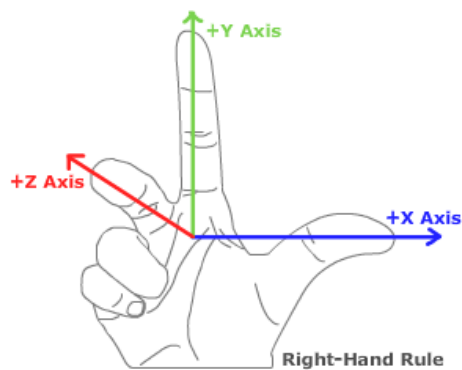


07. Kinematics, inverse kinematics, Programming of a simulated robotic arm

Repetition

3D transformations

- **Position:** 3 element offset vector



- **Orientation:** 3 x 3 rotation matrix
 - additional orientation representations: Euler angles, RPY, angle axis, quaternion
- **Pose** (pose): 4 x 4 transformation matrix
- **Coordinate system** (frame): zero point, 3 axis, 3 base vector, right-hand rule
- **Homogeneous transformations:** rotation and translation together
 - e.g. \mathbf{R} for rotation and \mathbf{v} for translation:

$$\begin{aligned} \mathbf{T} &= \begin{bmatrix} \mathbf{R} & \mathbf{v} \\ \mathbf{0} & 1 \end{bmatrix} \\ &= \begin{bmatrix} r_{1,1} & r_{1,2} & r_{1,3} & v_x \\ r_{2,1} & r_{2,2} & r_{2,3} & v_y \\ r_{3,1} & r_{3,2} & r_{3,3} & v_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

- **Homogeneous coordinates:**

- **Vector:** add 0, $\mathbf{a}_H = \begin{bmatrix} \mathbf{a} \\ 0 \end{bmatrix} = \begin{bmatrix} a_x \\ a_y \\ a_z \\ 0 \end{bmatrix}$
- **Point:** add 1, $\mathbf{p}_H = \begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$
- Using transformations is simpler:

$$\mathbf{q} = \mathbf{R}\mathbf{p} + \mathbf{v} \rightarrow \begin{bmatrix} \mathbf{q} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & \mathbf{v} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix}$$

- **Degree of freedom (DoF):** number of independent quantities.

Robotics basics



- Robot structure: **segments** (link) and **joints**
- **Task space** (cartesian space):
 - Three-dimensional space where the task, trajectories, obstacles, etc. are defined.

- **TCP** (Tool Center Point): coordinate frame fixed to the end effector
- **Base/world frame**
- **Joint space:**
 - Quantities assigned to the robot's joints, which can be interpreted by the robot's low-level control system.
 - Joint coordinates, velocities, accelerations, torques...

Lecture

Kinematics, inverse kinematics

Kinematics

Def. Kinematics

Calculating the position of the TCP (or anything else) from the hinge coordinates.

- Kinematic model
 - Denavit--Hartenberg (DH) convention
 - URDF (Unified Robotics Description Format, XML-based)

If the coordinate systems assigned to the segments are $(\text{base}, 1, 2, 3, \dots, \text{TCP})$, the transformations between adjacent segments (i) and $(i+1)$ are $(T_{i+1,i}(q_{i+1}))$ (which is a function of the angle of the joint between them), the transformations between the base frame and TCP can be written up (for a robot with (n) joints):

$$T_{\text{TCP,base}}(q_1, \dots, q_n) = T_{\text{TCP},n-1}(q_n) \cdot T_{n-1,n-2}(q_{n-1}) \cdot T_{n-2,n-3}(q_{n-2}) \cdot \dots \cdot T_{2,1}(q_2) \cdot T_{1,\text{base}}(q_1) \cdot T_{\text{base}}$$

Inverse kinematics

Def. Inverse kinematics

Compute joint coordinates to achieve (desired) TCP (or any other) pose.

Differential inverse kinematics

Def. Differential inverse kinematics

Which change in the wrist coordinates achieves the desired **small change** in the TCP pose (rotation and translation).

- **Jacobi matrix** (Jacobian): a matrix of first-order partial derivatives of a vector-valued function.

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x_1}{\partial q_1} & \frac{\partial x_1}{\partial q_2} & \frac{\partial x_1}{\partial q_3} & \dots & \frac{\partial x_1}{\partial q_n} \\ \frac{\partial x_2}{\partial q_1} & \frac{\partial x_2}{\partial q_2} & \frac{\partial x_2}{\partial q_3} & \dots & \frac{\partial x_2}{\partial q_n} \\ \frac{\partial x_3}{\partial q_1} & \frac{\partial x_3}{\partial q_2} & \frac{\partial x_3}{\partial q_3} & \dots & \frac{\partial x_3}{\partial q_n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial x_m}{\partial q_1} & \frac{\partial x_m}{\partial q_2} & \frac{\partial x_m}{\partial q_3} & \dots & \frac{\partial x_m}{\partial q_n} \end{bmatrix}$$

- **Jacobi matrix significance in robotics:** gives the relationship between joint velocities and TCP velocity.

$$\left[\begin{matrix} \mathbf{v} \\ \boldsymbol{\omega} \end{matrix} \right] = \mathbf{J} \dot{\mathbf{q}}$$

Inverse kinematics using Jacobian inverse

1. Calculate the difference between the desired and the current position: $\Delta \mathbf{r} = \mathbf{r}_{\text{desired}} - \mathbf{r}_0$
2. Calculate the difference in rotations: $(\Delta \mathbf{R} = \mathbf{R}_{\text{desired}} \mathbf{R}_0^T)$, then convert to axis angle representation (\mathbf{t}, ϕ)
3. Compute $\Delta \mathbf{q} = \mathbf{J}^{-1}(\mathbf{q}_0) \cdot \left[\begin{matrix} \mathbf{k}_1 \cdot \Delta \mathbf{r} \\ \mathbf{k}_2 \cdot \begin{bmatrix} \mathbf{t} \\ \phi \end{bmatrix} \end{matrix} \right]$, where the inverse can be pseudo-inverse or transposed
4. $\mathbf{q}_{\text{better}} = \mathbf{q}_0 + \Delta \mathbf{q}$

Exercise

1: Doosan2 install

Reset the `~/.bashrc` file to ROS2 default.

1. Install the dependencies.



```
sudo apt update
sudo apt-get install libpoco-dev
sudo apt-get install ros-foxy-control-msgs ros-foxy-realtime-tools ros-foxy-xacro ros-foxy-
joint-state-publisher-gui
pip3 install kinpy
```

Tip

Also download the source of the `kinpy` package, it may be useful for understanding the API: <https://pypi.org/project/kinpy/>

2. Clone and build the repo.

```
mkdir -p ~/doosan2_ws/src
cd ~/doosan2_ws/src
git clone https://github.com/TamasDNagy/doosan-robot2.git
```

```

git clone https://github.com/ros-controls/ros2_control.git
git clone https://github.com/ros-controls/ros2_controllers.git
git clone https://github.com/ros-simulation/gazebo_ros2_control.git
cd ros2_control && git reset --hard 3dc62e28e3bc8cf636275825526c11d13b554bb6
&& cd ..
cd ros2_controllers && git reset --hard 83c494f460f1c8675f4fdd6fb8707b87e81cb197
&& cd ...
cd cd gazebo_ros2_control && git reset --hard
3dfe04d412d5be4540752e9c1165ccf25d7c51fb && cd ..
git clone -b ros2 --single-branch https://github.com/ros-planning/moveit_msgs
cd ~/doosan2_ws
rosdep update
rosdep install --from-paths src --ignore-src --rosdistro foxy -r -y
colcon build --cmake-args -DCMAKE_EXPORT_COMPILE_COMMANDS=ON
. install/setup.bash
rosdep update

```

Warning

Already installed on VMs, but update the repo here too:

```

cd ~/doosan2_ws/src/doosan-robot2
git pull
cd ~/doosan2_ws
colcon build --cmake-args -DCMAKE_EXPORT_COMPILE_COMMANDS=ON

```

Add the following line to the `~/bashrc` file:

```
source ~/doosan2_ws/install/setup.bash
```

3. Test the simulator in a new window:

```
ros2 launch dsr_launcher2 single_robot_rviz_topic.launch.py model:=a0912 color:=blue
```

2: Move robot in articulated space

1. Create a new python source file named `doosan2_controller.py` in `~/ros2_ws/src/ros2_course/ros2_course` folder. Specify the new entry point in `setup.py` in the usual way. Subscribe to the topic publishing the robot's articulation angles (configuration). Create publisher for the topic that can be used to configure the wrist angles.

```
/joint_states  
/joint_cmd
```

2. Move the robot to the configuration $q = [0.24, -0.3, 1.55, 0.03, 1.8, 0.5]$.

3. Kinematics

1. Import the `kinpy` package and read the urdf file describing the robot:

```
import kinpy as kp  
  
self.chain = kp.build_serial_chain_from_urdf(open(  
    "/home/<USERNAME>/doosan2_ws/src/doosan-robot2/dsr_description2/urdf/  
    a0912.blue.urdf").read(),  
    "link6")  
print(self.chain.get_joint_parameter_names())  
print(self.chain)
```

2. Calculate and print the TCP position in the given configuration using the `kinpy` package.

```
tg = chain.forward_kinematics(th1)
```

4: Inverse kinematics with Jacobian inverse method

Write a method that implements the inverse kinematics problem on the robot using the Jacobian inverse method presented in the lecture. The orientation is ignored. Move the TCP to the position $(0.55, 0.05, 0.45)$. Let us diagram the TCP trajectory of TCP using Matplotlib.

1. Write a loop with a stop condition of the appropriate size of `delta_r` and `rclpy.ok()`.

2. Calculate the difference between the desired and the current TCP positions (`delta_r`). Scale with the constant `k_1`.
3. Set `omega` to `[0.0, 0.0, 0.0]` (ignore orientation).
4. Concatenate `delta_r` and `omega`.
5. Calculate the Jacobian matrix in the given configuration using the function `kp.jacobian.calc_jacobian(...)`.
6. Calculate the pseudo-inverse of the Jacobian matrix `np.linalg.pinv(...)`.
7. Calculate `delta_q` using the above formula.
8. Increment the joint angles with the obtained values.

Bonus: Inverse kinematics with orientation

Complete the solution to the previous problem by including orientation in the inverse kinematics calculation.

Useful links

- [doosan-robot2 github](#)
- <https://pypi.org/project/kinpy/>
- https://en.wikipedia.org/wiki/Axis%E2%80%93angle_representation
- <https://www.rosroboticslearning.com/jacobian>