



**DEPARTMENT OF ELECTRICAL
AND ELECTRONICS
ENGINEERING**

EE492 PROJECT REPORT

**LIMITED FEEDBACK CHANNEL
ESTIMATION IN MASSIVE MIMO
SYSTEMS**

OZGE OZALTIN 210206019

Assoc. Dr. BERNA OZBEK

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ABSTRACT

This paper proposes an adaptive sparse channel estimation with limited feedback scheme to estimate accurate and timely downlink channel state information (CSI) for massive multi input-output (MIMO) systems which is based on frequency division duplex (FDD) systems. The limited feedback scheme need to feedback burden which is linearly proportional the base station antennas. This work proposes limited feedback algorithm which lift the burden by double directional (DD) MIMO channel representation using uniform dictionaries which are associated with angle of arrival (AoA) and angle of departure (AoD). The proposed algorithms provide acceptable channel estimation accuracy using a small number of feedback bits, even though the number of transmission antennas in the BS is large, making them ideal for 5G massive MIMO. Simulation results show that the proposed algorithm can approach the performance limit.

TABLE OF CONTENTS

| | |
|--|-----------|
| ABBREVIATIONS | 3 |
| LIST OF FIGURES | 4 |
| LIST OF TABLES | 5 |
| 1. INTRODUCTION | 6 |
| 2. PROBLEM DEFINITION | 7 |
| 2.1. Physical Channel Model | 8 |
| 2.2 Angular Dictionaries | 9 |
| 2.3 Channel Estimation | 9 |
| 2.4 Estimated Channel Reconstruction at BS | 10 |
| 2.5 Feedback Channel Recovery Performance Evaluation | 10 |
| 3. PROPOSED SOLUTION | 11 |
| 3.1. CSI with OMP | 11 |
| 3.2. Lloyd Scalar Quantization for limitation | 13 |
| 4. RESULTS AND DISCUSSIONS | 14 |
| 5. CONCLUSIONS | 16 |
| REFERENCES | 17 |

ABBREVIATIONS

MIMO: Multiple Input Multiple Output

AoA: Angle of Arrival

AoD: Angle of Departure

SNR: Signal to Noise Ratio

FDD: Frequency Division Duplexing

UE: User Equipment

BS: Base Station

DD: Directional Dictionaries

OMP: Orthogonal Matching Pursuit

NMSE: Normalized Mean Square Error

LIST OF FIGURES

| | |
|--|----|
| Figure 1 The System Model | 7 |
| Figure 2 NMSE vs Number of Transmitter Antennas (M_T) | 15 |
| Figure 3 SNR vs Capacity | 15 |

LIST OF TABLES

Table 1 Simulation Parameters.....14

1. INTRODUCTION

Massive Multiple-Input Multiple-Output (MIMO) systems are large scale antenna systems, are being considered for next generation communication systems. Nowadays, the interest in MIMO systems is increasing because, for these systems, the base stations (BS) contain excessive number of antennas which is more than the user equipment (UE) antennas. The reason for more interest of MIMO researches is that system provides leaps in spectral efficiency, decreasing intercell interference [1]. Even though MIMO systems use large number of BS antennas, they are energy efficient as these antennas operate with low energy [2].

To benefit from large scale BS antennas, the essential thing is knowledge of channel state information (CSI) because of high data rate [3]. The adversity for MIMO systems is to obtain accurate and timely downlink CSI at the BS with sending only few feedback bits from the UE. Thus, we must solve this adversity with limitation of feedback bits because without limitation applied approaches are wasteful and impractical and this leads to deterioration of the overall system efficiency. Consequently, our main goal is estimating the channel with low feedback rate.

Generally, the feedback-based channel estimation literature investigates various quantization techniques; as a [9] for a comprehensive explanation. Furthermore, feedback based channel estimation literature looks for distinctive sorts of vector quantization (VQ) techniques and the codebook of VQ is known both the BS and the UE. A large portion of limited feedback approximates in MIMO systems allow a Rayleigh fading channel model [10], [11], [12], [13]. Under this channel model, the VQ feedback bits need to guarantee acceptable performance is linear in the transmit antenna numbers at the BS [5] because these types of antennas are costly in a massive MIMO systems. However, the designer is not limited to using VQ-based approaches and maybe MIMO channels may be far from Rayleigh. Thus, we adopted Double Dictionaries model.

1.1 Adopted Channel Model for Massive MIMO Systems

Today, plenty of cellular systems use frequency-division duplexing (FDD) systems which are more effective for symmetric traffic and delay-sensitive applications [4]. Thus, we are using FDD in Massive MIMO. However, there are some difficulties in FDD systems as downlink training for CSI while training and feedback overhead is proportional to the antenna numbers at BS. We can implement DD model to overcome the problem of limited feedback scheme of downlink channel. Under this model we will utilise the virtual sparse representation of downlink channel. The DD model parameterizes each channel path using the angle of departure (AoD) at the BS and the angle of arrival (AoA) at the UE [1]. While quantizing the AoD and AoA, we can create over complete dictionaries that include steering vectors related to the actual arrival angle and departure angle. After that, using these parametrization, we will estimate the channel state information with the greedy orthogonal matching pursuit (OMP) algorithm at UE.

After the instant downlink CSI has been estimated at the UE, the UE sends the index of the best matched code block for minimizing the probability of error and maximizing the link capacity over a limited feedback channel [1]. The codebooks for spatially correlated channels are related to the Lloyd algorithm which will be helpful to find the non-zero elements of sparse vector with known thresholds at the BS.

2. PROBLEM DEFINITION

In wireless communications systems, channel state information (CSI) define the known properties of channel. CSI includes signal propagation from the transmitter to the receiver and a signifies the combined scattering and fading affect with distance. This method is called Channel estimation. CSI makes it possible to adapt transmissions to existing channel conditions. Thus, CSI is very important for reliable communication with high data rates in MIMO systems.

Our major challenge is to get timely and accurate downlink CSI at BS with limited feedback bits from UE and we need to estimate limited feedback channel at this point. To create limited feedback channel, we need to create DD Dictionaries and we need to apply compress sensing. However, while we are applying compress sensing, we can obtain optimization problem so, we need to maximum correlation to the active paths with minimum noise with using the Orthogonal Matching Pursuit (OMP) and we can compress the sparse channel using the Lloyd quantization algorithm.

The limited feedback channel estimation while overcoming the feedback burden problems was explained step by step with section 2.1 ,2.2 ,2.3 and 2.4 and our block diagram of the channel estimation is given figure 1.

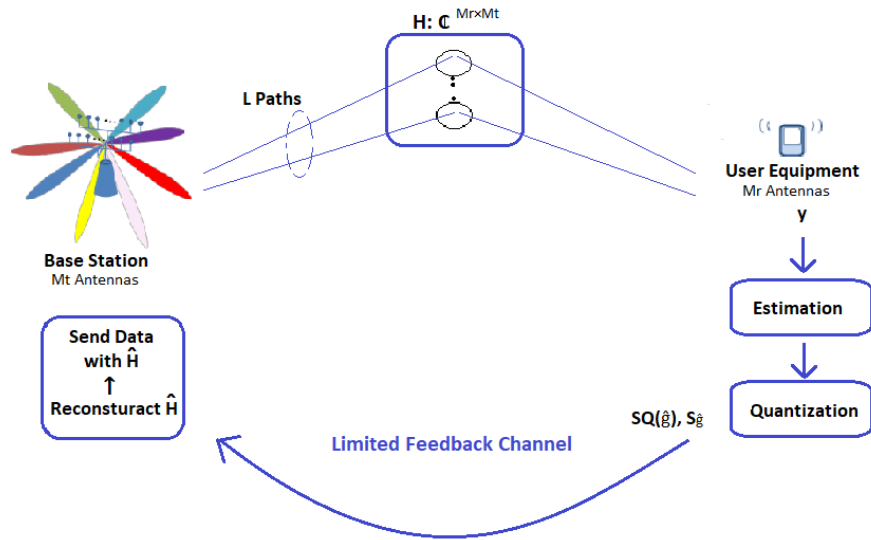


Figure 1: The block diagram of the Limited Feedback Channel Estimation

For the estimation of the channel, we firstly focused on the physical channel model which was constructed perfectly.

2.1 PHYSICAL CHANNEL MODEL

We were focus on FDD cellular systems. It is constituted with BS antennas which is serving K active UE terminals. The downlink channel is estimated at BS via the feedback channel from UE. The BS includes M_T antennas and UE includes M_R antennas. For the downlink transmission, BS obtains CSI through feedback from active UE terminals. Then, the transmitted signal is designed with N_{tr} training symbols. Below equation gives received vector

$$\mathbf{Y}_n = \mathbf{H} \mathbf{s}_n + \mathbf{n}_n \quad n = 1, 2, \dots, N_{tr}, \quad (1)$$

where \mathbf{s}_n is transmitted training signal and \mathbf{H} is complex baseband channel [1].

The main target is to estimate accurate channel with a few feedback bits. To carry out this idea, we prefer to apply DD model containing L paths. The parametrization of L paths can implement with virtual sparse representation. However, this sparse representation could cause a feedback scheme that is highly complex both in terms of overall and computational complexity. The downlink channel can be expressed as a below equation:

$$\mathbf{H} = \sqrt{\frac{M_T M_R}{L}} \sum_{l=1}^L \alpha_l \mathbf{a}_R(\phi_l) \mathbf{a}_T^H(\phi_l') e^{j\varphi_l} \quad (2)$$

The downlink channel parameters for channel are α_l which represents small scale fading effects with Rician parameters, ϕ_l and ϕ_l' are azimuth angle of arrival (AoA) and azimuth angle of departure (AoD), the steering vectors are transmit and receive signals are $\mathbf{a}_T(\cdot)$ and $\mathbf{a}_R(\cdot)$, φ is the delay of l th path M_T .

The BS steering vector is given by with carrier wavelength λ and distance between the antenna elements d_y through the y axis.

$$\mathbf{a}_T(\phi) = \sqrt{\frac{1}{M_T}} \left[1 e^{\frac{-j2\pi d_y}{\lambda} \sin(\phi)} \dots e^{\frac{-j2\pi d_y (M_T-1)}{\lambda} \sin(\phi)} \right] \quad (3)$$

This channel parameters can represent more compact form as a below.

$$\mathbf{A}_R = [\mathbf{a}_R(\phi_1) \dots \dots \mathbf{a}_R(\phi_L)] : \text{receive steering matrix form with its vectors} \quad (4)$$

$$\mathbf{A}_T = [\mathbf{a}_T(\phi_1') \dots \dots \mathbf{a}_T(\phi_L')] : \text{transmit steering matrix form with its vectors} \quad (5)$$

Also, we can simply arrange the path loss and phase shift component combination as;

$$\boldsymbol{\alpha} = \sqrt{\frac{M_T M_R}{L}} [\alpha_1 e^{-j\varphi_1} \dots \alpha_L e^{-j\varphi_L}]^T \quad (6)$$

Finally, we can get the more compact form rather than channel in equation 2 via the above simplest forms.

$$\mathbf{H} = \mathbf{A}_R \text{diag}(\boldsymbol{\alpha}) \mathbf{A}_T^H \quad (7)$$

2.2 ANGULAR DICTIONARIES

To estimate the channel, the double directional dictionaries was constructed. From the equation 7, we could obtain sparse channel representation. Firstly, we quantized AoA and AoD with angular space discretization as dictionaries. These dictionaries are defined over uniformly in an angular sector $[a, b) \subseteq [-\pi, \pi)$. G_T and G_R are the members of dictionaries.

$$\mathbf{P}_T = \left\{ a + \frac{j(b-a)}{G_R+1} \right\}_{j=1}^{G_T} \quad \mathbf{P}_R = \left\{ a + \frac{j(b-a)}{G_R+1} \right\}_{j=1}^{G_R} \quad (8)$$

The dictionary matrices which are given below are quantized approximation of matrix of \mathbf{A}_R and \mathbf{A}_T .

$$\tilde{\mathbf{A}}_R = \{a_R(\phi) : \phi \in \mathbf{P}_R\} \in \mathcal{C}^{M_R \times G_R} \quad (9)$$

$$\tilde{\mathbf{A}}_T = \{a_T(\phi) : \phi \in \mathbf{P}_T\} \in \mathcal{C}^{M_T \times G_T} \quad (10)$$

2.3 CHANNEL ESTIMATION

The training sequence is transmitted via the non-perfect channel and the received signal at UE includes error channel parameters. However, at the UE we have the measurement matrix which includes the information of the channel steering vectors and true training sequence.

The Error channel parameters was given below:

$$\phi_{er} = \phi + \beta \quad (11)$$

$$\tilde{\mathbf{A}}_{Rer} = \{a_R(\phi_{er}) : \phi_{er} \in \mathbf{P}_{Rer}\} \in \mathcal{C}^{M_R \times G_R} \quad (12)$$

$$\tilde{\mathbf{A}}_{Ter} = \{a_T(\phi_{er}) : \phi_{er} \in \mathbf{P}_{Ter}\} \in \mathcal{C}^{M_T \times G_T} \quad (13)$$

$$\mathbf{H}_{er} \approx \tilde{\mathbf{A}}_{Rer} \mathbf{G} \tilde{\mathbf{A}}_{Ter}^H \quad (14)$$

where β is the deflection angle, $\tilde{\mathbf{A}}_{Rer}$ and $\tilde{\mathbf{A}}_{Ter}$ are steering vectors the error channel.

Thus, we can apply OMP algorithm to estimate the channel state information which vectors of the channel can be active and we can send this information from UE to BS with limitation of feedback bits with respect to the active paths using the compression with eliminating the inactive paths. After that, we can send the CSI from UE through the BS, and we can reconstruct the estimated limited feedback channel at BS to adaptive communication.

2.4 ESTIMATED CHANNEL RECONSTRUCTION at BS

We could arrange the channel form at the equation (7) via the estimation of interaction matrix up to some quantization errors.

$$\mathbf{H} \approx \tilde{\mathbf{A}}_R \mathbf{G} \tilde{\mathbf{A}}_T^H \quad (15)$$

The matrix $\mathbf{G} \in \mathbb{C}^{G_R \times G_T}$ and its interaction matrix which the matrix elements related with a quantized matrix of \mathbf{A}_R and \mathbf{A}_T . Therefore, if the interaction matrix is not equal zero, we can say the k th angle \mathbf{P}_T and j th angle \mathbf{P}_R is active. When the active paths are less, \mathbf{G} matrix is mostly sparse.

After that, the baseband signal can be ordered according to channel estimation.

$$\mathbf{Y} = \tilde{\mathbf{A}}_R \mathbf{G} \tilde{\mathbf{A}}_T^H \mathbf{S} + \mathbf{N} \quad (16)$$

We need to apply vectorization property because the analysis of the vectors is easier than the matrices. When the vectorization property applied as $\text{vec}(ABC) = (\mathbf{C}^T \otimes \mathbf{A})\text{vec}(B)$ in Eq. (12), the received baseband signal can be written as;

$$\mathbf{y} = \left((\mathbf{S}^T \tilde{\mathbf{A}}_T^*) \otimes \tilde{\mathbf{A}}_R \right) \mathbf{g} + \mathbf{n} = \mathbf{Q} \mathbf{g} + \mathbf{n} \quad (17)$$

where $\mathbf{y} \cong \text{vec}(\mathbf{Y}) \in \mathbb{C}^{M_R N_{tr}}$, $\mathbf{g} \cong \text{vec}(\mathbf{G}) \in \mathbb{C}^{G_T \times G_R}$, $\mathbf{n} \cong \text{vec}(\mathbf{N}) \in \mathbb{C}^{G_R M_R}$, $\mathbf{Q} \cong (\mathbf{S}^T \tilde{\mathbf{A}}_T^*) \otimes \tilde{\mathbf{A}}_R \in \mathbb{C}^{M_R N_{tr} \times G_T G_R}$.

2.5 FEEDBACK CHANNEL RECOVERY PERFORMANCE EVALUATION

2.5.1 Normalized Mean Square Error(NMSE) Calculation of Reconstructed and Perfect Channel for Performance Analysis

In this work, to study the recovery performance of the CS based OMP-SQ technique, average Normalized Mean Squared Error (NMSE) and sum capacity are investigated under different quantization bits.

In literature, there are several calculation techniques for NMSE. One of them calculates NMSE between perfect channel and the reconstructed channel. And, the other one calculates NMSE regarding reconstructed channel. $\|\cdot\|$ represents the L2-norm.

NMSE between the estimated channel and perfect channel was found below formula [9]:

$$NMSE = \frac{\|(\tilde{\mathbf{H}} - \mathbf{H})\|_2}{(\tilde{\mathbf{H}}_m \times \mathbf{H}_m)} \quad (18)$$

$$\|(\tilde{\mathbf{H}} - \mathbf{H})\|_2 = \frac{1}{N} \sum_{i=1}^N (\tilde{\mathbf{H}} - \mathbf{H})_2 \quad (19)$$

where $\tilde{\mathbf{H}}$ is reconstructed channel and \mathbf{H} is perfect channel, $\tilde{\mathbf{H}}_m$ represented average reconstructed channel and \mathbf{H}_m represented average of perfect channel.

2.5.2 Capacity Calculation of Reconstructed and Perfect Channel for Performance Analysis

Shannon Capacity of a MIMO Channel [9]:

$$C_r = \log \left[\det \left(I_{M_R} + \frac{SNR}{M_T} \tilde{\mathbf{H}} \times \tilde{\mathbf{H}}^H \right) \right] \quad (20)$$

$$C_p = \log \left[\det \left(I_{M_R} + \frac{SNR}{M_T} \mathbf{H} \times \mathbf{H}^H \right) \right] \quad (21)$$

$$C_{er} = \log \left[\det \left(I_{M_R} + \frac{SNR}{M_T} \mathbf{H} \times \mathbf{H}_{er}^H \right) \right] \quad (22)$$

where C_r is estimated channel capacity and C_p is perfect channel capacity and C_{er} is error channel capacity, I_{M_R} is the $M_R \times M_R$ identity matrix.

Furthermore, h_{ij} , an element of the matrix \mathbf{H} defines the complex channel coefficient between the i th receive antenna and j th transmit antenna. It is obvious that the channel capacity (in bits/sec/Hz) is highly dependent on the structure of matrix \mathbf{H} . The equation (20), (21) and (22) were used to calculate perfect, estimated and error channel capacities and directly related to the SNR.

3. PROPOSED SOLUTION

We are offering the OMP and SQ algorithms for compress sensing to find active paths both transmitter and receiver side. Thus, the section 3.1 for OMP and 3.2 for SQ was explained below.

3.1 CSI with OMP

Our aim is to solve the problem of the sparse vector maximum estimation approach with minimum noise. To solve it in practice; some compress sensing approximation algorithm will need to be used such as OMP based.

OMP has high capabilities of reliably recover of a high-dimensional sparse signal based on a small number of noisy linear measurements a signal with nonzero entries. OMP is a recursive greedy algorithm. At each step of it, the column which is most correlated with the residual is chosen. The OMP algorithm has rules to limit the feedback bits and recover the received signal. According to (5), it indicates that OMP algorithm with high possibility would estimate the sparse vector, under these conditions on the reciprocal incoherence and the minimum magnitude of the non-zero components of the signal [5].

Proposed Algorithm: OMP**Input:** Q, y, \bar{L}

Step 1: $t = 0$ Initialize: $r = y$ $S_{\hat{g}} = \emptyset$
 Step 2: **While** $\|Q^H r_t\|_{\infty} > \epsilon$ **and** $t < \bar{L}$ **do**
 Step 3: $t = t + 1$
 Step 4: $p_t = Q^H r_t$
 Step 5: $n_t^* = \operatorname{argmax}_{j=1,2 \dots G} (|p_{t,j}|)$
 Step 6: $S_{\hat{g}} = S_{\hat{g}} \cup n_t^*$
 Step 7: $\hat{g}_{S_{\hat{g}}} = 0$ **and** $\hat{g}_{S_{\hat{g}}} = Q_{:,S_{\hat{g}}}^H y$
 Step 8: $r = y - Q \hat{g}$
 Step 9: **end while**

Output: $\hat{g}, S_{\hat{g}}$ **Inputs:**

Q : measurement matrix
 y : received signal
 \bar{L} : iteration number

Outputs:

$S_{\hat{g}}$: index set
 \hat{g} : estimated sparse vector

Initialize:

t : iteration counter
 r : residual

The procedure of iterative algorithm OMP can be explained step by step;

Step 1: We initialized the residual $r = y$ and index set $S_{\hat{g}} = \emptyset$ and $t = 0$;

Step 2: We increased the iteration of algorithm.

Step 3: While the norm of residual is bigger than ϵ and t is less than \bar{L} , we iterated the residuals and continued the other steps. " ϵ " will be a boundary to get small error when finding acceptable measurement matrix. After that, OMP would recover original signal with high probability [6]. Moreover, we limited the iteration number with feedback overload.

Step 4: We applied the QR decomposition to find sparse \hat{g} because the matrix Q has knowledge of the dictionary at which the signal is received.

$$p_t = Q^H r_t$$

Step 5: Using sparse \hat{g} vector, we found max probability of active path indexes. Its mean, maximum correlation and minimum noise could be provided.

$$n_t^* = \operatorname{argmax}_{j=1,2 \dots G} (|p_{t,j}|)$$

Step 6: The indices of active paths was added in an index set and index set was augmented.

$$S_{\hat{g}} = S_{\hat{g}} \cup n_t^*$$

Step 7: The estimated sparse vector was initialized, and estimation could continue until the iteration end.

$$\begin{aligned}\hat{\mathbf{g}}_{\mathcal{S}_{\hat{\mathbf{g}}}} &= 0 \\ \hat{\mathbf{g}}_{\mathcal{S}_{\hat{\mathbf{g}}}} &= \mathbf{Q}_{:, \mathcal{S}_{\hat{\mathbf{g}}}}^H \mathbf{y}\end{aligned}$$

Step 8: The new approximation of the received signal and the new residual was calculated.

$$\mathbf{r} = \mathbf{y} - \mathbf{Q}\hat{\mathbf{g}}$$

Step 9: While $t < \bar{L}$, algorithm will return to Step 3 during this process, when the limit is exceeded the loop could break and end the process.

Also, we shouldn't forget that the residual \mathbf{r} is always orthogonal to the columns of \mathbf{Q} .

After the estimation sparse vector which is associated with sparse interaction matrix $\hat{\mathbf{G}}$ we applied the feedback technique with quantizing non-zero elements of $\hat{\mathbf{g}}$ with index set. For quantization of $\hat{\mathbf{g}}$, we can apply max Lloyd scalar quantizing technique as a compress sensing, and after we would obtain $\hat{\mathbf{G}}$ with reshaping the sparse $\hat{\mathbf{g}}$ vector. Then, receiving bits which are related to non-zero indices of $\hat{\mathbf{g}}$, BS can reconstruct the channel according to equation (7).

$$\mathbf{H} \approx \tilde{\mathbf{A}}_R \mathbf{G} \tilde{\mathbf{A}}_T^H \quad (23)$$

3.2 LLOYD SCALAR QUANTIZATION FOR LIMITATION

The input field, which is associated with each quantizer, is divided into the regions expressing around the code word. Designing quantities to find the codebook and portion rule which is making a minimization the overall average distortion measure.

Two necessary conditions prove that it is necessary for the design of the quantizer. First, it is the so-called centroid condition that is necessary for the optimization of the codebook, which means that for each region, the average decay measure over that region or the optimal codeword must be selected to minimize the local mean distortion. Second is the nearest neighbor rule that is required for the optimization of the channel space partition, which allows all input vectors closer to the code word to be assigned to more neighbors or regions than another code word. The generalized Lloyd algorithm reexamines the two conditions necessary to find the optimal codebook and channel space partition [7].

Lloyd Algorithm

- Step 1: Initialize the valid codebook ($\hat{\mathbf{g}}$).
- Step 2: Apply the nearest neighbor rule to find the optimal regions for $\hat{\mathbf{g}}$.
- Step 3: Apply the centroid condition to determine the optimal codewords for optimal regions.
- Step 4: Continue these steps until convergence.

Because of the centroid condition and the nearest neighbor rule, the overall average distortion reduces monotonically. This means that in each iteration, we can estimate the non-zero elements of the sparse channel. After quantization, we can dequantize the sparse vector at BS using known thresholds.

The number of feedback bits is the non-zero elements of the \hat{g} :

$$\bar{L} = \log_2 G + 2Q \quad (24)$$

where $2Q$ is the quantization bit number for one Q is real part another imaginary part of the CSI, and G is the dictionary members multiplication ($\mathbf{G}^R \mathbf{G}^T$) and \bar{L} is related to directly OMP algorithm for limitation of feedback bits.

4. RESULTS AND DISCUSSIONS

To understand difference between perfect, non-perfect and estimated channel we examined the normalized mean-squared error (NMSE) because the matrix dimensions are not the same and we calculated the channel capacities. The uplink feedback channel is considered error-free. Also, below the table we can see simulation parameters.

| SIMULATION PARAMETERS | |
|--|--|
| Parameters | Values |
| Carrier Frequency (f_c) | 2 GHz |
| Carrier Wavelength (λ) | $\lambda = c/f_c$ |
| Noise power (σ^2) | 10^{-10} Watts |
| Path number (L) | Discrete uniform in $[5, 6, \dots, 19, 20]$ |
| AoA and AoD (ϕ_l and ϕ_l') | $\phi_l, \phi_l' \sim U[-\pi/2, \pi/2]$ |
| Path loss exponent (η) | $\eta \sim N(2.8, 0.12)$ |
| Rician parameter (κ_l) | $\kappa_l \sim U[0, 50]$ |
| Multipath gain (α_l) | $\alpha_l \sim CN(\sqrt{\frac{\kappa_l}{\kappa_l + 1}}, \frac{1}{\kappa_l + 1})$ |
| Path delay (φ_l) | $\varphi_l \sim U[0, 2\pi]$ |
| Deflection angle (β) | $\beta \sim U[-5, 5]$ |

Table 1: Simulation parameters

NMSE vs NUMBER OF TRANSMITTER ANTENNAS (M_t)

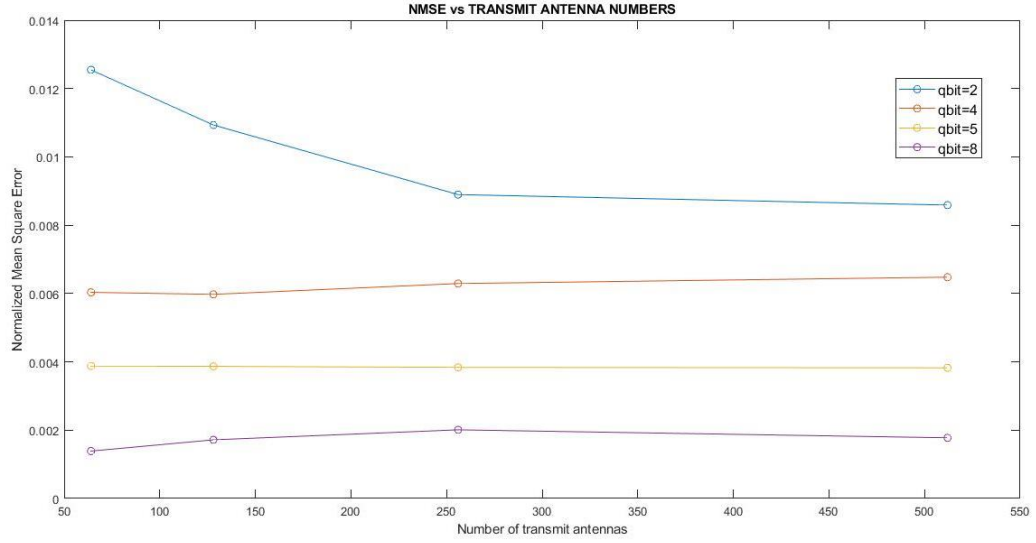


Figure 2: NMSE vs M_t .

The Channel was constructed according to M_t and it is directly related NMSE so, we can see the above graph the relation of NMSE and M_t . While increasing the M_t , NMSE is decreasing as it is expected according to equation (18) and (19). Also, after 256 transmitter antennas number NMSE is keeping the same NMSE value so 128 is optimum number of antennas.

Furthermore, we can see the effect of Lloyd algorithm quantization level. Lloyd is limited the feedback channel and it is over the feedback burden and it decrease the NMSE.

CAPACITY vs SNR

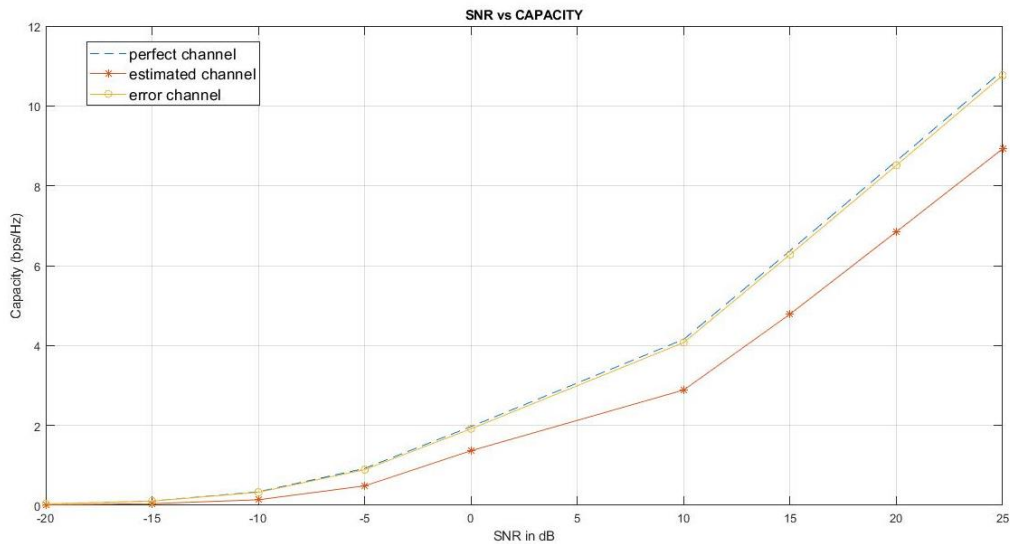


Figure 3: SNR vs Capacity

The graph is provided that the increased SNR values capacity is increasing as an expected according to equation (20), (21) and (22). The estimated channel capacity is shown with red line and its capacity less than perfect and error channel's. Also, perfect channel capacity is higher than the error channel's because error decrease the capacity of the channel.

5. CONCLUSION

The limited feedback system was constructed using dictionary-based sparse channel estimation algorithms. The dictionaries explain the antenna direction and they can proposal high capacity while requiring less feedback burden. The feedback bits number should increase with the BS antennas number proportionally to keep a certain performance level. The number of feedback bits for the OMP is under designer control, and they can achieve better performance using a significantly lower bit budget. The proposed OMP-SQ algorithm reaches the expected capacity performance when the number of transmit antennas is reasonable and SNR is high in the massive MIMO regime.

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