

1 Homework sheet 4 - Evolutionary games, games with incomplete information and stochastic games

1. Consider the pairwise contest games with the following associated two player games:

$$\begin{pmatrix} (2, 2) & (4, 5) \\ (5, 4) & (1, 1) \end{pmatrix}$$

$$\begin{pmatrix} (1, 1) & (0, 0) \\ (0, 0) & (1, 1) \end{pmatrix}$$

$$\begin{pmatrix} (\alpha, \alpha) & (1, \beta) \\ (\beta, 1) & (0, 0) \end{pmatrix}$$

(Assume $\alpha, \beta > 0$ and $\alpha \neq \beta$)

Identify all evolutionary stable strategies.

2. Consider the following game:

In a mathematics department, researchers can choose to use one of two systems for typesetting their research papers: LaTeX or Word. We will refer to these two strategies as L and W respectively. A user of W receives a basic utility of 1 and as L is more widely used by mathematicians out of the department and is in general considered to be a better system a user of L gets a basic utility of $\alpha > 1$. Members of the mathematics department often collaborate and as such it is beneficial for the researchers to use the same typesetting system. If we let μ represent the proportion of users of L we let:

$$u(L, \chi) = \alpha + 2\mu$$

$$u(W, \chi) = 1 + 2(1 - \mu)$$

What are the evolutionary stable strategies?

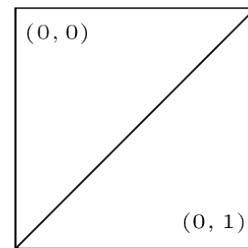
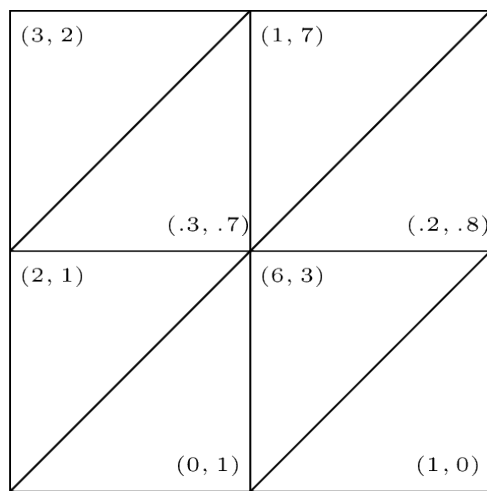
3. Consider the following two normal form games:

$$A = \begin{pmatrix} (3, 0) & (-1, -1) & (1, 2) \\ (1, 0) & (-1, 1) & (2, 0) \end{pmatrix}$$

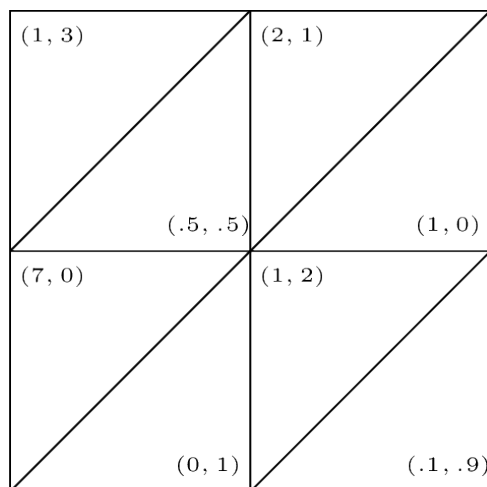
$$B = \begin{pmatrix} (2, 2) & (1, 1) & (1, 3) \\ (1, 3) & (-2, -3) & (4, 2) \end{pmatrix}$$

Assume both players play either game A or game B with probability $1/2$, neither player knows which game is played. Obtain the Nash equilibrium for this game.

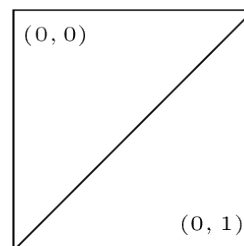
4. Repeat the analysis of the principal agent game assuming that p is the probability of the project being successful in case of a high level of effort by the employee.
 - i. What are the expected utilities to the employer and the employee?
 - ii. Obtain a condition for which the employer should offer a bonus.
5. Obtain the Markov Nash equilibrium for the following games assuming $\delta = 1/4$.



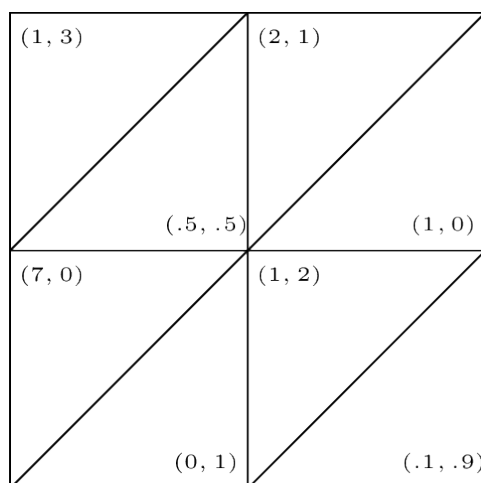
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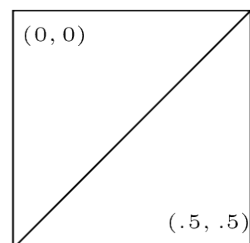
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