

Receive Antenna Selection in MIMO Systems

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Introduction

Multiple-Input and Multiple-Output(MIMO) is a method for multiplying the capacity of a radio link using multiple transmission and receiving antennas to exploit multipath propagation

MIMO has become an essential element of wireless communication standards like WiFi,3G,4G and 4G-LTE.

A limiting factor in it's deployment is the cost of multiple analog chains at the receiver end.

By Antenna selection,a limited number of transmit/receive chains are used, thereby bringing the cost

System Model

Received signal can be represented as

$$\mathbf{x}(k) = \sqrt{E_s} \mathbf{H}_s(k) + \mathbf{n}(k)$$

where

$\mathbf{x}(k)$ is a vector of size $M \times 1$. It represents the k^{th} sample of the signals collected at the M receive antennas.

$\mathbf{s}(k)$ is a vector of size $N \times 1$. It represents the k^{th} sample of the signals transmitted at the N transmit antennas.

E_s is the average energy per receive antenna and per channel use

$\mathbf{n}(k)$ is AWGN with energy $\frac{N_0}{2}$

\mathbf{H} is the $M \times N$ channel matrix. The entries of \mathbf{H} are assumed to be zero-mean circularly symmetric complex Gaussian (ZMCSCG), such that the covariance matrix of any two columns of \mathbf{H} is a scaled identity matrix.

Receive Antenna Selection In MIMO Systems

Objective - Select receive antenna to maximize capacity.

The capacity of a system is given by the formula

$$C(\mathbf{H}) = \log_2 \det(\mathbf{I}_N + \gamma \mathbf{R}_{ss} \mathbf{H}^H \mathbf{H})$$

where

$$\gamma = \frac{E_s}{N_0}$$

$\mathbf{R}_{ss} = E[s(k)s(k)^H]$ is the co-variance matrix of the transmitted signals with $\text{trace}(\mathbf{R}_{ss})=1$

When only $M' < M$ receive antennas are used, the capacity becomes a function of the antennas chosen.

We represent the indices of the selected antennas by $r = [r_1, \dots, r_{M'}]$

The effective channel matrix becomes \mathbf{H}_r which is a $M' \times N$ matrix.

The channel capacity with antenna selection is given by

$$C_r(\mathbf{H}_r) = \log_2 \det(\mathbf{I}_N + \gamma \mathbf{R}_{ss} \mathbf{H}_r^H \mathbf{H}_r)$$

Antenna Selection as an Optimization Problem

‘ We define Δ_i

$$\Delta_i = \begin{cases} 1, & \text{if } i^{\text{th}} \text{ antenna is selected} \\ 0, & \text{otherwise.} \end{cases}$$

Now, consider an $M \times M$ diagonal matrix $\mathbf{\Delta}$ that has Δ_i as its diagonal entries.

Let us denote $\mathbf{F} = \mathbf{\Delta H}$.

\mathbf{F} can be written as \mathbf{H}_r with $(M-M')$ zero rows appended to it and left multiplied by a $M \times M$ row-permutation matrix \mathbf{P} . Thus,

$$\mathbf{F} = \mathbf{P} \begin{bmatrix} \mathbf{H}_r \\ \mathbf{0}_{(M-M') \times N} \end{bmatrix} = \mathbf{P} \widetilde{\mathbf{H}}_r$$

Since \mathbf{P} is a permutation matrix, $\mathbf{P}^H \mathbf{P} = \mathbf{I}_M$.

$$\begin{aligned}\mathbf{F}^H \mathbf{F} &= \widetilde{\mathbf{H}}_r^H \mathbf{P}^H \mathbf{P} \widetilde{\mathbf{H}}_r \\ &= \widetilde{\mathbf{H}}_r^H \widetilde{\mathbf{H}}_r \\ &= \mathbf{H}_r^H \mathbf{H}_r\end{aligned}$$

The channel capacity as a function of Δ is

$$\begin{aligned}C_r(\Delta) &= \log_2 \det(\mathbf{I}_N + \gamma \mathbf{F}^H \mathbf{F}) \\ &= \log_2 \det(\mathbf{I}_N + \gamma \mathbf{H}^H \Delta^H \Delta \mathbf{H})\end{aligned}$$

Since $\mathbf{\Delta}$ is a diagonal matrix with entries either 0 or 1

$$\mathbf{\Delta}^H \mathbf{\Delta} = \mathbf{\Delta}$$

The MIMO channel capacity with antenna selection can be re-written as

$$C_r(\mathbf{\Delta}) = \log_2 \det(\mathbf{I}_N + \gamma \mathbf{H}^H \mathbf{\Delta} \mathbf{H})$$

Using the identity $\det(\mathbf{I}_m + \mathbf{A}\mathbf{B}) = \det(\mathbf{I}_n + \mathbf{B}\mathbf{A})$ we get

$$C_r(\mathbf{\Delta}) = \log_2 \det(\mathbf{I}_M + \gamma \mathbf{\Delta} \mathbf{H} \mathbf{H}^H)$$

Maximize

$$C_r(\mathbf{\Delta}) = \log_2 \det(\mathbf{I}_M + \gamma \mathbf{\Delta} \mathbf{H} \mathbf{H}^H)$$

subject to

$$0 \leq \Delta_i \leq 1, \quad i = 1, 2, \dots, M$$

$$\text{trace}(\mathbf{\Delta}) = \sum_{i=1}^M \Delta_i = M'$$

Optimization in JMMSE receiver

The capacity achievable is $C_J(\mathbf{H})$ is

$$C_J(\mathbf{H}) = \sum_{k=1}^N \log_2(1 + \rho_k^2)$$

where

$$\rho_k^2 = \left[(\mathbf{I}_N + \gamma \mathbf{H}^H \mathbf{H})^{-1} \right]_{k,k}^{-1}$$

is the SINR of the k th output stream of the MMSE receiver.

Since

$$\mathbf{H}_r^H \mathbf{H}_r = \mathbf{H}^H \mathbf{\Delta} \mathbf{H}$$

The capacity of the JMMSE architecture with receive antenna selection is given by

$$C_{J_r}(\mathbf{H}) = \sum_{k=1}^N \log_2(1 + \tilde{\rho}_k^2)$$

where

$$\tilde{\rho}_k^2 = \left[(\mathbf{I}_N + \gamma \mathbf{H}^H \mathbf{\Delta} \mathbf{H})^{-1} \right]_{k,k}^{-1}$$

Maximize

$$C_{J_r}(\mathbf{H}) = \sum_{k=1}^N \log_2(1 + \tilde{\rho}_k^2)$$

where

$$\tilde{\rho}_k^2 = \left[(\mathbf{I}_N + \gamma \mathbf{H}^H \mathbf{\Delta} \mathbf{H})^{-1} \right]_{k,k}^{-1}$$

subject to

$$0 \leq \Delta_i \leq 1, \quad i = 1, 2, \dots, M$$

$$\text{trace}(\mathbf{\Delta}) = \sum_{i=1}^M \Delta_i = M'$$

Result

Receive antenna subset selection has been approximated by a constrained convex relaxation that can be solved using standard low complexity techniques

The selection algorithm gives an Ergodic capacity which is very close to the optimal one

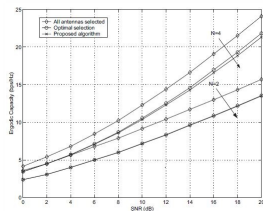


Figure: Ergodic capacity v/s SNR (N_γ) for JMMSE, $M = 6$, $N = 2, 4$, $M' = N$