



# Probabilistic Graphical Models

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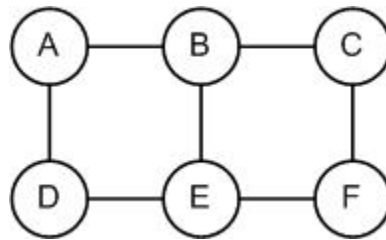
## Feedback — Inference: Belief Propagation

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You achieved a score of **10.42** out of **12.00**

### Question 1

**Cluster Graph Construction.** Consider the pairwise MRF,  $H$ , shown below. The pairwise potentials are  $\psi_{B,C}$ ,  $\psi_{A,D}$ ,  $\psi_{B,E}$ ,  $\psi_{C,F}$ ,  $\psi_{D,E}$  and  $\psi_{E,F}$ .



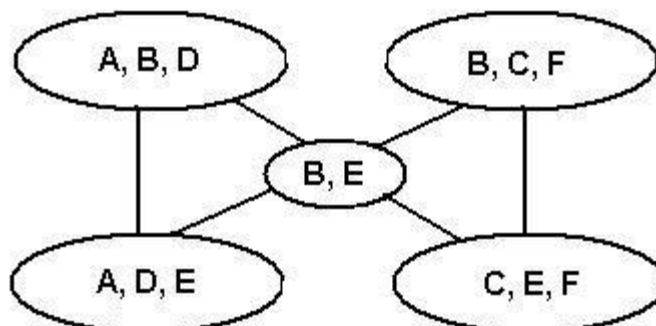
Which of the following is/are valid cluster graph(s) for  $H$ ? (A cluster graph must satisfy the intersection property and family preservation. You may select 1 or more options. If you think none apply.)

Your Answer

Score

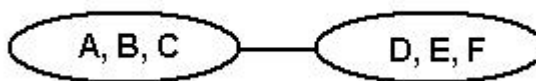


0.25

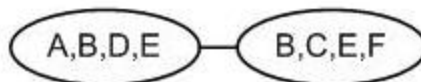




0.25



0.00



0.25



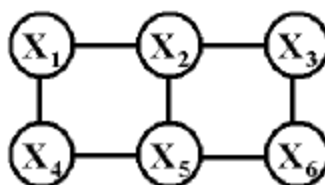
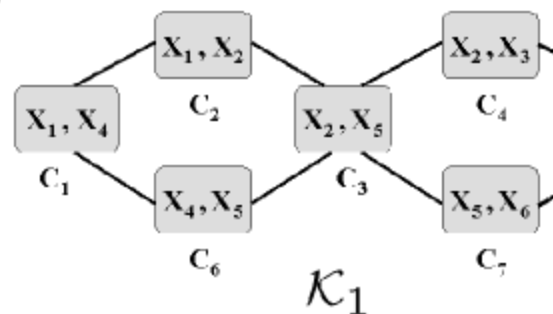
Total

0.75

## Question 2

### Message Passing in a Cluster Graph.

Suppose we wish to perform inference over the Markov network  $M$  as shown below, and the only potentials in the network are the pairwise potentials  $\phi_{i,j}(X_i, X_j)$  of variables  $X_i, X_j$  connected by an edge in  $M$ . Which of the following expresses  $\delta_{3 \rightarrow 6}$  that cluster  $C_3$  will send to cluster  $C_6$  during belief propagation? Assume that the message is equal to the intersection of the variables in the adjacent cliques.

 $\mathcal{M}$ 

$$\delta_{3 \rightarrow 6}(X_5) = \sum_{X_2} \phi_{2,5}(X_2, X_5) \delta_{2 \rightarrow 3}(X_2) \delta_{4 \rightarrow 3}(X_2) \delta_{7 \rightarrow 3}(X_5)$$

### Question 3

You may separate the entries of the message by commas. Order the entries in lexicographic order: for example, if the message is over one variable  $X_i$ , then enter in order  $\delta_{3 \rightarrow 6}(X_i = 1)$ . If the message is over two variables  $X_i, X_j$ , where  $i < j$ , then enter in order  $\delta_{3 \rightarrow 6}(X_i = 0, X_j = 0)$ ,  $\delta_{3 \rightarrow 6}(X_i = 0, X_j = 1)$ ,  $\delta_{3 \rightarrow 6}(X_i = 1, X_j = 0)$ ,  $\delta_{3 \rightarrow 6}(X_i = 1, X_j = 1)$ .

$X_i$	$X_j$	$\phi(X_i, X_j)$
1	1	10
1	0	1
0	1	1

0

0

10

11, 11

Your Answer		Score	
11		0.50	
11		0.50	
Total		1.00	

Question 4

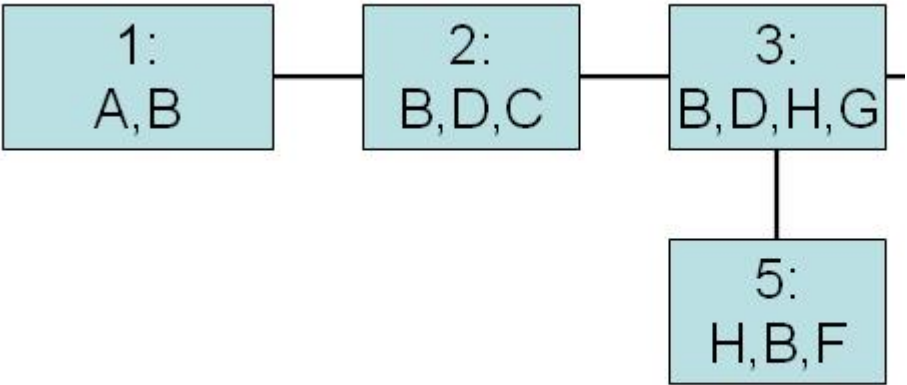
**\*Extracting Marginals at Convergence.** Given that you can renormalize during belief propagation and still obtain correct marginals, consider the  $\delta_{i \rightarrow j}$  computed. Use this observation to compute the final and possibly approximate  $P(X_4 = 1, X_5 = 1)$  ( $X_4$  and  $X_5$  are the variables in the previous question). Give your answer to 2 decimal places (as extracted from the cluster beliefs), giving your answer to

0.45

Your Answer		Score	Explanation
0.45		1.00	Since $\delta_{3 \rightarrow 6}$ is proportional to a uniform factor, the same holds for $\delta_{1 \rightarrow 6}$ . Thus the final cluster beliefs are proportional to the initial beliefs, giving us a probability of $\frac{10}{22}$ .
Total		1.00	

Question 5

**Message Ordering.** In the clique tree below which of the following starting points is/are valid? (Note: These are not necessarily full sweeps that result in cycles, but more options, or none of them, if you think none apply.)



Your Answer	Score	Explainer
<input checked="" type="checkbox"/> $C_1 \rightarrow C_2, C_2 \rightarrow C_3, C_5 \rightarrow C_3, C_3 \rightarrow C_4$	<input checked="" type="checkbox"/> 0.25	This is a valid starting point for a message passing algorithm.
<input checked="" type="checkbox"/> $C_4 \rightarrow C_3, C_5 \rightarrow C_3, C_3 \rightarrow C_2, C_1 \rightarrow C_2$	<input checked="" type="checkbox"/> 0.25	This is a valid starting point for a message passing algorithm.
<input type="checkbox"/> $C_1 \rightarrow C_2, C_2 \rightarrow C_3, C_3 \rightarrow C_4, C_3 \rightarrow C_5$	<input checked="" type="checkbox"/> 0.25	$C_3$ needs to pass a message to $C_5$ .
<input type="checkbox"/> $C_4 \rightarrow C_3, C_3 \rightarrow C_5, C_3 \rightarrow C_2, C_1 \rightarrow C_2$	<input checked="" type="checkbox"/> 0.25	$C_3$ needs to pass a message to $C_5$ .
Total	1.00	

Question 6

**Message Passing in a Clique Tree.** In the clique tree above, what is the

from clique 3 to clique 2,  $\delta_{3 \rightarrow 2}$ , where  $\psi_i(C_i)$  is the initial potential of cl

Your Answer	Score	Explanation
<div><div><div></div></div><div><math>\sum_{G,H} \psi_3(C_3) \times \delta_{4 \rightarrow 3} \times \delta_{5 \rightarrow 3}</math></div></div>	<div><div></div><div>1.00</div></div>	This is correct; to co multiply the initial po incoming messages and eliminate the va sepsset.
Total	1.00	

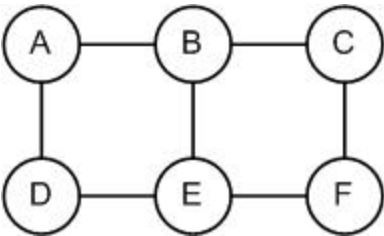
Question 7

**Family Preservation.** Suppose we have a factor  $P(A \mid C)$  that we wish message passing inference. We should:

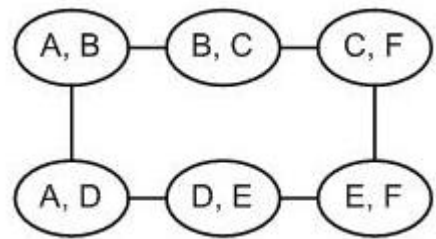
Your Answer	Score	Explanation
<div><div><div></div></div><div>Assign the factor to <b>one</b> clique that contain <b>A and C</b></div></div>	<div><div></div><div>1.00</div></div>	Family Preservation explains th clique tree (cluster graph) requir cluster whose scope contains th
Total	1.00	

Question 8

**Clique Tree Properties.** Consider the following Markov Network over pc  $\phi_{A,B}, \phi_{B,C}, \phi_{A,D}, \phi_{B,E}, \phi_{C,F}, \phi_{D,E}$ , and  $\phi_{E,F}$ :



Which of the following properties are necessary for a valid clique tree for satisfied by this graph:



You may select 1 or more options (or none of them, if you think none apply)

Your Answer	Score	Expl
<input checked="" type="checkbox"/> Family preservation	0.25	Fami beca anyw
<input type="checkbox"/> Node degree less than or equal to 2	0.25	This a clus
<input type="checkbox"/> The number of nodes in a clique tree containing a variable should be exactly the number of factors in the Markov network that contain the same variable	0.25	Multip the s provi
<input type="checkbox"/> Running intersection property	0.25	The c inters the s inters adjac
Total	1.00	

Question 9

**Cluster Graphs vs. Clique Trees.** Suppose that we ran sum-product message passing on a graph  $G$  for a Markov network  $M$  and that the algorithm converged. Which of the following is true **only if**  $G$  is a clique tree and is **not** necessarily true otherwise?

Your Answer	Score	Explanation
<input checked="" type="radio"/> If there are $E$ edges in $G$ , there exists a message ordering that guarantees	1.00	This is a property specific to cliques. The root clique is the root of the tree. Messages from the root clique to all other cliques are guaranteed to have cal


convergence after passing  $2E$  messages.

graph however, depending on  $n$  may take longer.

Total	1.00
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## Question 10

**\*Numerical Issues in Belief Propagation.** In practice, one of the issues with belief propagation is that when we multiply many small numbers, we run into the limits of floating-point numbers, resulting in arithmetic underflow. One possible solution to this problem is to renormalize each message, as it's passed, such that its norm is 1. If we do not store the renormalization factor at each step. Which of the following is a consequence of this approach?



Your Answer		Score	Explanation
<input checked="" type="radio"/> This does not change the results of the algorithm: when the clique tree is calibrated, we can obtain from it both the partition function and the correct marginals.		0.00	Think about the effect of renormalizing messages on the partition function of the partition function. In the case where we do not renormalize, the partition function is not normalized.
Total		0.00	

## Question 11

**\*Numerical Issues in Belief Propagation.** The same numerical issues arise with belief propagation on messages in a *cluster graph with loops*. Which of the following statements are true about this approach? You may select 1 or more options (or none of them, if you want).



Your Answer		Score	Explanation
<input type="checkbox"/> At convergence, the marginals that we extract from each of the beliefs in the clusters will now be the marginals of the original joint distribution.		0.33	While renormalizing messages, the numerical errors do not disappear (when renormalizing the same message multiple times).



			general, $\mu$ marginals yields on
<input type="checkbox"/> Assuming nothing else changes (e.g., message passing order), at convergence, the approximate marginals that are obtained from renormalizing the beliefs at each cluster will be the same as in the original algorithm (ignoring differences arising from numerical precision issues).		0.00	This is true for clique renormalization: a cluster once we multiply
<input checked="" type="checkbox"/> At convergence, the cluster graph will (in general) not satisfy the cluster graph invariant, i.e., the product of cluster beliefs divided by the product of sepset beliefs will not be equal to the original unnormalized distribution.		0.33	This is true written as unnormalized can recover the normalized function in the cluster change equation
Total		0.67	

## Question 12

**Convergence in Belief Propagation.** Suppose we ran belief propagation on a clique tree  $T$  for the same Markov network that is a perfect map for a distribution  $P$ . If  $P$  and  $T$  are valid, i.e., they satisfy family preservation and the running intersection property, which of the following statements regarding the algorithm are true? You may select 1 or more answers (or none, if you think none apply).

Your Answer	Score	Explanation
<input checked="" type="checkbox"/> Assuming the algorithm converges, if a variable $X$ appears in two cliques in $T$ , the marginals $P(X)$ computed from the the two clique beliefs must agree.	 0.25	This is true due to the convergence and the fact that they satisfy the running intersection property: variable $X$ is in two cliques connected by a path for which $X$ is in the sepset (by the RIP), and must agree on the belief.
<input checked="" type="checkbox"/> Assuming the algorithm converges, if a variable $X$ appears in two clusters in $G$ , the marginals $P(X)$	 0.25	This is true due to the convergence and the fact that they satisfy the running intersection property: variable $X$ is in two clusters

computed from the two  
cluster beliefs must agree.

connected by a path for  
sepset (by the RIP), and  
must agree on the belie

■ If the algorithm  
converges, the final *cluster*  
beliefs in  $G$ , when  
renormalized to sum to 1,  
are true marginals of  $P$ .



0.25

This is not true, because  
loops. One consequence  
about a variable may be  
cluster graphs only return

■ Belief propagation always  
converges on  $G$ .



0.25

This is not always true,  
strong opposing potentials  
graph.

Total

1.00