

07. Kinematics, inverse kinematics. Programming a simulated robot in joint space and task space

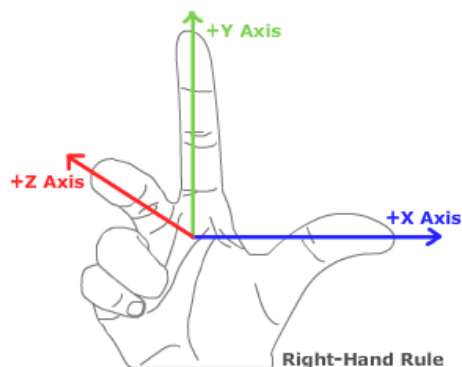
Warning

ZH2 (Roslaunch, ROS parameter server, ROS service, ROS action, Kinematics, inverse kinematics) and **project presentation: December 6.**

Rehearsal

3D transformations

•



Position: 3D offset vector

- **Orientation:** 3 x 3 rotation matrix
 - further orientation representations: Euler-angles, RPY, angle axis, quaternion
- **Pose:** 4 x 4 (homogenous) transformation matrix
- **Frame:** origin, 3 axes, 3 base vectors, right hand rule
- **Homogenous transformation:** rotation and translation in one transformation
 - e.g., for the rotation \mathbf{R} and translation \mathbf{v} :

$$\begin{aligned} \mathbf{T} &= \begin{bmatrix} \mathbf{R} & \mathbf{v} \\ \mathbf{0} & 1 \end{bmatrix} \\ &= \begin{bmatrix} r_{1,1} & r_{1,2} & r_{1,3} & v_x \\ r_{2,1} & r_{2,2} & r_{2,3} & v_y \\ r_{3,1} & r_{3,2} & r_{3,3} & v_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

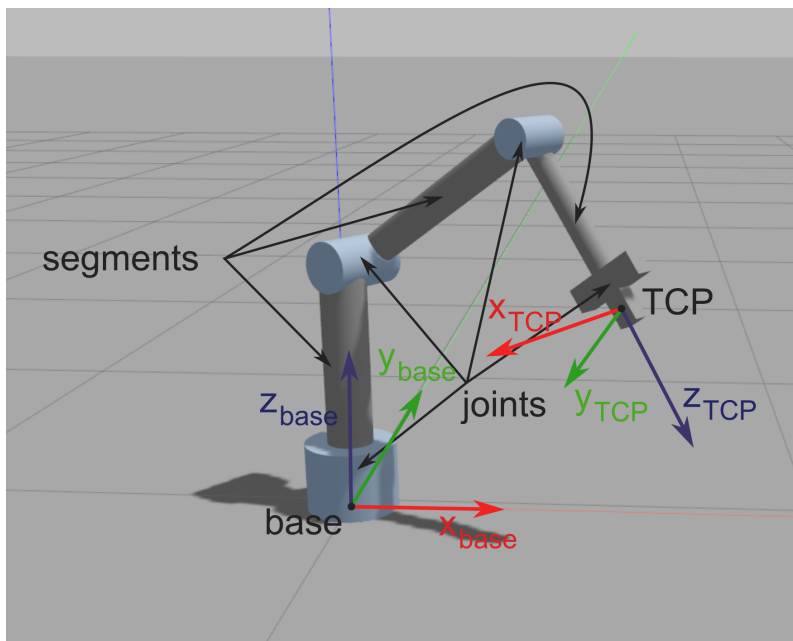
- **Homogenous coordinates:**

- **Vector:** extended with 0, $\mathbf{a}_H = \begin{bmatrix} \mathbf{a} \\ 0 \end{bmatrix} = \begin{bmatrix} a_x \\ a_y \\ a_z \\ 0 \end{bmatrix}$
- **Point:** extended by 1, $\mathbf{p}_H = \begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$
- Applying transformations is much easier:

$$\mathbf{q} = \mathbf{R}\mathbf{p} + \mathbf{v} \rightarrow \begin{bmatrix} \mathbf{q} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & \mathbf{v} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix}$$

- **Degrees of Freedom (DoF):** the number of independent parameters.

Principles of robotics



- Robots are built of: **segments** (or links) és **joints**
- **Task space** (or cartesian space):
 - 3D space around us, where the task, endpoint trajectories, obstacles are defined.

- **TCP** (Tool Center Point): Frame fixed to the end effector of the robot.
- **Base frame, world frame**
- **Joint space:**
 - Properties or values regarding the joints.
 - Low-level controller.
 - Joint angles, joint velocities, accelerations, torques....

Lecture

Kinematics, inverse kinematics

Def. Kinematics

Calculation of the pose of the TCP from joint angles. (From joint space to task space)

- Kinematic model
 - Denavit--Hartenberg (HD) convention
 - URDF (Unified Robotics Description Format, XML-based)

If the frames attached to each segment are named $(base, 1, 2, 3, \dots, TCP)$, transformations between two neighboring segments (i) and $(i+1)$ ---dependent on the angle of the joint enclosed by them---are named $(T_{i+1,i}(q_{i+1}))$, the transformation from the base frame to the TCP can be calculated as follows for a robot with (n) joints:

$$[T_{TCP,base}(q_1, \dots, q_n) = T_{TCP,n-1}(q_n) \cdot T_{n-1,n-2}(q_{n-1}) \cdot \dots \cdot T_{2,1}(q_2) \cdot T_{1,base}(q_1) \cdot base]$$

Def. Inverse kinematics

Calculation of the joint angles in order to reach desired (or any) TCP pose. (From task space to joint space)

Differential inverse kinematics

Def. Differential inverse kinematics

How to change the joint angles to achieve the desired **small** change in TCP pose (including rotation and translation).

- **Jacobian matrix** (Jacobian): The Jacobian matrix of a vector-valued function of several variables is the matrix of all its first-order partial derivatives.

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x_1}{\partial q_1} & \frac{\partial x_1}{\partial q_2} & \frac{\partial x_1}{\partial q_3} & \dots & \frac{\partial x_1}{\partial q_n} \\ \frac{\partial x_2}{\partial q_1} & \frac{\partial x_2}{\partial q_2} & \frac{\partial x_2}{\partial q_3} & \dots & \frac{\partial x_2}{\partial q_n} \\ \frac{\partial x_3}{\partial q_1} & \frac{\partial x_3}{\partial q_2} & \frac{\partial x_3}{\partial q_3} & \dots & \frac{\partial x_3}{\partial q_n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial x_m}{\partial q_1} & \frac{\partial x_m}{\partial q_2} & \frac{\partial x_m}{\partial q_3} & \dots & \frac{\partial x_m}{\partial q_n} \end{bmatrix}$$

- **Jacobian matrix in robotics:** defines the relationship between the joint velocities and the velocity of the TCP:

$$\begin{bmatrix} \mathbf{v} \\ \boldsymbol{\omega} \end{bmatrix} = \mathbf{J}(\mathbf{q}) \dot{\mathbf{q}}$$

If Δt is small enough:

$$\begin{aligned} & \frac{\Delta \mathbf{r}}{\Delta t} \approx \mathbf{J}(\mathbf{q}) \frac{\Delta \mathbf{q}}{\Delta t} \end{aligned}$$

Differential inverse kinematics using Jacobian inverse

1. Calculate the difference of the desired and the current position: $\Delta \mathbf{r} = \mathbf{r}_{\text{desired}} - \mathbf{r}_0$
2. Calculate the rotation between the current orientation and the desired orientation: $\Delta \mathbf{R} = \mathbf{R}_{\text{desired}} \mathbf{R}_0^T$, majd konvertáljuk át axis angle reprezentációba (\mathbf{t}, ϕ)
3. Calculate $\Delta \mathbf{q} = \mathbf{J}^{-1}(\mathbf{q}_0) \begin{bmatrix} \mathbf{v} \\ \phi \end{bmatrix}$

$\right]$), where the inverse could be substituted by pseudo-inverse or transpose

4. Change joint angles: $\mathbf{q}_{\text{better}} = \mathbf{q}_0 + \Delta \mathbf{q}$

Practice

1: Install rrr-arm

1. Install dependencies:

```
sudo apt update
sudo apt-get install ros-noetic-effort-controllers
sudo apt-get install ros-noetic-position-controllers
sudo apt-get install ros-noetic-gazebo-ros-pkgs
sudo apt-get install ros-noetic-gazebo-ros-control
sudo apt-get install ros-noetic-gazebo-ros
pip3 install kinpy
rosdep update
```

Tip

We will use the package `kinpy` to calculate forward kinematics. Download the source of the package `kinpy` and study the API: <https://pypi.org/project/kinpy/>

2. Clone and build the repo:

```
cd ~/catkin_ws/src
git clone https://github.com/Robotawi/rrr-arm.git
cd ..
catkin build
```

3. Launch the simulator and move the arm:

```
roslaunch rrr_arm view_arm_gazebo_control_empty_world.launch
```

```
rostopic pub /rrr_arm/joint1_position_controller/command std_msgs/Float64 "data: 1.0"
& rostopic pub /rrr_arm/joint2_position_controller/command std_msgs/Float64 "data:
```

```
1.0" & rostopic pub /rrr_arm/joint3_position_controller/command std_msgs/Float64  
"data: 1.5" & rostopic pub /rrr_arm/joint4_position_controller/command std_msgs/  
Float64 "data: 1.5"
```

Tip

The simulator might raise errors like "No p gain specified for pid...", but those can be ignored as won't cause any issues.

4. Build the URDF file describes the robot:

```
cd ~/catkin_ws/src/rrr-arm/urdf  
roslaunch xacro xacro rrr_arm.xacro > rrr_arm.xacro.urdf
```

2: Move the arm in joint space

1. Create a new file named `rrr_arm_node` in the `scripts` folder. Add it to the `CMakeLists.txt`, as usual. Subscribe to the topic which publishes the joint angles (configuration) of the robot. Create publishers to the 4 topics setting the joint angles of the arm. Use the previous Python scripts as a template.

Warning

Gazebo and `kinpy` organized the joints in different orders: **1.**

[gripper_joint_1, gripper_joint_2, joint_1, joint_2, joint_3, joint_4] - topic `/rrr_arm/joint_states` - method `kp.jacobian.calc_jacobian(...)`

2. [joint_1, joint_2, joint_3, joint_4, gripper_joint_1, gripper_joint_2] - method `chain.forward_kinematics(...)` - method `chain.inverse_kinematics(...)`

2. Send the arm to the configuration [1.0, 1.0, 1.5, 1.5].

3. Kinematic task

1. Import `kinpy` and read the URDF of the robot:

```
import kinpy as kp

chain = kp.build_serial_chain_from_urdf(open("/home/<USERNAME>/catkin_ws/src/
rrr-arm/urdf/rrr_arm.xacro.urdf").read(), "gripper_frame_cp")
print(chain)
print(chain.get_joint_parameter_names())
```

2. Calculate the TCP pose in the current configuration using `kinpy`. The example at <https://pypi.org/project/kinpy/> is wrong, use the following example:

```
th1 = np.random.rand(2)
tg = chain.forward_kinematics(th1)
th2 = chain.inverse_kinematics(tg)
self.assertTrue(np.allclose(th1, th2, atol=1.0e-6))
```

4: Inverse kinematics using Jacobian inverse

Implement a method calculating inverse kinematics using Jacobian inverse for the robot. Ignore the orientation for now. Move the TCP to position `(0.59840159, -0.21191189, 0.42244937)`.

1. Write a while loop stopping if the length of `delta_r` is below threshold or `rospy.is_shutdown()`.
2. Calculate the difference of the desired and current TCP positions (`delta_r`). Scale by constant `k_1`.
3. Let `phi_dot_t` be `[0.0, 0.0, 0.0]` (ignore the orientation).
4. Concatenate `delta_r` and `phi_dot_t`.

5. Calculate the Jacobian matrix in the current configuration using the method `kp.jacobian.calc_jacobian(...)`.
6. Calculate the pseudo inverse of the Jacobian using `np.linalg.pinv(...)`.
7. Calculate `delta_q`, use the `.dot(...)` method from Numpy.
8. Increase the joint angles by `delta_q`.

Bonus exercise: Inverse kinematics with orientation

Extend the previous exercise by calculating both the TCP position and orientation.

Useful links

- [rrr-arm model](#)
- <https://pypi.org/project/kinpy/>
- https://en.wikipedia.org/wiki/Axis%E2%80%93angle_representation
- <https://www.rosroboticslearning.com/jacobian>