Section 7 Problem

In this problem, you will implement a symbolic computation for a fluid dynamics equation, known as the unsteady, two-dimensional Euler equations:

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x}(U) + \frac{\partial G}{\partial x}(U) = 0,$$

where $U = [\rho, \rho u, \rho v, e]^T, V = [\rho, u, v, p]^T$

$F = [\rho u, \rho u^2 + p, \rho uv, (e + p)u]^T$, $G = [\rho v, \rho vu, \rho v^2 + p, (e + p)v]^T$, $p = (\gamma - 1)\left(e - \rho\frac{u^2 + v^2}{2}\right)$, and γ is a constant. The vector U is known as the *conservative* form of the flow variables and V is the *primitive* form.

One may rewrite the conservation form of the Euler equations above in wave speed form as

$$\frac{\partial V}{\partial t} + A\frac{\partial V}{\partial x} + B\frac{\partial V}{\partial x} = 0.$$

where

$$A = \left[\frac{\partial U}{\partial V}\right]^{-1}\frac{\partial F}{\partial V} \qquad B = \left[\frac{\partial U}{\partial V}\right]^{-1}\frac{\partial G}{\partial V}.$$

The *symbolic* eigenvalue decomposition of A and B plays an important role in the construction of some CFD schemes.

Task 1

Use MATLAB's symbolic toolbox to compute A and B symbolically. The expressions for A and B should *only* involve the terms of ρ, u, v, p, γ.

Task 2

Use the expression $\gamma p = \rho c^2$ to eliminate p and γ from these expressions for A and B.

Task 3

Finally, compute the eigenvalue decomposition of A and B symbolically.

Checkpoint

Please answer the following questions and put the answers in the EdX page:

(A) What is the second diagonal term for A after being formed in Task 1?

(B) What is the third diagonal term for B after being formed in Task 2?

(C) What is component (3,2) of the eigenvector matrix for A? Provide the exact expression from MATLAB.

(D) What is component (1,4) of the eigenvector matrix for B? Provide the exact expression from MATLAB.

(E) What are the unique eigenvalues for A? Provide in list format [x, x, x] with spaces in between variables.

(F) What are the unique eigenvalues for B? Provide in list format [x, x, x] with spaces in between variables.