Let $X \in \mathbb{R}^{n \times d}$ be our data and let $\tilde{X}$ be the data representation in the rotated Principal component space. That is, if $X = U\Sigma V^T$ and svd decomposition of $X$, we get $\tilde{X} = XV = U\Sigma$.

Let $p_i$ denote the $i$'th element of the standard basis in the PC space. And let $A, B \subset \tilde{X}$ such that $A = \{\tilde{x} = (\tilde{x}_1, ..., \tilde{x}_d) \, | \tilde{x}_i < m\}$ and $B = \tilde{X} \setminus A$. This division is defined by a hyperplane perpendicular to $p_i$, and can be mathematically formulated by the equation $\chi_{p_i,A}(\tilde{x}) = \begin{cases} 1 & \langle p_i, \tilde{x} \rangle < m \\ 0 & \text{otherwise} \end{cases}$.

Since $V$ is orthogonal, we have $\langle p_i, \tilde{x} \rangle < m \iff \langle Vp_i, V\tilde{x} \rangle < m$, thus $\chi_{Vp_i,A}(V\tilde{x}) = \chi_{p_i,A}(\tilde{x})$.

So the hyper-plane in the original plane is actually $\langle Vp_i, x \rangle < m$.

What we previously did was $\langle V(m \cdot p_i), x \rangle < 0$