

4. (a) Provide definitions for the following terms:

- Normal form game.
- Strictly dominated strategy.
- Weakly dominated strategy.
- Best response strategy.
- Nash equilibrium.

[5]

(b) Consider the following game:

$$\begin{pmatrix} (1, \alpha) & (0, 2) \\ (0, 0) & (\alpha, 1) \end{pmatrix}$$

(i) Prove that a pure Nash equilibrium exists for all values of $\alpha \in \mathbb{R}$.

[7]

(ii) State the equality of payoffs theorem. Using this theorem obtain the value of α (if it exists) for which the following (σ_1, σ_2) are mixed Nash equilibria for the game.

- A. $(\sigma_1, \sigma_2) = ((1/2, 1/2), (1/2, 1/2))$
- B. $(\sigma_1, \sigma_2) = ((1/2, 1/2), (3/4, 1/4))$
- C. $(\sigma_1, \sigma_2) = ((1/5, 4/5), (3/4, 1/4))$

[13]

5. (a) Define a (finitely) repeated game.

[4]

- (b) Define a strategy in a repeated game.

[2]

- (c) Prove that for any repeated game, any sequence of stage Nash profiles gives the outcome of a subgame perfect Nash equilibrium.

[7]

- (d) For the following stage games, plot the possible outcomes for a repetition of $T = 2$ periods and obtain a Nash equilibria **that is not a sequence of stage Nash profiles**:

$$\begin{pmatrix} (3, 4) & (1, 2) & (2, 5) \\ (-1, 1) & (1, 2) & (-1, -1) \end{pmatrix} \quad \begin{pmatrix} (3, 1) & (1, 1) \\ (-1, 1) & (1, 0) \\ (1, 3) & (.5, 1) \end{pmatrix}$$

$$\begin{pmatrix} (2, 9) & (3, 1) \\ (3, 1) & (6, 1) \\ (3, 3) & (5, 1) \end{pmatrix} \quad \begin{pmatrix} (1, 1) & (1, 0) & (1, 1) \\ (1, 2) & (3, 2) & (2, 5) \end{pmatrix}$$

[12]

6. (a) Define a stochastic game.

[4]

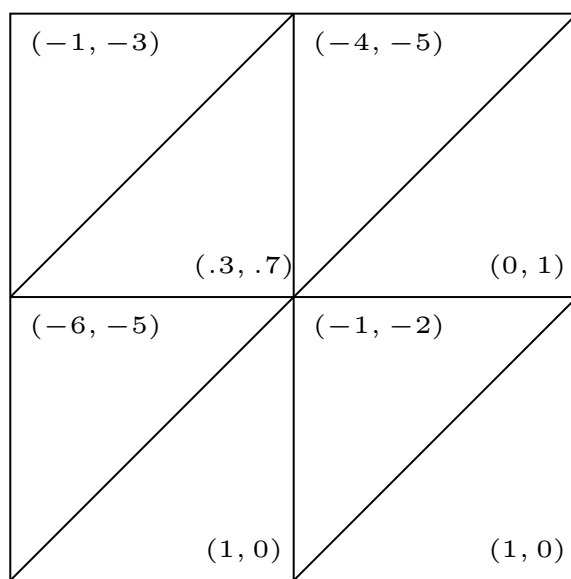
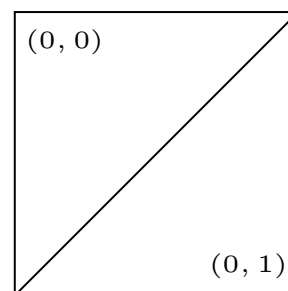
(b) Define a Markov strategy.

[2]

(c) Give the conditions for Nash equilibrium in a stochastic game.

[3]

(d) Obtain the pure strategy Nash equilibria (if any exist) for the following game with $\delta = .3$:

 x  y

[16]