Receive Antenna Selection in MIMO Systems

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Objective of the Paper

- The paper aims to maximize the channel capacity of the MIMO system.
- Reduce the cost of the system
- Optimization problem is solved using low complexity techniques

Convex Optimization Problem

Maximize

$$C_r(\mathbf{\Delta}) = \log_2 \det(\mathbf{I}_M + \gamma \mathbf{\Delta} \mathbf{H} \mathbf{H}^H)$$

subject to

$$0 \leq \Delta_i \leq 1, \quad i = 1, 2, ..., M$$

$$trace(\mathbf{\Delta}) = \sum_{i=1}^{M} \Delta_i = M'$$

where

$C_r(\mathbf{\Delta})$ is channel capacity
H is the M$\times$N channel matrix.

Proof of Concavity

We define g(t) $= \log|\mathbf{X} + t\mathbf{V}|$ such that $\mathbf{X} \in S_{++}^N$ and $\mathbf{V} \in S^N$

$$g(t) = \log|\mathbf{X} + t\mathbf{V}|$$
$$= \log|\mathbf{X}| + \sum_{i=1}^{N} \log(1 + t\lambda_i)$$

$$g''(t) = \sum_{i=1}^{N} \frac{-\lambda_i^2}{(1 + t\lambda_i)^2}$$

Since $\mathrm{g}''(t) < 0 \mathrm{g}(\mathrm{t})$ is concave

$\Rightarrow \log|\mathbf{X}|$ is concave

Result

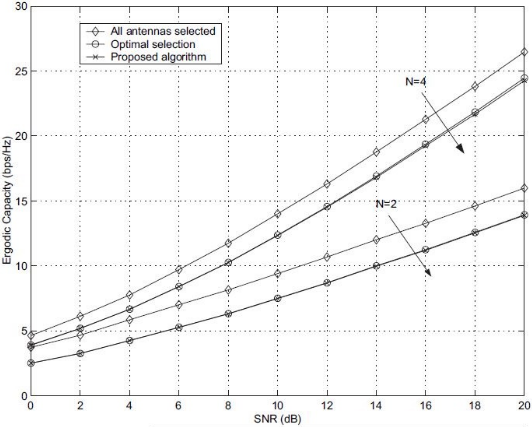

Figure: Ergodic capacity v/s SNR (N_γ), M = 6, N = 2,4 M' = N

System Model

Received signal can be represented as

$$x(k) = \sqrt{E_s}\boldsymbol{H}s(k) + \boldsymbol{n}(k)$$

where

x(k) is the k^{th} sample of the received signal.

s(k) is the k^{th} sample of the transmitted signal.

E_s is the average energy per receive antenna and per channel use

n(k) is AWGN with energy $\frac{N_0}{2}$

H is the M$\times$N channel matrix.

Receive Antenna Selection In MIMO Systems

'**Objective** - Select receive antenna to maximize capacity.
The capacity of a system is given by the formula

$$C(\boldsymbol{H}) = \log_2 \det(\boldsymbol{I}_N + \gamma \boldsymbol{R}_{ss} \boldsymbol{H}^H \boldsymbol{H})$$

where

$\gamma = \frac{E_s}{N_0}$

$\mathbf{R}_{ss} = \mathrm{E}[\mathrm{s(k)s(k)}^H]$ is the co-variance matrix of the transmitted signals with trace($\mathbf{R}_{ss}$)=1

When only M'<M receive antennas are used, the capacity becomes a function of the antennas chosen.

We represent the indices of the selected antennas by $\mathrm{r} = [\mathrm{r}_1, \ldots, \mathrm{r}_{M'}]$

The effective channel matrix becomes $\mathbf{H}_r$ which is a M'×N matrix.

The channel capacity with antenna selection is given by

$$C_r(\boldsymbol{H}_r) = \log_2 \det(\boldsymbol{I}_N + \gamma \boldsymbol{R}_{ss} \boldsymbol{H}_r^H \boldsymbol{H}_r)$$

Antenna Selection as an Optimization Problem

‘ We define Δ_i

$$\Delta_i = \begin{cases} 1, & \text{if} \quad i^{th} \text{ antenna is selected} \\ 0, & \text{otherwise.} \end{cases}$$

Now, consider an M$\times$M diagonal matrix $\mathbf{\Delta}$ that has Δ_i as its diagonal entries.

Let us denote F $= \mathbf{\Delta}$H.

F can be written as $\mathbf{H}_r$ with (M-M’) zero rows appended to it and left multiplied by a M M row-permutation matrix P. Thus,

$$\boldsymbol{F} = \boldsymbol{P} \begin{bmatrix} \boldsymbol{H}_r \\ 0_{(M-M')\times N} \end{bmatrix} = \boldsymbol{P}\widetilde{\boldsymbol{H}_r}$$

Since P is a permutation matrix, $\mathbf{P}^H\mathbf{P} = \mathbf{I}_M$.

$$\begin{aligned}\mathbf{F}^H\mathbf{F} &= \widetilde{\boldsymbol{H}_r}^H\mathbf{P}^H\mathbf{P}\widetilde{\boldsymbol{H}_r} \\ &= \widetilde{\boldsymbol{H}_r}^H\widetilde{\boldsymbol{H}_r} \\ &= \mathbf{H}_r^H\mathbf{H}_r\end{aligned}$$

The channel capacity as a function of $\boldsymbol{\Delta}$ is

$$\begin{aligned}C_r(\boldsymbol{\Delta}) &= \log_2\det(\mathbf{I}_N + \gamma\mathbf{F}^H\mathbf{F}) \\ &= \log_2\det(\mathbf{I}_N + \gamma\mathbf{H}^H\boldsymbol{\Delta}^H\boldsymbol{\Delta}\mathbf{H})\end{aligned}$$

Since $\mathbf{\Delta}$ is a diagonal matrix with entries either 0 or 1

$$\mathbf{\Delta}^H \mathbf{\Delta} = \mathbf{\Delta}$$

The MIMO channel capacity with antenna selection can be re-written as

$$C_r(\mathbf{\Delta}) = \log_2 \det(\mathbf{I}_N + \gamma \mathbf{H}^H \mathbf{\Delta} \mathbf{H})$$

Using the identity $\det(\mathbf{I}_m + \mathbf{AB}) = \det(\mathbf{I}_n + \mathbf{BA})$ we get

$$C_r(\mathbf{\Delta}) = \log_2 \det(\mathbf{I}_M + \gamma \mathbf{\Delta} \mathbf{H} \mathbf{H}^H)$$

Maximize

$$C_r(\mathbf{\Delta}) = \log_2 \det(\mathbf{I}_M + \gamma \mathbf{\Delta} \mathbf{H}\mathbf{H}^H)$$

subject to

$$0 \leq \Delta_i \leq 1, \quad i = 1, 2, ..., M$$

$$trace(\mathbf{\Delta}) = \sum_{i=1}^{M} \Delta_i = M'$$

Proof of Concavity

We define g(t) = $\log|\mathbf{X} + t\mathbf{V}|$ such that $\mathbf{X} \in S^N_{++}$ and $\mathbf{V} \in S^N$

$$
\begin{aligned}
g(t) &= \log|\mathbf{X} + t\mathbf{V}| \\
&= \log|\mathbf{X}^{\frac{1}{2}}\mathbf{X}^{\frac{1}{2}} + t\mathbf{X}^{\frac{1}{2}}\mathbf{X}^{\frac{-1}{2}}\mathbf{V}\mathbf{X}^{\frac{-1}{2}}\mathbf{X}^{\frac{1}{2}}| \\
&= \log|\mathbf{X}^{\frac{1}{2}}(\mathbf{I} + t\mathbf{X}^{\frac{-1}{2}}\mathbf{V}\mathbf{X}^{\frac{-1}{2}})\mathbf{X}^{\frac{1}{2}}| \\
&= \log|\mathbf{X}^{\frac{1}{2}}(\mathbf{I} + t\mathbf{U}\mathbf{\Lambda}\mathbf{U}^T)\mathbf{X}^{\frac{1}{2}}| \\
&= \log|\mathbf{X}^{\frac{1}{2}}\mathbf{U}(\mathbf{I} + t\mathbf{\Lambda})\mathbf{U}^T\mathbf{X}^{\frac{1}{2}}| \\
&= \log|\mathbf{X}^{\frac{1}{2}}||\mathbf{U}||\mathbf{I} + t\mathbf{\Lambda}||\mathbf{U}^T||\mathbf{X}^{\frac{1}{2}}| \\
&= \log|\mathbf{X}||\mathbf{I} + t\mathbf{\Lambda}| \\
&= \log|\mathbf{X}| + \log|\mathbf{I} + t\mathbf{\Lambda}|
\end{aligned}
$$

$$\begin{aligned} g(t) &= \log|\mathbf{X}| + \log|\mathbf{I} + t\mathbf{\Lambda}| \\ &= \log|\mathbf{X}| + \log(\prod_{i=1}^{N} 1 + t\lambda_i) \\ &= \log|\mathbf{X}| + \sum_{i=1}^{N} \log(1 + t\lambda_i) \end{aligned}$$

$$g''(t) = \sum_{i=1}^{N} \frac{-\lambda_i^2}{(1 + t\lambda_i)^2}$$

Since $\mathrm{g}''(t) < 0 \mathrm{g}(\mathrm{t})$ is concave

$\Rightarrow \log|\mathbf{X}|$ is concave

Result

Receive antenna selection has been approximated to a convex relaxation that can be solved using low complexity techniques. It is of order $O(M^{3.5})$

The selection algorithm gives an Ergodic capacity which is very close to the optimal one

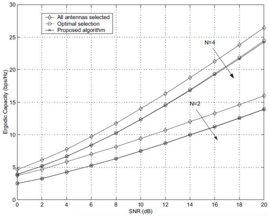

Figure: Ergodic capacity v/s SNR (N_γ), M = 6, N = 2,4 M' = N