Receive Antenna Selection in MIMO Systems

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Introduction

Multiple-Input and Multiple-Output(MIMO) is a method for multiplying the capacity of a radio link using multiple transmission and receiving antennas to exploit multipath propagation

MIMO has become an essential element of wireless communication standards like WiFi,3G,4G and 4G-LTE.

A limiting factor in it's deployment is the cost of multiple analog chains at the receiver end.

By Antenna selection,a limited number of transmit/receive chains are used, thereby bringing the cost

System Model

Received signal can be represented as

$$x(k) = \sqrt{E_s}\boldsymbol{H}s(k) + \boldsymbol{n}(k)$$

where
x(k) is a vector of size M×1. It represents the k^{th} sample of the signals collected at the M receive antennas.

s(k) is a vector of size N×1. It represents the k^{th} sample of the signals transmitted at the N transmit antennas.

E_s is the average energy per receive antenna and per channel use

n(k) is AWGN with energy $\frac{N_0}{2}$

H is the M×N channel matrix.The entries of **H** are assumed to be zero-mean circularly symmetric complex Gaussian (ZMCSCG), such that the covariance matrix of any two columns of **H** is a scaled identity matrix.

Receive Antenna Selection In MIMO Systems

'**Objective** - Select receive antenna to maximize capacity.
The capacity of a system is given by the formula

$$C(\boldsymbol{H}) = \log_2 \det(\boldsymbol{I}_N + \gamma \boldsymbol{R}_{ss} \boldsymbol{H}^H \boldsymbol{H})$$

where

$\gamma = \frac{E_s}{N_0}$

$\mathbf{R}_{ss} = \mathrm{E}[\mathrm{s(k)s(k)}^H]$ is the co-variance matrix of the transmitted signals with trace($\mathbf{R}_{ss}$)=1

When only M'<M receive antennas are used, the capacity becomes a function of the antennas chosen.

We represent the indices of the selected antennas by r = $[r_1, \ldots, r_{M'}]$

The effective channel matrix becomes $\mathbf{H}_r$ which is a M'×N matrix.

The channel capacity with antenna selection is given by

$$C_r(\boldsymbol{H}_r) = \log_2 \det(\boldsymbol{I}_N + \gamma \boldsymbol{R}_{ss} \boldsymbol{H}_r^H \boldsymbol{H}_r)$$

Antenna Selection as an Optimization Problem

‘ We define Δ_i

$$\Delta_i = \begin{cases} 1, & \text{if} \quad i^{th} \text{ antenna is selected} \\ 0, & \text{otherwise.} \end{cases}$$

Now, consider an M$\times$M diagonal matrix $\mathbf{\Delta}$ that has Δ_i as its diagonal entries.

Let us denote F $= \mathbf{\Delta}$H.

F can be written as $\mathbf{H}_r$ with (M-M’) zero rows appended to it and left multiplied by a M M row-permutation matrix P. Thus,

$$\boldsymbol{F} = \boldsymbol{P}\begin{bmatrix} \boldsymbol{H}_r \\ 0_{(M-M')\times N} \end{bmatrix} = \boldsymbol{P}\widetilde{\boldsymbol{H}_r}$$

Since P is a permutation matrix, $\mathbf{P}^H\mathbf{P} = \mathbf{I}_M$.

$$\begin{aligned}\mathbf{F}^H\mathbf{F} &= \widetilde{\boldsymbol{H}_r}^H\mathbf{P}^H\mathbf{P}\widetilde{\boldsymbol{H}_r} \\ &= \widetilde{\boldsymbol{H}_r}^H\widetilde{\boldsymbol{H}_r} \\ &= \mathbf{H}_r^H\mathbf{H}_r\end{aligned}$$

The channel capacity as a function of $\boldsymbol{\Delta}$ is

$$\begin{aligned}C_r(\boldsymbol{\Delta}) &= \log_2 \det(\mathbf{I}_N + \gamma\mathbf{F}^H\mathbf{F}) \\ &= \log_2 \det(\mathbf{I}_N + \gamma\mathbf{H}^H\boldsymbol{\Delta}^H\boldsymbol{\Delta}\mathbf{H})\end{aligned}$$

Since $\boldsymbol{\Delta}$ is a diagonal matrix with entries either 0 or 1

$$\boldsymbol{\Delta}^H\boldsymbol{\Delta} = \boldsymbol{\Delta}$$

The MIMO channel capacity with antenna selection can be re-written as

$$C_r(\boldsymbol{\Delta}) = \log_2 \det(\mathbf{I}_N + \gamma\mathbf{H}^H\boldsymbol{\Delta}\mathbf{H})$$

Using the identity $\det(\mathbf{I}_m + \mathbf{AB}) = \det(\mathbf{I}_n + \mathbf{BA})$ we get

$$C_r(\boldsymbol{\Delta}) = \log_2 \det(\mathbf{I}_M + \gamma\boldsymbol{\Delta}\mathbf{H}\mathbf{H}^H)$$

Maximize

$$C_r(\boldsymbol{\Delta}) = \log_2 \det(\mathbf{I}_M + \gamma \boldsymbol{\Delta} \mathbf{H} \mathbf{H}^H)$$

subject to

$$0 \leq \Delta_i \leq 1, \quad i = 1, 2, ..., M$$

$$trace(\boldsymbol{\Delta}) = \sum_{i=1}^{M} \Delta_i = M'$$

Optimization in JMMSE receiver

The capacity acheivable is $C_J(\mathbf{H})$ is

$$C_J(\mathbf{H}) = \sum_{k=1}^{N} \log_2(1 + \rho_k^2)$$

where

$$\rho_k^2 = \left[\left(\mathbf{I}_N + \gamma \mathbf{H}^H \mathbf{H}\right)^{-1}\right]_{k,k}^{-1}$$

is the SINR of the kth output stream of the MMSE receiver.

Since

$$\mathbf{H}_r^H \mathbf{H}_r = \mathbf{H}^H \mathbf{\Delta} \mathbf{H}$$

The capacity of the JMMSE architecture with receive antenna selection is given by

$$C_{J_r}(\mathbf{H}) = \sum_{k=1}^{N} \log_2(1 + \tilde{\rho_k}^2)$$

where

$$\tilde{\rho_k}^2 = \left[\left(\mathbf{I}_N + \gamma \mathbf{H}^H \mathbf{\Delta} \mathbf{H} \right)^{-1} \right]_{k,k}^{-1}$$

Maximize

$$C_{J_r}(\mathbf{H}) = \sum_{k=1}^{N} \log_2(1 + \tilde{\rho_k}^2)$$

where

$$\tilde{\rho_k}^2 = \left[\left(\mathbf{I}_N + \gamma \mathbf{H}^H \boldsymbol{\Delta} \mathbf{H}\right)^{-1}\right]^{-1}_{k,k}$$

subject to

$$0 \leq \Delta_i \leq 1, \;\; i = 1, 2, ..., M$$

$$trace(\boldsymbol{\Delta}) = \sum_{i=1}^{M} \Delta_i = M'$$

Result

Receive antenna subset selection has been approximated by a constrained convex relaxation that can be solved using standard low complexity techniques

The selection algorithm gives an Ergodic capacity which is very close to the optimal one

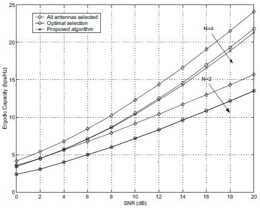

Figure: Ergodic capacity v/s SNR (N_γ) for JMMSE, M = 6, N = 2,4 M' = N