1 Chapter 15: Applications of the Holographic Correspondence

1. Taking $U = r/\ell_s^2$ and $g_{YM}^2 = g_s(2\pi)^{p-2}\ell_s^{p-3}$ fixed as $\ell_s \to 0$, we have that at the scale U,
$$e^{2\Phi} = g_s^2 H^{(3-p)/2} \Rightarrow g_{eff}^2 = g_{YM}^2 N U^{p-3}.$$
In the extremal case the electric field is:
$$F_{r01\ldots p} = -\frac{H'}{H^2} = \frac{g_s N}{\Omega_{8-p} H^2}\frac{(2\pi\ell_s)^{7-p}}{r^{8-p}}$$
$$\to \frac{g_s N}{\Omega_{8-p}}\left(\frac{2\pi\ell_s}{L^2}\right)^{7-p} r^{6-p} = \frac{(7-p)^2\Omega_{8-p}}{(2\pi\ell_s)^{7-p} g_s N} r^{6-p} = \frac{\ell_s^2(7-p)^2(2\pi)^{2p-9}\Omega_{8-p}}{g_{YM}^2 N} U^{6-p}$$

2. The original near-horizon metric is:
$$\ell_s^2\left[\frac{U^{(7-p)/2}}{g_{YM}\sqrt{d_p N}}(-dt^2 + dx\cdot dx) + \frac{g_{YM}\sqrt{d_p N}}{U^{(7-p)/2}}(dU^2 + U^2 d\Omega_{8-p}^2)\right]$$
The sphere factor is direct and yields:
$$\ell_s^2\sqrt{d_p N}U^{(p-3)/2} g_{YM} d\Omega_{8-p}^2$$
The other factor will require our change of variables. Pulling out the same overall factor as before, we are left with:
$$\ell_s^2\sqrt{d_p N}U^{(p-3)/2} g_{YM}\left[\frac{U^{5-p}}{g_{YM}^2 d_p N}(-dt^2 + dx\cdot dx) + \frac{dU^2}{U^2}\right]$$
Upon making the substitution:
$$U^{5-p} = \left(\frac{2 g_{YM}\sqrt{d_p N}}{(5-p)u}\right)^2 \Rightarrow \frac{dU}{U} = -\frac{2}{5-p}\frac{du}{u}$$
we get:
$$\frac{4}{(5-p)^2}\left[\frac{1}{u^2}(du^2 - dt^2 + dx\cdot dx)\right]$$
Exactly AdS with radius $4/(5-p)$. **I'm not sure how Kiritsis is absorbing the g_{YM} - strictly speaking the metric in 15.1.17 is off by that factor is the $d\Omega_5$ is to be unital.**

3. For an extremal brane it is straightforward to get the curvature in terms of the dilaton EOM, and indeed we've done this in an exercise for chapter 8, as well as having it written explicitly in **8.8.31**.
Schematically:
$$\ell_s^2 R \sim \frac{\ell_s^2}{r^{(p-3)/2}L^{(7-p)/2}} \sim \frac{1}{\sqrt{g_s\ell_s^{p-3}U^{p-3}N}} \sim \frac{1}{g_{eff}} \sim \sqrt{\frac{U^{3-p}}{g_{YM}^2 N}}$$
as required.

4. Ok here the limits are subtle and worth discussing. I'm following section 13.7. There are two horizons. Near-horizon means near the *outer* horizon. In order to take this limit successfully, we must take $r_0 \ll L$. In fact, we must take $r_0 \to 0$ in a controlled way. Expectedly, we must hold $U_0 = r_0/\ell_s^2$ fixed alongside $U = r/\ell_s^2$ while taking r_0, r, ℓ_s^2 to zero at the same rate.

For this reason, it is safe to replace H by L^{7-p}/r^{7-p} as before, and also to replace f by $1 - \frac{U_0^{7-p}}{U^{7-p}}$ in the nonextremal solution. We then recover exactly the near-horizon extremal solution with the dt^2 and dU^2 terms modified by f:
$$ds^2 = \frac{-f(r)dt^2 + dx\cdot dx}{\sqrt{H}(r)} + \sqrt{H(r)}\left[\frac{dr^2}{f(r)} + r^{8-p}d\Omega_{8-p}\right]$$
$$\to \ell_s^2\left[\frac{U^{(7-p)/2}}{g_{YM}\sqrt{d_p N}}(-f(U)dt^2 + dx\cdot dx) + \frac{g_{YM}\sqrt{d_p N}}{U^{(7-p)/2}}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_{8-p}\right)\right].$$

with

$$f(U) = 1 - \frac{U_0^{7-p}}{U^{7-p}}.$$

5. Let's start with the Hawking temperature. From excercise 13.1 it is simply

$$T_H = \frac{C'(r_0)}{4\pi} = \frac{(7-p)U_0^{(5-p)/2}}{4\pi g_{YM}\sqrt{d_p N}}$$

The ADM mass above extremality is given by **(again, I think there must be something wrong with equation 8.8.14)**

$$\frac{V_p}{2\kappa_{10}^2}(9-p)r_0^{7-p} = V_p \frac{2^{-10+2p}(9-p)\pi^{\frac{-13+3p}{2}}}{g_{YM}^4 \Gamma(\frac{9-p}{2})} U_0^{7-p}$$

as required.

The entropy density will come from the area of the horizon at $U = U_0$.

$$\frac{V_p}{4G_{10}}(g_{YM}\sqrt{d_p N})^{4-p} U_0^{p(7-p)/4} U_0^{-(8-p)(7-p)/4} U_0^{8-p} = \frac{V_p}{2^5 \pi^6 \ell_s^8 g_s^2}$$

By straightfoward algebra, this is equal to the messy expression that Kirtisis has.

6. Area law behavior is indicative of confinement, which is what we would qualitatively expect in