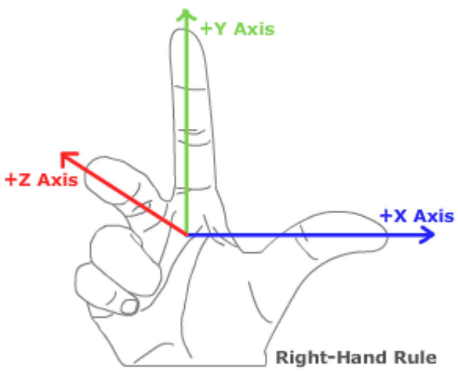
09. Kinematics, inverse kinematics, Programming of a simulated robotic arm

Rehearsal

> **Warning**
>
> **Test 2** on **December 8.**

3D transformations

- **Position:** 3 element offset vector



- **Orientation:** 3 x 3 rotation matrix
    - additional orientation representations: Euler angles, RPY, angle axis, quaternion
- **Pose:** 4 × 4 transformation matrix
- **Coordinate frame:** origin, 3 axis, 3 base vector, right-hand rule
- **Homogeneous transformations:** rotation and translation together
    - e.g. \(\mathbf{R}\) for rotation and \(\mathbf{v}\) for translation:

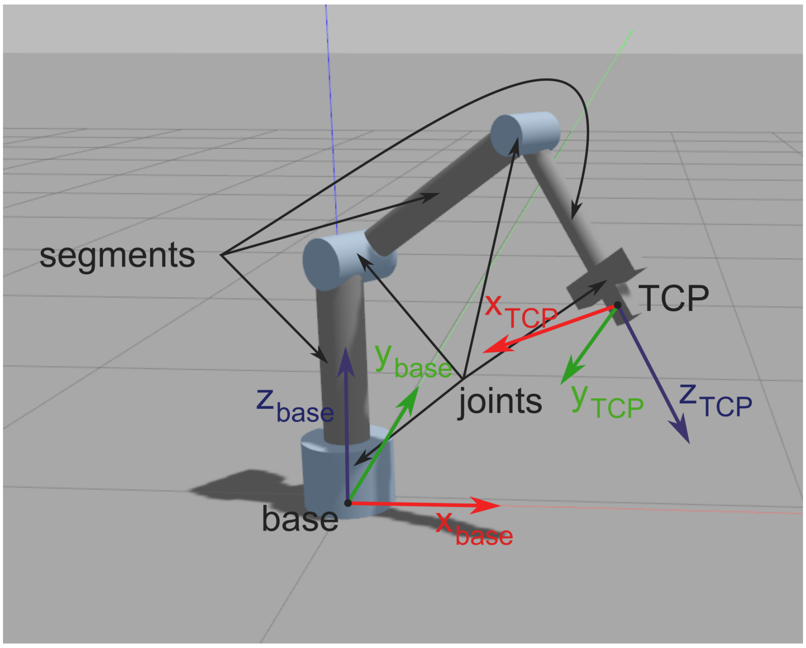
$$ \mathbf{T} = \left[\matrix{\mathbf{R} & \mathbf{v}\\\mathbf{0} & 1 }\right] = \left[\matrix{r_{1,1} & r_{1,2} & r_{1,3} & v_x\\r_{2,1} & r_{2,2} & r_{2,3} & v_y\\r_{3,1} & r_{3,2} & r_{3,3} & v_z\\ 0 & 0 & 0 & 1 }\right] $$

- **Homogeneous coordinates:**
  - **Vector:** add 0, $\mathbf{a_H}=\left[\matrix{\mathbf{a} \\ 0}\right]=\left[\matrix{a_x \\ a_y \\ a_z \\ 0}\right]$
  - **Point:** add 1, $\mathbf{p_H}=\left[\matrix{\mathbf{p} \\ 1}\right]=\left[\matrix{p_x \\ p_y \\ p_z \\ 1}\right]$
  - Applying transformations is simpler:

$$ \mathbf{q} = \mathbf{R}\mathbf{p} + \mathbf{v} \to \left[\matrix{\mathbf{q} \\ 1}\right] = \left[\matrix{\mathbf{R} & \mathbf{v}\\\mathbf{0} & 1 }\right] \left[\matrix{\mathbf{p} \\ 1}\right] $$

- **Degree of freedom** (DoF): number of independent variables (quantities).

Robotics basics



- Robot structure: **segments** (link) and **joints**
- **Task space** (Cartesian space):
  - Three-dimensional space where the task, trajectories, obstacles, etc. are defined.

    - **TCP** (Tool Center Point): coordinate frame fixed to the end effector
    - **Base/world frame**
- **Joint space**:
    - Quantities assigned to the robot's joints, which can be interpreted by the robot's low-level control system.
    - Joint angles, velocities, accelerations, torques...

Lecture

Kinematics, inverse kinematics

Kinematics

> **Def. Kinematics**
>
> Calculating the pose of the TCP (or anything else) from the joint coordinates.

- Kinematic model
    - Denavit--Hartenberg (DH) convention
    - URDF (Unified Robotics Description Format, XML-based)

If the coordinate systems assigned to the segments are $base, 1, 2, 3, ..., TCP$, the transfomrms between adjacent segments i and $i+1$ are $T_{i+1,i}(q_{i+1})$ (which is a function of the angle of the joint between them), the transfomrs between the base frame and TCP can be written as (for a robot with n joints):

$$
T_{TCP,base}(q_1, \cdots, q_n) = T_{TCP,n-1}(q_{n}) \cdot T_{n-1,n-2}(q_{n-1}) \cdots T_{2,1}(q_2) \cdot T_{1,base}(q_1) \cdot base
$$

Inverse kinematics

Def. Inverse kinematics

Calculate the joint coordinates to achieve (desired) TCP (or any other) pose.

Differential inverse kinematics

Def. Differential inverse kinematics

Which change in the joint coordinates achieves the desired **small change** in the TCP pose (rotation and translation).

- **Jacobi matrix** (Jacobian): a matrix of first-order partial derivatives of a vector-valued function.

  $$ \mathbf{J} = \left[\matrix{\frac{\partial x_1}{\partial q_1} & \frac{\partial x_1}{\partial q_2} &\frac{\partial x_1}{\partial q_3} & \dots &\frac{\partial x_1}{\partial q_n} \\ \frac{\partial x_2}{\partial q_1} & \frac{\partial x_2}{\partial q_2} &\frac{\partial x_2} {\partial q_3} & \dots &\frac{\partial x_2}{\partial q_n} \\ \frac{\partial x_3}{\partial q_1} & \frac{\partial x_3}{\partial q_2} &\frac{\partial x_3}{\partial q_3} & \dots &\frac{\partial x_3}{\partial q_n} \\ \vdots &\vdots &\vdots &\ddots &\vdots \\ \frac{\partial x_m}{\partial q_1} & \frac{\partial x_m}{\partial q_2} &\frac{\partial x_m}{\partial q_3} & \dots &\frac{\partial x_m}{\partial q_n} \\}\right] $$

- **Jacobi matrix significance in robotics**: gives the relationship between joint velocities and TCP velocity.

  $$ \left[\matrix{\mathbf{v} \\ \mathbf{\omega}}\right] =\mathbf{J}(\mathbf{q})\cdot \mathbf{\dot{q}} $$

  ,where $\mathbf{v}$ is the linear velocity of the TCP, $\mathbf{\omega}$ is the angular velocity of the TCP, and $\mathbf{q}$ is the configuration of the robot.

Def. Configuration

The vector or array containing the current joint angles of the robot.

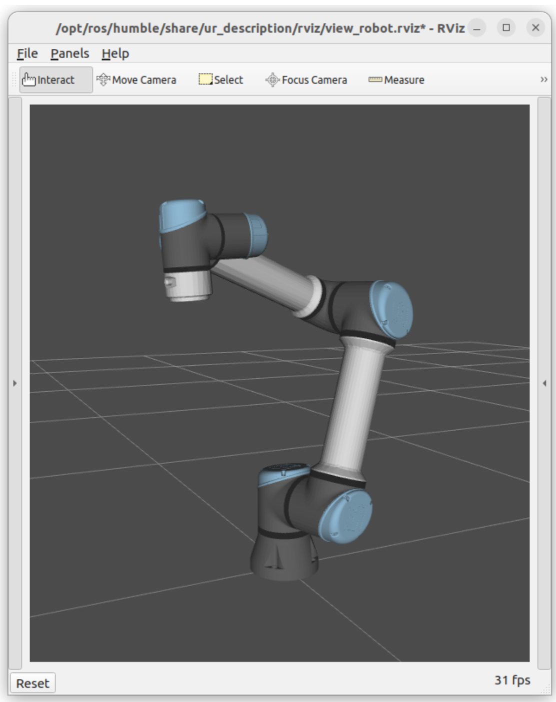
Inverse kinematics using Jacobian inverse

1. Calculate the difference between the desired and the current position: $\Delta\mathbf{r} = \mathbf{r}_{desired} - \mathbf{r}_0$
2. Calculate the difference in rotations: $\Delta\mathbf{R} = \mathbf{R}_{desired}\mathbf{R}_{0}^{T}$, then convert to axis angle representation $(\mathbf{t},\phi)$
3. Compute $\Delta\mathbf{ q}=\mathbf{J}^{-1}(\mathbf{q_0})\cdot \left[\begin{matrix}k_1 \cdot \Delta\mathbf{r} \\ k_2 \cdot \mathbf{\omega}\end{matrix}\right]$, where the inverse can be pseudo-inverse or transposed
4. $\mathbf{q}_{better} = \mathbf{q}_{0} + \Delta\mathbf{q}$

Exercise

1: UR install

1. Install the dependencies and the UR driver.




```bash
sudo apt update
sudo apt upgrade
```

```
sudo apt-get install ros-humble-ur python3-pip
pip3 install kinpy
```
```

!!! tip
  Also download the source of the `kinpy` package, it might be useful for understanding the API: [https://pypi.org/project/kinpy/]()



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```

1. Download the zip containing your source files from Moodle (`ur_ros2_course.zip`). Copy the `view_ur.launch.py` file to the `ros2_course/launch` folder, and `topic_latcher.py` to `ros2_course/ros2_course`. Add the following lines to `setup.py` (launch and entry point):

   ```
   import os
   from glob import glob

   # ...

   data_files=[
       ('share/ament_index/resource_index/packages',
           ['resource/' + package_name]),
       ('share/' + package_name, ['package.xml']),
       # Include all launch files.
       (os.path.join('share', package_name),
           glob('launch/*launch.[pxy][yma]*'))
   ],

   # ...

   entry_points={
   'console_scripts': [
       # ...
       'topic_latcher = ros2_course.topic_latcher:main',
   ],
   ```

2. Start the simulator, move the wrists using the Joint State Publisher GUI.

   ```
   ros2 launch ros2_course view_ur.launch.py ur_type:=ur5e
   ```

   Tip

   Try other robots using argument `ur_type` (ur3, ur3e, ur5, ur5e, ur10, ur10e, ur16e, ur20).

2: Move the robot in joint space

1. Create a new python source file named `ur_controller.py` in `~/ros2_ws/src/ros2_course/ros2_course` folder. Specify the new entry point in `setup.py` in the usual way. Subscribe to the topic publishing the robot's joint angles (configuration). Create publisher for the topic that can be used to set the joint angles.

   ```
   /joint_states
   /set_joint_states
   ```

2. Move the robot to the configuration `q = [-1.28, 4.41, 1.54, -1.16, -1.56, 0.0]`.

3. Kinematics

1. The simulator publishes the urdf description of the robot in a topic. Subscribe to this topic.

```
/robot_description_latch
```

2. Import the `kinpy` package and create the kinematic chain based on the urdf describing the robot in the callback function just implemented:

```
import kinpy as kp

# ...


self.chain = kp.build_serial_chain_from_urdf(self.desc, 'tool0')
print(self.chain.get_joint_parameter_names())
print(self.chain)
```

3. Calculate and print the TCP position in the given configuration using the `kinpy` package.

```
p = chain.forward_kinematics(q)
```

4: Inverse kinematics with Jacobian inverse method

Write a method that implements the inverse kinematics problem on the robot using the Jacobian inverse method presented in the lecture. The orientation is ignored. Move the TCP to the position `(0.50, -0.60, 0.20)`. Let us diagram the TCP trajectory of TCP using Matplotlib.

1. Write a loop with a stop condition of the appropriate size of `delta_r` and `rclpy.ok()`.

2. Calculate the difference between the desired and the current TCP positions (`delta_r`). Scale with the constant `k_1`.

3. Set `omega` to `[0.0, 0.0, 0.0]` (ignore orientation).

4. Concatenate `delta_r` and `omega`.

5. Calculate the Jacobian matrix in the given configuration using the function `kp.jacobian.calc_jacobian(...)`.

6. Calculate the pseudo-inverse of the Jacobian matrix `np.linalg.pinv(...)`.

7. Calculate `delta_q` using the above formula.

8. Increment the joint angles with the obtained values.

Plot the TCP trajectory using Matplotlib.

```
import matplotlib.pyplot as plt

# ...

# Plot trajectory
ax = plt.figure().add_subplot(projection='3d')
ax.plot(x, y, z, label='TCP trajectory',  ls='-', marker='.')
ax.legend()
ax.set_xlabel('x [m]')
ax.set_ylabel('y [m]')
ax.set_zlabel('z [m]')
plt.show()
```

Bonus: Inverse kinematics with orientation

Complete the solution to the previous problem by including orientation in the inverse kinematics calculation.

Useful links

- https://github.com/UniversalRobots/Universal_Robots_ROS2_Driver/tree/humble
- https://docs.ros.org/en/ros2_packages/humble/api/ur_robot_driver/usage.html#usage-with-official-ur-simulator
- https://github.com/UniversalRobots/Universal_Robots_Client_Library
- https://pypi.org/project/kinpy/
- https://en.wikipedia.org/wiki/Axis%E2%80%93angle_representation
- https://www.rosroboticslearning.com/jacobian