

Maximizing Minimum Phase Difference Based Hybrid Beamforming for Multiuser mmWave Massive MIMO systems

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Abstract—In this paper, we propose a hybrid beamforming algorithm based on maximizing the minimum phase difference for millimeter-wave (mmWave) massive multiple-input multiple-output (MIMO) multiuser systems. The proposed algorithm uses the channel gain threshold and the correlation characteristic of the array response vectors to select vector sets that provide higher signal power from analog beamforming vector codebook. Then, inter-user interference is measured by the phase difference between the analog beamforming vector and the main propagation path array response vector. Additionally, the criterion of maximizing the minimum phase difference is proposed to suppress the strongest inter-user interference. Analysis and simulation results show that the proposed algorithm can provide higher system sum rate with lower computational complexity than traditional approaches.

Keywords—multiple-input multiple-output (MIMO); millimeter-wave (mmWave); multiuser; interference; complexity;

I. INTRODUCTION

With the rapid development of wireless communication, low-band spectrum cannot meet the growing needs of communication [1]. Researchers have shifted their attention to underutilized millimeter-wave (mmWave) bands [2]. MmWave communication bandwidth is generally up to 8 GHz, which can provide high capacity [3]. However, the signal transmission in mmWave band endures a high path loss. For example, signals at 60 GHz suffer from 28 dB higher path loss than that at 2.4 GHz band [4]. Fortunately, the short wavelength facilitates the deployment of large arrays to compensate for path loss. Thus, the combination of mmWave and massive multiple-input multiple-output (MIMO) has great research prospects.

Conventional MIMO systems typically use full digital processing for optimal performance. However, full digital processing requires an independent radio frequency (RF) chain for each antenna. The large number of antennas in MmWave massive MIMO systems require a large number of RF chains, which will result in high power consumption and high complexity [5]. For single-user mmWave MIMO systems, researchers have proposed some hybrid beamforming (HBF) techniques that include analog beamforming (ABF) and digital beamforming (DBF). In [6], authors generalize orthogonal matching pursuit (OMP) algorithm, which iteratively finds the codebook vector with the strongest correlation with the rest of the channel. In [7], a

greedy algorithm is proposed, in which only one element in the ABF matrix is optimized in each iteration. In [8], authors propose a beamforming algorithm without any codebook, in which the signal space generated by the singular value decomposition of the channel matrix constitutes the ABF matrix.

For multiuser mmWave MIMO systems, there are also several HBF schemes. In [9], a beam steering algorithm is proposed, in which the transmitter selects the codebook vectors with the strongest correlation with the channel vectors as the ABF vectors. However, this method only maximizes the signal power and cannot effectively suppress inter-user interference. In order to reduce inter-user interference, [10] proposes a Gram-Schmidt based method. The method orthogonalizes the channel vectors of multiple users, then forms ABF vectors with orthogonalized vectors such that the ABF vectors are orthogonal to the partial channel vectors. But phase shifts of arbitrary accuracy is necessary, which is hard in practice.

In this paper, a HBF algorithm based on maximizing the minimum phase difference is proposed for uplink multiuser mmWave massive MIMO systems with limited array response vectors as the ABF vector codebook. For each user, the method first selects a path with path loss lower than the average path loss value. Then, a candidate ABF vector set is formed by selecting vectors from the codebook which result into smallest phase difference value. Finally, the minimum value of the phase differences between the phases corresponding to the vectors in the candidate ABF vector set and the direction-of-arrival (DOA) of other user channels are calculated, and the vector that maximizes the minimum value is selected as the ABF vector for each user. The main contributions of this paper are as follows. 1) The candidate ABF vector set ensures a high received signal power. In addition, maximizing the minimum phase difference avoids the strongest inter-user interference. As a result, the proposed approach can form a more reasonable compromise between improving power and suppressing interference than the approaches in [9] and [10]. 2) Since only the phase differences need to be calculated, the computational complexity of the proposed approach is lower than that of the methods in [9] and [10].

Notations: Matrices and vectors are in boldface, with uppercase letters for matrices and lower case letters for vectors. $[\bullet]_{m,n}$ denotes the element in the m th row and the

n th column of a matrix. $[\cdot]_m$ representd the m th element of a vector or the m th column of a matrix. \mathbf{A}^H denotes the conjugate transpose of \mathbf{A} . $\mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$ is used to denote a vector of length M and each element is a complex Gaussian variable with zero mean and covariance 1. $\mathbb{E}[\cdot]$ is used to denote expectation. $\|\cdot\|_2$ denotes the Euclidean norm of a vector. Finally, $O(x)$ represents the same order infinity as x .

II. SYSTEM MODEL

Consider an mmWave massive MIMO system that composes multiple single-antenna users and a base station (BS), as shown in Fig. 1. We consider uplink transmission, in which K single-antenna users simultaneously transmit data symbols to the BS, which is equipped with M antennas and K RF chains. Each RF chain is connected to all antennas of the BS by a RF combiner in a fully connected manner. The RF combiner is composed of an analog phase shift network, and each RF chain is independent.

The received signal vector at the BS can be expressed as

$$\mathbf{r} = \sqrt{\rho} \mathbf{H} \mathbf{s} + \mathbf{n} \in \mathbb{C}^{M \times 1}, \quad (1)$$

where ρ is transmission power, $\mathbf{H} \in \mathbb{C}^{M \times K}$ is channel matrix between the K users and the BS, $\mathbf{s} \in \mathbb{C}^{K \times 1}$ is the signal vector, whose elements satisfy $\mathbb{E}[\mathbf{s} \mathbf{s}^H] = \mathbf{I}_K$, $\mathbf{n} \in \mathbb{C}^{M \times 1}$ is the noise vector, its elements obey $\mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$. Furthermore, we denote the channel matrix as $\mathbf{H} = [\mathbf{h}_1, \dots, \mathbf{h}_K]$, where $\mathbf{h}_k \in \mathbb{C}^{M \times 1}$ denotes the channel vector between the k th user and the BS.

At the BS, signals received at the antennas pass the RF combiner and the baseband combiner, which can be expressed as the ABF matrix $\mathbf{W}_{\text{RF}} \in \mathbb{C}^{M \times K}$ and the DBF matrix $\mathbf{W}_{\text{BB}} \in \mathbb{C}^{K \times K}$. They are used to recover the signals sent by users, the recovered signal vector can be expressed as

$$\mathbf{y} = \mathbf{W}_{\text{BB}}^H \mathbf{W}_{\text{RF}}^H \mathbf{r} \in \mathbb{C}^{K \times 1}, \quad (2)$$

Since the ABF matrix is formed by phase shifters, its element is of constant-magnitude, and is given by

$$[\mathbf{W}_{\text{RF}}]_{m,n} = \frac{1}{\sqrt{M}}, \forall m = 1, \dots, M, \forall n = 1, \dots, N, \quad (3)$$

Assume that the channel state information (CSI) is known perfectly at the BS. Then, \mathbf{W}_{BB} can be designed with the minimum mean square error (MMSE) criterion as

$$\mathbf{W}_{\text{BB}} = \tilde{\mathbf{H}} (\tilde{\mathbf{H}}^H \tilde{\mathbf{H}} + \mathbf{I}_K)^{-1}, \quad (4)$$

where $\tilde{\mathbf{H}} = \mathbf{W}_{\text{RF}}^H \mathbf{H} \in \mathbb{C}^{K \times K}$. Substituting (1) into (2) yields

$$\mathbf{y} = \sqrt{\rho} \mathbf{W}_{\text{BB}}^H \mathbf{W}_{\text{RF}}^H \mathbf{H} \mathbf{s} + \mathbf{W}_{\text{BB}}^H \mathbf{W}_{\text{RF}}^H \mathbf{n}, \quad (5)$$

According to (2), the signal-to-interference-plus-noise ratio (SINR) of user k is

$$\eta_k = \frac{|\left[\mathbf{W}_{\text{RF}}^H \mathbf{W}_{\text{BB}} \right]_k^H \mathbf{h}_k|^2}{\sum_{m \neq k} |\left[\mathbf{W}_{\text{RF}}^H \mathbf{W}_{\text{BB}} \right]_k^H \mathbf{h}_m|^2 + \frac{1}{\rho} \left\| \left[\mathbf{W}_{\text{RF}}^H \mathbf{W}_{\text{BB}} \right]_k^H \right\|_2^2}, \quad (6)$$

Then, the system sum rate can be expressed as

$$R = \sum_{k=1}^K \log_2 (1 + \eta_k). \quad (7)$$

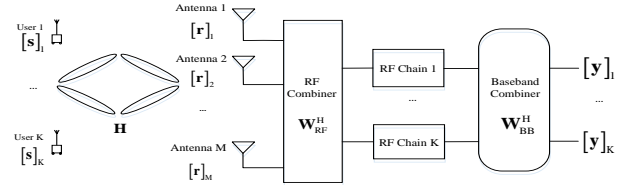


Figure 1. The architecture of system.

A. Channel Model

Because mmWave channel attenuation is serious during and the attenuation after reflection is also serious, the number of multipaths is small. This paper uses the channel model of the [5-6]. Assuming that user k has L_k propagation paths to the BS, the channel vector of user k can be expressed as

$$\mathbf{h}_k = \sqrt{\frac{M}{L_k}} \sum_{l=1}^{L_k} \alpha_{kl} \mathbf{a}(\varphi_{kl}), \quad (8)$$

where $\alpha_{kl} \in \mathcal{CN}(0, p_{kl})$ is the complex channel gain of the l th path from user k to the BS, with $\frac{1}{L_k} \sum_{l=1}^{L_k} p_{kl} = 1$.

$\varphi_{kl} \in [0, 2\pi]$ denotes the azimuth DOA of the l th path from user k to the BS. Assume that a uniform linear array (ULA) is equipped at BS, then the antenna array response vector $\mathbf{a}(\varphi_{kl}) \in \mathbb{C}^{M \times 1}$ can be expressed as

$$\mathbf{a}(\varphi_{kl}) = \frac{1}{\sqrt{M}} \left[1, e^{-j \frac{2\pi}{\lambda} d \sin(\varphi_{kl})}, \dots, e^{-j (M-1) \frac{2\pi}{\lambda} d \sin(\varphi_{kl})} \right]^T, \quad (9)$$

where d is distance between adjacent antennas in the array, and λ is the carrier wavelength.

B. Problem Formulation

In this paper, the object is to maximize the system sum rate by designing the ABF, and can be written as

$$\max_{\mathbf{W}_{\text{RF}}} R \quad \text{s.t.} \quad [\mathbf{W}_{\text{RF}}]_k \in \mathcal{F}_{\text{RF}}, k = 1, \dots, K, \quad (10)$$

where \mathcal{F}_{RF} stands for the set of feasible ABF vectors that satisfy (3). Since the cardinality of \mathcal{F}_{RF} is infinite, as [8], we further limit \mathcal{F}_{RF} to a set of array response vectors with finite angles as

$$\mathcal{F}_{\text{RF}} = \left\{ \mathbf{a}(\phi_c) \mid \phi_c = \frac{c\pi}{M}, c = 0, \dots, M-1 \right\}. \quad (11)$$

For (10), the exhaustive search method can results into the optimal solution. However, the exhaustive search operation complexity is M^K . When K and M increase, the computational complexity increases dramatically. In mmWave massive MIMO systems, M is usually very large. When $M = 100, K = 5$, its complexity is 10^{10} . To solve this problem, we will propose a method with lower computational complexity, which will be described in detail in Section III.

III. MAXIMIZING THE MINIMUM PHASE DIFFERENCE BASED ALGORITHM

The existing HBF methods for mmWave massive MIMO multiuser systems still have shortcomings. The beam control method in [9] only maximizes signal power but does not suppress interference. The Gram-Schmidt based method in [10] requires an infinite number of vectors in the ABF codebook, and the interference suppression may be worsen when modified for the optimization problem (10). In this section, we present a HBF method that compromises between increasing signal power and suppressing interference.

Since the sum rate expression in (7) is directly related to the SINR η_k , we can replace it with the maximization of (6). Then, the object in (10) is transformed into

$$\max_{[\mathbf{W}_{\text{RF}}]_k} \eta_k \quad \text{s.t.} \quad [\mathbf{W}_{\text{RF}}]_k \in \mathcal{F}_{\text{RF}}, k = 1, \dots, K. \quad (12)$$

There are two aspects in maximizing η_k , which are the received signal power and the inter-user interference, i.e., $[\mathbf{W}_{\text{RF}} \mathbf{W}_{\text{BB}}]_k^H \mathbf{h}_k$ and $\sum_{m \neq k} [\mathbf{W}_{\text{RF}} \mathbf{W}_{\text{BB}}]_k^H \mathbf{h}_m$ in (6). The design idea of this paper is to compromise between maximizing the signal power and minimizing the

interference. Specifically, we first ensure a high signal power and then minimize the inter-user interference.

A. Signal Power Optimization

Consequently, the first object question is to ensure that

$[\mathbf{W}_{\text{RF}} \mathbf{W}_{\text{BB}}]_k^H \mathbf{h}_k$ is a large value. It is known that

$$\left| [\mathbf{W}_{\text{RF}} \mathbf{W}_{\text{BB}}]_k^H \mathbf{h}_k \right|^2 = \left| [\mathbf{W}_{\text{BB}}]_k^H \mathbf{W}_{\text{RF}}^H \mathbf{h}_k \right|^2, \quad (13)$$

Moreover, we need to guarantee that the signal power is high for every k . Thus, the design goal is to make $[\mathbf{W}_{\text{RF}}]_k^H \mathbf{h}_k$ a large value. For convenience, we term it as the power factor

$$b_k = \left| [\mathbf{W}_{\text{RF}}]_k^H \mathbf{h}_k \right|^2, \quad (14)$$

Substituting (8) into (14) results into

$$b_k = \left| \sqrt{\frac{M}{L_k}} \sum_{l=1}^{L_k} \alpha_{kl} [\mathbf{W}_{\text{RF}}]_k^H \mathbf{a}(\varphi_{kl}) \right|^2, \quad (15)$$

According to (10) and (11), $[\mathbf{W}_{\text{RF}}]_k \in \mathcal{F}_{\text{RF}}$ can be expressed as $[\mathbf{W}_{\text{RF}}]_k = \mathbf{a}(\theta_k)$, where $\theta_k \in \mathbf{G}_k$, $\mathbf{G}_k = \left\{ \phi_c \mid \phi_c = \frac{c\pi}{M}, c = 0, \dots, M-1 \right\}$. Substituting the above formula into (15) yields

$$b_k = \left| \sqrt{\frac{M}{L_k}} \sum_{l=1}^{L_k} \alpha_{kl} \mathbf{a}^H(\theta_k) \mathbf{a}(\varphi_{kl}) \right|^2, \quad (16)$$

From (16), it is known that b_k is determined by α_{kl} and $\left| \mathbf{a}^H(\theta_k) \mathbf{a}(\varphi_{kl}) \right|^2$. As known to all, when θ_k gets closer to φ_{kl} , the value of $\left| \mathbf{a}^H(\theta_k) \mathbf{a}(\varphi_{kl}) \right|^2$ generally gets larger. Thus, if $|\theta_k - \varphi_{kl}| < \frac{\pi}{M}$, the requirement of large receiving power may be satisfied. For the path gain, we define a path gain threshold e_k . Moreover, we set e_k as the mean of path gain, i.e., $e_k = \frac{1}{L_k} \sum_{l=1}^{L_k} |\alpha_{kl}|$. If $|\alpha_{kl}| \geq e_k$, we category α_{kl} as a large value. If we only choose to transmit through paths with path gains larger than e_k , we may achieve high received power requirements. Accordingly, we record the set of θ_k that meets the above two requirements as

$$\bar{G}_k = \left\{ \theta_k \left| \theta_k = \frac{c\pi\pi}{M}, |\theta_k - \varphi_{kl}| < \frac{\pi}{M}, |\alpha_{kl}| \geq e_k \right. \right\},$$

$$s.t. \ c = 0, \dots, M-1, l = 1, \dots, \bar{L}_k, \quad (17)$$

where \bar{L}_k is the number of paths meeting the condition $|\alpha_{kl}| \geq e_k$. Then, for user k , the set for $[\mathbf{W}_{RF}]_k$ vector is changed from \mathcal{F}_{RF} to

$$\mathcal{F}_k = \left\{ \mathbf{a}(\theta_{kl}) \left| \theta_{kl} \in \bar{G}_k \right. \right\}. \quad (18)$$

By comparing (11) and (18), it can be seen that the number of elements in (18) is smaller than that in (11).

B. Inter-User Interference Suppression

With (18), the signal power is made high. The other aspect related to the SINR is to make $\sum_{m \neq k} \left| [\mathbf{W}_{RF} \mathbf{W}_{BB}]_k^H \mathbf{h}_m \right|^2$ small. Similar to (14), the goal is to make $\sum_{m \neq k} \left| [\mathbf{W}_{RF}]_k^H \mathbf{h}_m \right|^2$ as small as possible with $[\mathbf{W}_{RF}]_k \in \mathcal{F}_k$. To solve this problem, the simplest method is to perform vector correlations based on the elements within \mathcal{F}_k . Then, we can get the vector $[\mathbf{W}_{RF}]_k$ that minimizes $\sum_{m \neq k} \left| [\mathbf{W}_{RF}]_k^H \mathbf{h}_m \right|^2$. But this requires $M\bar{L}_k(K-1)$ complex multiplications. In order to further reduce computational complexity, we should avoid vector correlations that cause these complex multiplications.

It is known that the phase difference calculation can be used to replace the vector correlation calculation and thus avoid complex multiplications if the two vectors are in the form of (9). From (8), \mathbf{h}_k is the sum of several array response vectors and $[\mathbf{W}_{RF}]_k$ is in the form (9). In order to reduce complexity, we can approximate \mathbf{h}_k with the path of the highest path gain, i.e.,

$$\mathbf{h}_k \approx \beta_k \mathbf{a}(\tilde{\theta}_k), \beta_k = \sqrt{\frac{M}{L_k}} \tilde{\alpha}_k, \quad (19)$$

where $\tilde{\alpha}_k$ is the maximum path gain coefficient of the paths of user k and $\tilde{\theta}_k$ is the corresponding DOA. Consequently, we have the approximation $\sum_{m \neq k} \left| [\mathbf{W}_{RF}]_k^H \mathbf{h}_m \right|^2 \approx \sum_{m \neq k} \left| \beta_m \mathbf{a}^H(\theta_{kl}) \mathbf{a}(\tilde{\theta}_m) \right|^2$ where $\theta_{kl} \in \bar{G}_k$ and the

phase difference calculation can be used to reduce complexity.

Define set $T_k = \{m | m \neq k, m = 1, \dots, K\}$, we calculate the distance

$$D_{kl} = \min_{m \in T_k} \left| \sin(\theta_{kl}) - \sin(\tilde{\theta}_m) \right|, \theta_{kl} \in \bar{G}_k, \quad (20)$$

Which is the minimum phase difference. According to (16), if D_{kl} is decreases, the correlation $\left| \mathbf{a}^H(\theta_{kl}) \mathbf{a}(\tilde{\theta}_m) \right|$ generally increases. Thus, we can use D_{kl} to approximately measure the correlation value $\left| [\mathbf{W}_{RF}]_k^H \mathbf{h}_m \right|$. Since D_{kl} reflects the strongest interference for each choice of $\theta_{kl} \in \bar{G}_k$, we should make D_{kl} as large as possible, i.e., $\max_{l=1,2,\dots,\bar{L}_k} D_{kl}$. As a result, the designed phase for user k is

$$\theta_k^{opt} = \arg \max_{\theta_{kl} \in \bar{G}_k} D_{kl}, \quad (21)$$

Correspondingly, the designed ABF vector is

$$[\mathbf{W}_{RF}]_k = \mathbf{a}(\theta_k^{opt}), \quad (22)$$

In fact, the algorithm is implemented sequentially. More specifically, we first calculate θ_1^{opt} , then calculate θ_2^{opt} and so on. Thus, \bar{G}_k should be replaced by

$$P_k = \bar{G}_k / \left\{ \theta_j^{opt} \left| j = 1, \dots, k-1 \right. \right\}. \quad (23)$$

This is to avoid the situation when two column vectors in \mathbf{W}_{RF} being same, which may make $(\tilde{\mathbf{H}}^H \tilde{\mathbf{H}} + \mathbf{I}_K)$ approximately singular.

IV. COMPLEXITY ANALYSIS

In this section, we will analyze the complexity of the proposed algorithm and the complexity of the Gram-Schmidt algorithm in [10]. Since the DBF matrix is the same in this paper and [10], we only analyze the complexity of the ABF part.

For the proposed algorithm, (17) contains $K\bar{L}_k$ real additions; (20) contains $K(K-1)\bar{L}_k$ real additions. Thus, the proposed algorithm only contains real additions, and the overall complexity is $O(KL_k + K^2\bar{L}_k)$.

For the Gram-Schmidt algorithm in [10], the Gram-Schmidt orthogonalization contains MK^2 complex multiplications and $K(K-1)/2$ real divisions; The calculation of \mathbf{W}_{RF} contains MK complex multiplications,

$2MK$ real divisions, and MK real square roots. Thus, the calculations in this algorithm are mostly complex multiplications and the overall complexity is $O(MK^2)$.

It is known that the complexity of one real addition is less than that of one complex multiplication. Moreover, we usually have $M \gg L_k$. Therefore, it can be seen that the proposed ABF algorithm is of much less complexity than the approach in [10].

V. SIMULATIONS

In order to evaluate the performance of the proposed approach, we will simulate the sum rate of the proposed approach and compare it with the sum rates of the full digital, the Gram-Schmidt in [10], the modification of the Gram-Schmidt in [10], and the beam steering in [9]. For the full digital algorithm, each antenna is connected to one RF chain, and the zero forcing (ZF) criterion is employed. For the modification of the Gram-Schmidt in [10], the one in (9) that is mostly correlated to the ABF vector obtained by the Gram-Schmidt algorithm in [10] is used as the ABF vector. The simulation parameters are as follows. φ_{kl} is uniformly distributed in $[0, 2\pi]$, the number of the BS antennas $M = 64$, the number of the users is $K = 6$, the antenna distance is $d = 0.5\lambda$, the number of path is $L_k = 3, \forall k$, the SNR is $10 \log_{10} \rho = 10$ dB.

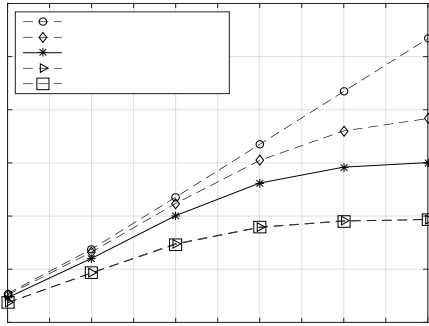


Figure 2. Sum rates versus SNR.

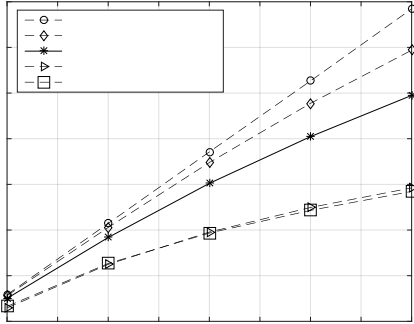


Figure 3. Sum rates versus number of users.

In Fig. 2, sum rates versus the SNR is shown. It can be seen that the full digital algorithm has the highest system sum rate. The Gram-Schmidt algorithm in [10] is inferior to it. The sum rate of the proposed algorithm is higher than that of the modification of the Gram-Schmidt in [10] and the beam control algorithm in [9] in the whole SNR range.

In Fig. 3, sum rates versus the number of users is shown. As can be seen, the sum rate of the proposed algorithm is always higher than the modification of the Gram-Schmidt in [10] and the beam control algorithm in [9]. Moreover, the sum rates of these two approaches saturate with the increase of the number of users. In contrast, the sum rate of the proposed algorithm increases steadily with the increase of the number of users. These results reflect the superiority of the proposed approach in reducing inter-user interference.

In Fig. 4, sum rates versus the number of the BS antennas is shown. It can be seen that the proposed algorithm performs better than the modification of the Gram-Schmidt in [10] and the beam control algorithm in [9]. Moreover, with the increase of the BS antennas, the sum rate of the proposed algorithm is getting close to that of the full digital algorithm.

In Fig. 5, sum rates versus the number of paths is shown. The sum rate of the proposed approach decreases faster than that of other approaches. This is because the increase of the number of paths leads to more severe inter-user interference. However, even with a large number of paths, the performance of proposed algorithm is still superior to the modification of the Gram-Schmidt in [10] and the beam control algorithm in [9].

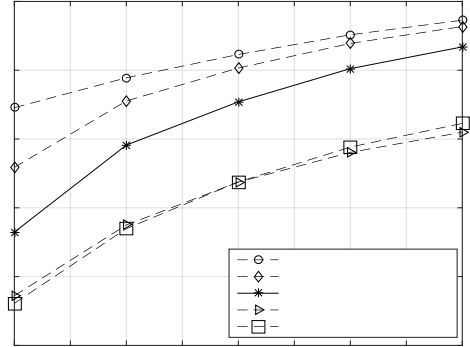


Figure 4. Sum rates versus number of BS antennas.

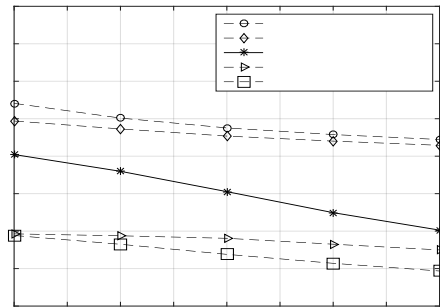


Figure 5. Sum rates versus number of paths.

VI. CONCLUSIONS

In this paper, we propose a HBF algorithm which maximizes the minimum phase difference. Using the characteristics of mmWave MIMO channels, we reduce the number of elements in ABF matrix codebook to be optimized, which reduces the computational complexity. We use a path gain threshold to ensure a high signal power. Moreover, the criterion of maximizing the minimum phase difference is proposed, which suppresses the inter-user interference. Simulation results show that the performance of the proposed algorithm can achieve a reasonable compromise between suppressing inter-user interference and enhancing signal power.

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