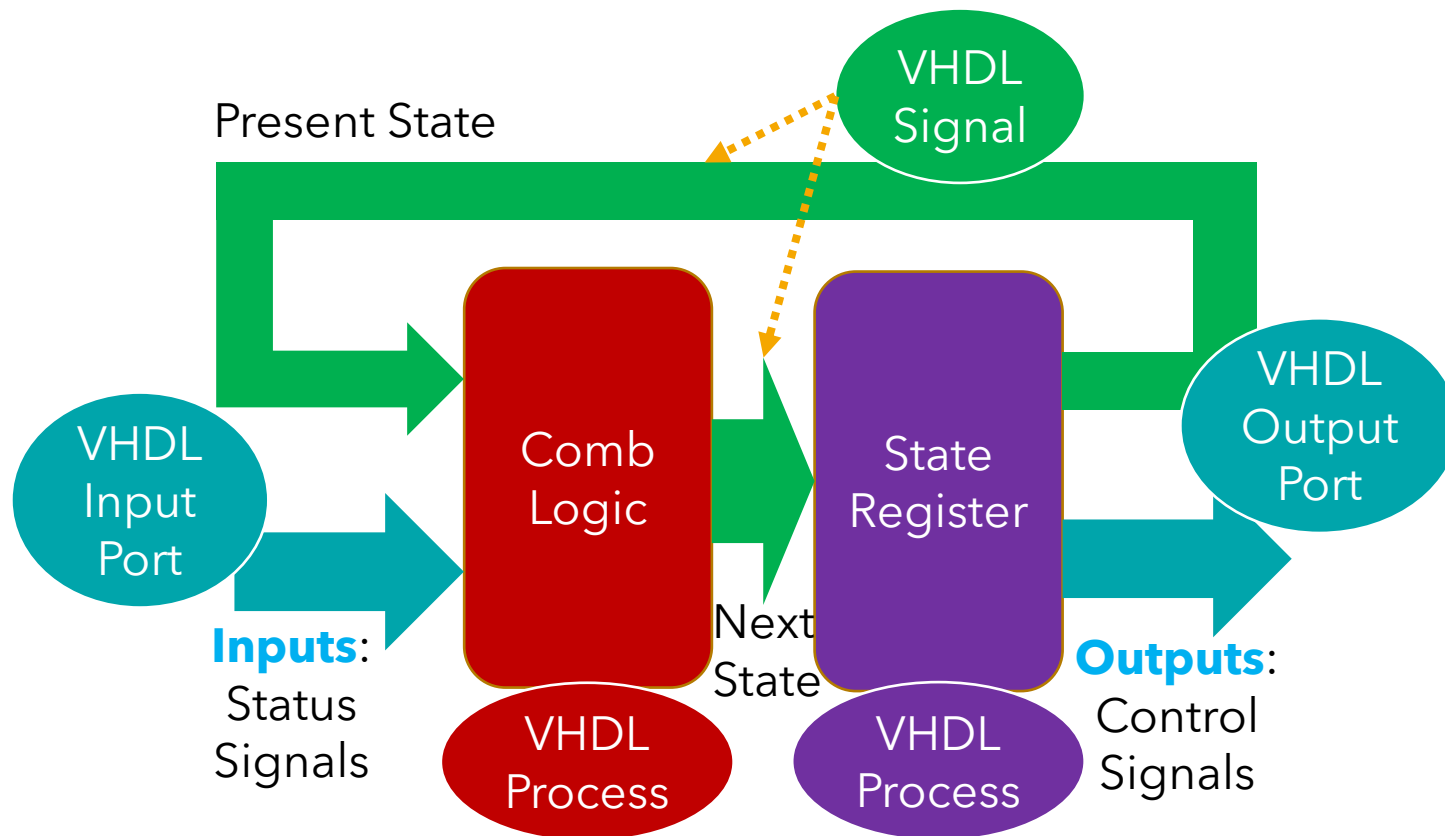
The 2 styles are equivalent

Organising complex designs

- Controller + Datapath
 - **Controller: FSM** (sends control signals)
 - MUX **select** inputs
 - **Load enable** signals to flip-flops/registers
 - **Datapath: Computation**
 - Adders, Multipliers, Muxes
 - May send **status** signals back to FSM
 - **results** of comparisons
 - overflow/exceptions



Specifying FSMs in HDL



VHDL Model

```
entity...  
port...  
end entity
```

```
architecture...
```

```
signal...
```

```
process...  
-- state register  
end process
```

```
process...  
-- comb. logic  
end process
```

```
end architecture
```


State Register in VHDL

State Register

```
type state_type is (ADD, SUB);  
signal cur_state : state_type := ADD;  
signal next_state : state_type := ADD;
```

```
process (clk)  
begin  
if (clk'EVENT AND clk = '1') then  
cur_state <= next_state;  
end if;  
end process;
```

states

initial state

rising edge

$Q \leq D$

State Register with Reset

```
process (clk, reset)  
begin  
if (reset = '1') then  
cur_state <= ADD;  
elsif (clk'EVENT AND clk = '1')  
then  
cur_state <= next_state;  
end if;  
end process;
```

initial/reset
state

normal
function

- Synthesised into set of D flip-flops
- #flip-flops (#bits) not specified here (automatically inferred)

Source: Hardware Assignment 3
Author: Naman Jain

Next State and Output Logic in VHDL

```
process (cur_state, M, Done)
begin
  next_state <= cur_state;
```

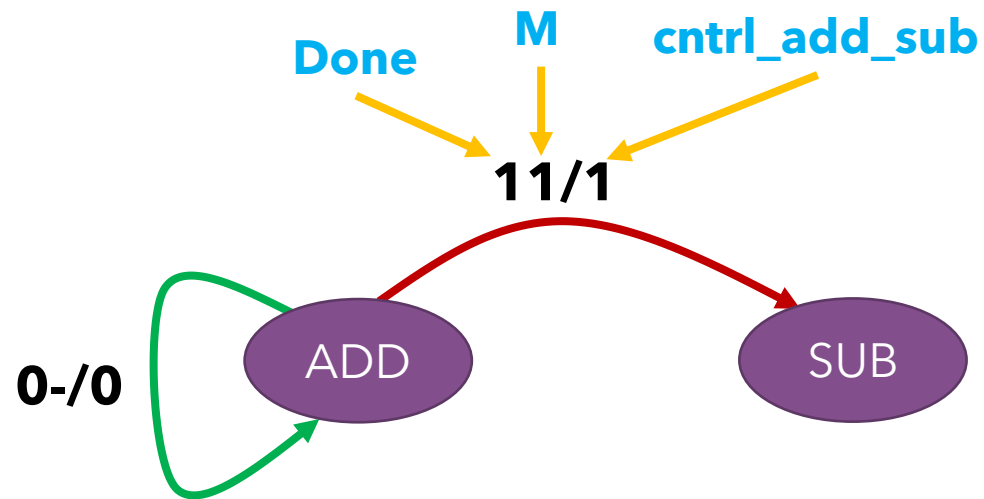
```
case cur_state is
when ADD =>
```

```
  if Done = '1' and M = '1' then
    next_state <= SUB;
    cntrl_add_sub <= '1';
```

```
  elsif Done = '0' then
    next_state <= ADD;
    cntrl_add_sub <= '0';
```

```
  end if;
```

```
when SUB => ...
end case;
end process;
```



Source: Hardware Assignment 3
Author: Naman Jain

Next State and Output Logic in VHDL

```
process (cur_state, M, Done)
begin
  next_state <= cur_state;
```

```
case cur_state is
when ADD =>
```

```
  if Done = '1' and M = '1' then
    next_state <= SUB;
    cntrl_add_sub <= '1';
```

```
  elsif Done = '0' then
    next_state <= ADD;
    cntrl_add_sub <= '0';
  end if;
```

...

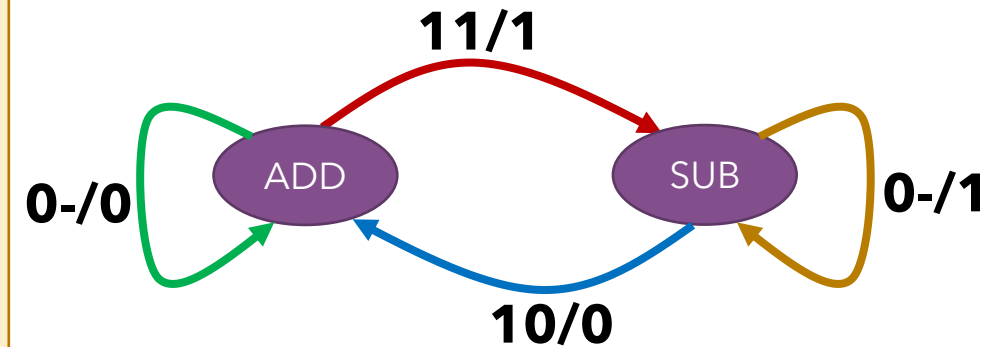
...

```
when SUB =>
```

```
  if Done = '1' and M = '0' then
    next_state <= ADD;
    cntrl_add_sub <= '0';
```

```
  elsif Done = '0' then
    next_state <= SUB;
    cntrl_add_sub <= '1';
```

```
  end if;
end case;
end process;
```



Source: Hardware Assignment 3
Author: Naman Jain