Secure Transmission for Massive MIMO with Statistical Channel State Information

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Abstract

This paper considers secure transmission with quality of service guarantee for massive MIMO system where only statistical channel state information of all legitimate UTs and eavesdropper are known at the BS. We introduce a lower bound on the achievable ergodic secrecy rate of each legitimate UT, then we adopt the system minimum secrecy rate as the optimization goal. We demonstrate it is optimal to transmit signals in the beam domain. Then we simplify the large-dimensional matrix-valued secure transmission design into a beam domain power allocation problem. Based on majorization-minimization procedure, we utilize large dimensional random matrix theory and derive the deterministic equivalents of the objective in each iteration. Simulation results show that XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

Index Terms

secure transmission, massive MIMO, beam domain, concave-convex procedure

I. Introduction

Due to the broadcast nature of wireless medium, secure transmission is always considered as a very important issue in wireless communication. Traditionally, secure transmission has been achieved by utilizing key-based encryption techniques. However, these approaches are potentially vulnerable because they are built on certain assumptions for computational complexity [1]. In addition to key-based encryption techniques, physical layer security has attracted tremendous research interest. As shown in the pioneering work on physical layer security [2], if the legitimate

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UT's channel conditions is better than the eavesdropper's conditions, the BS can reliably send private message to the legitimate UT. And a typical approach to enhance the security of data transmission is the use of artificial noise (AN) to disturbs the signal to interference-plus-noise ratio (SINR) and the decoding process at the eavesdropper [3]. Much recent research has considered physical layer secure transmission of multi-antenna systems [4]–[6]. For MIMO wiretap channels, the work in [7], [8] determine the optimal input covariance matrix to maximize the ergodic secrecy rate in the case only statistical channel state information (CSI) is required.

Massive multiple-input multiple-output (MIMO) which employs a large number of antennas at the base stations (BSs) to simultaneously serve a relatively large number of user terminals (UTs) to improve spectral efficiency and power efficiency [9]. And Massive MIMO has been considered as a promising technology to improve the physical layer security, the great number of antennas can not only increase the signal strength towards the legitimate UTs, but also focus the AN energy into direction of eavesdropper [10]. Massive MIMO enables simple UL detection and DL precoding to eliminate the inter-user interference [9], [11], [12]. There are some works have been dedicated to investigating physical layer security in massive MIMO systems. The work in [10] dedicated to secure downlink transmission in multi-cell massive MIMO system in the presence of a multi-antenna eavesdropper, where only imperfect CSI of the legitimate UTs is available at the BS. Article [13] proposed three secure transmission schemes for single-cell multi-user massive MIMO systems. The research in [14], [15] based on the assumption that instantaneous CSI of the single-antenna legitimate UTs is available at the BS.

The accuracy of instantaneous CSI at the BS plays an significance role in most existing works. However, there are some challenge in the acquisition of instantaneous CSI. Exploiting the statistical CSI provides a practical way to overcome this challenge for statistical CSI varies over much larger time scales than instantaneous CSI. Article [16] proposed the method of beam domain transmission only utilize statistic CSI at the BS, and this transmission method was proven optimal for a sum-rate upper bound maximization. The work in [17] was further extended to solve the problem of beam domain secure transmission. It optimized for secure transmission sum-rate lower bound. However, such optimization results may result in some UTs' secrecy ergodic secure rate very small, even negative numbers. This is very inapplicable for some scenarios where quality of service (QoS) is required.

In this paper, we investigate the secure transmission with QoS guarantee for massive MIMO systems with an eavesdropper. We assume only the statistical CSI of legitimate UTs and the eavesdropper is acquired at the BS. Jointly correlated MIMO channel model is considered here. We utilize the method of beam domain transmission proposed in [16] to perform secure transmission with QoS guarantee.

A. Contributions

The major contributions of our works are summarized as follows:

- In order to guarantee QoS, we use the system minimum UT's secrecy rate as the optimization goal to perform power allocation to the BS unicast signal. And we introduce a lower bound of above optimization goal. Based on this lower bound, we formulate a power allocation problem to maximize the achievable ergodic system unicast secrecy rate.
- We show the closed-form optimal secure transmit directions, i.e., the eigenvectors of transmit signal covariance matrices for secure transmission. We demonstrate that it is optimal to transmit signals in beam domain.
- We proposed an iterative beam domain power allocation algorithm for secure transmission via invoking the majorization-minimization (MM) procedure. Our proposed algorithm is guaranteed to converge to a stationary point.
- We derive the DE of the objectives in each iteration of the proposed iterative algorithm to further reduce the computational complexity of the algorithm.

B. Notations

The following notations are used.

- Upper and lower case boldface letters denote matrices and column vectors, respectively.
- $\mathbb{C}^{M\times N}$ denote the $M \times N$ dimensional complex-valued vector matrix.
- $\mathbf{I}_M$ denotes the $M \times M$ dimensional identity matrix.
- $(\cdot)^H$,$(\cdot)^*$ and $(\cdot)^T$ denote conjugate-transpose, conjugate, and transpose operations, respectively.
- $\odot$ denotes the Hadamard product.
- The operations $\mathrm{tr}\left\{\cdot\right\}$ and $\det\left\{\cdot\right\}$ denote the matrix trace and determinant operations, respectively.
- $[\mathbf{A}]_{m,n}$ denotes the (m, n)th element of matrix $\mathbf{A}$.
- $\mathbf{A} \succeq \mathbf{0}$ denotes that $\mathbf{A}$ is positive semidefinite.

II. Massive MIMO Channel Model

Consider massive MIMO secure-cell secure transmission with one M-antennas BS, K legitimate UTs, each with N_r antennas, and one eavesdropper with N_{eve} antennas. The BS transmits private and independent messages to each legitimate user. We note that neither the BS nor the UTs are assumed to know which UT is eavesdropper, hence, we assume that any UT may be potentially targeted by the eavesdropper.

Let $\mathbf{H}_k \in \mathbb{C}^{N_r \times M}$ and $\mathbf{H}_{\text{eve}} \in \mathbb{C}^{N_{\text{eve}} \times M}$ denote the downlink channel matrices from the BS to legitimate UT k and the eavesdropper, respectively. The received signals at the kth UT and at the eavesdropper are denoted by $\mathbf{y}_k \in \mathbb{C}^{N_k \times 1}$ and $\mathbf{y}_k \in \mathbb{C}^{N_{\text{eve}} \times 1}$, respectively, and can be written as:

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \sum_{i \neq k} \mathbf{H}_k \mathbf{x}_i + \mathbf{n}_k, \tag{1}$$

$$\mathbf{y}_{\text{eve}} = \sum_{i=1}^{K} \mathbf{H}_{\text{eve}} \mathbf{x}_i + \mathbf{n}_{\text{eve}}, \tag{2}$$

where $\mathbf{x}_k \in \mathbb{C}^{M \times 1}$ denotes the signal vector transmitted to the kth UT which satisfies $\mathbb{E}\left\{\mathbf{x}_k\right\} = \mathbf{0}$, $\mathbb{E}\left\{\mathbf{x}_k \mathbf{x}_{k'}^H\right\} = 0\,(k \neq k')$, and $\mathbb{E}\left\{\mathbf{x}_k \mathbf{x}_k^H\right\} = \mathbf{Q}_k \in \mathbb{C}^{M \times M}$. $\mathbf{n}_k \in \mathbb{C}^{N_r \times 1}$ and $\mathbf{n}_{\text{eve}} \in \mathbb{C}^{N_{\text{eve}} \times 1}$ are zero-mean circularly symmetric complex Gaussian noise with covariance matrices $\mathbf{I}_{N_r}$ and $\mathbf{I}_{N_{\text{eve}}}$ respectively. Here, without loss of generality, we consider a unit variance and assume that the BS has the power constraint

$$\sum_{k} \operatorname{tr}\left(\mathbf{Q}_k\right) \leq P, \tag{3}$$

where $P \leq 0$ depends on the BS power budget.

In this work, we consider the joint spatially correlated Rayleigh fading MIMO channel model [18], [19], which captures the joint correlation characteristics between the transmitter and the receiver. In particular, the downlink channel matrices from the BS to legitimate UT k and the eavesdropper in (1) and (2) can be modeled as:

$$\mathbf{H}_k = \mathbf{U}_{r,k} \mathbf{G}_k \mathbf{V}_{t,k}^H \in \mathbb{C}^{N_r \times M}, \tag{4}$$

$$\mathbf{H}_{\text{eve}} = \mathbf{U}_{r,\text{eve}} \mathbf{G}_{\text{eve}} \mathbf{V}_{t,\text{eve}}^H \in \mathbb{C}^{N_{\text{eve}} \times M}, \tag{5}$$

where $\mathbf{U}_{r,k} \in \mathbb{C}^{N_r \times N_r}$, $\mathbf{U}_{r,\text{eve}} \in \mathbb{C}^{N_{\text{eve}} \times N_{\text{eve}}}$, $\mathbf{V}_{t,k} \in \mathbb{C}^{M \times M}$, and $\mathbf{V}_{t,\text{eve}} \in \mathbb{C}^{M \times M}$ are deterministic unitary matrices, $\mathbf{G}_k \in \mathbb{C}^{N_r \times M}$ and $\mathbf{G}_{\text{eve}} \in \mathbb{C}^{N_{\text{eve}} \times M}$ are random matrices with zero-mean independent elements. Note that $\mathbf{G}_k$ and $\mathbf{G}_{\text{eve}}$ are referred to as the downlink beam domain channel matrices between the BS and legitimate UT k and the eavesdropper, respectively [16], [20]. The statistics of the beam domain channels $\mathbf{G}_k$ and $\mathbf{G}_{\text{eve}}$ can be described as

$$\boldsymbol{\Omega}_k = \mathbb{E}\left\{\mathbf{G}_k \odot \mathbf{G}_k^*\right\} \in \mathbb{R}^{N_r \times M}, \tag{6}$$

$$\boldsymbol{\Omega}_{\text{eve}} = \mathbb{E}\left\{\mathbf{G}_{\text{eve}} \odot \mathbf{G}_{\text{eve}}^*\right\} \in \mathbb{R}^{N_{\text{eve}} \times M}, \tag{7}$$

respectively.

For massive MIMO channels, as the number of BS antennas M tends to infinity, the eigenvector matrices of the BS correlation matrices of different legitimate UTs and the eavesdropper tend to be equal to a deterministic unitary matrix $\mathbf{V}$, which only depends on the BS array topology [20]–[22]. Thus the channel matrices can be well approximated by

$$\mathbf{H}_k = \mathbf{U}_{r,k}\mathbf{G}_k\mathbf{V}^H, \tag{8}$$

$$\mathbf{H}_{\text{eve}} = \mathbf{U}_{r,k}\mathbf{G}_{\text{eve}}\mathbf{V}^H, \tag{9}$$

respectively. Note that the above approximations have been widely adopted in previous works and shown to be quire accurate for a practical number of antennas [16], [21]. Thus, we will adopt the massive MIMO channel model in (8) and (9) in this work.

III. Secure transmission design

We assume that the BS only has statistical CSI of all legitimate UTs as well as the eavesdropper. We also assume that the legitimate UTs and the eavesdropper have instantaneous CSI of their corresponding channel matrices. At each legitimate UT, we treat the aggregate interference-plus-noise $\mathbf{n}_k' = \sum_{i \neq k} \mathbf{H}_k \mathbf{x}_i + \mathbf{n}_k$ as Gaussian noise with covariance matrix

$$\mathbf{K}_k = \mathbf{I}_{N_r} + \sum_{i \neq k} \mathbb{E}\left\{\mathbf{H}_k \mathbf{Q}_i \mathbf{H}_k^H\right\} \in \mathbb{C}^{N_r \times N_r}. \tag{10}$$

Here, we assume the covariance matrix $\mathbf{K}_k$ is known at the kth UT. Besides, we make the pessimistic assumption that, at the eavesdropper, signals of all legitimate UTs can be decoded and cancelled from the received signal $\mathbf{y}_{\text{eve}}$ except the signal transmitted to the UT of interest. Since each UT in the system has the risk of being eavesdropped, the unicast secrecy rate of UT k can be expressed as [17]

$$R_k^{\text{sec}} = \left[R_k - R_k^{\text{eve}}\right]^+ , \tag{11}$$

and the system minimum secrecy rate can be expressed as

$$R_{\text{sec}} = \min_k R_k^{\text{sec}} = \min_k \left[R_k - R_k^{\text{eve}}\right]^+ , \tag{12}$$

where

$$\begin{aligned} R_k &= \mathbb{E}\left\{\text{logdet}\left(\mathbf{I}_{N_r} + \mathbf{K}_k^{-1}\mathbf{H}_k\mathbf{Q}_k\mathbf{H}_k^H\right)\right\} \\ &= \mathbb{E}\left\{\text{logdet}\left(\mathbf{K}_k + \mathbf{H}_k\mathbf{Q}_k\mathbf{H}_k^H\right)\right\} - \text{logdet}\left\{\mathbf{K}_k\right\} \\ &= \mathbb{E}\left\{\text{logdet}\left(\bar{\mathbf{K}}_k + \mathbf{G}_k\mathbf{V}^H\mathbf{Q}_k\mathbf{V}\mathbf{G}_k^H\right)\right\} - \text{logdet}\left\{\bar{\mathbf{K}}_k\right\} , \end{aligned} \tag{13}$$

here

$$\begin{aligned} \bar{\mathbf{K}}_k &= \mathbf{U}_{r,k}^H\mathbf{K}_k\mathbf{U}_{r,k} \\ &= \mathbf{I}_{N_r} + \sum_{i\neq k}\mathbb{E}\left\{\mathbf{G}_k\mathbf{V}^H\mathbf{Q}_i\mathbf{V}\mathbf{G}_k^H\right\} , \end{aligned} \tag{14}$$

and

$$\begin{aligned} R_k^{\text{eve}} &= \mathbb{E}\left\{\text{logdet}\left(\mathbf{I}_{N_{\text{eve}}} + \mathbf{H}_{\text{eve}}\mathbf{Q}_k\mathbf{H}_{\text{eve}}^H\right)\right\} \\ &= \mathbb{E}\left\{\text{logdet}\left(\mathbf{I}_{N_{\text{eve}}} + \mathbf{G}_{\text{eve}}\mathbf{V}^H\mathbf{Q}_k\mathbf{V}\mathbf{G}_{\text{eve}}^H\right)\right\} . \end{aligned} \tag{15}$$

where R_k denotes an achievable ergodic rate between the BS and the kth UT, and R_k denotes the ergodic capacity between the BS and the eavesdropper, which seeks to decode the private messages intended for the kth UT. And the Sylvester's determinant identity $\det\left\{\mathbf{I}+\mathbf{AB}\right\} = \det\left\{\mathbf{I}+\mathbf{BA}\right\}$ is exploited in (13) and (15).

Notice that, in practical system, it is difficult to acquire instantaneous $\mathbf{H}_k\mathbf{Q}_i\mathbf{H}_k$ $(i \neq k)$ at the kth UT in massive MIMO system. Thus, we make an assumption that each legitimate UT treats $\mathbf{n}_k'$ as a Gaussian noise and the covariance matrix with expectation over $\mathbf{H}_k$ is known at each UT's side. With this assumption, the matrix $\mathbf{K}_k$ defined in (10) is the covariance matrix of $\mathbf{n}_k'$. Therefore, the ergodic rate defined in (13) is reasonable for practice.

In general, the system minimum unicast secrecy rate given by (12) is a non-concave function with respect to $(\mathbf{Q}_1, ..., \mathbf{Q}_K)$. Hence, it is difficult to determine the optimal input covariance matrices to maximize the system minimum secrecy rate. From Jensen's inequality, R_k^{eve} in (15) can be upper bounded by

$$R_k^{\text{eve}} \leq R_{k,\text{ub}}^{\text{eve}} = \text{logdet}\left(\mathbf{I}_{N_{\text{eve}}} + \mathbb{E}\left\{\mathbf{G}_{\text{eve}}\mathbf{V}^H\mathbf{Q}_k\mathbf{V}\mathbf{G}_{\text{eve}}^H\right\}\right). \tag{16}$$

Thus the lower bound of security rate for UT k can be ontained as

$$R_{k,\text{lb}}^{\text{sec}} = \left[R_k - R_{k,\text{ub}}^{\text{eve}}\right]^+, \tag{17}$$

Then the lower bound of system minimum unicast security rate in (12) can be obtained as follows

$$R_{\text{sec,lb}} = \min_k \left[R_k - R_{k,\text{ub}}^{\text{eve}}\right]^+. \tag{18}$$

Then, we design the secure transmission strategies by optimizing the lower bound of system minimum unicast security rate. Our main objective is to design the input covariance matrices $\mathbf{Q}_1, ..., \mathbf{Q}_K$ maximizing (18), which can be formulated as the following optimization problem

$$\begin{aligned} [\mathbf{Q}_1^{\text{op}}, ..., \mathbf{Q}_K^{\text{op}}] = & \underset{\mathbf{Q}_1,...,\mathbf{Q}_K}{\arg\max} \min_k \left(R_k - R_{k,\text{ub}}^{\text{eve}}\right) \\ \text{subject to} \quad & \text{tr}\left(\sum_{k=1}^K \mathbf{Q}_k\right) \leq P \\ & \mathbf{Q}_k \succeq \mathbf{0}, \quad k = 1, ..., K, \end{aligned} \tag{19}$$

where $\mathbf{Q}_1^{\text{op}}, ..., \mathbf{Q}_K^{\text{op}}$ is the optimal solution of the problem in (19). Because any negative term in the summation could increase to zero by setting the corresponding $\mathbf{Q}_k = \mathbf{0}, k = 1, ..., K$, the notation $[\cdot]^+$ is ignored when solving the problem in (19).

Let $\mathbf{Q}_k = \mathbf{\Phi}_k\mathbf{\Lambda_k}\mathbf{\Phi}_k^H$, where $\mathbf{\Phi}_k$ is the eigenmatrix and $\mathbf{\Lambda}_k$ is a diagonal matrix of the corresponding eigenvalues. Note that eigenvectors and the eigenvalues of the transmit covariance matrix have practical engineering meaning. Specifically, the eigenvectors of the transmit covariance matrix represent the directions of the transmit signals, while the eigenvalues represent the powers allocated onto each direction. And for the beam domain transmission proposed in [16], $\mathbf{\Phi}_k$ is set to be $\mathbf{V}, k = 1, ..., K$.

We start our investigation of the optimal transmit covariance $\mathbf{Q}_k, k = 1, ..., K$ by focusing on its eigenvectors. In particular, we present the eigenvectors of the optimal transmit covariance matrix in the following proposition.

Proposition 1: The eigenvector of the optimal input covariance matrix of each legitimate UT, maximizing the lower bound of system minimum unicast security rate as given by (18) are given by

$$\mathbf{Q}_k^{\mathrm{op}} = \mathbf{V}\boldsymbol{\Lambda}_k\mathbf{V}^H, \quad k = 1, ..., K. \tag{20}$$

Proof: Please refer to the Appendix. ■

Proposition 1 shows the eigenmatrices of the input signals maximizing the low bound of system minimum unicast are given by the columns of $\mathbf{V}$, which implies that the optimal secure transmission should be performed in the beam domain.

Inspired by Proposition 1, we know focus on the beam domain secure transmission. Thus the optimization problem in (19) can be simplified to

$$\begin{aligned}
[\boldsymbol{\Lambda}_1^{\mathrm{op}}, ..., \boldsymbol{\Lambda}_K^{\mathrm{op}}] = \arg\max_{\boldsymbol{\Lambda}_1,...,\boldsymbol{\Lambda}_K} \min_k & \left(R_k\left(\boldsymbol{\Lambda}_1, ..., \boldsymbol{\Lambda}_K\right) - R_{k,\mathrm{ub}}^{\mathrm{eve}}\left(\boldsymbol{\Lambda}_1, ..., \boldsymbol{\Lambda}_K\right)\right) \\
\text{subject} \quad \text{to} \quad & \mathrm{tr}\left(\sum_{k=1}^{K}\boldsymbol{\Lambda}_k\right) \le P \\
& \boldsymbol{\Lambda}_k \succeq \mathbf{0}, \quad k = 1, ..., K,
\end{aligned} \tag{21}$$

where

$$R_k\left(\boldsymbol{\Lambda}_1, ..., \boldsymbol{\Lambda}_K\right) = \mathbb{E}\left\{\mathrm{logdet}\left(\bar{\mathbf{K}}_k + \mathbf{G}_k\boldsymbol{\Lambda}_k\mathbf{G}_k^H\right)\right\} - \mathrm{logdet}\left(\bar{\mathbf{K}}_k\right), \tag{22}$$

$$R_{k,\mathrm{ub}}^{\mathrm{eve}}\left(\boldsymbol{\Lambda}_1, ..., \boldsymbol{\Lambda}_K\right) = \mathrm{logdet}\left(\bar{\mathbf{K}}_{\mathrm{eve},k}\right), \tag{23}$$

and

$$\bar{\mathbf{K}}_k = \mathbf{I}_{N_r} + \sum_{i\neq k}\mathbb{E}\left\{\mathbf{G}_k\boldsymbol{\Lambda}_i\mathbf{G}_k^H\right\}, \tag{24}$$

$$\bar{\mathbf{K}}_{\mathrm{eve},k} = \mathbf{I}_{N_{\mathrm{eve}}} + \mathbb{E}\left\{\mathbf{G}_{\mathrm{eve}}\boldsymbol{\Lambda}_k\mathbf{G}_{\mathrm{eve}}^H\right\}. \tag{25}$$

We define

$$R_{k,1}\left(\mathbf{\Lambda}_1,...,\mathbf{\Lambda}_K\right)=\mathbb{E}\left\{\operatorname{logdet}\left(\bar{\mathbf{K}}_k+\mathbf{G}_k\mathbf{\Lambda}_k\mathbf{G}_k^H\right)\right\}, \tag{26}$$

and

$$R_{k,2}\left(\mathbf{\Lambda}_1,...,\mathbf{\Lambda}_K\right)=\operatorname{logdet}\left(\bar{\mathbf{K}}_k\right)+\operatorname{logdet}\left(\bar{\mathbf{K}}_{\text{eve},k}\right) \tag{27}$$

Then we can rewrite (21) as follows

$$\begin{aligned}\left[\mathbf{\Lambda}_1^{\text{op}},...,\mathbf{\Lambda}_K^{\text{op}}\right]=\underset{\mathbf{\Lambda}_1,...,\mathbf{\Lambda}_K}{\arg\max}\min_k&\left(R_{k,1}\left(\mathbf{\Lambda}_1,...,\mathbf{\Lambda}_K\right)-R_{k,2}\left(\mathbf{\Lambda}_1,...,\mathbf{\Lambda}_K\right)\right)\\ \text{subject}\quad\text{to}\quad&\operatorname{tr}\left(\sum_{k=1}^K\mathbf{\Lambda}_k\right)\le P\\ &\mathbf{\Lambda}_k\succeq\mathbf{0},\quad k=1,...,K,\end{aligned} \tag{28}$$

We observe that $R_{k,1}\left(\mathbf{\Lambda}_1,...,\mathbf{\Lambda}_K\right)$ and $R_{k,2}\left(\mathbf{\Lambda}_1,...,\mathbf{\Lambda}_K\right)$ in the objective function of (28) are both concave functions with respect to $(\mathbf{\Lambda}_1,...,\mathbf{\Lambda}_K)$, thus (28) is an Non-convex problem. We note that the objective function can be lower-bounded by the concave function obtained replacing $R_{k,2}\left(\mathbf{\Lambda}_1,...,\mathbf{\Lambda}_K\right)$ with its first-order Taylor expansions at the given point. Then we exploit the MM procedure [23], [24] which is a sequential optimization approach to solve (28). We first form a convex optimization program by replacing the second term $R_{k,2}\left(\mathbf{\Lambda}_1,...,\Lambda_K\right)$ with its first-order Taylor expansions at the current iteration and then solve it, which further yields the next generation. In particular, the problem in (21) is tackled via iteratively solving the following sequence of convex optimization problems.

$$\begin{aligned}\left[\mathbf{\Lambda}_1^{(\ell+1)},...,\mathbf{\Lambda}_K^{(\ell+1)}\right]=\underset{\mathbf{\Lambda}_1,...,\mathbf{\Lambda}_K}{\arg\max}\min_k&\left\{R_{k,1}\left(\mathbf{\Lambda}_1,...,\mathbf{\Lambda}_K\right)\right.\\ -R_{k,2}\left(\mathbf{\Lambda}_1^{(\ell)},...,\mathbf{\Lambda}_K^{(\ell)}\right)-\sum_{i=1}^K\operatorname{tr}&\left.\left\{\left(\frac{\partial}{\partial\mathbf{\Lambda}_i}R_{k,2}\left(\mathbf{\Lambda}_1^{(\ell)},...,\mathbf{\Lambda}_K^{(\ell)}\right)\right)^T\left(\mathbf{\Lambda}_i-\mathbf{\Lambda}_i^{(\ell)}\right)\right\}\right\}\\ \text{subject}\quad\text{to}\quad&\operatorname{tr}\left(\sum_{k=1}^K\mathbf{\Lambda}_k\right)\le P\\ &\mathbf{\Lambda}_k\succeq\mathbf{0},\quad k=1,...,K,\end{aligned} \tag{29}$$

where ℓ is the iteration index. We define

$$\begin{aligned} f_k\left(\mathbf{\Lambda}_1, ..., \mathbf{\Lambda}_K\right) &= \sum_{i\neq k} \operatorname{tr}\left\{\left(\frac{\partial}{\partial \mathbf{\Lambda}_i} R_{k,2}\left(\mathbf{\Lambda}_1^{(\ell)}, ..., \mathbf{\Lambda}_K^{(\ell)}\right)\right)^T \left(\mathbf{\Lambda}_i - \mathbf{\Lambda}_i^{(\ell)}\right)\right\} \\ &= \sum_{i\neq k} \operatorname{tr}\left\{\left(\frac{\partial}{\partial \mathbf{\Lambda}_i} \operatorname{logdet}\left(\bar{\mathbf{K}}_k\right)\right)^T \left(\mathbf{\Lambda}_i - \mathbf{\Lambda}_i^{(\ell)}\right)\right\}, \end{aligned} \tag{30}$$

$$\begin{aligned} g_k\left(\mathbf{\Lambda}_1, ..., \mathbf{\Lambda}_K\right) &= \operatorname{tr}\left\{\left(\frac{\partial}{\partial \mathbf{\Lambda}_k} R_{k,2}\left(\mathbf{\Lambda}_1^{(\ell)}, ..., \mathbf{\Lambda}_K^{(\ell)}\right)\right)^T \left(\mathbf{\Lambda}_k - \mathbf{\Lambda}_k^{(\ell)}\right)\right\} \\ &= \operatorname{tr}\left\{\left(\frac{\partial}{\partial \mathbf{\Lambda}_k} \operatorname{logdet}\left(\bar{\mathbf{K}}_{\mathrm{eve},k}\right)\right)^T \left(\mathbf{\Lambda}_k - \mathbf{\Lambda}_k^{(\ell)}\right)\right\}. \end{aligned} \tag{31}$$

Moreover, the the derivative part of (30) and (31) is a diagonal matrix, whose mth element is given by

$$\left[\frac{\partial}{\partial \mathbf{\Lambda}_i} \operatorname{logdet}\left(\bar{\mathbf{K}}_k\right)\right]_{m,m} = \sum_{n=1}^{N_r} \frac{\left[\mathbf{\Omega}_k\right]_{n,m}}{1 + \sum_{j\neq k} \sum_{q=1}^{M} \left[\mathbf{\Omega}_k\right]_{n,q} \left[\mathbf{\Lambda}_j^{(\ell)}\right]_{q,q}} \tag{32}$$

and

$$\left[\frac{\partial}{\partial \mathbf{\Lambda}_k} \operatorname{logdet}\left(\bar{\mathbf{K}}_{\mathrm{eve},k}\right)\right]_{m,m} = \sum_{n=1}^{N_{\mathrm{eve}}} \frac{\left[\mathbf{\Omega}_{\mathrm{eve}}\right]_{n,m}}{1 + \sum_{q=1}^{M} \left[\mathbf{\Omega}_{\mathrm{eve}}\right]_{n,q} \left[\mathbf{\Lambda}_k^{(\ell)}\right]_{q,q}} \tag{33}$$

respectively.

According to [23], [25], the solution sequence $\left\{\Lambda_1^{(\ell)}, ..., \Lambda_K^{(\ell)}\right\}_{\ell=0}^{\infty}$ generated by the proposed approach in (29) is proven to be convergent and approximately optimal of the original problem in (21).

To reduce the computational complexity of the expectation operation, we further employ the large dimensional random matrix theory [26]–[28] to calculate the deterministic equivalent (DE) of $R_{k,1}\left(\mathbf{\Lambda}_1, ..., \mathbf{\Lambda}_K\right)$ in each iteration, rather than utilize Monte-Carlo method averaging over the channels. In particular, the DE of $R_{k,1}\left(\mathbf{\Lambda}_1, ..., \mathbf{\Lambda}_K\right)$ in the ℓth iteration is given by

$$\bar{R}_{k,1}^{(\ell)}\left(\mathbf{\Lambda}_1, ..., \mathbf{\Lambda}_K\right) = \operatorname{logdet}\left(\mathbf{I}_M + \mathbf{\Gamma}_k^{(\ell)} \mathbf{\Lambda}_k\right) + \operatorname{logdet}\left(\widetilde{\mathbf{\Gamma}}_k^{(\ell)} + \bar{\mathbf{K}}_k^{(\ell)}\right) - \operatorname{tr}\left(\mathbf{I}_{N_r} - \left(\widetilde{\mathbf{\Phi}}_k^{(\ell)}\right)^{-1}\right), \tag{34}$$

where $\mathbf{\Gamma}_k^{(\ell)} \in \mathbb{C}^{M\times M}$, $\widetilde{\mathbf{\Gamma}}_k^{(\ell)} \in \mathbb{C}^{N_r\times N_r}$ are given by

$$\mathbf{\Gamma}_k^{(\ell)} = \mathbf{\Pi}_k\left(\left(\bar{\mathbf{K}}_k^{(\ell)}\widetilde{\mathbf{\Phi}}_k^{(\ell)}\right)^{-1}\right), \tag{35}$$

$$\widetilde{\mathbf{\Gamma}}_k^{(\ell)} = \mathbf{\Xi}_k\left(\left(\mathbf{\Phi}_k^{(\ell)}\right)^{-1}\mathbf{\Lambda}_k^{(\ell)}\right). \tag{36}$$

$\mathbf{\Phi}_k^{(\ell)} \in \mathbb{C}^{M\times M}$ and $\widetilde{\mathbf{\Phi}}_k^{(\ell)} \in \mathbb{C}^{N_r\times N_r}$ are given by the iterative equations

$$\mathbf{\Phi}_k^{(\ell)} = \mathbf{I}_M + \mathbf{\Pi}_k\left(\left(\bar{\mathbf{K}}_k^{(\ell)}\widetilde{\mathbf{\Phi}}_k^{(\ell)}\right)^{-1}\right)\mathbf{\Lambda}_k^{(\ell)} \tag{37}$$

$$\widetilde{\mathbf{\Phi}}_k^{(\ell)} = \mathbf{I}_{N_r} + \mathbf{\Xi}_k\left(\left(\mathbf{\Phi}_k^{(\ell)}\right)^{-1}\mathbf{\Lambda}_k^{(\ell)}\right)\left(\bar{\mathbf{K}}_k^{\ell}\right)^{-1} \tag{38}$$

where $\mathbf{\Pi}_k\left(\mathbf{X}\right) \triangleq \mathbb{E}\left\{\mathbf{G}_k^H\mathbf{X}\mathbf{G}_k\right\} \in \mathbb{C}^{M\times M}$ and $\mathbf{\Xi}_k\left(\mathbf{Y}\right) \triangleq \mathbb{E}\left\{\mathbf{G}_k\mathbf{Y}\mathbf{G}_k^H\right\} \in \mathbb{C}^{N_r\times N_r}$ are both both matrix operations that produce diagonal matrices and the diagonal entries are given by

$$\left[\mathbf{\Pi}_k\left(\mathbf{X}\right)\right]_{m,m} = \mathrm{tr}\left\{\mathrm{diag}\left\{\left[\mathbf{\Omega}_k\right]_{:,m}\right\}\mathbf{X}\right\}, \tag{39}$$

$$\left[\mathbf{\Xi}_k\left(\mathbf{Y}\right)\right]_{n,n} = \mathrm{tr}\left\{\mathrm{diag}\left\{\left(\left[\mathbf{\Omega}_k\right]_{n,:}\right)^T\right\}\mathbf{Y}\right\}, \tag{40}$$

respectively.

Compared with utilizing Monte-Carlo method to average over the channels fo expectation operation, the DE can be calculated in a fer iterations with a quite tight accuracy. In addition $\bar{R}_k^{(\ell)}\left(\mathbf{\Lambda}_1, ..., \mathbf{\Lambda}_K\right)$ is strictly concave on $\left(\mathbf{\Lambda}_1, ..., \mathbf{\Lambda}_K\right)$ [29], [30]. Via replacing $R_{k,1}\left(\mathbf{\Lambda}_1, ..., \mathbf{\Lambda}_K\right)$ with its DE in (34) in each iteration, we turn to consider the following series of convex programs instead of (29)

$$\begin{aligned}\left[\mathbf{\Lambda}_1^{(\ell+1)}, ..., \mathbf{\Lambda}_K^{(\ell+1)}\right] = \underset{\mathbf{\Lambda}_1,...,\mathbf{\Lambda}_K}{\arg\max}\ \min_k \Bigg\{ & \bar{R}_{k,1}^{(\ell)}\left(\mathbf{\Lambda}_1, ..., \mathbf{\Lambda}_K\right) \\ & -R_{k,2}\left(\mathbf{\Lambda}_1^{(\ell)}, ..., \mathbf{\Lambda}_K^{(\ell)}\right) - \sum_{i=1}^{K}\mathrm{tr}\left\{\left(\frac{\partial}{\partial\mathbf{\Lambda}_i}R_{k,2}\left(\mathbf{\Lambda}_1^{(\ell)}, ..., \mathbf{\Lambda}_K^{(\ell)}\right)\right)^T\left(\mathbf{\Lambda}_i - \mathbf{\Lambda}_i^{(\ell)}\right)\right\}\Bigg\} \\ \text{subject to}\quad & \mathrm{tr}\left(\sum_{k=1}^{K}\mathbf{\Lambda}_k\right) \le P \\ & \mathbf{\Lambda}_k \succeq \mathbf{0}, \quad k = 1, ..., K,\end{aligned} \tag{41}$$

Algorithm 1 Beam Domain Secure Transmission Power Allocation Algorithm

```
Initialize $\left(\mathbf{\Lambda}_1^{(0)},\ldots,\mathbf{\Lambda}_K^{(0)}\right)$, $\bar{\mathcal{R}}\left(\mathbf{\Lambda}_1^{(-1)},\ldots,\mathbf{\Lambda}_K^{(-1)}\right)=0$, threshold $\epsilon$, and iteration index $\ell=-1$.
repeat
    $\ell=\ell+1$;
    Calculate DE $\bar{\mathcal{R}}_{k,1}\left(\mathbf{\Lambda}_1^{(\ell)},\ldots,\mathbf{\Lambda}_U^{(\ell)}\right)$ by (34).
    Calculate $\bar{\mathcal{R}}\left(\mathbf{\Lambda}_1^{(\ell)},\ldots,\mathbf{\Lambda}_U^{(\ell)}\right)=\min_k\left(\bar{R}_{k,1}\left(\mathbf{\Lambda}_1^{(\ell)},...,\mathbf{\Lambda}_K^{(\ell)}\right)-R_{k,2}\left(\mathbf{\Lambda}_1^{(\ell)},...,\mathbf{\Lambda}_K^{(\ell)}\right)\right)$.
    Calculate the gradient of $\mathcal{R}_{k,2}\left(\mathbf{\Lambda}_1^{(\ell)},\ldots,\mathbf{\Lambda}_K^{(\ell)}\right)$ by (32) and (33).
    Calculate $\left(\mathbf{\Lambda}_1^{(\ell+1)},\ldots,\mathbf{\Lambda}_K^{(\ell+1)}\right)$ via solving (41)
until $\left|\bar{\mathcal{R}}^{(\ell)}-\bar{\mathcal{R}}^{(\ell-1)}\right|\geq\epsilon$.
Return $(\mathbf{\Lambda}_1,\ldots,\mathbf{\Lambda}_U)=\left(\mathbf{\Lambda}_1^{(\ell)},\ldots,\mathbf{\Lambda}_U^{(\ell)}\right)$
```

The proposed beam domain secure transmission power allocation algorithm is described in Algorithm 1.

IV. Simulation Results

V. Conclusion

Appendix

Proof of Proposition 1

From (24) (25) and (40), we can observe that for all k, the off-diagonal entries of $\mathbf{V}^H\mathbf{Q}_k\mathbf{V}$ do not affect the value of $\bar{\mathbf{K}}_k$ and $\bar{\mathbf{K}}_{\text{eve},k}$. Use the similar technique in [31], we can prove $\mathbf{V}^H\mathbf{Q}_k\mathbf{V}_{\forall k}$ should be diagonal to maximize $R_{k,\text{lb}}^{\text{sec}}$ in (17) for all UT k. Moreover, the transmit power $\text{tr}\left(\sum_{k=1}^K\mathbf{\Lambda}_k\right)$ is only related to the diagonal entries of $\mathbf{V}^H\mathbf{Q}_k\mathbf{V}$ for all k. Therefore we can obtain the conclusion that the objective (12) can be maximized when $\mathbf{V}^H\mathbf{Q}_k\mathbf{V}_{\forall k}$ is diagonal. This concludes the proof.

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