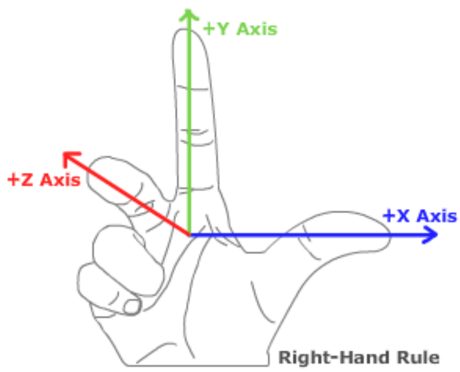
07. Kinematics, inverse kinematics. Programming a simulated robot in joint space and task space

> **Warning**
>
> **ZH2** (Roslaunch, ROS parameter szerver. ROS service. ROS action. Kinematics, inverse kinematics) and **project presentation**: **December 6.**

Rehearsal

3D transformations

- 

  Position: 3D offset vector
- **Orientation:** 3 x 3 rotation matrix
  - further orientation representations: Euler-angles, RPY, angle axis, quaternion
- **Pose**: 4 × 4 (homogenous) transformation matrix
- **Frame**: origin, 3 axes, 3 base vectors, right hand rule
- **Homogenous transformation:** rotation and translation in one transfromation
  - e.g., for the rotation \(\mathbf{R}\) and translation \(\mathbf{v}\):

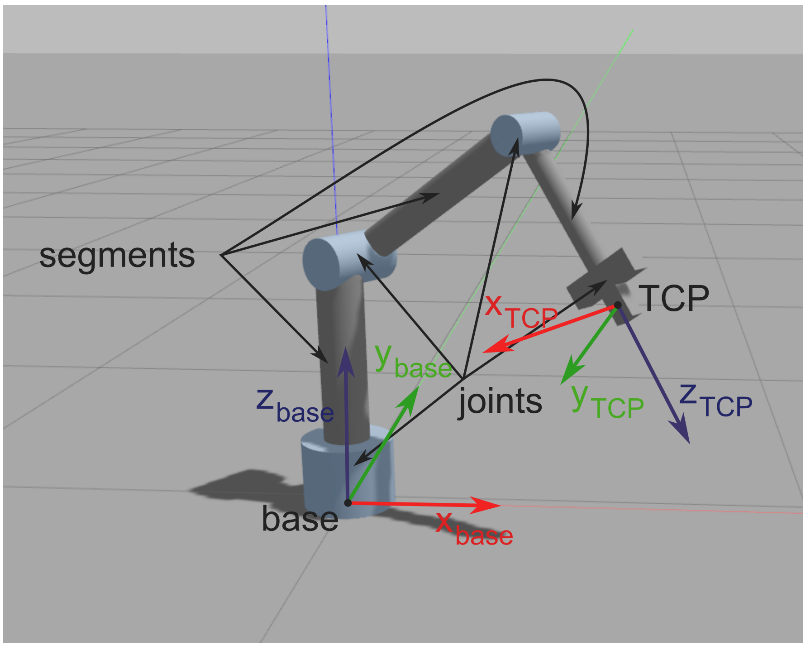
$$ \mathbf{T} = \left[\begin{matrix}\mathbf{R} & \mathbf{v}\\\mathbf{0} & 1 \end{matrix}\right] = \left[\begin{matrix}r_{1,1} & r_{1,2} & r_{1,3} & v_x\\r_{2,1} & r_{2,2} & r_{2,3} & v_y\\r_{3,1} & r_{3,2} & r_{3,3} & v_z\\ 0 & 0 & 0 & 1 \end{matrix}\right] $$

- **Homogenous coordinates:**
  - **Vector:** extended with 0, $\mathbf{a_H}=\left[\begin{matrix}\mathbf{a} \\ 0\end{matrix}\right]=\left[\begin{matrix}a_x \\ a_y \\ a_z \\ 0\end{matrix}\right]$
  - **Point:** extended by 1, $\mathbf{p_H}=\left[\begin{matrix}\mathbf{p} \\ 1\end{matrix}\right]=\left[\begin{matrix}p_x \\ p_y \\ p_z \\ 1\end{matrix}\right]$
  - Applying transfomrations is much easier:

$$ \mathbf{q} = \mathbf{R}\mathbf{p} + \mathbf{v} \to \left[\begin{matrix}\mathbf{q} \\ 1\end{matrix}\right] = \left[\begin{matrix}\mathbf{R} & \mathbf{v}\\\mathbf{0} & 1 \end{matrix}\right] \left[\begin{matrix}\mathbf{p} \\ 1\end{matrix}\right] $$

- **Degrees of Freedom** (DoF): the number of independent parameters.

Principles of robotics



- Robots are built of: **segments** (or links) és **joints**
- **Task space** (or cartesian space):
  - 3D space around us, where the task, endpoint trajectories, obstacles are defined.

    - **TCP** (Tool Center Point): Frame fixed to the end effector of the robot.
    - **Base frame**, **world frame**
- **Joint space**:
    - Properties or values regarding the joints.
    - Low-level controller.
    - Joint angles, joint velocities, accelerations, torques....

Lecture

Kinematics, inverse kinematics

> **Def. Kinematics**
>
> Calculation of the pose of the TCP from joint angles. (From joint space to task space)

If the frames attached to each segment are named $base, 1, 2, 3, ..., TCP$, transformations between two neighboring segments i and $i+1$---dependent on the angle of the joint enclosed by them---are named $T_{i+1,i}(q_{i+1})$, the transformation from the base frame to the TCP can be calculated as follows for a robot with n joints:

$$ T_{TCP,base}(q_1, \cdots, q_n) = T_{TCP,n-1}(q_{n}) \cdot T_{n-1,n-2}(q_{n-1}) \cdots T_{2,1}(q_2) \cdot T_{1,base}(q_1) \cdot base $$

> **Def. Inverse kinematics**
>
> Calculation of the joint angles in order to reach desired (or any) TCP pose. (From task space to joint space)

Differential inverse kinematics

> **Def. Differential inverse kinematics**
>
> How to change the joint angles to achieve the desired **small** change in TCP pose (including rotation and translation).

- **Jacobian matrix** (Jacobian): The Jacobian matrix of a vector-valued function of several variables is the matrix of all its first-order partial derivatives.

  $$ \mathbf{J} = \left[\matrix{\frac{\partial x_1}{\partial q_1} & \frac{\partial x_1}{\partial q_2} &\frac{\partial x_1}{\partial q_3} & \dots &\frac{\partial x_1}{\partial q_n} \\ \frac{\partial x_2}{\partial q_1} & \frac{\partial x_2}{\partial q_2} &\frac{\partial x_2} {\partial q_3} & \dots &\frac{\partial x_2}{\partial q_n} \\ \frac{\partial x_3}{\partial q_1} & \frac{\partial x_3}{\partial q_2} &\frac{\partial x_3}{\partial q_3} & \dots &\frac{\partial x_3}{\partial q_n} \\ \vdots &\vdots &\vdots &\ddots &\vdots \\ \frac{\partial x_m}{\partial q_1} & \frac{\partial x_m}{\partial q_2} &\frac{\partial x_m}{\partial q_3} & \dots &\frac{\partial x_m}{\partial q_n} \\}\right] $$

- **Jacobian matrix in robotics**: defines the relationship between the joint velocities and the velocity of the TCP:

  $$ \left[\matrix{\mathbf{v} \\ \mathbf{\omega}}\right] =\mathbf{J}(\mathbf{q})\cdot \mathbf{\dot{q}} $$

Differential inverse kinematics using Jacobian inverse

1. Calculate the difference of the desired and the current position: $\Delta\mathbf{r} = \mathbf{r}_{desired} - \mathbf{r}_0$
2. Calculate the rotation between the current orientation and the desired orientation: $\Delta\mathbf{R} = \mathbf{R}_{desired}\mathbf{R}_{0}^{T}$, majd konvertáljuk át axis angle reprezentációba $(\mathbf{t},\phi)$
3. Calculate $\Delta\mathbf{ q}=\mathbf{J}^{-1}(\mathbf{q_0})\cdot \left[\matrix{k_1 \cdot \Delta\mathbf{r} \\ k_2 \cdot \phi \cdot \mathbf{t}}\right]$, where the inverse could be substituted by pseudo-inverse or transpose
4. Change joint angles: $\mathbf{q}_{better} = \mathbf{q}_{0} + \Delta\mathbf{q}$

Practice

1: Install rrr-arm

1. Install dependencies:

```
sudo apt update
sudo apt-get install ros-noetic-effort-controllers
sudo apt-get install ros-noetic-position-controllers
sudo apt-get install ros-noetic-gazebo-ros-pkgs
sudo apt-get install ros-noetic-gazebo-ros-control
sudo apt-get install ros-noetic-gazebo-ros
pip3 install kinpy
rosdep update
```

> **Tip**
>
> We will use the package `kinpy` to calculate forward kinematics. Download the source of the package `kinpy` and study the API: https://pypi.org/project/kinpy/

2. Clone and build the repo:

```
cd ~/catkin_ws/src
git clone https://github.com/Robotawi/rrr-arm.git
cd ..
catkin build
```

3. Launch the simulator and move the arm:

```
roslaunch rrr_arm view_arm_gazebo_control_empty_world.launch
```

```
rostopic pub /rrr_arm/joint1_position_controller/command  std_msgs/Float64 "data: 1.0"
&    rostopic pub /rrr_arm/joint2_position_controller/command  std_msgs/Float64 "data:
1.0" & rostopic pub /rrr_arm/joint3_position_controller/command  std_msgs/Float64
"data: 1.5" & rostopic pub /rrr_arm/joint4_position_controller/command std_msgs/
Float64 "data: 1.5"
```

> **Tip**
>
> The simulator might raise errors like "No p gain specified for pid...", but those can be ignored as won't cause any issues.

4. Build the URDF file describes the robot:

```
cd ~/catkin_ws/src/rrr-arm/urdf
rosrun xacro xacro rrr_arm.xacro > rrr_arm.xacro.urdf
```

2: Move the arm in joint space

1. Create a new file named `rrr_arm_node` in the `scripts` folder. Add it to the `CMakeLists.txt`, as usual. Subscribe to the topic which publishes the joint angles (configuration) of the robot. Create publishers to the 4 topics setting the joint angles of the arm. Use the previous Python scripts as a template.

> **Warning**
>
> Gazebo and `kinpy` organized the joints in different orders: **1. [gripper_joint_1, gripper_joint_2, joint_1, joint_2, joint_3, joint_4]** - topic `/rrr_arm/joint_states` - method `kp.jacobian.calc_jacobian(...)`
>
> **2. [joint_1, joint_2, joint_3, joint_4, gripper_joint_1, gripper_joint_2]** - method `chain.forward_kinematics(...)` - method `chain.inverse_kinematics(...)`

2. Send the arm to the configuration [1.0, 1.0, 1.5, 1.5].

3. Kinematic task

1. Import `kinpy` and read the URDF of the robot:

```
import kinpy as kp
```

```
chain = kp.build_serial_chain_from_urdf(open("/home/<USERNAME>/catkin_ws/src/rrr-arm/urdf/rrr_arm.xacro.urdf").read(), "gripper_frame_cp")
print(chain)
print(chain.get_joint_parameter_names())
```

2. Calculate the TCP pose in the current configuration using `kinpy`. The example at https://pypi.org/project/kinpy/ is wrong, use the following example:

```
th1 = np.random.rand(2)
tg = chain.forward_kinematics(th1)
th2 = chain.inverse_kinematics(tg)
self.assertTrue(np.allclose(th1, th2, atol=1.0e-6))
```

4: Inverse kinematics using Jacobian inverse

Implement a method calculating inverse kinematics using Jacobian inverse for the robot. Ignore the orientation for now. Move the TCP to position `(0.59840159, -0.21191189, 0.42244937)`.

1. Write a while loop stopping if the length of `delta_r` is below threshold or `rospy.is_shutdown()`.

2. Calculate the difference of the desired and current TCP positions (`delta_r`). Scale by constant `k_1`.

3. Let `phi_dot_t` be `[0.0, 0.0, 0.0]` (ignore the orientation).

4. Concatenate `delta_r` and `phi_dot_t`-t.

5. Calculate the Jacobian matrix in the current configuration using the method `kp.jacobian.calc_jacobian(...)`.

6. Calculate the pseudo inverse of the Jacobian using `np.linalg.pinv(...)`.

7. Calculate `delta_q`, use the `.dot(...)` method from Numpy.

8. Increase the joint angles by `delta_q`.

Bonus exercise: Inverse kinematics with orientation

Extend the previous exercise by calculating both the TCP position and orientation.

Useful links

- rrr-arm model
- https://pypi.org/project/kinpy/
- https://en.wikipedia.org/wiki/Axis%E2%80%93angle_representation
- https://www.rosroboticslearning.com/jacobian