A Suboptimal Scheme for Uplink NOMA in 5G Systems

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***Abstract*—Non-orthogonal multiple access (NOMA) scheme is a key technique towards the 5th generation (5G) of wireless communication networks due to its higher spectral efficiency comparing to its orthogonal multiple access (OMA) counterpart. However, the complexity of conventional NOMA scheme is extremely high in massive-access scenes.**

In this paper, we propose a user-pairing based sub-optimal scheme for uplink multiple-access by combining NOMA and OMA to facilitate the massive-access scenes. Furthermore, a user-pairing algorithm with $O(n \log n)$ complexity is also derived. According to the Monte Carlo simulation, the proposed scheme shows only a little throughput loss comparing to the optimal transmission scheme in information-theoretic perspective.

***Index Terms*—5G, non-orthogonal multiple access, NOMA, user-pairing strategy**

I. Introduction

In the past decades, the demand of data transmission over cellular network is increased considerably due to the success of smartphones and tablets, which has motivated the design of the 5th generation (5G) of wireless communication networks [1]. In physical layer of 5G, multiple-access techniques play an important role for supporting the requirement of massive connectivity and high spectral efficiency, which is called massive-access scenes in this paper. In conventional wireless communication networks, e.g. the 4th generation (4G) of wireless communication networks, the multiple access scheme is mainly based on orthogonal multiple access (OMA) [2], e.g. time-division multiple access (TDMA) and orthogonal frequency division multiple access (OFDMA), etc. However, towards 5G, non-orthogonal multiple access (NOMA) has gained a lot of attentions in both academic and industrial circles as a key technique due to its inherent higher spectral efficiency comparing to its OMA counterpart [3]–[5].

In a NOMA system, signals from different users are allowed to be superimposed at the receiver, which is a multiple access channel (MAC) in information-theoretic perspective. According to the achievable rate region of MAC established by network information theory [6], [7], it is capacity-approaching to allow signals from all users to superimpose directly without orthogonal resource allocation, which is called direct-superimposition scheme. In this way, multi-user detection at receiver is much more important. However, even aided by successive interference cancellation or joint decoding, the implementation complexity of optimal multi-user detection in massive-access scenes is still extremely high.

Recently, many simplified NOMA scheme has been proposed to reduce the complexity of multi-user detection. For example, in [8], an elementary signal estimator was introduced for interleave-division multiple access to achieve multi-user iterative detection with linear complexity. And in [9]–[11], signals from different users can be detected separately and therefore easily by adding redundancy via coding or spreading. However, all these schemes are not capacity-approaching due to simplification of multi-user detection or decoding algorithm.

On the other hand, when the number of accessed users is very small, e.g. only 2, many practical capacity-approaching schemes have been proposed. In [12], sum-rate capacity-approaching low-density parity-check (LDPC) coded scheme was proposed for binary-input two-user Gaussian MAC (GMAC). In [13], SC-LDPC was proposed as a universal code for approaching any point in achievable rate region of any binary-input two-user MAC. However, the number of accessed users might be far more than 2 in practical systems and high-order inputs are expected for the case of high spectrum efficiency, therefore these capacity-approaching schemes cannot be adopted directly.

For achieving throughput gain of NOMA comparing to OMA with practical capacity-approaching schemes,

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in this paper, we propose a user-pairing based suboptimal scheme for uplink multiple access by combining NOMA and OMA to facilitate the deployment of massive accesses. Furthermore, a user-pairing algorithm with $O(n\log n)$ complexity is also derived, where n denotes the number of accessed users. According to the Monte Carlo simulation, the proposed scheme shows only a little throughput loss comparing to the optimal direct-superimposition transmission scheme in information-theoretic perspective.

The rest of this paper is organized as follows. In section II, the system model of multiple access channel and its achievable rate region are revealed. The motivation, description and analysis of the proposed scheme is discussed in section III. Some numeric results are shown in section IV to reveal the performance of the proposed scheme. Section V concludes the paper.

II. System Model

The system model of n user uplink multiple access is shown in Fig. 1, where g_1, g_2, ..., g_n denotes the channel gain of users and Z denotes the additive white Gaussian noise (AWGN) at receiver.

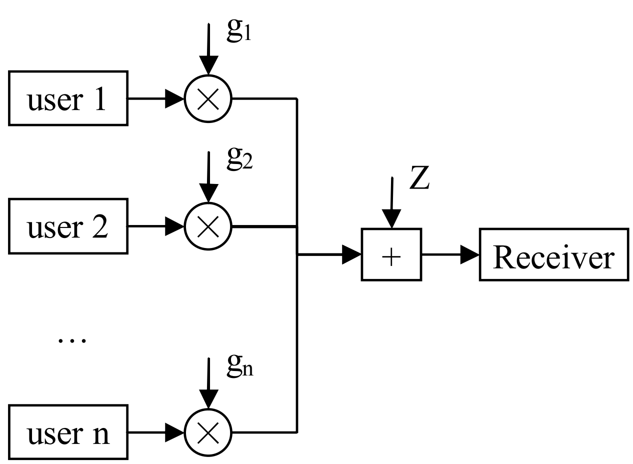

Fig. 1. System model of n user uplink multiple access.

In this paper, full channel state information at receiver as well as symbol-level synchronization is assumed. Therefore, the channel gain of different users can be treated as equivalent transmission power. For example, if the transmission power of user 1 is P_1 and the channel gain is g_1, we can treat user 1 as a user whose transmission power is $P_1 \cdot g_1$. Thus, at a certain complex symbol, the i-th user sends equivalent symbol X_i and the receiver receives the superimposed symbol:

$$Y = \sum_i X_i + Z, \tag{1}$$

where Z $N(0,N_0)$ denotes the AWGN.

According to the achievable rate region of MAC [6], the maximum throughput of such system is restricted by

$$\begin{aligned}\sum_i R_i &\leq I(X_1X_2...X_n;Y)\\ &= \log_2(1+\frac{\sum_i P_i}{N_0}),\end{aligned} \tag{2}$$

where P_i denotes the equivalent transmission power of the i-th user.

III. Proposed User-Pairing Scheme

A. Motivation

Although conventional direct superimposition scheme is able to be optimal in information-theoretic perspective, the construction and decoding complexity is extremely high when the number of user is large. Therefore, we proposed a user-pairing scheme for uplink multiple access. First, the channel resource is split orthogonally into several identically sub-channels via OMA techniques, e.g. FDMA, TDMA or OFDMA. In each channel, only up to two users can transmit their signal simultaneously, i.e. the received signal is only the superposition of two user's signals. Thus, the users in different sub-channel are orthogonal and can be decoded independently. While in the same sub-channel, the number of superimposed users is no more than 2, in which the decoding complexity is acceptable.

For example, we assume a multiple access scenario with 4 activated users $\chi = \{1,2,3,4\}$. The equivalent transmission powers of 4 users are assumed as P_1 = 20 dBm, P_2 = 15 dBm, P_3 = 10 dBm, P_4 = 5 dBm and the power of AWGN is assumed as N_0 = 0 dBm. Hence, the theoretical throughput achieved via direct superposition scheme can be calculated as

$$\sum_{i=1}^{4} R_i \leq \log_2(1+\frac{\sum_{i=1}^{4} P_i}{N_0}) = 7.19 bits/s/Hz. \tag{3}$$

In the case of OMA only, the channel is split into 4 identical sub-channels to allow accesses from 4 different users. Therefore, the transmission resources, e.g. time or bandwidth, as well as the power of AWGN at each sub-channel is reduced to 1/4, and the achievable throughput is

$$\sum_{i=1}^{4} R_i \leq \frac{1}{4}\sum_{i=1}^{4} \log_2(1+\frac{P_i}{N_0/4}) = 6.19 bits/s/Hz. \tag{4}$$

In the proposed scheme, 4 users are partitioned into 2 groups $G_1 = \{1,4\}$ and $G_2 = \{2,3\}$, and then transmit their signals over two identical sub-channels with half transmission resources and the power of AWGN. Similarly, the achievable throughput of the proposed scheme is

$$\sum_{i=1}^{4} R_i \leq \frac{1}{2}\sum_{i=1}^{2}\log_2(1+\frac{\sum_{j\in G_i} P_j}{N_0/2}) = 7.05 bits/s/Hz. \quad (5)$$

As calculated above, in this simple case, the throughput gain of NOMA comparing to OMA only is about 1 bits/s/Hz while the proposed suboptimal scheme can achieve most of the throughput gain. Furthermore, as the number of superimposed users is only 2, the construction and decoding complexity of the proposed scheme is much lower than that of direct-superimposition scheme, in which the number of superimposed users is 4.

B. Extension for multiple users

Generally, the proposed scheme for multi-user access can be described as follows.

Step 1: the activated users are partitioned into k groups according to the equivalent transmission powers of different users, where the number of users in each group is no more than 2. The available channel resources are also split orthogonally into k identically sub-channels and assigned to each group. The number of groups as well as the user-pairing strategy will be discussed in section III-C.

Step 2: the users within the same group transmit their messages over the assigned sub-channels simultaneously.

Step 3: as all sub-channels are pairwise orthogonal, the receiver could decode messages from each sub-channel independently.

In the proposed scheme, as all sub-channel are identical, if the activated users are $\chi = \{1,2,...,n\}$, we can use set partition to represent a specific user-pairing scheme $S = \{G_1, G_2, ..., G_k\}$, where $\bigcup_{G\in S} G = \chi$, $|G_i| \leq 2$ for all $G_i \in S$ and $G_i \bigcap G_j = \emptyset$ for all unique $G_i, G_j \in S$, k denotes the number of sub-channels. For accesses of more users, k is minimized to be $k = \lceil n/2 \rceil$.

As the channel is split into k sub-channel, the transmission resources and the power of AWGN at each sub-channel is one k-th. Hence, for a given sub-channel and group G_i, the throughput of the sub-channel can be calculated according to the achievable rate region of MAC as:

$$\begin{aligned} R(G_i) &\leq I(X_j \text{ for } j\in G_i; Y) \\ &= \frac{1}{k}\log_2(1+\frac{\sum_{j\in G_i} P_j}{N_0/k}), \end{aligned} \quad (6)$$

where P_j denotes the transmission power of j-th user. And the throughput of all sub-channels can be calculated as:

$$\begin{aligned} R \leq R(S) &\triangleq \sum_{G_i\in S} R(G_i) \\ &= \frac{1}{k}\sum_{G_i\in S}\log_2(1+\frac{\sum_{j\in G_i} P_j}{N_0/k}). \end{aligned} \quad (7)$$

Hence, the throughput of the user-pairing scheme is also depends on the user-pairing strategy, which will be discussed in the next section.

C. User-pairing Algorithm

In section III-A, a simple example is given to show that the proposed user-pairing scheme is able to achieve most of the throughput gain of NOMA comparing to OMA. However, the performance of the user-pairing scheme highly depends on the user-pairing strategy. In this section, we will propose an $O(n\log n)$ user-pairing algorithm, where n denotes the number of the activated users.

The proposed user-pairing algorithm can be described as follows. First, the equivalent transmission powers of all users are sorted in ascending order. Then, all users are grouped with head-to-tail pairing, which can be described in detail as follow.

Algorithm 1 Proposed user-pairing algorithm.

Require: $P_1, P_2, ..., P_n, N_0$.

Ensure: The number of split sub-channels k, the user-pairing strategy $S = \{G_1, G_2, ..., G_k\}$.

1: Sort the equivalent transmission powers of all users in ascending order. Without loss of generality, we assume $P_1 < P_2 < ... < P_n$.
2: **if** n is odd **then**
3: $\quad S = \{\{n\}\} \cup \{\{i, 2t-i+1\} | 0 < i \leq \lfloor n/2 \rfloor\}$
4: **else**
5: $\quad S = \{\{i, 2t-i+1\} | 0 < i \leq \lfloor n/2 \rfloor\}$
6: **end if**

As described below, the complexity of the proposed algorithm is $O(n\log n)$, which is mainly determined by the sorting procedure. Furthermore, the following theorem shows the rationality and optimality of the proposed user-pairing algorithm.

Theorem 1 (Optimality of the proposed algorithm)**.** *If the number of split sub-channels $k = \lceil n/2 \rceil$, then for any noise power N_0:*

$$S_0 = S_{max} \triangleq \arg\max_S R(S), \quad (8)$$

where S_0 denotes the user-pairing strategy given by the proposed algorithm.

Proof. See Appendix A. □

IV. Numeric Result

In this section, we compare the proposed scheme to the case of OMA and the theoretical throughput bound of MAC to analyzing the performance of the proposed transmission scheme. As an example, the distribution of users are referenced in [14], wherein all users are assumed uniformly located in a circular region around the receiver between d_{min} and d_{max} to simulating the access between handheld/mobile devices and a base station. Hence, probability density function of the distance from a user to base station can be given by:

$$f_D(d) = \begin{cases} \frac{2d}{d_{max}^2 - d_{min}^2} & d_{min} \le d \le d_{max} \\ 0 & \text{otherwise} \end{cases}. \quad (9)$$

And the signal-to-noise ratio (SNR) of the user with given distance d can be obtained as:

$$P_i/N_0 = L_2 d^{-L_1}, \quad (10)$$

where L_1 denotes the pass loss exponent and L_2 is a constant value related to the transmission power and the power of AWGN at receiver. As an example, we use d_{min} = 1, d_{max} = 10, L_1 = 3 and $L_2 = 10^3$ in our simulations.

As shown in Fig. 2, the theoretical throughput bound of MAC, the achievable throughput of OMA and the proposed scheme in information-theoretic perspective are compared.

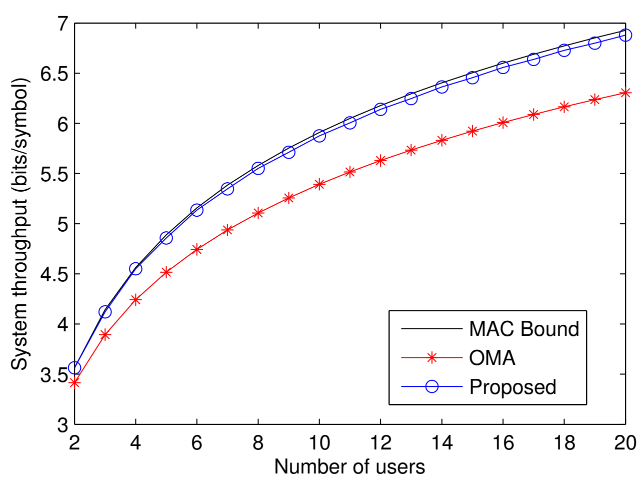

Fig. 2. Theoretical throughput achieved by different schemes.

It is shown in Fig. 2 that the proposed scheme can achieve most of throughput gain of NOMA comparing to OMA. For example, when the number of users is 12, the spectral efficiency is 6.18 bits/s/Hz in optimal NOMA scheme, 5.63 bits/s/Hz in OMA scheme, and 6.14 bits/s/Hz in proposed scheme. Hence, NOMA scheme shows about 9.8% throughput gain comparing to OMA scheme, while the proposed suboptimal scheme also shows about 9.1% throughput gain comparing to OMA scheme and less than 1% throughput loss comparing to optimal NOMA scheme.

V. Conclusion

In this paper, we propose a user-pairing based suboptimal scheme for uplink multiple-access by combining NOMA and OMA to facilitate the implementation. Furthermore, a user-pairing algorithm is also derived. According to the Monte Carlo simulation, the proposed scheme shows only a little throughput loss comparing to the optimal transmission scheme in information-theoretic perspective.

Appendix A
Proof of Theorem 1

Without loss of generality, we assume $n > 2$ and $P_1 < P_2 < \ldots < P_n$.

Lemma 1. *If n is odd, then $\{n\} \in S_{max}$ for any N_0.*

Proof. If $\{n\} \notin S_{max}$, then there exist unique $s, t \neq n$, $\{\{s\}, \{t, n\}\} \subset S_{max}$. In this case, we can construct another strategy:

$$S' = S_{max} \setminus \{\{s\}, \{t, n\}\} \cup \{\{n\}, \{s, t\}\},$$

such that:

$$\begin{aligned} &R(S') - R(S_{max}) \\ =&\frac{1}{k}\log_2(1 + \frac{kP_n}{N_0}) + \frac{1}{k}\log_2(1 + \frac{k(P_s + P_t)}{N_0}) \\ &- \frac{1}{k}\log_2(1 + \frac{kP_s}{N_0}) + \frac{1}{k}\log_2(1 + \frac{k(P_t + P_n)}{N_0}) \\ =&\frac{1}{k}\log_2(\frac{(N_0/k + P_n)(N_0/k + P_s + P_t)}{(N_0/k + P_s)(N_0/k + P_t + P_n)}). \end{aligned}$$

However, as

$$\begin{aligned} &(N_0/k + P_n)(N_0/k + P_s + P_t) \\ &- (N_0/k + P_s)(N_0/k + P_t + P_n) \\ =&(P_n - P_s)P_t > 0 \\ \Rightarrow&\log_2(\frac{(N_0/k + P_n)(N_0/k + P_s + P_t)}{(N_0/k + P_s)(N_0/k + P_t + P_n)}) > 0 \\ \Rightarrow&R(S') > R(S_{max}), \end{aligned}$$

which gives a contradiction. □

Lemma 2. *If n is even, then $\{1,n\} \in S_{max}$ for any N_0.*

Proof. If $\{1,n\} \notin S_{max}$, then there exist unique $s,t \notin \{1,n\}$, $\{\{1,s\},\{t,n\}\} \subset S_{max}$. In this case, we can construct another strategy:

$$S' = S_{max} \setminus \{\{1,s\},\{t,n\}\} \cup \{\{1,n\},\{s,t\}\},$$

such that:

$$\begin{aligned}
&R(S') - R(S_{max}) \\
=&\frac{1}{k}\log_2(1+\frac{k(P_1+P_n)}{N_0}) + \frac{1}{k}\log_2(1+\frac{k(P_s+P_t)}{N_0}) \\
&-\frac{1}{k}\log_2(1+\frac{k(P_1+P_s)}{N_0}) + \frac{1}{k}\log_2(1+\frac{k(P_t+P_n)}{N_0}) \\
=&\frac{1}{k}\log_2(\frac{(N_0/k+P_1+P_n)(N_0/k+P_s+P_t)}{(N_0/k+P_1+P_s)(N_0/k+P_t+P_n)}).
\end{aligned}$$

However, as

$$\begin{aligned}
&(N_0/k+P_1+P_n)(N_0/k+P_s+P_t) \\
&-(N_0/k+P_1+P_s)(N_0/k+P_t+P_n) \\
=&(P_1-P_t)(P_s-P_n) > 0 \\
\Rightarrow& R(S') > R(S_{max}),
\end{aligned}$$

which gives a contradiction. □

Proof of Theorem 1. The theorem 1 is proved by induction on the number of users n. For $n = 3,4$, the theorem is obvious according to lemma 1 and lemma 2. Suppose the theorem holds for some $5 \le n_1 < n$ and let $k = \lceil n/2 \rceil$.

If n is odd, then $\{n\} \in S_{max}$ according to lemma 1, and we have

$$\begin{aligned}
R(S) &= \frac{1}{k}\sum_{G_i\in S}\log_2(1+k\cdot\frac{\sum_{j\in G_i}P_j}{N_0}) \\
&= \frac{1}{k}(\log_2(1+\frac{kP_n}{N_0}) + \sum_{G_i\in S\setminus\{n\}}\log_2(1+k\cdot\frac{\sum_{j\in G_i}P_j}{N_0})) \\
&= \frac{1}{k}\log_2(1+\frac{kP_n}{N_0}) \\
&+ \frac{k-1}{k}(\frac{1}{k-1}\sum_{G_i\in S\setminus\{n\}}\log_2(1+(k-1)\cdot\frac{\sum_{j\in G_i}P_j}{(k-1)/k\cdot N_0})).
\end{aligned}$$

If we define S^* as the strategy for the case of $n-1$ users whose $P_i^* = P_i$ for $i \in \{1,2,...,n-1\}$ and $N_0^* = (k-1)/k \cdot N_0$, and

$$R(S^*) = \frac{1}{k-1}\sum_{G_i\in S\setminus\{n\}}\log_2(1+(k-1)\cdot\frac{\sum_{j\in G_i}P_j}{(k-1)/k\cdot N_0},$$

then, S can be derived according to the induction hypothesis by

$$\begin{aligned}
S_{max} &= \arg\max_S R(S) \\
&= \{\{n\}\} \cup \arg\max_{S^*} R(S^*) \\
&= \{\{n\}\} \cup \{\{i, 2t-i+1\} | 0 < i \le \lfloor n/2 \rfloor\}.
\end{aligned}$$

If n is even, then $\{1,n\} \in S_{max}$ according to lemma 1, similarly we have

$$\begin{aligned}
R(S) &= \frac{1}{k}\sum_{G_i\in S}\log_2(1+k\cdot\frac{\sum_{j\in G_i}P_j}{N_0}) \\
&= \frac{1}{k}(\log_2(1+\frac{k(P_1+P_n)}{N_0}) \\
&\quad + \sum_{G_i\in S\setminus\{1,n\}}\log_2(1+k\cdot\frac{\sum_{j\in G_i}P_j}{N_0})) \\
&= \frac{1}{k}\log_2(1+\frac{k(P_1+P_n)}{N_0}) \\
&+ \frac{k-1}{k}(\frac{1}{k-1}\sum_{G_i\in S\setminus\{1,n\}}\log_2(1+(k-1)\cdot\frac{\sum_{j\in G_i}P_j}{(k-1)/k\cdot N_0})).
\end{aligned}$$

By defining S^* as the strategy for the case of $n-2$ users whose $P_i^* = P_i$ for $i \in \{2,3,...,n-1\}$ and $N_0^* = (k-1)/k \cdot N_0$, we have

$$R(S^*) = \frac{1}{k-1}\sum_{G_i\in S\setminus\{1,n\}}\log_2(1+(k-1)\cdot\frac{\sum_{j\in G_i}P_j}{(k-1)/k\cdot N_0}.$$

According to the induction hypothesis we have

$$\begin{aligned}
S_{max} &= \arg\max_S R(S) \\
&= \{\{1,n\}\} \cup \arg\max_{S^*} R(S^*) \\
&= \{\{1,n\}\} \cup \{\{i+1, 2t-i+2\} | 0 < i \le k-1\} \\
&= \{\{i, 2t-i+1\} | 0 < i \le \lfloor n/2 \rfloor\}.
\end{aligned}$$

□

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