

# Population Games and Normal Form Games

## Game Theory

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## Pairwise Contest Games

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$$u(\sigma, \chi) = \sum_{s, s' \in S} \sigma(s) \chi(s') u(s, s')$$

## Pairwise Contest Games

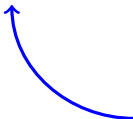
Population:  $\chi$ .

$$u(\sigma, \chi) = \sum_{s, s' \in S} \sigma(s) \chi(s') u(s, s')$$

Prob of  $\sigma$   
playing  $s$



Prob of meet-  
ing  $s'$



$$\begin{pmatrix} u(s, s), u(s, s) & u(s, s'), u(s, s') \\ u(s', s), u(s', s) & u(s', s'), u(s', s') \end{pmatrix}$$

### Theorem.

If  $\sigma^*$  is an ESS in a pairwise contest population game then for all  $\sigma \neq \sigma^*$ :

1.  $u(\sigma^*, \sigma^*) > u(\sigma, \sigma^*)$  OR
2.  $u(\sigma^*, \sigma^*) = u(\sigma, \sigma^*)$  and  $u(\sigma^*, \sigma) > u(\sigma, \sigma)$

Conversely, if either (1) or (2) holds for all  $\sigma \neq \sigma^*$  in a two player normal form game then  $\sigma$  is an ESS.