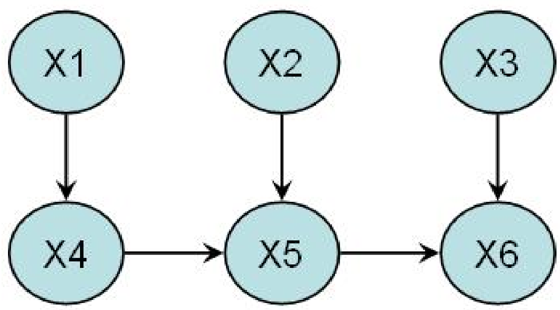
Feedback — Inference: Variable Elimination

You achieved a score of 10.00 out of 10.00

Question 1

Intermediate Factors. Consider running variable elimination on the following Bayesian network over binary variables. Which of the nodes, if eliminated first, results in the largest intermediate factor? By largest factor we mean the factor with the largest number of entries.



Your Answer		Score	Explanation
X_5	✔	1.00	Eliminating X_5 results in the intermediate factor $\tau(X_2, X_4, X_6)$, whic larger than for any of the other options.
Total		1.00	

Question 2

Elimination Orderings. Which of the following characteristics of the variable elimination algorithm a affected by the choice of elimination ordering? You may select 1 or more options (or none of them, if think none apply).

Your Answer		Score	Explanation
Correctness of the algorithm	✔	0.25	The correctness of variable elimination is independent the elimination ordering.
Which marginals can be computed correctly	✔	0.25	The correctness of the algorithm is independent of the elimination ordering.

✔ Runtime of the algorithm	✔	0.25	The elimination ordering affects the size of the largest factor created, which determines the runtime of the algorithm.
✔ Size of the largest intermediate factor	✔	0.25	The elimination ordering can affect the size of the large intermediate factor.
Total		1.00	

Question 3

Uses of Variable Elimination. Which of the following quantities can be computed using the sum-pro variable elimination algorithm? (In the options, let X be a set of query variables, and E be a set of evidence variables in the respective networks.) You may select 1 or more options (or none of them, i think none apply).

Your Answer		Score	Explanation
✔ $P(X \mid E = e)$ in a Markov network	✔	0.25	This is a standard use of the variable elimination algorithm.
The most likely assignment to the variables in a Markov network.	✔	0.25	We cannot do this with sum-product variable elimination, wh is what was discussed. However, we can do this with max-product variable elimination, a modification to the algorithm which will be discussed later in the course.
✔ The partition function for a Markov network	✔	0.25	We can do this by eliminating all of the variables.
✔ $P(X)$ in a Bayesian network	✔	0.25	This is a standard use of the variable elimination algorithm.
Total		1.00	

Question 4

Marginalization. Suppose we run variable elimination on a Bayesian network where we eliminate al variables in the network. What number will the algorithm produce?

1

Your Answer		Score	Explanation
1	✔	1.00	Bayesian networks represent valid probability distributions, and so summ up all the possible states will always return 1.
Total		1.00	

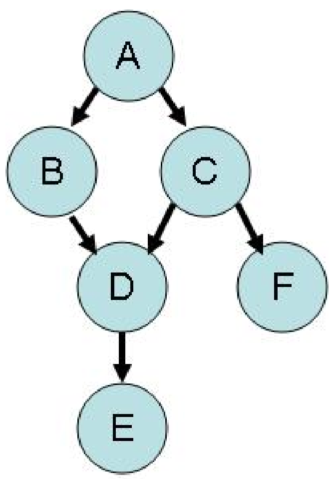
Question 5

Marginalization. Suppose we run variable elimination on a Markov network where we eliminate all tl variables in the network. What number will the algorithm produce?

Your Answer	Score	Explanation
Z, the partition function for the network. ✔	1.00	Eliminating all the variables yields the partition function for the network.
Total	1.00	

Question 6

Intermediate Factors. If we perform variable elimination on the graph shown below with the variable ordering B, A, C, F, E, D, what is the intermediate factor produced by the third step (just before summing out C)?



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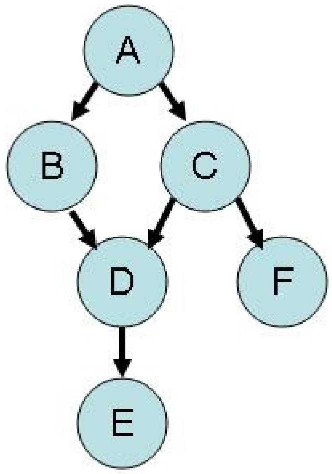
Your Answer	Score	Explanation
$\psi(C, D, F)$ ✔	1.00	After eliminating B we have a new factor $\tau_1(A, D, C)$, and after eliminating A, the factor becomes $\tau_2(D, C)$, then when eliminating the intermediate factor is $\psi(C, D, F) = \tau_2(D, C)P(F\|C)$. This i because the only factors involving C at this point are $\tau_2(D, C)$ and $P(F\|C)$. The only other factor involving C, $P(C\|A)$ was already us to compute $\tau_2(D, C)$ when eliminating A, so including it again wou be incorporating information from this factor twice.
Total	1.00	

Question 7

Induced Graphs. If we perform variable elimination on the graph shown below with the variable orde
B, A, C, F, E, D, what is the induced graph for the run?



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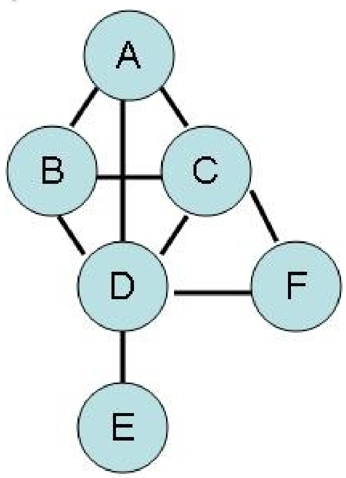
Your Answer		Score	Explanation
	✔	1.00	This is correct. There is an edge in the induced graph between every pair of variables that is pres together in a factor during a run of variable elimination.
Total		1.00	

Question 8

***Time Complexity of Variable Elimination.** Consider a Bayesian network taking the form of a chair variables, $X_1 \to X_2 \to \cdots \to X_n$, where each of the X_i can take on k values. What is the computational cost of running variable elimination on this network if we eliminate the X_i in order (i.e first X_1, then X_2 and so on)?

Your Answer		Score	Explanation
$O(nk^2)$	✔	1.00	When eliminating X_1, we sum out X_1 from $P(X_1)P(X_2\|X_1)$ to ob $\phi_1(X_2)$. For each value of X_2, we have to do k multiplications and $k-1$ summations, which is $O(k)$. Since X_2 can take k different valu to compute $\phi_1(X_2)$, the computational cost is $O(k^2)$ operations. The process continues for each X_i, so in total the cost is $O(nk^2)$.
Total		1.00	

Question 9

***Time Complexity of Variable Elimination.** Now take the same chain as in the previous question, b eliminate the X_i starting from X_2, going to $X_3, \ldots, X_n$ and then back to X_1. What is the computational cost of running variable elimination with this ordering?

Your Answer		Score	Explanation
$O(nk^3)$	✔	1.00	If we start by eliminating X_2, we create an intermediate factor over X_1, X_2, X_3, and continue from X_3 to the end and then X_1. Since th scope of the intermediate factor involves three variables, the complexity would be $O(nk^3)$ instead of $O(nk^2)$ as in the previous question.
Total		1.00	

Question 10

Time Complexity of Variable Elimination. Suppose we eliminate all the variables in a Markov netw using the variable elimination algorithm. Which of the following could affect the runtime of the algorit You may select 1 or more options (or none of them, if you think none apply).

Your Answer		Score	Explanation
☑ Number of variables in the network	✔	0.33	The runtime is affected by the number of variables in the netw as discussed in the textbook.
☑ The variable elimination ordering	✔	0.33	This potentially affects the size of the largest factor in the network, which is a key component of the algorithm's runtime.

✔ Whether there is evidence	✔	0.33	If there is evidence, the number of variables in the network is effectively reduced, since factors involving the evidence variab are reduced.
Total		1.00	