4-Linear-Algebra-Refresher

September 5, 2016

1 Linear Algebra Refresher

Today we'll review the essentials of linear algebra. Given the prerequisites for this course, I assume that you learned all of this once. What I want to do today is bring the material back into your mind fresh.

1.1 Vectors and Matrices

A **matrix** is a rectangular array of numbers, for example:

$$X = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -8 \\ 6 & 5 & 9 \end{bmatrix}$$

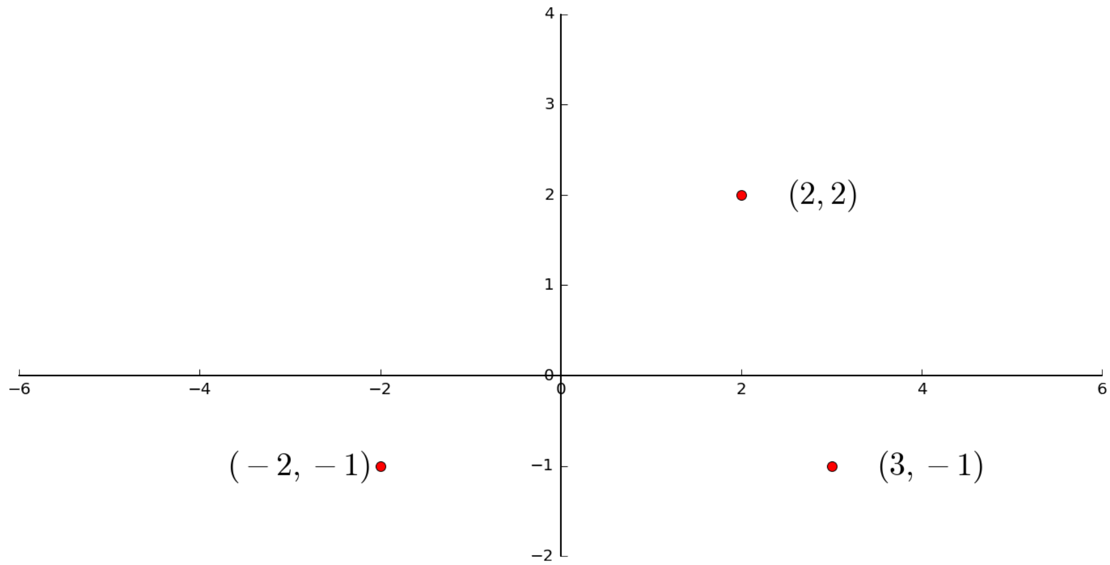
A matrix with only one column is called a **column vector**, or simply a **vector**.
Here are some examples.
These are vectors in $\mathbb{R}^2$:

$$\mathbf{u} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} .2 \\ .3 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

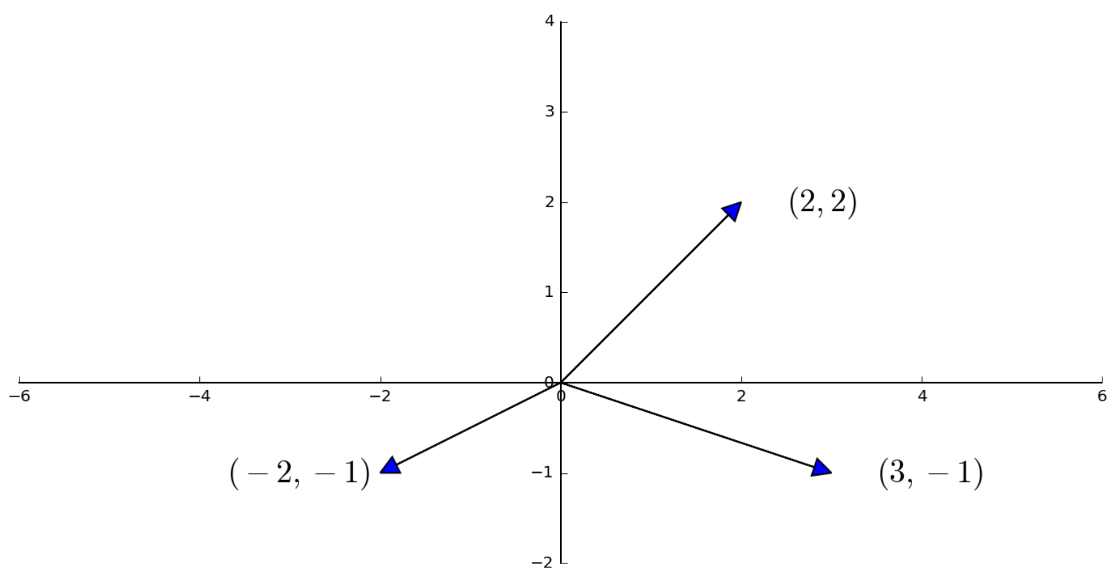
and these are vectors in $\mathbb{R}^3$:

$$\mathbf{u} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

We will use uppercase letters (X) for matrices and lowercase **bold** leters for vectors ($\mathbf{u}$).
A vector like $\begin{bmatrix} -2 \\ -1 \end{bmatrix}$ (also denoted $(-2, -1)$) can be thought of as a point on the plane.

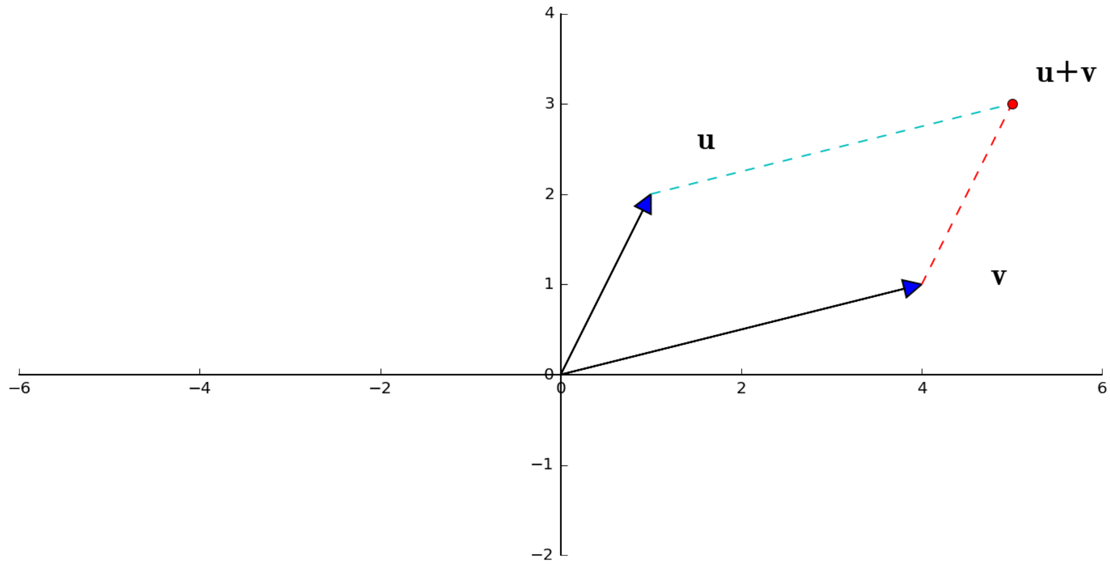


Sometimes we draw an arrow from the origin to the point. This comes from physics, but can be a helpful visualization in any case.



1.2 Vector Addition, Geometrically

A geometric interpretation of vector sum is as a parallelogram. If $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^2$ are represented as points in the plane, then $\mathbf{u} + \mathbf{v}$ corresponds to the fourth vertex of the parallelogram whose other vertices are $\mathbf{u}, 0$, and $\mathbf{v}$.



Optimization. Linear algebra – L2 norm, vectors, least squares, minimizing the L2 norm.
Need to review matrix decomposition - eigendecomposition (prep for SVD)
Markov Chains?
Most important: length and distance