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# Statistical Beamforming for Interference Mitigation in Multi-cell Massive MIMO Systems

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**Abstract:** This paper proposes a statistical beamforming approach to mitigate interference in multi-cell massive multiple-input multiple-output systems. The proposed approach projects the signal subspace onto the null space of the interference to form the weight vector with the statistical channel state information. In contrast to the existing methods which only utilize the partial null space of the interference, the proposed approach can collect more signal when mitigating the interference. In addition, the large system analysis demonstrates that the interference is eliminated with uniform rectangular arrays. Numerical results show that the proposed approach can achieve higher sum rate than the existing methods and verify the performance analysis.

**Keywords:** statistical beamforming, massive multiple-input multiple-output (MIMO), interference, uniform rectangular array (URA), direction-of-arrival (DOA)

## 1 Introduction

Multiple-input multiple-output (MIMO) technology has been widely employed for its capability of improving spectral efficiency [1]. Recently, large-scale or massive MIMO technology has been proposed. In typical massive MIMO systems, the base station (BS) is equipped with a hundred or a few hundred antennas, and serves tens of user terminals (UTs) simultaneously. Theoretical analysis demonstrates that tremendous capacity gains can be achieved and energy efficiency can be largely improved [2], [3]. However, the performance of multi-cell massive MIMO is constrained by the inter-cell interference (ICI). Since the channel coherence interval is limited, the pilots employed are correlated. Thus, the channel estimates as well as the signal detection are affected by the ICI. As the number of the BS antennas increases, the ICI remains and other effects become weak. This phenomenon is also known as the pilot contamination effect, and has been extensively studied [3]–[5].

A few approaches have been proposed to deal with the ICI in multi-cell massive MIMO systems. It is shown that subspace-based approaches [5]–[7] and cooperation-based approaches [8]–[10] are capable of mitigating the ICI in massive MIMO systems. However, these approaches either rely on the independence of the channel vectors [5]–[9] or necessitate the employment of linear antenna arrays [10]. In fact, the channel responses are correlated [11] and two-dimensional (2D) arrays (i.e., the BS antennas are placed in both the horizontal and the vertical domains) are more feasible than linear arrays [12] in massive MIMO systems.

Recently, several beamforming approaches have been proposed to mitigate the interference or achieve array gain in massive MIMO systems [13]–[21]. In these approaches, instantaneous channel state information (CSI) is essential in mitigating the interference [15]–[21]. Meanwhile, the statistical CSI can also be utilized to mitigate the interference. Moreover, the statistical CSI can be acquired in a simpler manner than the instantaneous CSI. However, the beamforming approaches that only rely on the statistical CSI cannot fully utilize the available CSI. For example, the approach in Refs [22], [23] maximizes the signal-to-interference-plus-noise-ratio (SINR) with the generalized eigenvector of the covariance matrix of the desired channel and that of the interference channel. In fact, the beamforming weight vector in Refs [22], [23] is the projection of the signal subspace onto the partial null space of the interference. Additionally, the approach in Refs [24], [25] does not take the column space of channels that cause little interference as the null space of the interference, which means the null space of the interference is not fully utilized. Since more signal power can be collected when more dimensions of the null space of the interference is utilized, the performance of the approaches in Refs [22]–[25] needs further improvement.

In this work, a statistical beamforming approach for interference mitigation in multi-cell massive MIMO systems is proposed. With the second-order statistical CSI, the proposed approach estimates the signal subspace and the null space of the interference. Then, the estimated signal subspace is projected onto the estimated null

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space to form the beamforming weight vector. More specifically, the main contributions of this work are three-fold.

(1) The proposed statistical beamforming approach only utilizes the second-order statistics of the channels to mitigate the interference in massive MIMO systems. In contrast, the known beamforming approaches that mitigate the interference for massive MIMO systems necessitate the instantaneous CSI [15]–[20]. Thus, the proposed approach is more feasible for massive MIMO systems than the approaches in Refs [15]–[20] when only the statistical CSI is available.

(2) The proposed statistical beamforming approach can collect more signal power than the existing statistical beamforming approaches in Refs [22]–[25] when mitigating the interference. This is because the null space of the interference is fully utilized in the proposed approach, while only part of the null space of the interference is utilized with the approaches in Refs [22]–[25].

(3) The asymptotic performance of the proposed approach with uniform rectangular arrays (URAs) is analyzed. Based on the relation between the channel covariance matrix and the distribution of the direction-of-arrivals (DOAs), it is shown that the column spaces of the channel covariance matrices of any two UTs with non-overlapping DOA regions are orthogonal in the large system limit. As a result, the interference tends to be completely eliminated as the number of the URA antennas increases.

This paper is organized as follows. Section 2 presents the system model and the assumptions. Section 3 contains the proposed statistical beamforming approach. In Section 4, the asymptotic performance of the proposed approach is analyzed. The simulation results are presented in Section 5 and the concluding remarks are provided in Section 6.

*Notations*: Lower-case (upper-case) boldface symbols denote vectors (matrices); $\mathbf{I}_K$ represents the $K \times K$ identity matrix; $(\cdot)^H$ and $\mathbb{E}\{\cdot\}$ denote the conjugate transpose and the expectation, respectively; $||\cdot||$ is the Euclidean norm of a vector; $[\cdot]_j$ is the $j$th element of a vector; and $i$ is the imaginary unit.

## 2 System model

The system considered consists of three rhombic cells. Each cell is served by one BS, which is equipped with a URA, as shown in Figure 1. The same frequency band is

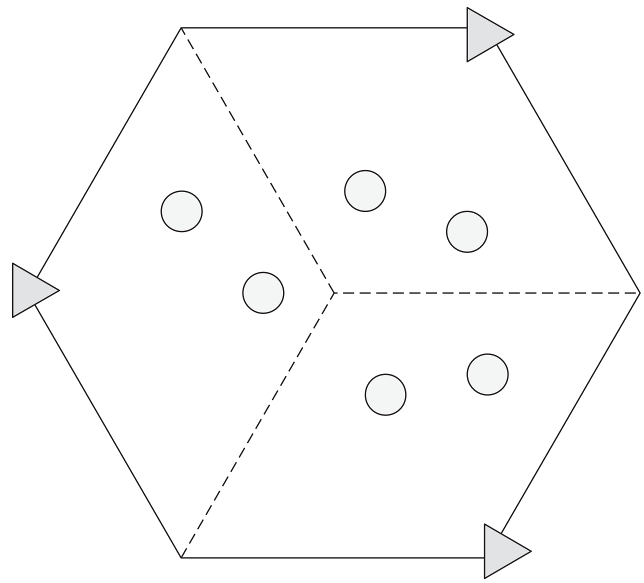

**Figure 1:** Cellular layout and cell geometry. The triangles represent the BSs, the circles represent the UTs.

reused in all the cells, i.e., reuse 1 is employed in this system. This cellular structure is the same as that in Ref. [26], and is similar to that in Ref. [27]. In addition, there is no cooperation among the BSs in two-way transmissions. To simplify the notations, the BS of the $j$th cell is denoted as the $j$th BS.

In the reverse-link transmission, the signal received by the $j$th BS at the $t$th time slot, $t = 1, 2, \ldots, T$, is

$$\mathbf{y}_j(t) = \sum_{l=1}^{L} \sum_{k=1}^{K} \mathbf{h}_{j,l_k}(t) s_{l,k}(t) + \mathbf{n}_j(t) \in \mathbb{C}^{M \times 1}, \tag{1}$$

where $K$ is the number of the UTs in each cell, $L = 3$ is the number of cells; $s_{l,k}(t)$ is the symbol transmitted by the $k$th UT in the $l$th cell at the $t$th time slot; $\mathbf{h}_{j,l_k}(t) \in \mathbb{C}^{M \times 1}$ is the channel from the $k$th UT in the $l$th cell to the $j$th BS at the $t$th time slot; $\mathbf{n}_j(t) \in \mathbb{C}^{M \times 1}$ is the noise received by the $j$th BS at the $t$th time slot. Obviously, the desired channel for the $k$th UT in the $j$th cell at the $t$th time slot is $\mathbf{h}_{j,j_k}(t)$, and the corresponding interference channel is

$$\mathbf{h}^{\mathrm{F}}_{j,j_k}(t) = \sum_{\substack{l=1 \\ l \neq j}}^{L} \sum_{k'=1}^{K} \mathbf{h}_{j,l_{k'}}(t) + \sum_{k'=1\, k' \neq k}^{K} \mathbf{h}_{j,j_{k'}}(t) \in \mathbb{C}^{M \times 1}.$$

In the following, only the reverse-link transmission is considered. The received signal vector, $\mathbf{y}_j(t)$ in eq. (1), is processed by the $j$th BS with the statistical beamforming, and the resulting signal is denoted as

$$\tilde{s}_{j,k}(t) = \mathbf{w}^H_{j,k} \mathbf{y}_j(t), \tag{2}$$

where $\mathbf{w}_{j,k} \in \mathbb{C}^{M\times 1}$ is the statistical beamforming weight vector for the $k$th UT in the $j$th cell, and is of unit norm, i.e., $||\mathbf{w}_{j,k}|| = 1$. Then, the resulting signal, $\tilde{s}_{j,k}(t)$, is employed for signal detection. To be specific, the focus of this work is the design of the statistical beamforming weight vector, $\mathbf{w}_{j,k}$ in eq. (2), to maximize the sum rate of the system.

Before presenting the proposed statistical beamforming approach, the initial assumptions are given as follows.

(1) The symbols, $s_{l,k}(t), l = 1, 2, 3, k = 1, 2, \ldots, K, t = 1, 2, \ldots, T$, are independent and identically distributed (i.i.d.) random variables with covariance one.
(2) The channel responses, $\mathbf{h}_{j,l_k}(t), l = 1, 2, 3, k = 1, 2, \ldots, K, t = 1, 2, \ldots, T$, are composed of large scale fading and small scale fading, and are i.i.d. random variables. The covariance matrices of $\mathbf{h}_{j,j_k}(t)$ and $\mathbf{h}^{\mathrm{F}}_{j,j_k}(t)$ are $\mathbf{S}_{j,k} \in \mathbb{C}^{M\times M}$ and $\mathbf{F}_{j,k} \in \mathbb{C}^{M\times M}$, respectively. The channel responses are unknown but the statistical CSI, $\mathbf{S}_{j,k}$ and $\mathbf{F}_{j,k}$, are known to the BS.
(3) The noise vectors, $\mathbf{n}_j(t), t = 1, 2, \ldots, T$, are composed of i.i.d. Gaussian random variables with zero mean, and the covariance matrix of these noise vectors is $\mathbf{I}_M$.
(4) The symbols, $s_{l,k}(t), l = 1, 2, 3, k = 1, 2, \ldots, K, t = 1, 2, \ldots, T$, and the elements of the noise vectors, $\mathbf{n}_j(t), t = 1, 2, \ldots, T$, are uncorrelated.

With the system model and the aim of this work presented, the proposed statistical beamforming approach will be presented in the next section.

# 3 The proposed statistical beamforming approach

With the knowledge of the statistical CSI, statistical beamforming can be employed to mitigate the interference. The known statistical beamforming approaches [22]–[25] only utilizes the partial null space of the interference to mitigate the interference. Thus, the approaches in Refs [22]–[25] cannot collect the signal power to the uttermost when mitigating the interference. In order to collect more signal power when mitigating the interference, a statistical beamforming approach that makes use of all the information available to form the weight vector is proposed. In this section, first the sum rate expression of the system is derived. Then, a statistical beamforming approach that maximizes the sum rate is proposed.

## 3.1 Sum rate analysis

The waves from the $k$th UT in the $j$th cell to the $j$th BS are taken as the signal, and the waves from other UTs constitute the interference. According to the expression of the received signal at the $j$th BS in eq. (1) and the beamforming process in eq. (2), the SINR of the signal transmitted from the $k$th UT in the $j$th cell to the $j$th BS at the $t$th time slot is

$$\xi_{j,k}(t) = \frac{\left|\left|\mathbf{w}^H_{j,k}\mathbf{h}_{j,j_k}(t)\right|\right|^2}{\left|\left|\mathbf{w}^H_{j,k}\mathbf{h}^{\mathrm{F}}_{j,j_k}(t)\right|\right|^2 + 1}. \tag{3}$$

Then, the ergodic sum rate of the system is

$$R = \sum_{j=1}^{L}\sum_{k=1}^{K}\mathbb{E}\left\{\log_2\left(1 + \xi_{j,k}(t)\right)\right\}. \tag{4}$$

Obviously, the instantaneous CSI is necessary to obtain the optimum beamforming weight vector which maximizes the sum rate. Instead, the statistical CSI, i.e., the channel covariance matrices, can be utilized to obtain a suboptimal beamforming weight vector which achieves a relatively large sum rate. Because of the randomness of the channel responses, the channels, $\mathbf{h}_{j,j_k}(t)$ and $\mathbf{h}^{\mathrm{F}}_{j,j_k}(t)$, span the partial $M$-dimensional Euclidean space randomly. Moreover, the subspace that may be spanned by these channels are the column spaces of $\mathbf{S}_{j,k}$ and $\mathbf{F}_{j,k}$, respectively. Then, it can be observed that the sum rate increases as the weight vector, $\mathbf{w}_{j,k}$, tends to be in the column space of the signal covariance matrix, $\mathbf{S}_{j,k}$, and be in the null space of the interference covariance matrix, $\mathbf{F}_{j,k}$.

In order to achieve a relatively large sum rate with the statistical CSI, a statistical beamforming approach is proposed and is presented in the following.

## 3.2 Statistical beamforming

It is known that the signal covariance matrix, $\mathbf{S}_{j,k}$, and the interference covariance matrix, $\mathbf{F}_{j,k}$, are Hermitian matrices. Thus, their eigenvalue decompositions (EVDs) can be expressed as

$$\mathbf{S}_{j,k} = \mathbf{E}_{\mathrm{S}_{j,k}}\mathbf{D}_{\mathrm{S}_{j,k}}\mathbf{E}^H_{\mathrm{S}_{j,k}}, \tag{5}$$

$$\mathbf{F}_{j,k} = \mathbf{E}_{\mathrm{F}_{j,k}}\mathbf{D}_{\mathrm{F}_{j,k}}\mathbf{E}^H_{\mathrm{F}_{j,k}}, \tag{6}$$

where $\mathbf{E}_{\mathrm{S}_{j,k}} \in \mathbb{C}^{M\times M}$ and $\mathbf{E}_{\mathrm{F}_{j,k}} \in \mathbb{C}^{M\times M}$ are composed of the eigenvectors of $\mathbf{S}_{j,k}$ and $\mathbf{F}_{j,k}$, respectively; $\mathbf{D}_{\mathrm{S}_{j,k}} \in \mathbb{R}^{M\times M}$ and $\mathbf{D}_{\mathrm{F}_{j,k}} \in \mathbb{R}^{M\times M}$ are diagonal matrices of the eigenvalues of $\mathbf{S}_{j,k}$ and $\mathbf{F}_{j,k}$, respectively. Additionally, the eigenvalues are sorted in descending order from the upper left corner of these diagonal matrices.

Then, in order to achieve a relatively large sum rate in eq. (4), the weight vector, $\mathbf{w}_{j,k}$ in eq. (3), should be in the column space of $\mathbf{E}_{\mathrm{S}_{j,k}}$ and in the null space of $\mathbf{E}_{\mathrm{F}_{j,k}}$. However, the overlapping space of the column space of $\mathbf{E}_{\mathrm{S}_{j,k}}$ and the null space of $\mathbf{E}_{\mathrm{F}_{j,k}}$ may be multidimensional. Therefore, the weight vector, $\mathbf{w}_{j,k}$, cannot occupy the whole overlapping space of the column space of $\mathbf{E}_{\mathrm{S}_{j,k}}$ and the null space of $\mathbf{E}_{\mathrm{F}_{j,k}}$. This problem can be solved by employing part of the column space instead of employing the whole column space to form the weight vector. Hence, the subspace spanned by the first column of $\mathbf{E}_{\mathrm{S}_{j,k}}$ is taken as the approximate column space of $\mathbf{E}_{\mathrm{S}_{j,k}}$, which is denoted as $\mathbf{c}_{\mathrm{S}_{j,k}} \in \mathbb{C}^{M\times 1}$. Meanwhile, the subspace spanned by the first $c_{j,k}$ columns of $\mathbf{E}_{\mathrm{F}_{j,k}}$ is denoted as the column space of $\mathbf{E}_{\mathrm{F}_{j,k}}$, where $c_{j,k}$ is the dimension of the column space of $\mathbf{E}_{\mathrm{F}_{j,k}}$ and satisfies $1 \le c_{j,k} \le M$. In accordance, the last $M - c_{j,k}$ columns of $\mathbf{E}_{\mathrm{F}_{j,k}}$ constitute the null space of $\mathbf{E}_{\mathrm{F}_{j,k}}$, which is denoted as $\mathbf{N}_{\mathrm{F}_{j,k}} \in \mathbb{C}^{M\times c_{j,k}}$. Obviously, the subspace spanned by the first column of $\mathbf{E}_{\mathrm{S}_{j,k}}$ contains most of the signal power from the $k$th UT in the $j$th cell, and the subspace spanned by the first $c_{j,k}$ columns of $\mathbf{E}_{\mathrm{F}_{j,k}}$ contains all the interference power to the signal from the $k$th UT in the $j$th cell.

After that, similar to the idea in Refs [22], [23], the projection of the approximate column space, $\mathbf{c}_{\mathrm{S}_{j,k}}$, on the null space, $\mathbf{N}_{\mathrm{F}_{j,k}}$, is employed as the weight vector, which is

$$\mathbf{w}_{j,k} = \frac{\mathbf{N}_{\mathrm{F}_{j,k}}\mathbf{N}_{\mathrm{F}_{j,k}}^{H}\mathbf{c}_{\mathrm{S}_{j,k}}}{\|\mathbf{N}_{\mathrm{F}_{j,k}}\mathbf{N}_{\mathrm{F}_{j,k}}^{H}\mathbf{c}_{\mathrm{S}_{j,k}}\|}. \tag{7}$$

Note that this is the proposed statistical beamforming weight vector. Figure 2 illustrates the projection of the approximate column space, $\mathbf{c}_{\mathrm{S}_{j,k}}$, on the null space, $\mathbf{N}_{\mathrm{F}_{j,k}}$. It can be seen that when the approximate column space is not in the null space, the projection removes the component of the approximate column space, $\mathbf{c}_{\mathrm{S}_{j,k}}$, that is orthogonal to the null space, $\mathbf{N}_{\mathrm{F}_{j,k}}$, and this leads to the loss of the array gain. In other words, the projection is a compromise between the array gain and the ability of mitigating the interference.

In contrast to the proposed approach which achieves a relatively large value in eq. (4), the approach in Refs [22], [23] maximizes

$$\frac{\mathbf{w}_{j,k}^{H}\mathbf{S}_{j,k}\mathbf{w}_{j,k}}{\mathbf{w}_{j,k}^{H}\mathbf{F}_{j,k}\mathbf{w}_{j,k}+1}.$$

Then, the weight vector is the projection of the signal subspace onto the partial null space of the interference. On the other hand, the approach in Refs [24], [25] estimates the null spaces of the interference of all the UTs, i.e., the null spaces of the channel covariance matrices of $\mathbf{h}_{j,l_{k'}}(t), l \neq j$ and $\mathbf{h}_{j,j_{k'}}(t), k' \neq k$. Consequently, the column spaces of channels that cause little interference are not included in the null space of the interference, which means the partial null space of the interference rather than the whole null space is utilized. However, the proposed approach utilizes the whole null space of the interference.

Figure 3 depicts the differences between the proposed approach and the approaches in Refs [22]–[25]. The left subgraph corresponds to the approaches in Refs [22]–[25], where the approximate column space, $\mathbf{c}_{\mathrm{S}_{j,k}}$, is projected into parts of the null space, $\mathbf{N}_{\mathrm{F}_{j,k}}$; The right subgraph corresponds to the proposed approach, in which the approximate column space is projected into the whole null space. As can be seen, the weight vector, $\mathbf{w}_{j,k}$, is in the null space in both subgraphs, which means the interference is eliminated with all the approaches. Meanwhile, the weight vector is more correlated with the approximate column vector in the right subgraph than in the left subgraph. Thus, the proposed approach can collect more signal power than the approaches in Refs [22]–[25].

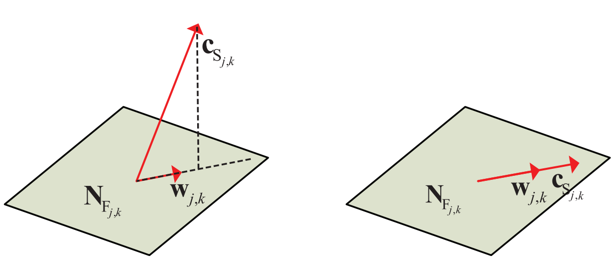


**Figure 2:** Illustration of the projection of the approximate column space, $\mathbf{c}_{\mathrm{S}_{j,k}}$, onto the null space, $\mathbf{N}_{\mathrm{F}_{j,k}}$. In the left subgraph, the approximate column space is not in the null space, the weight vector, $\mathbf{w}_{j,k}$, is not in the approximate column space. In the right subgraph, the approximate column space is in the null space, the weight vector is in the approximate column space.

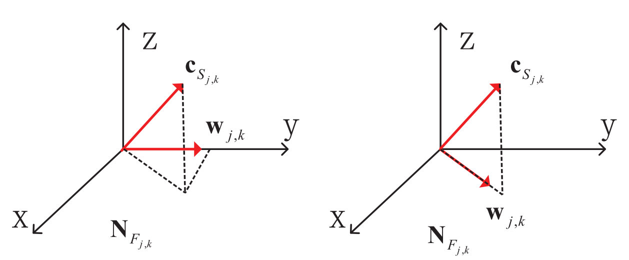


**Figure 3:** Comparison of the proposed approach and the approaches in Refs [22]–[25]. In the left subgraph, the y axis, i.e., the partial null space, is utilized to obtain the weight vector, and this corresponds to the approaches in Refs [22]–[25]; in the right subgraph, the x-y plane, i.e., the whole null space, is utilized to obtain the weight vector, and this corresponds to the proposed approach.

In comparison with those existing beamforming approaches for massive MIMO systems in Refs [15]–[20], the main novelty of the proposed approach is that the instantaneous CSI is not necessary for mitigating the interference. For example, the approach in Ref. [18] first estimates the instantaneous channel vector. Then, the channel estimate and the null space of the interference are utilized to mitigate the interference. The approach in Ref. [20] utilizes the instantaneous CSI to form the zero forcing beamforming matrix, which is employed to mitigate the interference. Although these approaches are efficient in mitigating the interference, they are not feasible for scenarios that only the statistical CSI is available. Therefore, the proposed approach is more feasible for mitigating the interference in massive MIMO systems than the approaches in Refs [15]–[20] when only the statistical CSI is available.

For clarity, the algorithm corresponding to the proposed statistical beamforming approach is presented as follows.

Algorithm 1: The proposed statistical beamforming approach

Step 1) Calculate the EVDs of $\mathbf{S}_{j,k}$ and $\mathbf{F}_{j,k}$ as eqs (5) and (6).

Step 2) Pick the first column of $\mathbf{E}_{\mathrm{S}_{j,k}}$ to form the approximate column space, $\mathbf{c}_{\mathrm{S}_{j,k}}$. Pick the last $M-c_{j,k}$ columns of $\mathbf{E}_{\mathrm{F}_{j,k}}$ to form the null space, $\mathbf{N}_{\mathrm{F}_{j,k}}$.

Step 3) Form the statistical beamforming weight vector using eq. (7).

Till now, it is shown that the proposed statistical beamforming approach can collect more signal power than the approaches in Refs [22]–[25] when mitigating the interference. Additionally, the performance of the proposed approach depends on the relation between the approximate column space, $\mathbf{c}_{\mathrm{S}_{j,k}}$, and the null space, $\mathbf{N}_{\mathrm{F}_{j,k}}$, as depicted in Figure 2. In the next section, the performance of the proposed approach in the large system limit will be presented.

## 4 Performance analysis

The proposed statistical beamforming approach in the last section is employed for mitigating the interference in massive MIMO systems. However, the beamforming performance in the large system limit is unclear. In this section, the limiting SINR of the proposed statistical beamforming approach with infinite number of the URA antennas is analyzed.

From the presentations in the last section, it is known that the proposed beamforming weight vector is the projection of the approximate column space of the desired signal onto the null space of the interference. In addition, the SINR depends on the relation between the approximate column space and the null space, as shown in Figure 2. Here, the asymptotic relation between the approximate column space and the null space in the large system limit is analyzed, which is employed to derive the asymptotic SINR.

Before presenting the result, the channel model should be re-expressed regarding the property of the URA. For BSs equipped with URAs, the channel vector in eq. (1) can be re-written as

$$\mathbf{h}_{j,l_k}(t)=\sum_{n=1}^{N}\frac{\gamma_{j,l_k,n}(t)}{\sqrt{N}}\,\mathbf{a}(\theta_{j,l_k,n}(t),\phi_{j,l_k,n}(t)), \tag{8}$$

where $N$ is the number of the paths from each UT; $\gamma_{j,l_k,n}(t)$, $\theta_{j,l_k,n}(t)$, and $\phi_{j,l_k,n}(t)$ are the channel response, the azimuth DOA, and the elevation DOA of the $n$th path from the $k$th UT in the $l$th cell to the $j$th BS at the $t$th time slot, respectively; $\mathbf{a}(\theta_{j,l_k,n}(t),\phi_{j,l_k,n}(t))\in\mathbb{C}^{M\times 1}$ is the array response vector of the URA, and is defined as Ref. [28]

$$\begin{aligned}&[\mathbf{a}(\theta_{\mathbf{j},\mathbf{l_k},\mathbf{n}}(t),\phi_{j,l_k,n}(t))]_m\\&\quad=\exp(iu\sin(\phi_{j,l_k,n}(t))[(m_{\mathrm{x}}-1)\times cos(\theta_{j,l_k,n}(t))\\&\qquad+(m_{\mathrm{y}}-1)\sin(\theta_{j,l_k,n}(t))]),\ m=(m_{\mathrm{y}}-1)M_{\mathrm{x}}\\&\qquad+m_{\mathrm{x}},\ m_{\mathrm{x}}=1,2,\ldots,M_{\mathrm{x}},\ m_{\mathrm{y}}=1,2,\ldots,M_{\mathrm{y}},\end{aligned} \tag{9}$$

where $u=2\pi d/\lambda$, $d$ is the distance between two adjacent antenna elements in the URA, and $\lambda$ is the wavelength of the signals transmitted by the UTs; $M_{\mathrm{x}}$ and $M_{\mathrm{y}}$ are the numbers of the antenna elements of the URA in the azimuth direction and the elevation direction, respectively. Obviously, $M_{\mathrm{x}}$ and $M_{\mathrm{y}}$ satisfy $M=M_{\mathrm{x}}M_{\mathrm{y}}$. Additionally, The DOAs, $\theta_{j,l_k,n}(t),\phi_{j,l_k,n}(t),n=1,2,\ldots,N,t=1,2,\ldots,T$, are i.i.d. random variables with means, $\bar{\theta}_{j,l_k},\bar{\phi}_{j,l_k}$, and standard deviations, $\sigma_{\theta_{j,l_k}},\sigma_{\phi_{j,l_k}}$. The means are also known as the nominal DOAs, and the standard deviations are usually named as the angular spreads.

Then, the following theorem gives the main result.

**Theorem 1:** When the DOA regions of any two UTs are not overlapping in either the azimuth direction or the elevation direction, the expectation of the SINR, $\xi_{j,k}(t)$ in eq. (3), tends to infinity as the numbers of the URA antennas in the azimuth direction and the elevation direction, $M_{\mathrm{x}}$ and $M_{\mathrm{y}}$, tend to infinity.

*Proof*: Refer to Appendix A for details.

**Remark 1:** For any two UTs not in the same position, they are either not in the same azimuth direction or the same elevation direction. With the proposed beamforming approach, the URA can steer the beam in both the azimuth direction and the elevation direction. Moreover, the precision of the steering of the beam improves as the number of the URA antennas increases. Therefore, the proposed beamforming approach can mitigate the interference more efficiently with larger URAs.

**Remark 2:** The proposed statistical beamforming approach may be extended to circular/cylindrical antenna arrays. Since the array steering vectors of circular/cylindrical antenna arrays can be transformed into virtual linear/rectangular arrays, the proposed approach can be applied on the virtual arrays. Meanwhile, the proposed approach may also be extended to downlink beamforming. For the downlink transmission, the maximization of the signal-to-leakage-plus-noise ratio (SLNR), which is the ratio of the power from the BS to the desired UT and that from the BS to other UTs, can be taken as the design criterion. It is known that this criterion results into suboptimal solutions but the corresponding algorithms are of low computational complexity. Because the expression of the downlink SLNR and that of the uplink SINR are similar, the proposed approach can also be employed in the downlink transmission for suboptimal solution.

The effect of increasing the number of the URA antennas can also be explained with Figure 2. When the number of the URA antennas is finite, the approximate column space, $\mathbf{c}_{S_{j,k}}$, is not in the null space, $\mathbf{N}_{F_{j,k}}$, and the projection of $\mathbf{c}_{S_{j,k}}$ onto $\mathbf{N}_{F_{j,k}}$ causes the loss of the array gain. This corresponds to the left subgraph of Figure 2. As the number of the URA antennas tends to infinity, the approximate column space tends to be in the null space, and the loss of the array gain caused by the projection tens to zero. This corresponds to the transition from the left subgraph of Figure 2 to the right subgraph of Figure 2.

The performance analysis shows that the proposed approach can achieve good performance in massive MIMO systems with URAs. In order to demonstrate the performance of the proposed approach more explicitly, numerical results will be given in the next section.

## 5 Simulation results

In this section, the performance of the proposed approach is evaluated and is compared with the performance of the existing approaches in Refs [22]–[25]. The system parameters are those listed in Table 1 unless additionally stated. Note that the DOAs of the multipaths from the UTs to the BSs in other cells are regarded as the same. Thus, the azimuth angular spread out of the cell, $\sigma_{\theta_{j,l_k}}, l \neq j$, and the elevation angular spread out of the cell, $\sigma_{\phi_{j,l_k}}, l \neq j$, are zeros in the simulations. The numbers of the URA antennas in the azimuth direction and the elevation direction satisfy $M_x = M_y = \sqrt{M}$. The UTs are distributed uniformly in each cell. According to eqs (1), (8), and the assumptions below them, the received signal-to-noise-ratio (SNR) at each URA antenna is $\sigma^2_{\gamma_{j,l_k}} N$. The azimuth DOAs and the elevation DOAs are Gaussian distributed. The means of the azimuth DOA and the

**Table 1:** System parameters.

| | |
|---|---|
| Cell radius (from center to edge) | 300 meters |
| Height of the BS | 30 meters |
| Decay exponent $v$ | 3.8 |
| Shadow fading standard deviation $\sigma_{\mathrm{shad}}$ | 8 dB |
| Minimal distance from the UT to the BS $r_{\min}$ | 30 meters |
| Maximal distance from the UT to the BS $r_{\max}$ | 240 meters |
| Number of cells $L$ | 3 |
| Number of the UTs per-cell $K$ | 10 |
| Number of the URA antennas $M$ | 100 |
| Number of multipaths $N$ | 50 |
| Distance between two adjacent URA antennas $d$ | 0.1 $\lambda$ meters |
| Received SNR $10\log_{10}(\sigma^2_{\gamma_{j,l_k}} N)$ | 10 dB |
| Number of time slots $T$ | 500 |
| Azimuth angular spread in the cell $\sigma_{\theta_{j,j_k}}$ | $\pi/180$ radians |
| Elevation angular spread in the cell $\sigma_{\phi_{j,j_k}}$ | $\pi/360$ radians |
| Azimuth angular spread out of the cell $\sigma_{\theta_{j,l_k}}$ | 0 radian |
| Elevation angular spread out of the cell $\sigma_{\phi_{j,l_k}}$ | 0 radian |

elevation DOA from a UT to a URA are the azimuth angle and the elevation angle between that UT and that URA. Moreover, the DOAs out of the range of the angular spreads are omitted. Each BS employs power control to compensate the large scale fading of the UTs in the coverage area of that BS.

Figure 4 shows the cumulative distribution functions of the SINRs of the proposed beamforming approach with various numbers of the BS antennas. The number of UTs per cell is set as $K = 2$ to decrease the overlapping regions of the DOAs. As can be seen, increasing the number of the BS antennas results in the increase of the SINR. This result verifies Theorem 1, which states that the expectation of the SINR tends to infinity as the numbers of URA antennas in the azimuth direction and the elevation direction tend to infinity.

Figure 5 depicts the sum rate versus the received SNR with the three approaches. As the received SNR increases, the influence of the noise decreases. Thus, the sum rates increase in Figure 5. Meanwhile, the interference becomes the dominant factor that influences the sum rate. Hence, the sum rates become relatively invariant when the received SNR is higher than 0 dB. Additionally, the sum rate with the proposed approach is much higher than that with the approaches in Refs [22]–[25]. This result verifies that the proposed approach performs better than the approaches in Refs [22]–[25] in massive MIMO systems.

In Figure 6, the sum rate versus the number of the URA antennas with the three approaches is shown. As the number of the URA antennas increases, the signal subspace and the interference subspace tend to be orthogonal. Thus, the sum rates of the three approaches increase

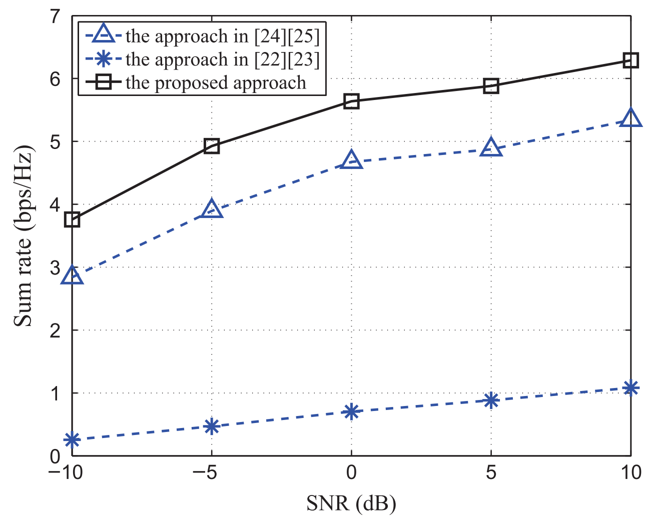


**Figure 5:** Illustration of the sum rate versus the received SNR with the proposed beamforming approach and the beamforming approaches in Refs [22]–[25].

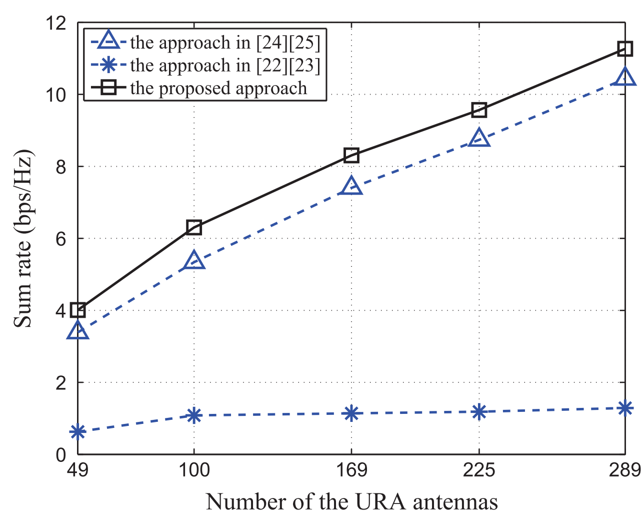


**Figure 6:** Illustration of the the sum rate versus the number of the URA antennas with the proposed beamforming approach and the beamforming approaches in Refs [22]–[25].

with the increase of the number of the URA antennas. This result also verifies the analysis result of the proposed beamforming approach in Theorem 1. Moreover, Figure 6 also shows that the sum rate with the proposed approach is much higher than that with the approaches in Refs [22]–[25].

In Figure 7, the sum rate versus the number of the UTs with the three approaches is illustrated. It can be seen that the sum rates decrease with the increase of the number of the UTs in each cell. This is because the interference increases as the number of the UTs increases. This figure also shows that the sum rate with the proposed approach

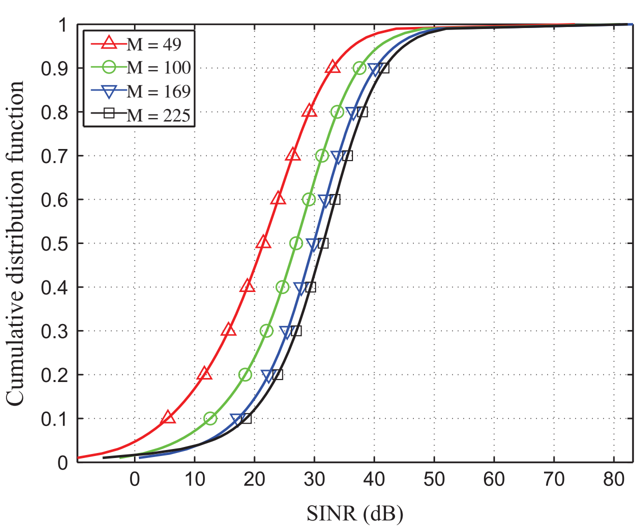


**Figure 4:** Illustration of the cumulative distribution functions of the proposed beamforming approach with various numbers of the BS antennas.

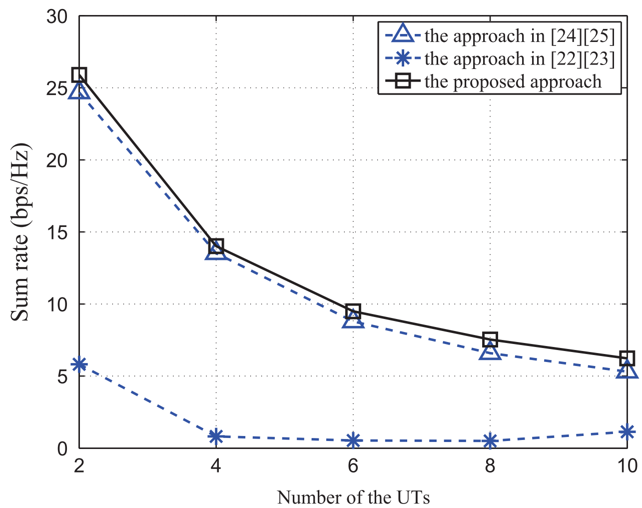


**Figure 7:** Illustration of the the sum rate versus the number of the UTs in each cell with the proposed beamforming approach and the beamforming approaches in Refs [22]–[25].

is higher than that with the approaches in Refs [22]–[25]. This result demonstrates that the proposed approach is able to serve more UTs than the approaches in Refs [22]–[25] in massive MIMO systems.

## 6 Conclusion

In this paper, the interference is tackled with the statistical beamforming in massive MIMO systems. The performance of multi-cell massive MIMO systems is constrained by the ICI. By employing the statistical beamforming with the URAs, the interference is mitigated according to the separation of the azimuth DOAs and the elevation DOAs of the UTs. The proposed approach utilizes the null space of the interference, while only the partial null space is taken into account in the existing methods. Thus, the proposed approach collects more signal power than the existing methods, which is crucial to the beamforming performance. Moreover, the performance of the proposed approach is analyzed, and it is shown that the interference tends to be eliminated as the number of the URA antennas increases. Simulations verify the analysis and demonstrate that the proposed approach performs better than the existing methods.

## Appendix A

Proof of Theorem 1

Denote the region of the azimuth DOA from the $k'$th UT in the $l$th cell to the $j$th BS as $\theta_{j,l_{k'},\min} \le \theta_{j,l_{k'},n}(t) \le \theta_{j,l_{k'},\max}$, and the region of the elevation DOA from the $k'$th UT in the $l$th cell to the $j$th BS as $\phi_{j,l_{k'},\min} \le \phi_{j,l_{k'},n}(t) \le \phi_{j,l_{k'},\max}$. Based on the channel model in eq. (9), the signal covariance matrix and the interference covariance matrix in eqs (5) and (6) can be expressed as

$$\begin{aligned}\mathbf{S}_{j,k} &= \mathbb{E}\left\{\mathbf{h}_{j,j_k}(t)\mathbf{h}^H_{j,j_k}(t)\right\}\\ &= \sigma_{\gamma_{j,j_k}} \int_{\theta_{j,j_k,\min}}^{\theta_{j,j_k,\max}} \int_{\varphi_{j,j_k,\min}}^{\phi_{j,j_k,\max}} \mathbf{a}(\theta,\phi)\mathbf{a}^H(\theta,\phi)\\ &\quad\times \rho_{j,j_k}(\theta,\phi)\mathrm{d}\theta\mathrm{d}\phi, \qquad (10)\end{aligned}$$

$$\begin{aligned}\mathbf{F}_{j,k} &= \mathbb{E}\left\{\mathbf{h}^{\mathrm{F}}_{j,j_k}(t)(\mathbf{h}^{\mathrm{F}}_{j,j_k}(t))^H\right\}\\ &= \sum_{\substack{l=1\\ l\neq j}}^{L}\sum_{k'=1}^{K} \sigma_{\gamma_{j,l_{k'}}} \int_{\theta_{j,l_{k'},\min}}^{\theta_{j,l_{k'},\max}} \int_{\phi_{j,l_{k'},\min}}^{\phi_{j,l_{k'},\max}} \mathbf{a}(\theta,\phi)\\ &\quad\times \mathbf{a}^H(\theta,\phi)\rho_{j,l_{k'}}(\theta,\phi)\mathrm{d}\theta\mathrm{d}\phi\\ &\quad+ \sum_{\substack{k'=1\\ k'\neq k}}^{L} \sigma_{\gamma_{j,j_{k'}}} \int_{\theta_{j,j_{k'},\min}}^{\theta_{j,j_{k'},\max}} \int_{\phi_{j,j_{k'},\min}}^{\phi_{j,j_{k'},\max}} \mathbf{a}(\theta,\phi)\\ &\quad\times \mathbf{a}^H(\theta,\phi)\rho_{j,j_{k'}}(\theta,\phi)\mathrm{d}\theta\mathrm{d}\phi. \qquad (11)\end{aligned}$$

According to eq. (10), for any $\tilde{\theta}, \tilde{\phi}$, there is

$$\begin{aligned}\frac{1}{M}\mathbf{a}^H(\tilde{\theta},\tilde{\phi})\mathbf{S}_{j,k}\mathbf{a}(\tilde{\theta},\tilde{\phi}) &= \frac{\sigma_{\gamma_{j,j_k}}}{M}\int_{\theta_{j,j_k,\min}}^{\theta_{j,j_k,\max}}\int_{\phi_{j,j_k,\min}}^{\phi_{j,j_k,\max}}\\ &\quad\left|\mathbf{a}^H(\tilde{\theta},\tilde{\phi})\mathbf{a}(\theta,\phi)\right|^2\rho_{j,j_k}(\theta,\phi)\mathrm{d}\theta\mathrm{d}\phi. \qquad (12)\end{aligned}$$

Before deriving the SINR, there is a special property of massive MIMO that should be presented first.

**Proposition 1** *When* $\theta \neq \tilde{\theta}$ *or* $\phi \neq \tilde{\phi}$, *there is* $\left|\mathbf{a}^H(\tilde{\theta},\tilde{\phi})\mathbf{a}(\theta,\phi)\right|/M \to 0$, *as* $M_{\mathrm{x}} \to \infty, M_{\mathrm{y}} \to \infty$.

*Proof*: Refer to appendix B.

When $\tilde{\theta}, \tilde{\phi}$ in eq. (12) satisfy $\tilde{\theta} < \theta_{j,l_{k'},\min}$, or $\tilde{\theta} > \theta_{j,l_{k'},\max}$, or $\tilde{\phi} < \phi_{j,l_{k'},\min}$, or $\tilde{\phi} > \phi_{j,l_{k'},\max}$, applying Lemma 1 on eq. (12) yields

$$\frac{1}{M}\mathbf{a}^H(\tilde{\theta},\tilde{\phi})\mathbf{S}_{j,k}\mathbf{a}(\tilde{\theta},\tilde{\phi}) \to 0, \quad \text{as} \quad M_{\mathrm{x}} \to \infty, M_{\mathrm{y}} \to \infty.$$

Similarly, when $\tilde{\theta}$ is not in the region of $\theta$, cf. eq. (11), or $\tilde{\phi}$ is not in the region of $\phi$, cf. eq. (11), applying Lemma 1 results into

$$\frac{1}{M}\mathbf{a}^H(\tilde{\theta},\tilde{\phi})\mathbf{F}_{j,k}\mathbf{a}(\tilde{\theta},\tilde{\phi}) \to 0, \quad \text{as} \quad M_{\mathrm{x}} \to \infty, M_{\mathrm{y}} \to \infty.$$

According to the above analysis, it can be inferred that $\mathbf{F}_{j,k}$ tends to be rank-deficient as $M_{\mathrm{x}} \to \infty, M_{\mathrm{y}} \to \infty$. Thus, the dimension of the null space of $\mathbf{F}_{j,k}$, i.e., $c_{j,k}$ is smaller than $M$, which means the weight vector $\mathbf{w}_{j,k}$ in eq. (7) satisfies $\mathbf{w}_{j,k}^H \mathbf{F}_{j,k} \mathbf{w}_{j,k} \to 0$ as $M_\mathrm{x} \to \infty, M_\mathrm{y} \to \infty$. In addition, the above analysis also demonstrates that when the DOA regions of any two UTs are not overlapping in either the azimuth direction or the elevation direction, the column space of $\mathbf{S}_{j,k}$ tends to be in the null space of $\mathbf{F}_{j,k}$ as $M_\mathrm{x} \to \infty, M_\mathrm{y} \to \infty$. Hence, $\mathbf{c}_{\mathrm{S}_{j,k}}$, which is the eigenvector that corresponds to the largest eigenvalue of $\mathbf{S}_{j,k}$, tends to be in the null space of $\mathbf{F}_{j,k}$. Then, $\mathbf{c}_{\mathrm{S}_{j,k}}^H \mathbf{N}_{\mathrm{F}_{j,k}} \mathbf{N}_{\mathrm{F}_{j,k}}^H \to \mathbf{c}_{\mathrm{S}_{j,k}}^H$ holds as $M_\mathrm{x} \to \infty, M_\mathrm{y} \to \infty$, which means the limit of $\mathbf{w}_{j,k}$ in eq. (7) can be expressed as

$$\mathbf{w}_{j,k} \to \frac{\mathbf{c}_{\mathrm{S}_{j,k}}}{||\mathbf{c}_{\mathrm{S}_{j,k}}||}, \quad \text{as} \quad M_\mathrm{x} \to \infty,\ M_\mathrm{y} \to \infty. \tag{13}$$

As a result, the limit of the expectation of the SINR in eq. (3) can be written as

$$\mathbb{E}\left\{\xi_{j,k}(t)\right\} \to \frac{\mathbf{c}_{\mathrm{S}_{j,k}}^H \mathbf{S}_{j,k} \mathbf{c}_{\mathrm{S}_{j,k}}}{||\mathbf{c}_{\mathrm{S}_{j,k}}||^2}, \quad \text{as} \quad M_\mathrm{x} \to \infty, M_\mathrm{y} \to \infty. \tag{14}$$

Since $\mathbf{c}_{\mathrm{S}_{j,k}}$ is in the column space of $\mathbf{S}_{j,k}$ in eq. (10), it can be expressed as

$$\mathbf{c}_{\mathrm{S}_{j,k}} = \int_{\theta_{j,j_k,\min}}^{\theta_{j,j_k,\max}} \int_{\phi_{j,j_k,\min}}^{\phi_{j,j_k,\max}} \mathbf{a}(\theta,\phi)\alpha_{j,j_k}(\theta,\phi)\mathrm{d}\theta\mathrm{d}\phi, \tag{15}$$

where $\alpha_{j,j_k}(\theta,\phi)$ is the unknown parameter that forms the eigenvector. Based on eqs (10), (15), and Lemma 1, there is

$$\mathbf{c}_{\mathrm{S}_{j,k}}^H \mathbf{S}_{j,k} \mathbf{c}_{\mathrm{S}_{j,k}} \to \sigma_{\gamma_{j,j_k}} \int_{\theta_{j,j_k,\min}}^{\theta_{j,j_k,\max}} \int_{\phi_{j,j_k,\min}}^{\phi_{j,j_k,\max}} M^2 \alpha_{j,j_k}^2(\theta,\phi) \times \rho_{j,j_k}(\theta,\phi)\mathrm{d}\theta\mathrm{d}\phi, \quad \text{as} \quad M_\mathrm{x} \to \infty, M_\mathrm{y} \to \infty. \tag{16}$$

It can be seen that $\mathbf{c}_{\mathrm{S}_{j,k}}^H \mathbf{S}_{j,k} \mathbf{c}_{\mathrm{S}_{j,k}}$ is proportional to $M^2$. Similarly, the square of the norm of $\mathbf{c}_{\mathrm{S}_{j,k}}$ can be denoted as

$$||\mathbf{c}_{\mathrm{S}_{j,k}}||^2 \to \int_{\theta_{j,j_k,\min}}^{\theta_{j,j_k,\max}} \int_{\phi_{j,j_k,\min}}^{\phi_{j,j_k,\max}} M \alpha_{j,j_k}^2(\theta,\phi)\mathrm{d}\theta\mathrm{d}\phi, \quad \text{as} \quad M_\mathrm{x} \to \infty,\ M_\mathrm{y} \to \infty. \tag{17}$$

It can be found that the square $||\mathbf{c}_{\mathrm{S}_{j,k}}||^2$ is proportional to $M$.

Then, substituting eqs (16) and (17) into eq. (14) yields

$$\mathbb{E}\left\{\xi_{j,k}(t)\right\} \to M\zeta \to \infty, \quad \text{as} \quad M_\mathrm{x} \to \infty, M_\mathrm{y} \to \infty,$$

where $\zeta$ is a variable that is not related to $M$. This result indicates that the expectation of the SINR tends to infinity as the numbers of the URA antennas in the azimuth direction and the elevation direction, $M_\mathrm{x}$ and $M_\mathrm{y}$, tend to infinity.

# Appendix B

Proof of Proposition 1

When $\theta \neq \tilde{\theta}$ or $\phi \neq \tilde{\phi}$, there is

$$\frac{1}{M}\left|\mathbf{a}^H(\tilde{\theta},\tilde{\phi})\mathbf{a}(\theta,\phi)\right| = \frac{1}{M_\mathrm{x}}\left|\sum_{m_\mathrm{x}=1}^{M_\mathrm{x}} \exp(iu(m_\mathrm{x}-1)\beta_\mathrm{x})\right| \times \frac{1}{M_\mathrm{y}}\left|\sum_{m_\mathrm{y}=1}^{M_\mathrm{y}} \exp(iu(m_\mathrm{y}-1)\beta_\mathrm{y})\right| \to 0, \quad \text{as} \quad M_\mathrm{x} \to \infty, M_\mathrm{y} \to \infty,$$

where the manifold vector $\mathbf{a}(\theta,\phi)$ is defined in eq. (9), and

$$\beta_\mathrm{x} = \sin(\phi)\cos(\theta) - \sin(\tilde{\phi})\cos(\tilde{\theta}),$$
$$\beta_\mathrm{y} = \sin(\phi)\sin(\theta) - \sin(\tilde{\phi})\sin(\tilde{\theta})$$

can both be zero only when $\theta = \tilde{\theta}$ and $\phi = \tilde{\phi}$. For the case $\phi = \tilde{\phi} = \pi/2$ and $M_\mathrm{y} = 0$, the limiting result becomes one of the lemmas in Ref. [10]. In other words, the lemma in Ref. [10] is a special case of the limiting result here.

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