L25SVD

November 28, 2015

1 The Singular Value Decomposition

Today we'll study the most useful decomposition in applied Linear Algebra.

Pretty exciting, eh?

The singular value decomposition is a matrix factorization.

EVERY matrix has a singular value decomposition.

The singular value decomposition (let's just call it SVD) is based on a very simple idea, which is closely related to eigendecomposition.

Recall that the absolute values of the eigenvalues of a symmetric matrix A measures the amount that A stretches or shrinks certain vectors (the eigenvectors).

For example, if $A\mathbf{x} = \lambda\mathbf{x}$ and $\|x\| = 1$, then

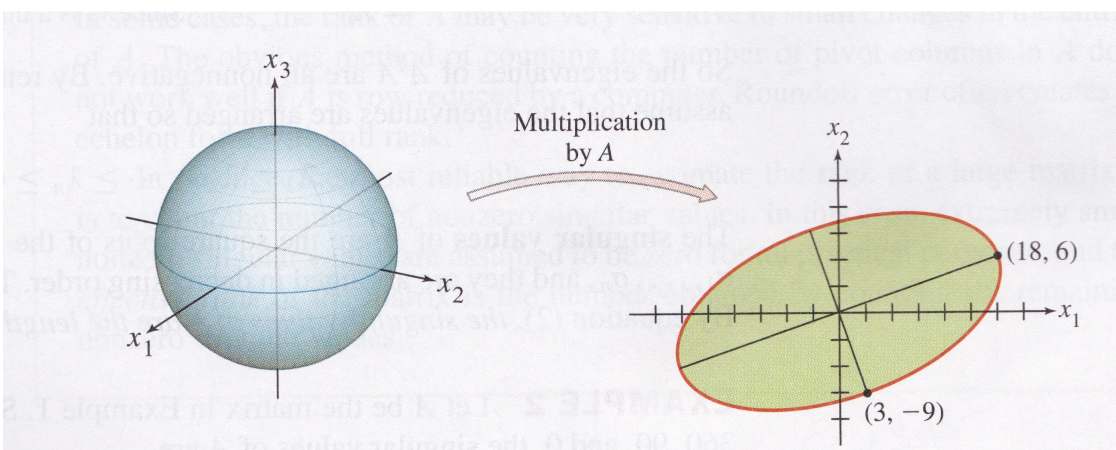
$$\|A\mathbf{x}\| = \|\lambda\mathbf{x}\| = |\lambda|\,\|\mathbf{x}\| = |\lambda|.$$

If λ_1 is the eigenvalue with the greatest magnitude, then a corresponding unit eigenvector $\mathbf{v}_1$ identifies a direction in which the stretching effect of A is greatest.

That is, the length of $A\mathbf{x}$ is maximized when $\mathbf{x} = \mathbf{v}_1$ and $\|A\mathbf{v}_1\| = |\lambda_1|$.

Now let's see by example how we can extend this idea to **arbitrary** (non-square) matrices.

Example.



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