

**1 (a). The crypto pledge (1 mark)**

*I promise that once I see how simple cryptographic constructions really are, I will not implement them in **production code** even though it will be really fun. This agreement will remain in effect until I learn all about side-channel attacks and countermeasures to the point where I lose all interest in implementing them myself.<sup>1</sup>*

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<sup>1</sup>Taken from the course textbook, originally due to Jeff Moser.

## 1 (b). True/False (4 marks)

For each of the following questions, indicate whether the statement is true or false, and **provide a one-line justification for your answer**.

1. (1 mark) The signing algorithms of all MAC systems are also secure pseudorandom functions.

No! Consider  $\text{MAC}'(k,m) = \text{MAC}(k,m) \parallel \text{MAC}(k,m)$

2. (1 mark) Let  $G : \{0,1\}^n \rightarrow \{0,1\}^{2n}$  be a secure pseudorandom generator. Since it is a secure PRG, the statistical distance of the following two distributions is bounded by a negligible function of  $n$ :

$$\mathcal{D}_0 := \left\{ \text{Sample uniformly random } s \leftarrow \{0,1\}^n, \text{ output } G(s) \right\}$$
$$\mathcal{D}_1 := \left\{ \text{Output uniformly random } u \leftarrow \{0,1\}^{2n} \right\}$$

No, the distance is in fact close to 1. Consider those points which are not in the range of  $G$

3. (1 mark) If  $(\text{Sign}, \text{Verify})$  is a secure MAC system, then at least some bits of  $m$  are guaranteed to be hidden if I give out  $\sigma = \text{Sign}(m, k)$ .

This is not necessary. We can even make  $\text{sign}(m,k) = m \parallel F(m,k)$

4. (1 marks) If  $G : \{0,1\}^n \rightarrow \{0,1\}^{2n}$  is a secure pseudorandom generator, then so is  $H : \{0,1\}^n \rightarrow \{0,1\}^{2n}$  where  $H(s) = G(s) \oplus G(0^n)$  for all  $s \in \{0,1\}^n$ .

Yes

## 1 (c). Multiple Choice Questions (8 marks)

For each of the following MCQs, select the correct option. **Provide a short justification.**

1. (2 marks) Consider a symmetric key encryption scheme  $\mathcal{E}$  that satisfies Semantic Security. Then,
- (A)  $\mathcal{E}$  is No-Query-Semantic-Security secure against **ALL (computationally unbounded)** adversaries.
  - (B) There exists a **computationally unbounded** adversary that wins the No-Query-Semantic-Security game against scheme  $\mathcal{E}$  with probability greater than  $1/2$ .
  - ☒ (C) Neither of the above. It depends on the scheme  $\mathcal{E}$ .

Let  $F : \mathcal{X} \times \mathcal{K} \rightarrow \mathcal{Y}$  be a secure pseudorandom function with  $\mathcal{X} = \mathcal{K} = \mathcal{Y} = \{0, 1\}^n$  (here  $\mathcal{K}$  is the key space). Consider the following keyed functions. For each of them, indicate whether the keyed function is a provably secure PRF, may be/may not be a secure PRF (depends on some additional properties of  $F$ ), or is definitely insecure.


2. (2 marks)  $F'(x, k) = F(x, k) \parallel F(x, F(x, k))$
- (A)  $F'$  is provably secure, assuming  $F$  is.
  - (B)  $F'$  may/may not be a secure PRF. It requires some additional properties of  $F$ .
  - ☒ (C)  $F'$  is an insecure PRF.

3. (2 marks)  $F'(x, k) = F(x, k) \oplus x$

- ☒ (A)  $F'$  is provably secure, assuming  $F$  is.
- (B)  $F'$  may/may not be a secure PRF. It requires some additional properties of  $F$ .
- (C)  $F'$  is an insecure PRF.

4. (2 marks)  $F'(x, k) = F(x, k) || F(x, x \oplus k)$

- (A)  $F'$  is provably secure, assuming  $F$  is.
- (B)  $F'$  may/may not be a secure PRF. It requires some additional properties of  $F$ .
- ☒ (C)  $F'$  is an insecure PRF.



## 2. PRGs with large expansion (4 marks)

Let  $G_1 : \{0,1\}^n \rightarrow \{0,1\}^{2n}$ , and  $G_2 : \{0,1\}^n \rightarrow \{0,1\}^{3n}$  be secure pseudorandom generators. Consider the following function  $H : \{0,1\}^n \rightarrow \{0,1\}^{6n}$  defined as follows:

$$\begin{aligned} H(s) &= y_1 \parallel y_2 \parallel \dots \parallel y_5 \parallel y_6 \\ &\quad \text{where} \\ G_1(s) &= s_0 \parallel s_1 \\ y_1 \parallel y_2 \parallel y_3 &= G_2(s_0) \\ y_4 \parallel y_5 \parallel y_6 &= G_2(s_1) \end{aligned}$$

Here, each  $y_i$ ,  $s_0$  and  $s_1$  are  $n$  bit strings.

We will show that  $H$  is a secure pseudorandom generator, assuming  $G_1$  and  $G_2$  are secure PRGs. The proof will go via a sequence of hybrids. In World-0, the challenger samples a uniformly random  $s \leftarrow \{0,1\}^n$  and sends  $H(s)$ . In World-1, the challenger chooses a  $6n$ -bit uniformly random string  $u \leftarrow \{0,1\}^{6n}$  and sends  $u$ .

**Define the intermediate hybrids required to prove security of  $H$ . No need to provide reductions for showing indistinguishability of hybrids.**

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### 3. A broken encryption scheme (4 marks)

Let  $P : \{0,1\}^n \times \{0,1\}^n \rightarrow \{0,1\}^n$  be a secure pseudorandom permutation, and let  $P^{-1}$  be its inverse (that is, for all  $x, k$ ,  $P^{-1}(P(x, k), k) = x$ ). Consider the following encryption scheme with message space  $\{0,1\}^{\ell \cdot n}$  for some  $\ell > 3$ :

- **KeyGen**: choose a random PRP key  $k \leftarrow \{0,1\}^n$ .
- **Enc** $\left(m = (m_1, \dots, m_\ell), k\right)$ : choose a uniformly random string  $x_0 \leftarrow \{0,1\}^n$  and set  $\text{ct}_0 = P(x_0, k)$ .

For  $i = 1$  to  $\ell$ , do the following:

- set  $x_i = m_i \oplus x_{i-1}$
- compute  $\text{ct}_i = P(x_i, k)$ .

Output  $\text{ct} = (\text{ct}_0, \text{ct}_1, \text{ct}_2, \dots, \text{ct}_\ell)$  as the final ciphertext.

- **Dec** $\left(\text{ct} = (\text{ct}_0, \text{ct}_1, \dots, \text{ct}_\ell), k\right)$ : Let  $y_0 = P^{-1}(\text{ct}_0, k)$ . For each  $i = 1$  to  $\ell$ , do the following:
  - compute  $y_i = P^{-1}(\text{ct}_i, k)$ .
  - set  $m_i = y_i \oplus y_{i-1}$

Output  $m = (m_1, m_2, \dots, m_\ell)$  as the final decryption.

**Show that the above scheme does not satisfy No-Query-Semantic-Security.**

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#### 4. A new MAC scheme (4+1 marks)

Let  $\text{MAC} = (\text{Sign}, \text{Verify})$  be a MAC scheme with message space  $\mathcal{M}$ , key space  $\mathcal{K}$  and signature space  $\mathcal{T}$ , satisfying **weak** unforgeability. Consider the following MAC scheme  $\text{MAC}' = (\text{Sign}', \text{Verify}')$ :

- $\text{Sign}'(m, k) = \text{Compute } \sigma_1 \leftarrow \text{Sign}(m, k), \sigma_2 \leftarrow \text{Sign}(m, k), \text{ output } (\sigma_1, \sigma_2).$
- $\text{Verify}'(m, (\sigma_1, \sigma_2), k)$ : Output 1 if either  $\text{Verify}(m, \sigma_1, k) = 1$  or  $\text{Verify}(m, \sigma_2, k) = 1$ .

1. Is  $\text{MAC}'$  a **weakly** unforgeable MAC scheme, assuming  $\text{MAC}$  is? Yes
2. Suppose  $\text{MAC}$  is a **strongly** unforgeable message auth. code. Can we conclude that  $\text{MAC}'$  is also **strongly** unforgeable?

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## Definitions

**Definition 04.01.** A function  $\mu : \mathbb{N} \rightarrow [0, 1]$  is said to be negligible if, for any polynomial  $p(\cdot)$ , there exists  $n_0 \in \mathbb{N}$  such that for all  $n > n_0$ ,  $\mu(n) < 1/p(n)$ .

**Definition 04.02.** An encryption scheme  $(\text{KeyGen}, \text{Enc}, \text{Dec})$  is said to satisfy no-query-semantic-security if, for any probabilistic polynomial time adversary  $\mathcal{A}$ , there exists a negligible function  $\mu(\cdot)$  such that for all  $n$ ,

$$\Pr \left[ \mathcal{A} \text{ wins the No-Query-Semantic-Security game} \right] \leq 1/2 + \mu(n)$$

where the No-Query-Semantic-Security game is defined in Figure 1.

No-Query-Semantic-Security
<ol style="list-style-type: none"> <li>1. Adversary sends two messages <math>m_0, m_1</math> to the challenger, such that <math> m_0  =  m_1 </math>.</li> <li>2. The challenger chooses a bit <math>b \leftarrow \{0, 1\}</math>, key <math>k \leftarrow \mathcal{K}</math> and sends <math>\text{Enc}(m_b, k)</math> to the adversary.</li> <li>3. The adversary sends its guess <math>b'</math>, and wins the security game if <math>b = b'</math>.</li> </ol>

Figure 1: The No-Query Semantic Security Game

**Definition 05.01.** A deterministic polynomial time computable function  $G : \{0, 1\}^n \rightarrow \{0, 1\}^\ell$  is a secure pseudorandom generator (PRG) if  $\ell > n$ , and for any prob. poly. time adversary  $\mathcal{A}$ , there exists a negligible function  $\mu(\cdot)$  such that for all  $n$ ,

$$\Pr [\mathcal{A} \text{ wins the PRG security game against } G] \leq 1/2 + \mu(n),$$

where the PRG game is defined in Figure 2.

PRG-Security
<ol style="list-style-type: none"> <li>1. The challenger chooses a bit <math>b \leftarrow \{0, 1\}</math>, string <math>s \leftarrow \{0, 1\}^n</math>, <math>u_1 \leftarrow \{0, 1\}^\ell</math>. It computes <math>u_0 = G(s)</math>, and sends <math>u_b</math> to the adversary.</li> <li>2. The adversary sends its guess <math>b'</math>, and wins the security game if <math>b = b'</math>.</li> </ol>

Figure 2: The PRG Security Game

**Definition 09.01.** A keyed function  $F : \mathcal{X} \times \mathcal{K} \rightarrow \mathcal{Y}$  is a pseudorandom function (PRF) if, for any p.p.t. adversary  $\mathcal{A}$ , there exists a negligible function  $\mu(\cdot)$  s.t. for all  $n$ ,

$$\Pr [\mathcal{A} \text{ wins the PRF security game}] \leq 1/2 + \mu(n),$$

where the PRF security game is defined in Figure 3.

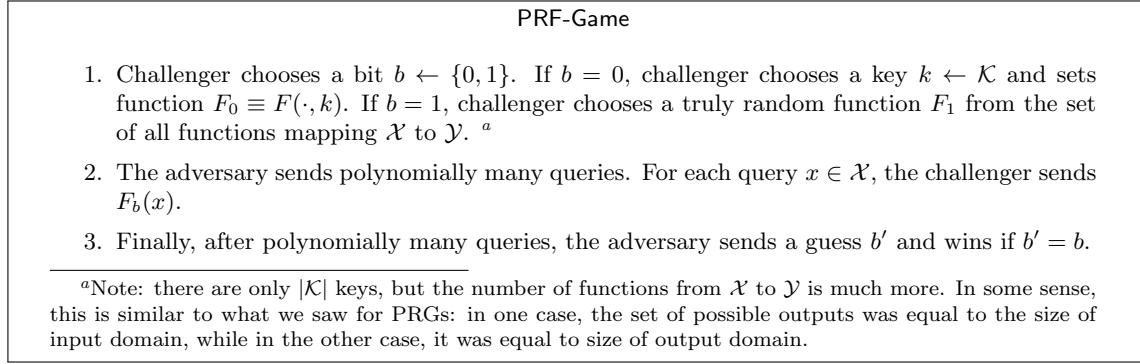


Figure 3: PRF Security Game

**Definition 13.01.** Given two distributions  $\mathcal{D}_0$  and  $\mathcal{D}_1$  over the same sample space  $\Omega$ , the statistical distance of  $\mathcal{D}_0$  and  $\mathcal{D}_1$  is defined as

$$\text{SD}(\mathcal{D}_0, \mathcal{D}_1) = \frac{1}{2} \left( \sum_{i \in \Omega} \left| \Pr_{x \leftarrow \mathcal{D}_0} [x = i] - \Pr_{x \leftarrow \mathcal{D}_1} [x = i] \right| \right)$$

**Definition 15.01.** An encryption scheme  $\mathcal{E} = (\text{KeyGen}, \text{Enc}, \text{Dec})$  is said to satisfy **Semantic Security** if, for any p.p.t. adversaries  $\mathcal{A}$ , there exists a negligible function  $\mu(\cdot)$  such that for all  $n$ ,

$$\Pr[\mathcal{A} \text{ wins the semantic security game}] \leq 1/2 + \mu(n)$$

where the probability is over the choice of key  $k$ , randomness used in  $\text{Enc}$ , and the adversary's randomness.

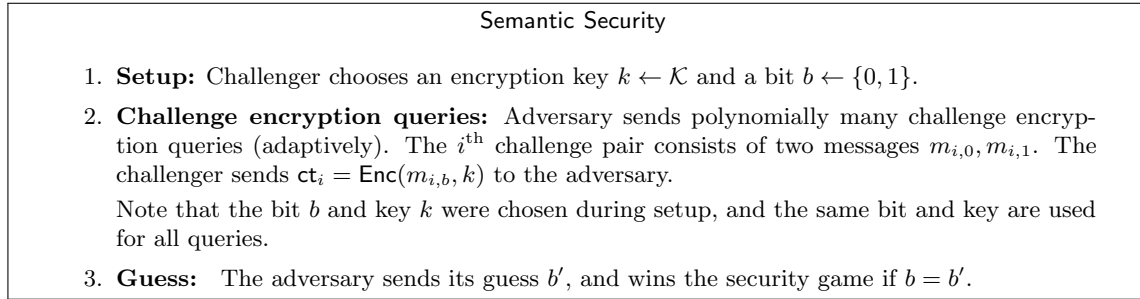


Figure 4: Semantic Security Game

**Definition 18.01.** A MAC scheme  $(\text{KeyGen}, \text{Sign}, \text{Verify})$  is said to satisfy **Strong-UF-CMA** if, for any p.p.t. adversary  $\mathcal{A}$ , there exists a negligible function  $\mu(\cdot)$  such that for all  $n$ ,

$$\Pr[\mathcal{A} \text{ wins the strong unforgeability game}] \leq \mu(n).$$

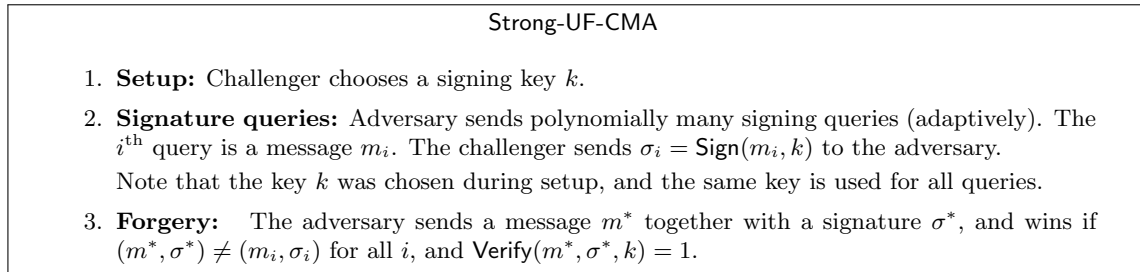


Figure 5: Security Game for MACs: Strong Unforgeability under Chosen Message Attack