An Introduction to Recurrent Neural Networks, Part 2

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April 24, 2018

Overview

Throughout this presentation I will be using the notation from the book by Ian Goodfellow, Yoshua Bengio, and Aaron Courville

Review

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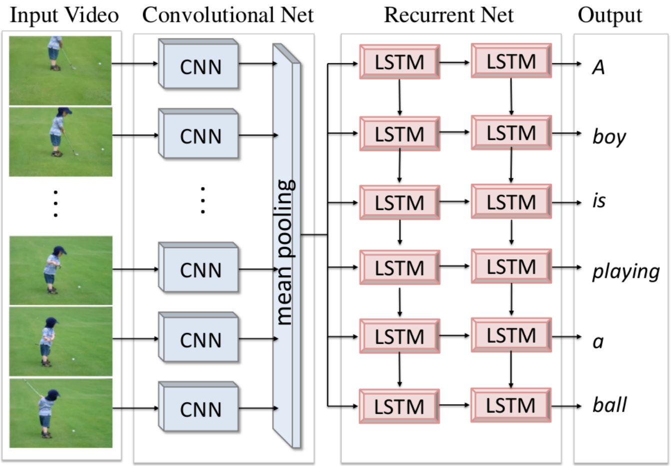
- A very general form of neural network
- Turing complete

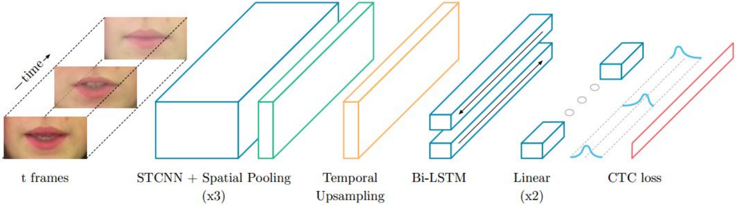
Review

We can combine RNNs with other networks we've seen before

Review

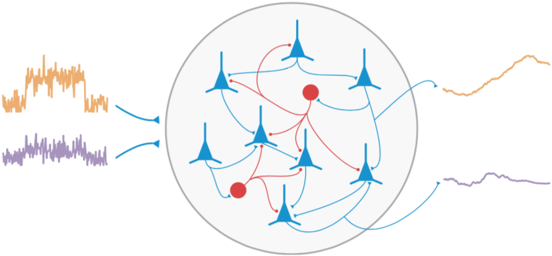

Input Video
Convolutional Net
Recurrent Net
Output
CNN
CNN
CNN
CNN
CNN
mean pooling
LSTM
A
boy
is
playing
a
ball

Review


time
t frames
STCNN + Spatial Pooling
(x3)
Temporal
Upsampling
Bi-LSTM
Linear
(x2)
CTC loss

Review

Depiction in neuroscience:



Equations of Evolution

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The time evolution of $\mathbf{h}^{(t)}, \mathbf{o}^{(t)}$ is given by:

$$\mathbf{h}^{(t)} = \tanh[\mathbf{U} \cdot \mathbf{x}^{(\mathbf{t})} + \mathbf{W} \cdot \mathbf{h}^{(\mathbf{t-1})} + \mathbf{b}]$$

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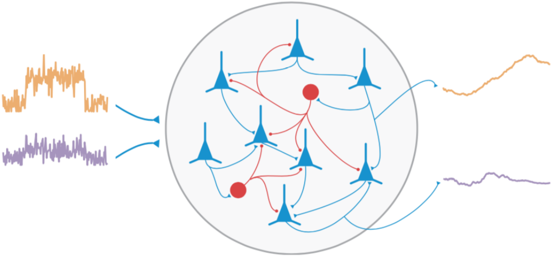
Often, we want a probability as our output, so our RNN output is

$$\hat{\mathbf{y}}^{(t)} = \mathrm{softmax}(\mathbf{o}^{(t)})$$

Equations of Evolution

So:



$\mathbf{x}^{(t)}$ $\quad$ $\mathbf{U}$ $\quad$ $\mathbf{h}^{(t)}$, $\mathbf{W}$, $\mathbf{b}$ $\quad$ $\mathbf{V}$, $\mathbf{c}$ $\quad$ $\mathbf{o}^{(t)}$

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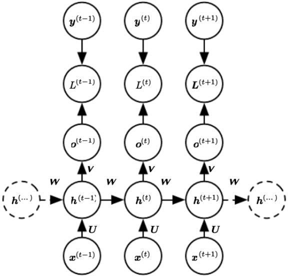
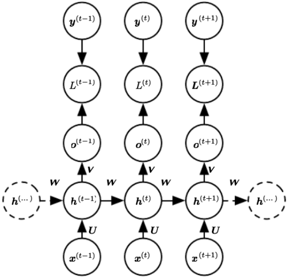


Gradients Descent on the Unrolled Graph

After unrolling the computational graph of an RNN, we can use the same gradient methods that we're familiar with for deep networks.



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$$\begin{aligned}
\nabla_{\boldsymbol{c}} L &= \sum_t \left(\frac{\partial \boldsymbol{o}^{(t)}}{\partial \boldsymbol{c}}\right)^\top \nabla_{\boldsymbol{o}^{(t)}} L = \sum_t \nabla_{\boldsymbol{o}^{(t)}} L, && (10.22)\\
\nabla_{\boldsymbol{b}} L &= \sum_t \left(\frac{\partial \boldsymbol{h}^{(t)}}{\partial \boldsymbol{b}^{(t)}}\right)^\top \nabla_{\boldsymbol{h}^{(t)}} L = \sum_t \operatorname{diag}\left(1-\left(\boldsymbol{h}^{(t)}\right)^2\right) \nabla_{\boldsymbol{h}^{(t)}} L, && (10.23)\\
\nabla_{\boldsymbol{V}} L &= \sum_t \sum_i \left(\frac{\partial L}{\partial o_i^{(t)}}\right) \nabla_{\boldsymbol{V}} o_i^{(t)} = \sum_t (\nabla_{\boldsymbol{o}^{(t)}} L)\, \boldsymbol{h}^{(t)^\top}, && (10.24)\\
\nabla_{\boldsymbol{W}} L &= \sum_t \sum_i \left(\frac{\partial L}{\partial h_i^{(t)}}\right) \nabla_{\boldsymbol{W}^{(t)}} h_i^{(t)} && (10.25)\\
&= \sum_t \operatorname{diag}\left(1-\left(\boldsymbol{h}^{(t)}\right)^2\right) (\nabla_{\boldsymbol{h}^{(t)}} L)\, \boldsymbol{h}^{(t-1)^\top}, && (10.26)\\
\nabla_{\boldsymbol{U}} L &= \sum_t \sum_i \left(\frac{\partial L}{\partial h_i^{(t)}}\right) \nabla_{\boldsymbol{U}^{(t)}} h_i^{(t)} && (10.27)\\
&= \sum_t \operatorname{diag}\left(1-\left(\boldsymbol{h}^{(t)}\right)^2\right) (\nabla_{\boldsymbol{h}^{(t)}} L)\, \boldsymbol{x}^{(t)^\top}, && (10.28)
\end{aligned}$$

Motivation for Vanishing/Exploding gradients

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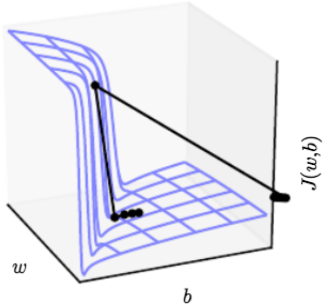
Consequently, they tend to have derivatives that can be either very large or very small in magnitude

Exploding Gradient



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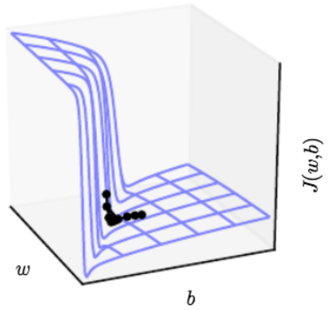
Example:

$$\text{if } ||\mathbf{g}|| > v, \qquad \mathbf{g} \leftarrow \frac{\mathbf{g}v}{||\mathbf{g}||}.$$

Clipped Gradient



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Another way to say this is we want:

$$|\nabla_{\mathbf{h}^{(t-1)}} L| = |(\nabla_{\mathbf{h}^{(t)}} L) \frac{\partial \mathbf{h}^{(t)}}{\partial \mathbf{h}^{(t-1)}}| \approx |\nabla_{\mathbf{h}^{(t)}} L|$$

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With this motivation, *Pascanu* came up with a regularizer to constrain the parameters of a given RNN:

$$\Omega = \sum_t \left(\frac{\left|(\nabla_{\mathbf{h}^{(t)}} L) \frac{\partial \mathbf{h}^{(t)}}{\partial \mathbf{h}^{(t-1)}}\right|}{|\nabla_{\mathbf{h}^{(t)}} L|} - 1 \right)$$

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Combined with gradient clipping, this regularizer can considerably increase the span of the dependencies that an RNN can learn. This is still not as effective as LSTMs in cases where training data is abundant.

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- Build in the context of theoretical neuroscience
- First order attempt at understanding the mechanism underlying associative memory
- Still an important object of study, primarily in neuroscience

Hopfield Networks: Mathematical Description

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$$s_i^{(t+1)} \leftarrow \begin{cases} +1 \text{ if } \sum_j w_{ij} s_j^{(t)} > \theta_i \\ -1 \text{ otherwise} \end{cases}$$

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This gives a model for associative memory in the human brain, and can be used to store many states of information in a single network.

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- Used for *Siri, Alexa, Apple's predictive typing*, and many more

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- This can be referred to as *fixing the time scale of integration* for neurons networks more generally
- Often causes vanishing gradients and (occasionally) exploding gradients to occur
- Perhaps there is a way to modify the individual neurons to make them more amenable to holding memory for longer periods?

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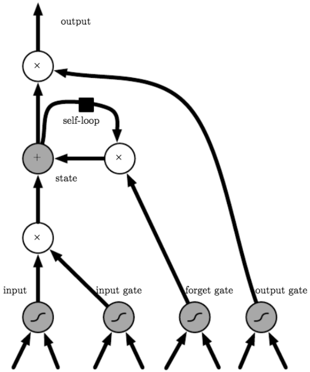
- This is controlled by a "forget gate", illustrated below:

LSTM Networks

A single LSTM "cell", to replace a unit neuron of a standard RNN:



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The internal "state" s_i of neuron i is updated as:

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and its output is given by:

$$h_i^{(t)} = \underline{q_i^{(t)}} \tanh\left(s_i^{(t)}\right)$$

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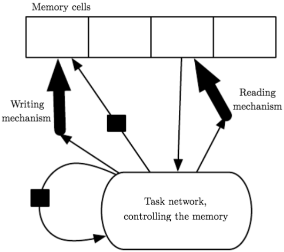

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- RNNs are powerful tools for processing sequential data containing context and long-term dependencies
- Their training involves vanishing/exploding gradients that need to be carefully handled
- LSTMs and other gated-type architectures are particularly powerful in being able to promote information flow and develop short-term memory