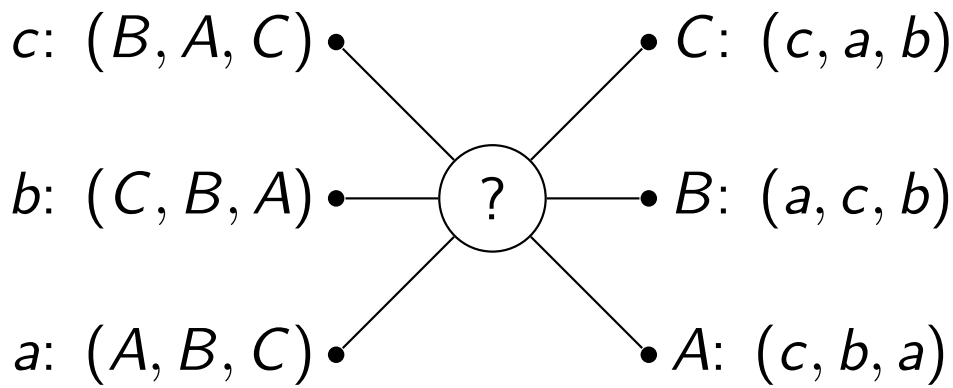
# Matching Games

## Game Theory

Vincent Knight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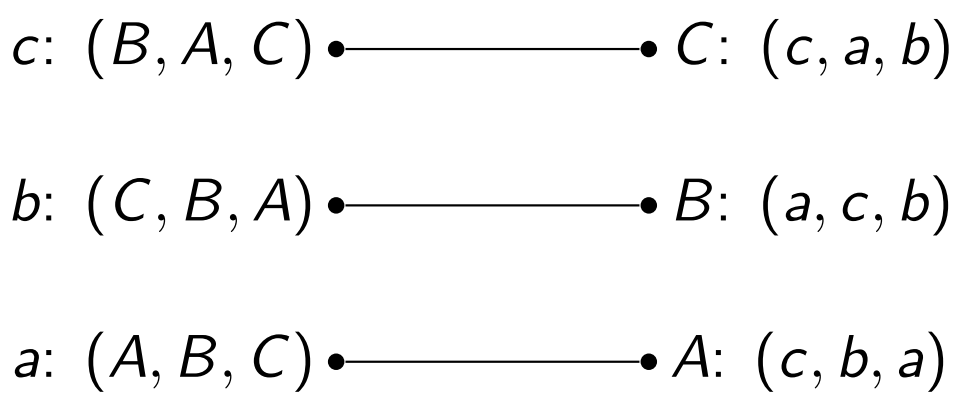
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**Gale-Shapley Algorithm:**

1. Assign every $s \in S$ and $r \in R$ to be unmatched
2. Pick some unmatched $s \in S$, let $r$ be the top of $s$'s preference list:
   - 2.1 If $r$ is unmatched set $M(s) = r$
   - 2.2 If $r$ is matched:
     - 2.2.1 If $r$ prefers $s$ to $M^{-1}(r)$ then set $M(r) = s$
     - 2.2.2 Otherwise $s$ remains unmatched and remove $r$ from $r$'s preference list.
3. Repeat step 2 until all $s \in S$ are matched.

$c\colon\ (B, A, C)\bullet$ $\qquad\qquad$ $\bullet\, C\colon\ (c, a, b)$

$b\colon\ (C, B, A)\bullet$ $\qquad\qquad$ $\bullet\, B\colon\ (a, c, b)$

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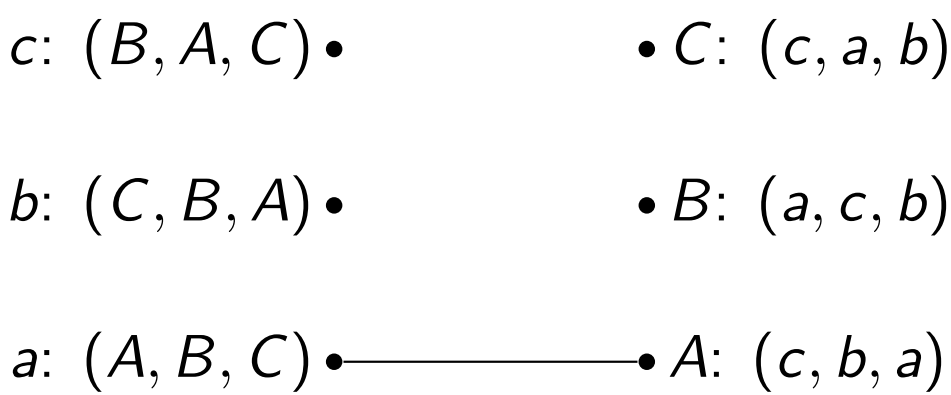
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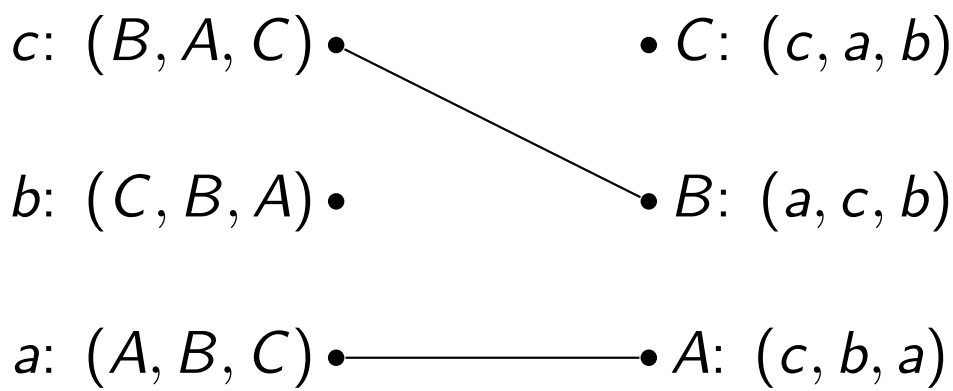
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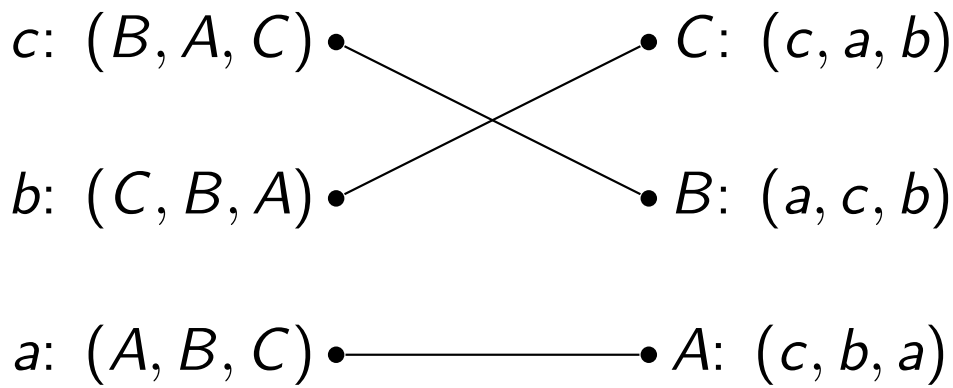
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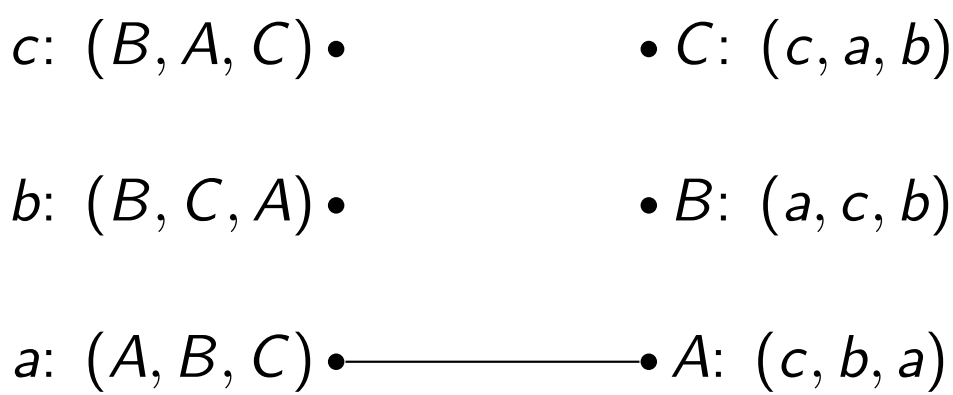
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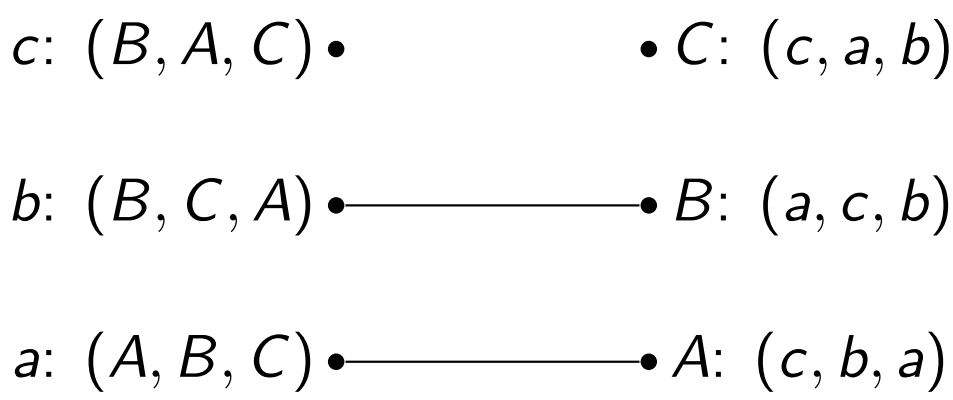
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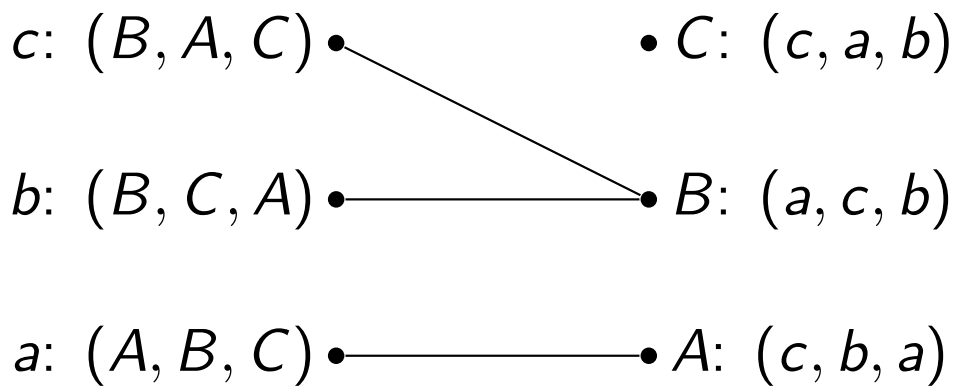
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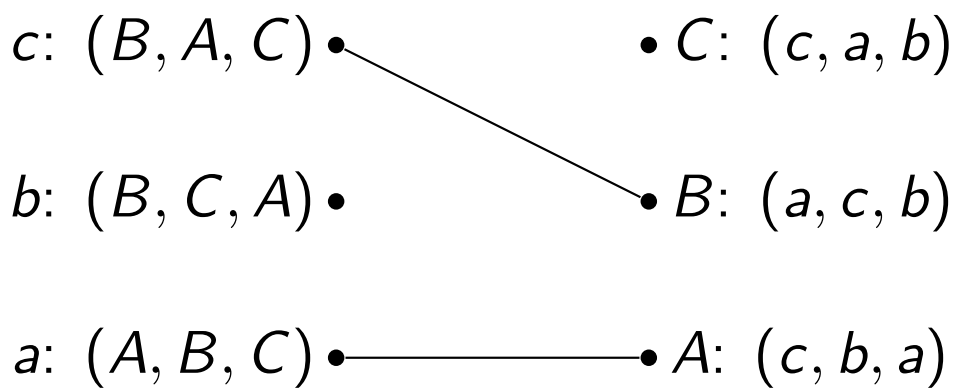
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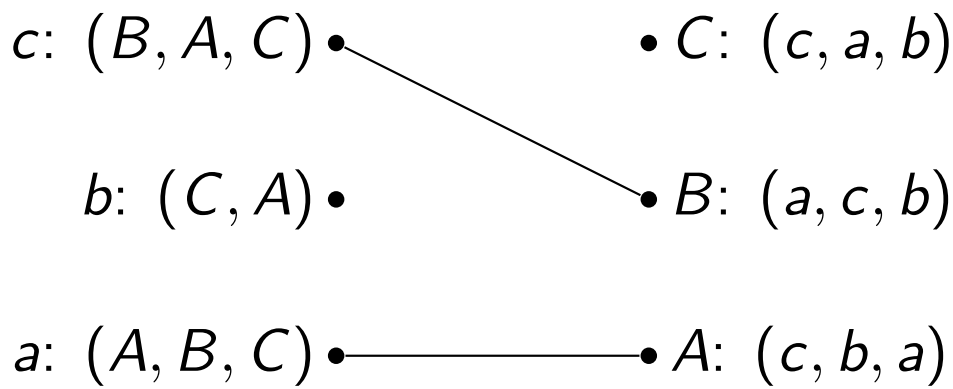
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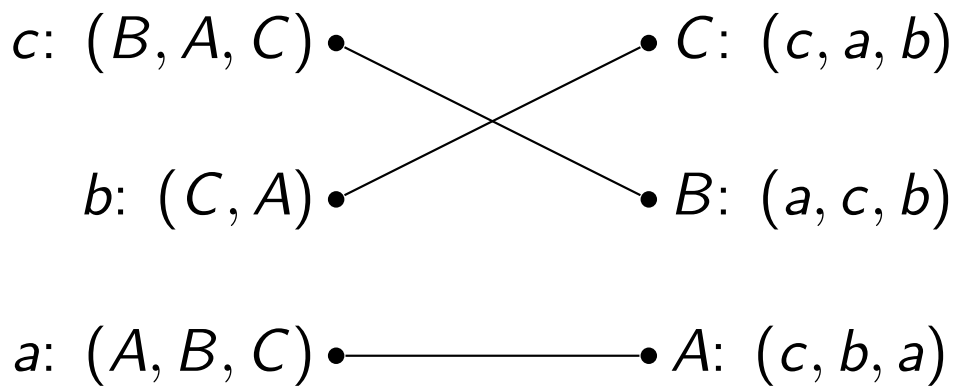
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