Image Processing

Image Restoration

Team 3

I. Introduction

Considering the importance of image restoration, we combine references and the content of our courses to implement the balanced regularization approach which bridges the synthesis-based and analysis-based approaches.

In this work, we focus on (1) Implement the algorithm of restoration with adjustable variables (Lambda(λ), gamma(γ)) which allow us to differentiate 3 ADMM(Alternating Direction Method of Multipliers) methods including ADMM-B, ADMM-A and ADMM-S (2) Produce various deteriorated and blurred images (such as adding noise in blurring image, random/hand-write deteriorated mask) to test our processing flow and evaluate the results (3)Use different blur kernel (Gaussian kernel with deviation 3 and 9*9 uniform) and adjust variables to analyze the outcomes and deduce the conclusion.

II. Method

The implement and evaluation of image restoration include three items, Simulate Blurring & Defect, Wavelet Frame Representation and ADMM Algorithms. For the part of ADMM Algorithms, we would further discuss for the reason that different values of variables in the algorithm could represent different approaches for solving a balanced regularization problem in the frame-based image restoration. The whole processing flow is like Fig.1

Fig.1 *Signal Processing Items*

A. *Simulate Blurring and Defect:*

- Goal: Produce various deteriorated and blurred images as inputs of our restoration processing. We implement (1.)Noise adding (2.)Gaussian/ Uniform blurring (3.) Random / hand-write mask inpainting.
- Base on y=Bu+n,

  Where B:blur kernel, u:fine image y:blur image, n:noise
- Noise adding: Adding random noise to the images is a choosable function in our project. With this function, we could compare the deblurring and inpainting results from different inputs with and without noise.
- Gaussian/ Uniform Blurring: We blur the original images with two different blur kernels(B).

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Gaussian Blurring:B = fspecial('gaussian', dim,sigma)
Uniform Blurring: B= ones(dim)/dim/dim
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B. *Wavelet Frame Representation*

- A signal u is said to have a sparse representation, if there exists a sparse vector $x \in R^d$ such that u = Wx.
- "u" can be represented by a linear combination of wavelet frame(W), with coefficient "x". Since the tight wavelet system is redundant, represent "u" by "x" is not unique. So use following constraint (***Eq.1***).

C. *Model the problem*

- Use different constraints to solve our problem, which is modeled as:

$$\min_x \frac{1}{2}||BWx - y||_2^2 + \frac{\gamma}{2}||(I - W^T W)x||_2^2 + \lambda^T |x|_1$$

(*Eq.1*)

The first term in ***Eq.1***: penalize on data fidelity
The second term in ***Eq.1***: penalize the distance between the frame coefficients x and the range of W^T
The last term in ***Eq.1***: penalize on the sparsity of coefficient vector

D. *Use ADMM (Alternating Direction Method of Multipliers)*

to solve the optimization problem:

- Consider a problem can be written as ***Eq.2***

$$\min_{u \in R^n} f_1(u) + f_2(Gu) \qquad \textbf{(Eq.2)}$$

Define a new variable v=Gu, that leads to a constrained problem:

$$\min_{u \in R^n} f_1(u) + f_2(v), \qquad \text{subject to } Gu = v \qquad \textbf{(Eq.3)}$$

We can introduce a Lagrange multiplier d_k to optimize it in several iterations:

$$(u_{k+1}, v_{k+1}) \in \arg\min_{u,v} f_1(u) + f_2(v) + \frac{\mu}{2}\|Gu - v - d_k\|_2^2$$
$$d_{k+1} = d_k - (Gu_{k+1} - v_{k+1}). \qquad \textbf{(Eq.4)}$$

And achieve the point u which has minimum E(u)
- For our problem, we can define:

$$f_1(x) = \frac{1}{2}\|BWx - y\|_2^2 + \frac{\gamma}{2}\|(I - W^T W)x\|_2^2 \qquad \textbf{(Eq.5)}$$

and

$$f_2(v) = \lambda^T |v|_1, \qquad G = I. \qquad \textbf{(Eq.6)}$$

And use ADMM algorithm to solve it.
- Synthesis-Based Approach:

We can obtain the ADMM-based algorithm for ADMM-A by $\gamma = 0$. Only the sparsity of the frame coefficients is penalized and the estimated image is synthesized by the sparsest coefficients.

- Analysis-based approach

  We can obtain the ADMM-based algorithm for ADMM-S by letting $\gamma = \infty$, force the term $\|I - W^T W\|^2$ must be 0 if the problem has a finite solution. This implies that x is in the range of W^T.

- ADMM for Balanced (analysis + synthesis approach) Regularization Approach (ADMM-B) :

  (1)Set $k = 0$, choose $\mu > 0$, v_0 and d_0

  (2) Repeat

  (3) $r_k = W^T B^T y + \mu(v_k + d_k)$

  (4) $x_k+1 = (1/\mu)(\alpha r_k + (1 - \alpha) W^T) W r_k - W^T \mathit{F} W r_k)$

  (5) $v_k +1 = \mathrm{soft}(v_k , \lambda/\mu)$

  (6) $d_k +1 = d_k - (x_k +1 - v_k +1)$

  (7) $k \leftarrow k + 1$

  (8) Until stopping criterion is satisfied.

  Where:

$$\alpha = \frac{\mu}{\mu + \gamma}$$
$$\mathcal{F} = B^T(\mu I + BB^T)^{-1}B. \qquad \textbf{(Eq.7)}$$

  a.) Use same steps but let $\alpha = 0$ =>ADMM-A

  b.) Use same steps but let $\alpha = 1$=>ADMM-S

- Accelerate blur operation in frequency domain:

  The blur operation:

$$B = U^T DU \qquad \textbf{(Eq.8)}$$

U: FFT and U^T: IFFT operator

D: a matrix with FFT coefficient of blur kernel on diagonal

We can have new F formulation:

$$\mathcal{F} = B^T(\mu I + BB^T)^{-1}B \rightarrow O(n^2)$$

$$= U^T D^*(|D|^2 + \mu I)^{-1} DU \rightarrow O(n\log n) + O(n) = O(n\log n)$$

As inpainting problem, in ADMM-B step(4), the matrix B represents the loss of some image pixels which satisfies $BB^T = I$, then we can obtain:

$$\mathcal{F} = \tfrac{1}{\mu+1} B^T B. \qquad \textbf{(Eq.9)}$$

- Stop Criterion:

$$|\mathrm{obj}(k+1) - \mathrm{obj}(k)|/\mathrm{obj}(k) \le \mathrm{Tol} \qquad \textbf{(Eq.10)}$$

III. EXPERIMENTS

Compare the inputs with different blurring and defects and handle the problem by three methods, finally evaluate the results by PSNR/MSE.

PSNR:Peak signal-to-noise ratio, is a term for the ratio between the maximum possible power of a signal and the power of corrupting noise that affects the fidelity of its representation.

$$PSNR = 10 \times \log_{10}\left(\frac{(2^n - 1)^2}{MSE}\right) \qquad \textbf{(Eq.11)}$$

MSE: Mean squared error

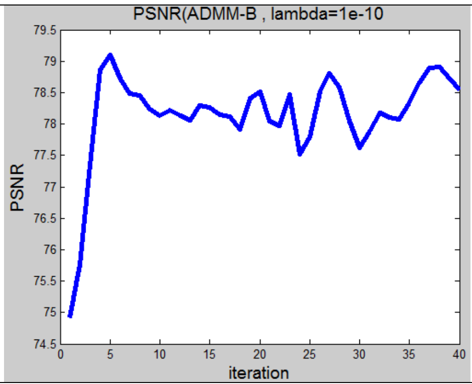
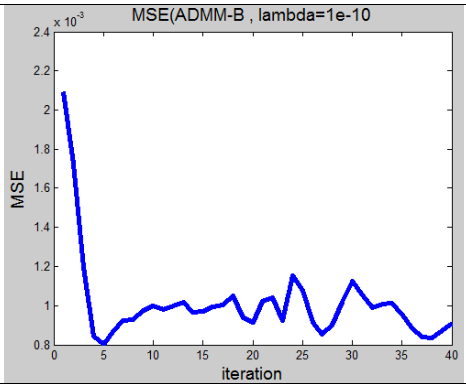
$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}[I(i,j) - K(i,j)]^2 \qquad \textbf{(Eq.12)}$$

A. Blurred Image without Noise (Gaussian blur kernel) .

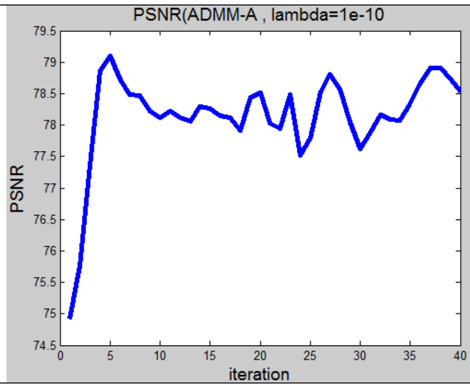
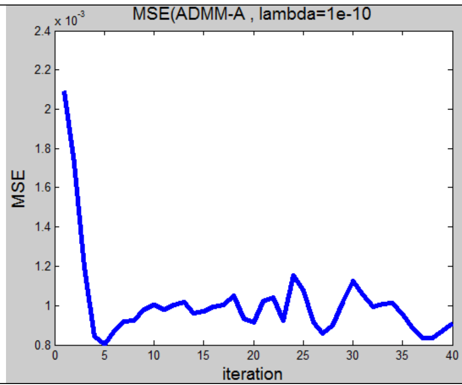
Original Image

Blurred Image

1) ADMM-B:

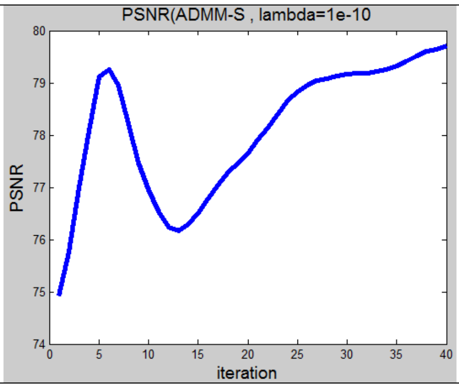
PSNR

MSE

Deblurred Image


2) ADMM-A:

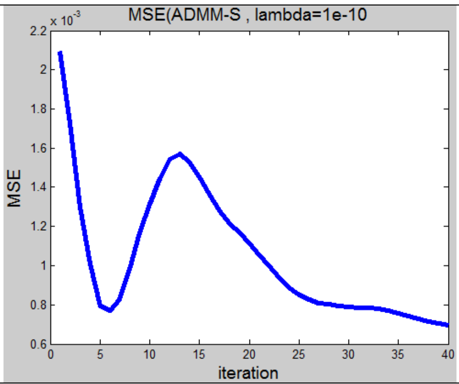
PSNR

MSE

Deblurred Image


3) ADMM-S:

PSNR



MSE



Deblurred Image

4) Comparison for Blurred Image without Noise Deblurred by 3 Methods:

As the rise of iteration number, PSNR will increase and MSE will decrease.

At the beginning, the improvements of PSNR and MSE are obvious, while ADMM-B and A will gradually converge to certain value when the iteration number over five and the convergence of ADMM-S is slower. Without the noise, deblurred images are close to the original one and the PSNR can reach 80dB (ADMM-S with 9*9 uniform blur kernel) and small MSE.

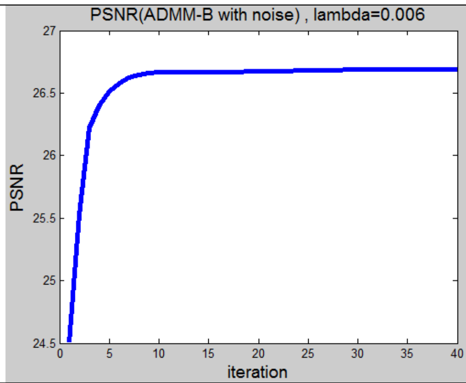
B. Blurred Image with Random Noise (Gaussian blur kernel)

Here, we change Lambda (λ) in ***Eq.1*** and analyze the results which is deblurred by ADMM-B method .

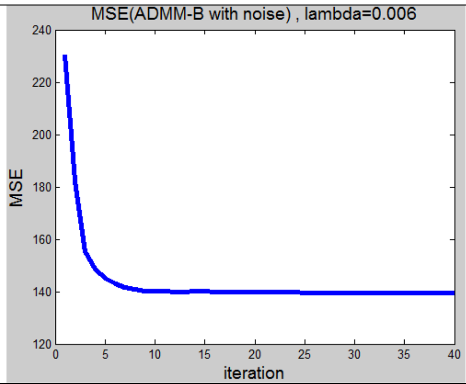
Blurred Image with Random Noise

1) **Lambda (λ)=0.06:**

PSNR



MSE

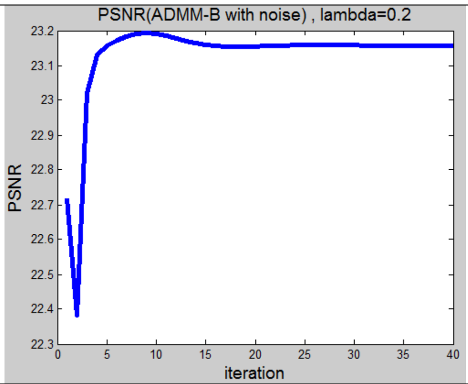


Deblurred Image

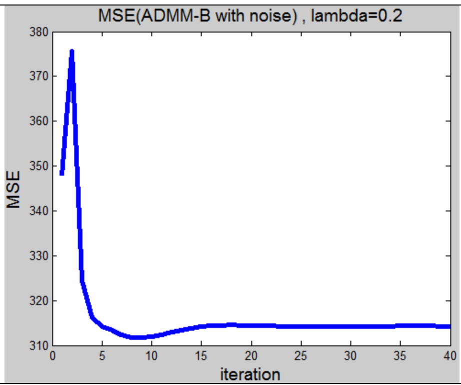
PSNR

2) Lambda (λ)=0.2:

PSNR



MSE



Deblurred Image

3) Comparison for Blurred Image with Random Noise Deblurred by ADMM-B method with different Lambda (λ):

The deblurred results are close related to the value of lambda. With small lambda, results are sharper while still with some noise. With large lambda, results are smoother and most of the noise is removed. PSNR can reach 27dB (the highest: ADMM-A with 9*9 uniform blur kernel) and MSE are about 111.

C. Inpainting Image

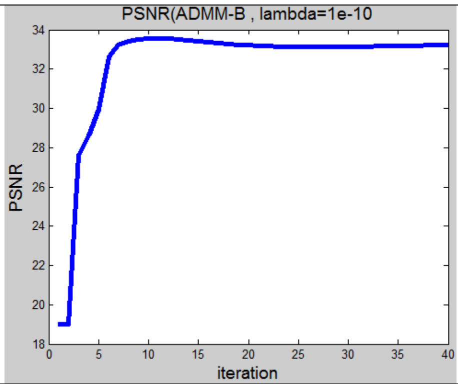
Original Image

Deteriorated Image (Random Mask)

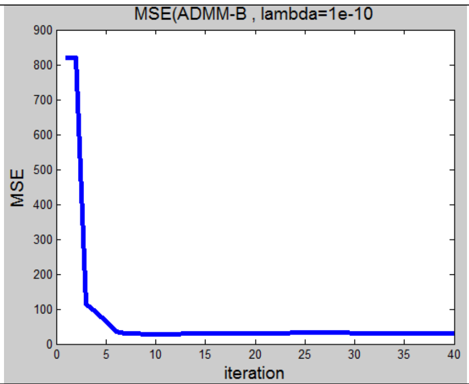
Deteriorated Image (Hand-Write Mask)

1) ADMM-B:

PSNR



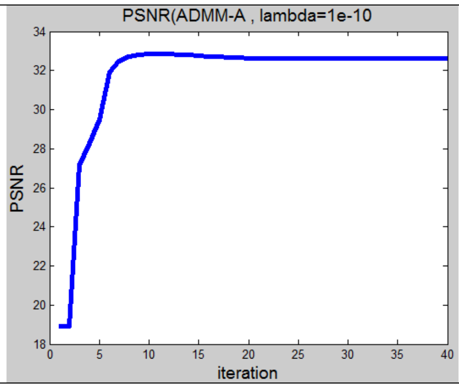
MSE



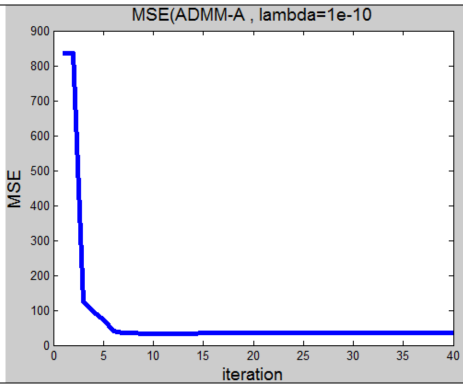
Deblurred Image

2) ADMM-A:

PSNR



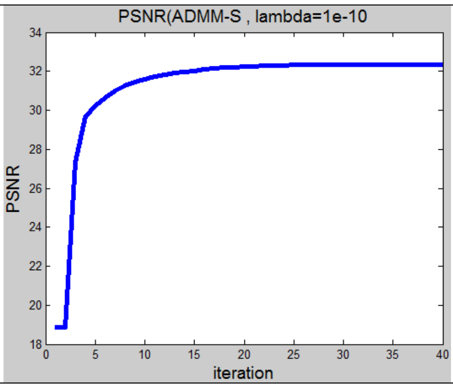
MSE



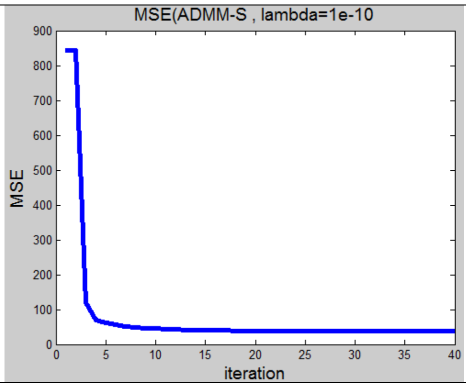
Deblurred Image

3) *ADMM-S:*

PSNR



MSE



Deblurred Image

4) *Comparison for Image Inpainted by 3 Methods:*

The PSNR and MSE of three method are fast-converging.For the most parts of the images, restorations are effective, while the edges are still with some defects. . PSNR can reach around 33.5dB (the highest: ADMM-B with 9*9 uniform blur kernel) and MSE are about 28.

IV. Discussion and Conclusion

A. *Comparison of the deblurred images(without noise)*

TABLE I

Blur kernel type	**9*9 uniform**		**Gaussian kernel with deviation 3**	
Algorithm	PSNR(dB)	MSE	PSNR(dB)	MSE
ADMM-B	70.8940	0.0053	79.0946	8.0098e-04
ADMM-A	70.8935	0.0053	79.0928	8.0130e-04
ADMM-S (slow)	71.0809	0.0051	79.7103	6.9511e-04

B. *Comparison of the deblurred images(with random noise)*

TABLE II

Blur kernel type	**9*9 uniform**		**Gaussian kernel with deviation 3**	
Algorithm	PSNR(dB)	MSE	PSNR(dB)	MSE
ADMM-B	27.6744	111.0811	26.6907	139.3196
ADMM-A	27.6746	111.0767	26.6910	139.3092
ADMM-S	24.9717	206.9733	26.2646	219.2641

C. *Comparison of the Inpainted images*

TABLE III

Algorithm	**PSNR(dB)**	**MSE**
ADMM-B	33.5660	28.6076
ADMM-A	32.8609	33.6506
ADMM-S	32.3601	37.7631

D. *Summary*

1) Deblurring without Noise:
Choose smaller lambda for sharper image.
ADMM-S> ADMM-B> ADMM-A;
However, ADMM-S is slow than ADMM-B.
2) Deblurring with Noise: choose proper lambda for removing noise and keep the image sharp.
ADMM-A> ADMM-B> ADMM-S;
The performance of ADMM-B is closer to ADMM-A.
3) Inpainting :
ADMM-B> ADMM-A> ADMM-S;
The performance of ADMM-B is best.

V. References

[1]Shoulie Xie, Senior Member, IEEE, and Susanto Rahardja, Fellow, IEEE "*Alternating Direction Method for Balanced Image Restoration*", IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 21, NO. 11, NOVEMBER 2012

[2] Liangtian He, Member, IEEE, and Yilun Wang, Member, IEEE, *"Iterative Support Detection-Based Split Bregman Method for Wavelet Frame-Based Image Inpainting"*,IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 23, NO. 12, DECEMBER 2014