

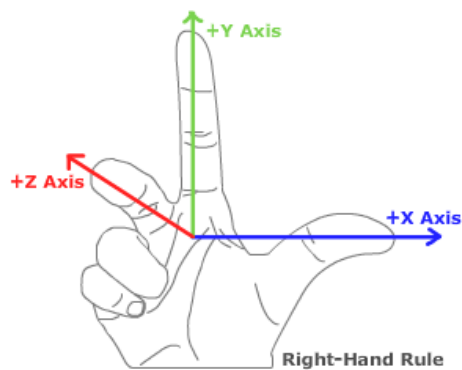


# 07. Kinematics, inverse kinematics, Programming of a simulated robotic arm

## Repetition

### 3D transformations

- **Position:** 3 element offset vector



- **Orientation:** 3 x 3 rotation matrix
  - additional orientation representations: Euler angles, RPY, angle axis, quaternion
- **Pose** (pose): 4 x 4 transformation matrix
- **Coordinate system** (frame): zero point, 3 axis, 3 base vector, right-hand rule
- **Homogeneous transformations:** rotation and translation together
  - e.g.  $\mathbf{R}$  for rotation and  $\mathbf{v}$  for translation:

$$\begin{aligned} \mathbf{T} &= \begin{bmatrix} \mathbf{R} & \mathbf{v} \\ \mathbf{0} & 1 \end{bmatrix} \\ &= \begin{bmatrix} r_{1,1} & r_{1,2} & r_{1,3} & v_x \\ r_{2,1} & r_{2,2} & r_{2,3} & v_y \\ r_{3,1} & r_{3,2} & r_{3,3} & v_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

- **Homogeneous coordinates:**

- **Vector:** add 0,  $\mathbf{a}_H = \begin{bmatrix} \mathbf{a} \\ 0 \end{bmatrix} = \begin{bmatrix} a_x \\ a_y \\ a_z \\ 0 \end{bmatrix}$
- **Point:** add 1,  $\mathbf{p}_H = \begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$
- Using transformations is simpler:

$$\mathbf{q} = \mathbf{R}\mathbf{p} + \mathbf{v} \rightarrow \begin{bmatrix} \mathbf{q} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & \mathbf{v} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix}$$

- **Degree of freedom (DoF):** number of independent quantities.

## Robotics basics



- Robot structure: **segments** (link) and **joints**
- **Task space** (cartesian space):
  - Three-dimensional space where the task, trajectories, obstacles, etc. are defined.

- **TCP** (Tool Center Point): coordinate frame fixed to the end effector
- **Base/world frame**
- **Joint space:**
  - Quantities assigned to the robot's joints, which can be interpreted by the robot's low-level control system.
  - Joint coordinates, velocities, accelerations, torques...

## Lecture

### Kinematics, inverse kinematics

#### Kinematics

##### Def. Kinematics

Calculating the position of the TCP (or anything else) from the joint coordinates.

- Kinematic model
  - Denavit--Hartenberg (DH) convention
  - URDF (Unified Robotics Description Format, XML-based)

If the coordinate systems assigned to the segments are  $(base, 1, 2, 3, \dots, TCP)$ , the transformations between adjacent segments  $(i)$  and  $(i+1)$  are  $(T_{i+1,i}(q_{i+1}))$  (which is a function of the angle of the joint between them), the transformations between the base frame and TCP can be written up (for a robot with  $(n)$  joints):

$$T_{TCP,base}(q_1, \dots, q_n) = T_{TCP,n-1}(q_n) \cdot T_{n-1,n-2}(q_{n-1}) \cdot T_{2,1}(q_2) \cdot T_{1,base}(q_1) \cdot T_{base}$$

#### Inverse kinematics

##### Def. Inverse kinematics

Compute joint coordinates to achieve (desired) TCP (or any other) pose.

## Differential inverse kinematics

### Def. Differential inverse kinematics

Which change in the wrist coordinates achieves the desired **small change** in the TCP pose (rotation and translation).

- **Jacobi matrix** (Jacobian): a matrix of first-order partial derivatives of a vector-valued function.

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x_1}{\partial q_1} & \frac{\partial x_1}{\partial q_2} & \frac{\partial x_1}{\partial q_3} & \dots & \frac{\partial x_1}{\partial q_n} \\ \frac{\partial x_2}{\partial q_1} & \frac{\partial x_2}{\partial q_2} & \frac{\partial x_2}{\partial q_3} & \dots & \frac{\partial x_2}{\partial q_n} \\ \frac{\partial x_3}{\partial q_1} & \frac{\partial x_3}{\partial q_2} & \frac{\partial x_3}{\partial q_3} & \dots & \frac{\partial x_3}{\partial q_n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial x_m}{\partial q_1} & \frac{\partial x_m}{\partial q_2} & \frac{\partial x_m}{\partial q_3} & \dots & \frac{\partial x_m}{\partial q_n} \end{bmatrix}$$

- **Jacobi matrix significance in robotics:** gives the relationship between joint velocities and TCP velocity.

$$\begin{bmatrix} \mathbf{v} \\ \boldsymbol{\omega} \end{bmatrix} = \mathbf{J} \dot{\mathbf{q}}$$

## Inverse kinematics using Jacobian inverse

1. Calculate the difference between the desired and the current position:  $\Delta \mathbf{r} = \mathbf{r}_{\text{desired}} - \mathbf{r}_0$
2. Calculate the difference in rotations:  $(\Delta \mathbf{R} = \mathbf{R}_{\text{desired}} \mathbf{R}_0^T)$ , then convert to axis angle representation  $(\mathbf{t}, \phi)$
3. Compute  $\Delta \mathbf{q} = \mathbf{J}^{-1}(\mathbf{q}_0) \cdot \begin{bmatrix} \mathbf{k}_1 \Delta \mathbf{r} \\ \mathbf{k}_2 \mathbf{t} \end{bmatrix}$ , where the inverse can be pseudo-inverse or transposed
4.  $\mathbf{q}_{\text{better}} = \mathbf{q}_0 + \Delta \mathbf{q}$

# Exercise

## 1: Doosan2 install

Reset the `~/bashrc` file to ROS2 default.

### 1. Install the dependencies.



```
sudo apt update
sudo apt-get install libpoco-dev
sudo apt-get install ros-foxy-control-msgs ros-foxy-realtime-tools ros-foxy-xacro ros-foxy-
joint-state-publisher-gui
pip3 install kinpy
```

#### Tip

Also download the source of the `kinpy` package, it may be useful for understanding the API: <https://pypi.org/project/kinpy/>

### 2. Clone and build the repo.

```
mkdir -p ~/doosan2_ws/src
cd ~/doosan2_ws/src
git clone https://github.com/TamasDNagy/doosan-robot2.git
```

```

git clone https://github.com/ros-controls/ros2_control.git
git clone https://github.com/ros-controls/ros2_controllers.git
git clone https://github.com/ros-simulation/gazebo_ros2_control.git
cd ros2_control && git reset --hard 3dc62e28e3bc8cf636275825526c11d13b554bb6
&& cd ..
cd ros2_controllers && git reset --hard 83c494f460f1c8675f4fdd6fb8707b87e81cb197
&& cd ...
cd cd gazebo_ros2_control && git reset --hard
3dfe04d412d5be4540752e9c1165ccf25d7c51fb && cd ..
git clone -b ros2 --single-branch https://github.com/ros-planning/moveit_msgs
cd ~/doosan2_ws
rosdep update
rosdep install --from-paths src --ignore-src --rosdistro foxy -r -y
colcon build --cmake-args -DCMAKE_EXPORT_COMPILE_COMMANDS=ON
. install/setup.bash
rosdep update

```

### Warning

Already installed on VMs, but update the repo here too:

```

cd ~/doosan2_ws/src/doosan-robot2
git pull
cd ~/doosan2_ws
colcon build --cmake-args -DCMAKE_EXPORT_COMPILE_COMMANDS=ON

```

Add the following line to the `~/bashrc` file:

```
source ~/doosan2_ws/install/setup.bash
```

3. Test the simulator in a new window:

```
ros2 launch dsr_launcher2 single_robot_rviz_topic.launch.py model:=a0912 color:=blue
```

## 2: Move robot in articulated space

1. Create a new python source file named `doosan2_controller.py` in `~/ros2_ws/src/ros2_course/ros2_course` folder. Specify the new entry point in `setup.py` in the usual way. Subscribe to the topic publishing the robot's articulation angles (configuration). Create publisher for the topic that can be used to configure the wrist angles.

```
/joint_states  
/joint_cmd
```

2. Move the robot to the configuration  $q = [0.24, -0.3, 1.55, 0.03, 1.8, 0.5]$ .

### 3. Kinematics

1. Import the `kinpy` package and read the urdf file describing the robot:

```
import kinpy as kp  
  
self.chain = kp.build_serial_chain_from_urdf(open(  
    "/home/<USERNAME>/doosan2_ws/src/doosan-robot2/dsr_description2/urdf/  
    a0912.blue.urdf").read(),  
    "link6")  
print(self.chain.get_joint_parameter_names())  
print(self.chain)
```

2. Calculate and print the TCP position in the given configuration using the `kinpy` package.

```
tg = chain.forward_kinematics(th1)
```

### 4: Inverse kinematics with Jacobian inverse method

Write a method that implements the inverse kinematics problem on the robot using the Jacobian inverse method presented in the lecture. The orientation is ignored. Move the TCP to the position  $(0.55, 0.05, 0.45)$ . Let us diagram the TCP trajectory of TCP using Matplotlib.

1. Write a loop with a stop condition of the appropriate size of `delta_r` and `rclpy.ok()`.



2. Calculate the difference between the desired and the current TCP positions (`delta_r`). Scale with the constant `k_1`.
3. Set `omega` to `[0.0, 0.0, 0.0]` (ignore orientation).
4. Concatenate `delta_r` and `omega`.
5. Calculate the Jacobian matrix in the given configuration using the function `kp.jacobian.calc_jacobian(...)`.
6. Calculate the pseudo-inverse of the Jacobian matrix `np.linalg.pinv(...)`.
7. Calculate `delta_q` using the above formula.
8. Increment the joint angles with the obtained values.

### *Bonus:* Inverse kinematics with orientation

Complete the solution to the previous problem by including orientation in the inverse kinematics calculation.

## Useful links

- [doosan-robot2 github](#)
- <https://pypi.org/project/kinpy/>
- [https://en.wikipedia.org/wiki/Axis%E2%80%93angle\\_representation](https://en.wikipedia.org/wiki/Axis%E2%80%93angle_representation)
- <https://www.rosroboticslearning.com/jacobian>