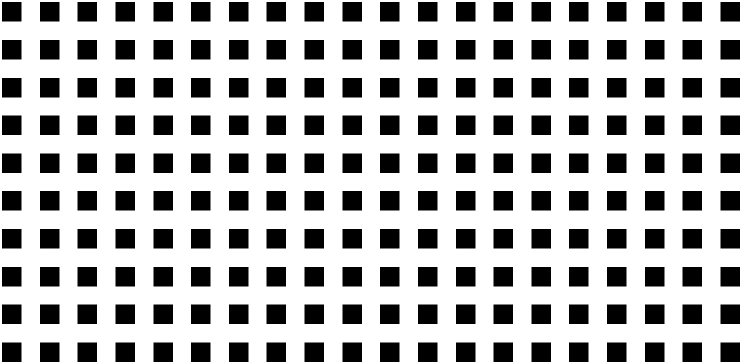
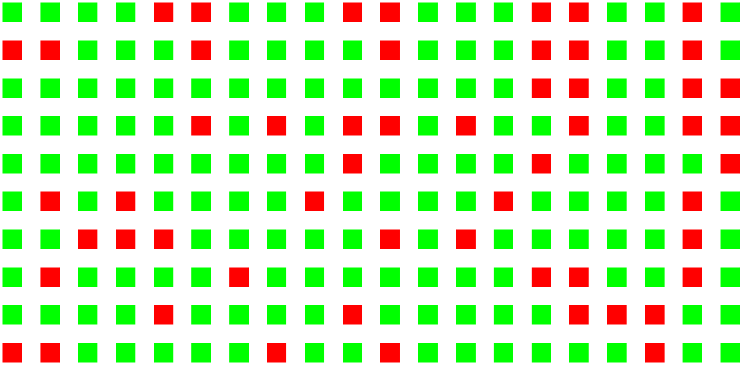
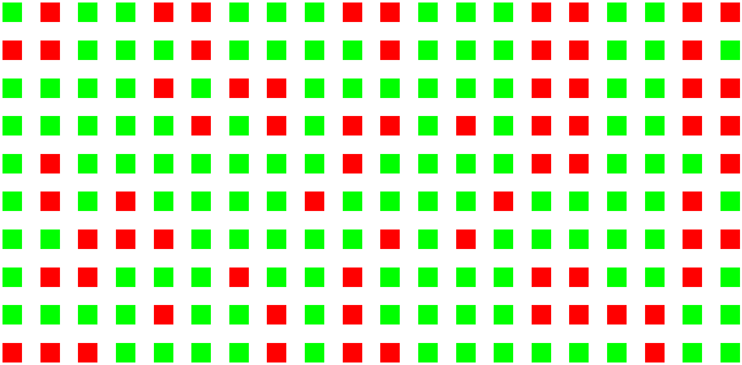
Population Games

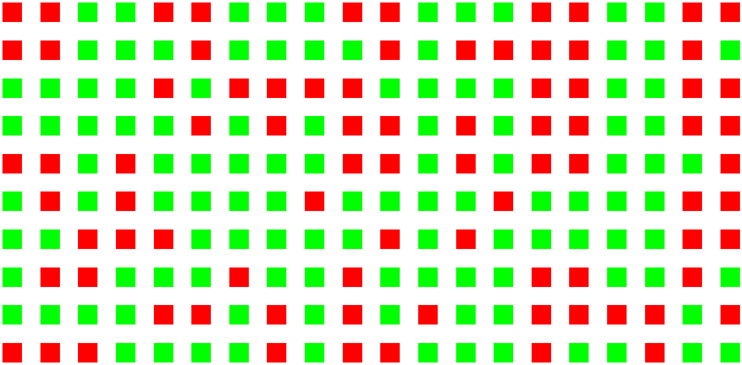
Game Theory

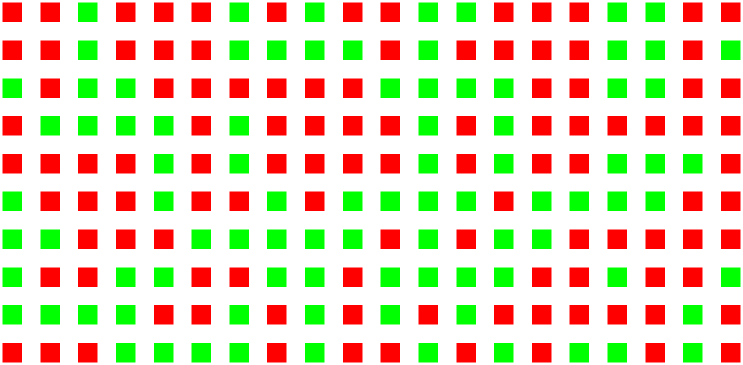
Vincent Knight

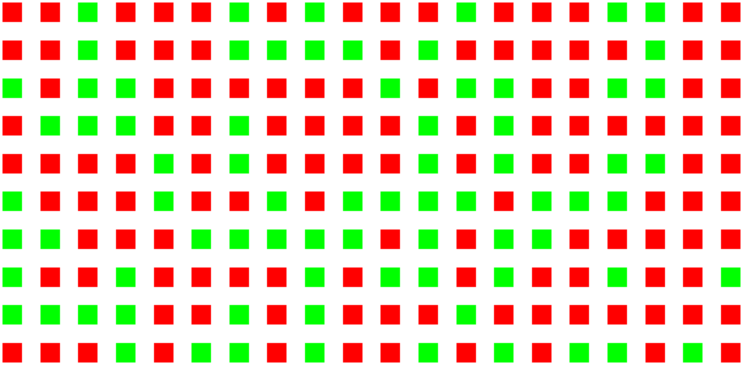


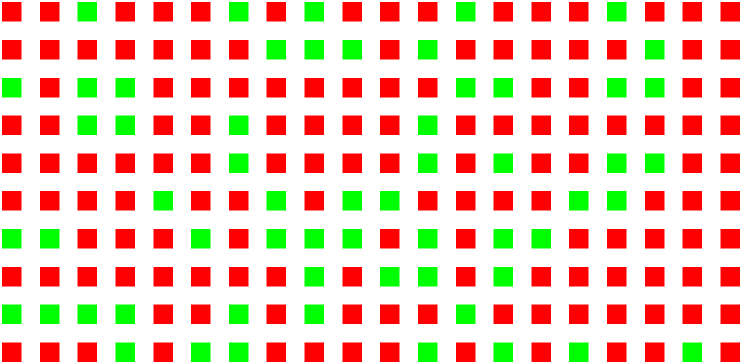


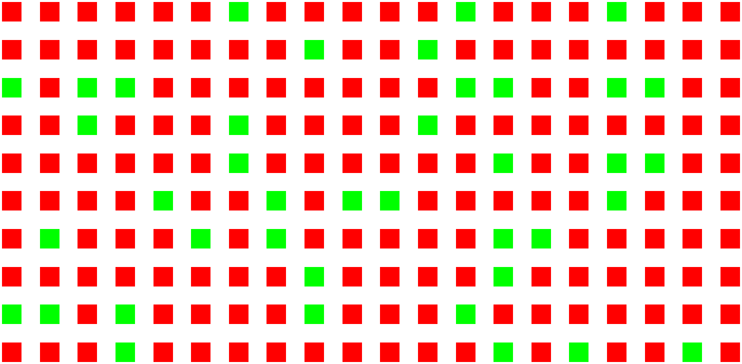


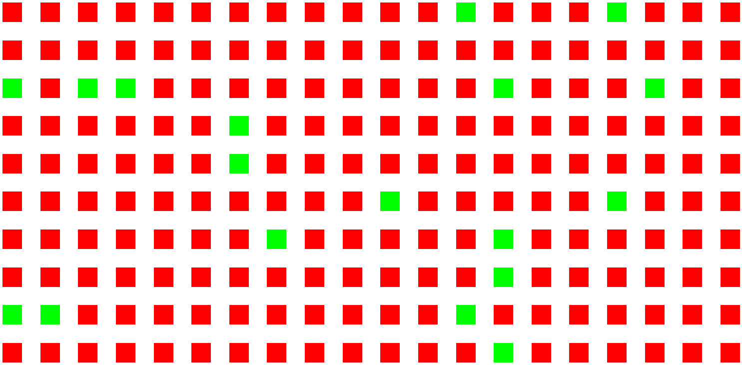


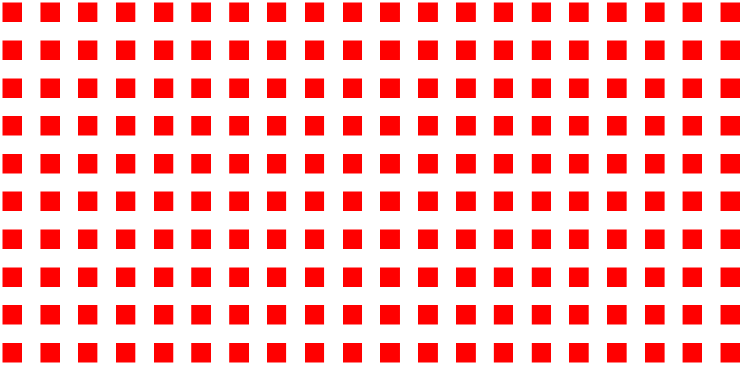












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χ is called a **population vector**

Utility of strategy $\sigma \in \Delta S$ in population χ:

$$u(\sigma, \chi) = \sum_{s \in S} \sigma(s) u(s, \chi)$$

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$\boxed{\textbf{Interpretation:}\text{ number of descendants of } \sigma.}$

Assume initial population $\chi = (.1, .9)$ and:

$$u(\text{red}, \chi) = 3|\text{red}| + 1 \text{ and } u(\text{green}, \chi) = 2/3|\text{green}|$$

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Consider a population where all individuals initially play σ^*. If we assume that a small proportion ϵ start playing σ. The new population is called the post entry population and will be denoted by χ_ϵ.

Evolutionary Stable Strategy:

A strategy $\sigma^* \in \Delta S$ is called an **Evolutionary Stable Strategy** if $\exists\ 0 < \bar{\epsilon} < 1$ such that for every $0 < \epsilon < \bar{\epsilon}$ and every $\sigma \neq \sigma^*$:

$$u(\sigma^*, \chi) > u(\sigma, \chi)$$