1 Homework sheet 1 - Normal form games

1. Represent the following game in normal form:

   Alice, Bob and Celine are childhood friends that would like to communicate online. Alive likes facebook, Bob likes twitter and Celine like G+.

Clearly state the players, strategy sets and interpretations of the utilities.

2. Represent the following game in normal form:

   Assume two neighbouring countries have at their disposal very destructive armies. If both countries attack each other the countries' civilian population will suffer 10 thousand casualties. If one country attacks whilst the other remains peaceful, the peaceful country will lose 15 thousand casualties but would also retaliate causing the offensive country 13 thousand casualties. If both countries remain peaceful then there are no casualties.

- Clearly state the players and strategy sets.
- Plot the utilities to both countries assuming that they play a mixed strategy while the other country remains peaceful.

3. Dominance

Attempt to predict rational behaviour **using iterated elimination of dominated strategies** for the games from 1. and 2. and the following:

- $$\begin{pmatrix} (2,1) & (1,1) \\ (1,1) & (1,3) \end{pmatrix}$$
- $$\begin{pmatrix} (2,11) & (1,9) & (3,10) & (17,22) \\ (27,0) & (3,1) & (1,1) & (1,0) \\ (4,2) & (6,10) & (7,12) & (18,0) \end{pmatrix}$$
- $$\begin{pmatrix} (3,2) & (3,1) & (2,3) \\ (2,2) & (1,3) & (3,2) \end{pmatrix}$$
- $$\begin{pmatrix} (3,-3) & (-1,1) \\ (2,1) & (7,-6) \end{pmatrix}$$

Explain when games occur that cannot be handled this way.

4. For all of the above games (including the games for questions 1 and 2), identify all best responses and attempt to predict rational behaviour.

Explain when games occur that cannot be handled this way.

5. Consider the following game:

$$\begin{pmatrix} (7,3) & (0,2) \\ (2,1) & (6,1) \\ (4,0) & (4,2) \end{pmatrix}$$

Compute directly B_1, B_2, UD_1 and UD_2.

6. In the notes the following theorem is given:

In a 2 player normal form game $B_i = UD_i$ for all $i \in \{1, 2\}$.

Prove the theorem for 2 player games with $|S_1| = |S_2| = 2$. I.e. prove the above result in the special case of 2×2 games.