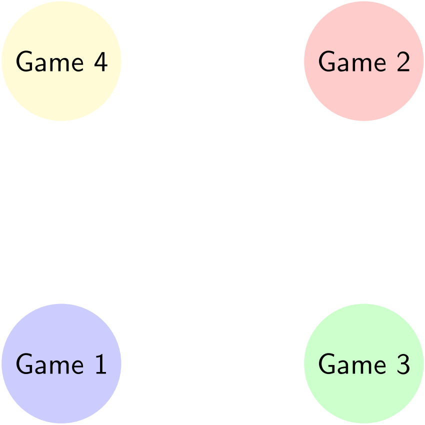
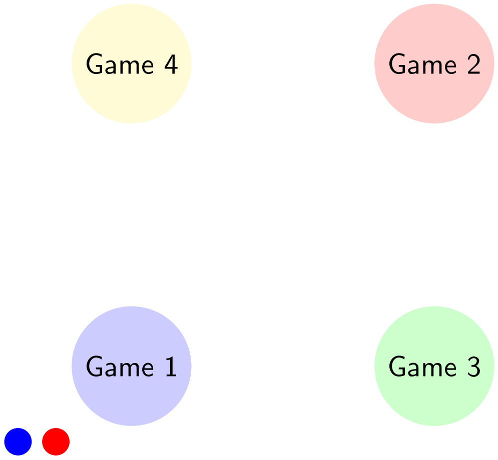
Stochastic Games

Game Theory

Vincent Knight

- X a set of states with a stage game defined for each state;
- A set of strategies $S_i(x)$ for each player for each state $x \in X$;
- A set of rewards dependant on the state and the actions of the other players: $u_i(x, s_1, s_2)$;
- A set of probabilities of transitioning to a future state: $\pi(x'|x, s_1, s_2)$;
- Each stage game is played at a set of discrete times t.


Game 4
Game 2
Game 1
Game 3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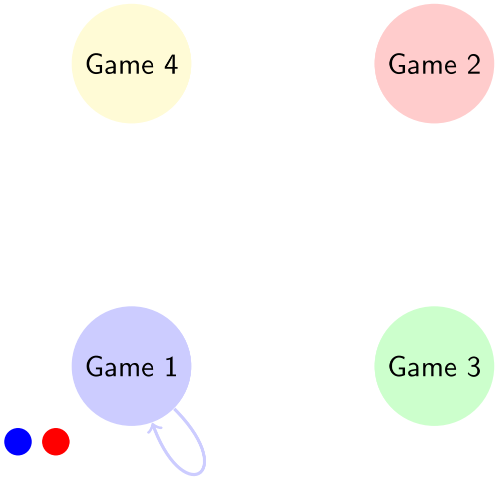
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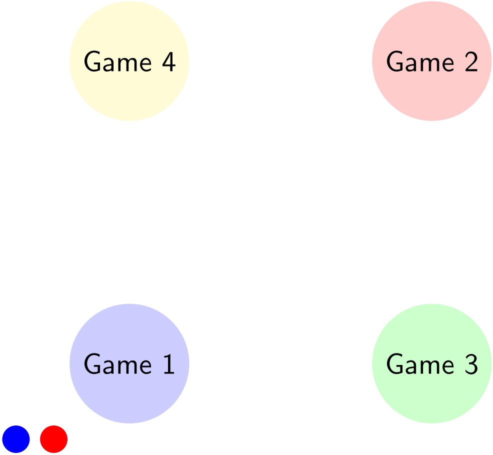
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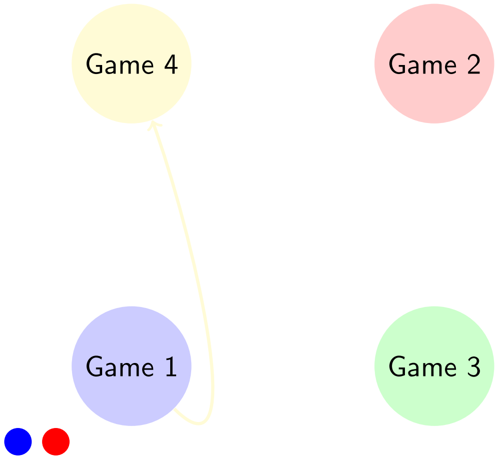
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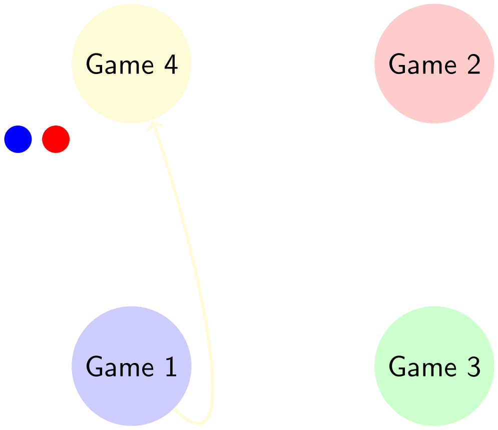
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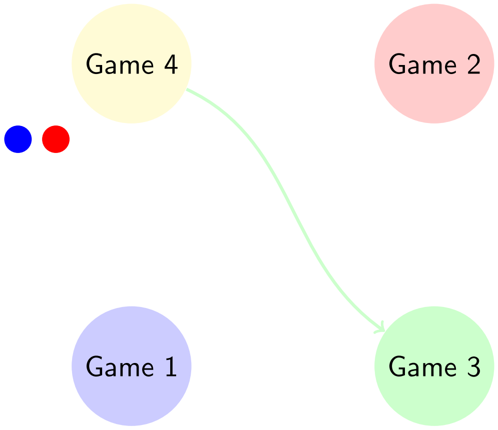
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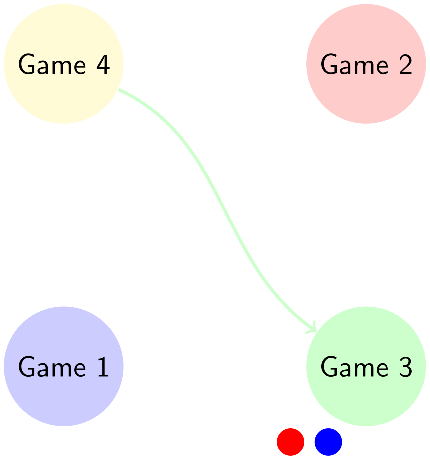
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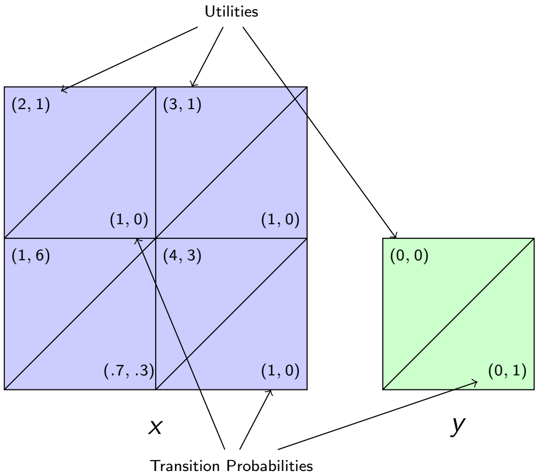
Utilities
(2, 1)
(3, 1)
(1, 0)
(1, 0)
(1, 6)
(4, 3)
(.7, .3)
(1, 0)
(0, 0)
(0, 1)
x
y
Transition Probabilities

$$U_i(r,s) = \left(u_i(x,r,s) + \delta \sum_{x' \in X} \pi(x'|x,r,s) U_i^*(x') \right)$$

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Thus a Nash equilibrium satisfies:

$$U_1^*(x) = \max_{r \in S_1(x)} (u_i(x,r,s^*) + \delta \sum_{x' \in X} \pi(x'|x,r,s^*) U_1^*(x')$$

$$U_2^*(x) = \max_{s \in S_2(x)} (u_i(x,r^*,s) + \delta \sum_{x' \in X} \pi(x'|x,r^*,s) U_2^*(x')$$