1 Homework sheet 2 - Nash equilibrium in normal form games

1. Compute the Nash equilibrium (if they exist) in pure strategies for the following games:

$$\begin{pmatrix} (5,3) & (70,-1) & (4,2) \\ (6,7) & (71,2) & (2,1) \end{pmatrix}$$

$$\begin{pmatrix} (6,7) & (2,1) & (4,6) \\ (0,4) & (3,8) & (2,3) \\ (1,2) & (1,5) & (1,1) \end{pmatrix}$$

$$\begin{pmatrix} (\pi, e) & (1-\pi, \sqrt(e)) \\ (\sqrt(2), 1/e) & (2,1) \end{pmatrix}$$

2. For what values of α does a Nash equilibrium exist in pure strategies for the following game:

$$\begin{pmatrix} (3,5) & (2-\alpha, \alpha) \\ (4\alpha, 6) & (\alpha, \alpha^2) \end{pmatrix}$$

3. Consider the following game:

   Suppose two vendors (of an identical product) must choose their location along a busy street. It is anticipated that their profit is directly related to their position on the street.

   If we allow their positions to be represented by a points x_1, x_2 on the $[0,1]_{\mathbb{R}}$ line segment then we have:

$$u_1(x_1, x_2) = \begin{cases} x_1 + (x_2 - x_1)/2, & \text{if } x_1 \leq x_2 \\ 1 - x_1 + (x_2 - x_1)/2, & \text{otherwise} \end{cases}$$

and

$$u_1(x_1, x_2) = \begin{cases} x_2 + (x_2 - x_1)/2, & \text{if } x_2 \leq x_1 \\ 1 - x_2 + (x_2 - x_1)/2, & \text{otherwise} \end{cases}$$

By considering best responses of each player, identify the Nash equilibrium for the game.

4. Consider the following game:

$$\begin{pmatrix} (3,2) & (6,5) \\ (1,4) & (2,3) \end{pmatrix}$$

Plot the expected utilities for each player against mixed strategies and use this to obtain the Nash Equilibria.

5. Assume a soccer player (player 1) is taking a penalty kick and has the option of shooting left or right: $S_1 = \{\text{SL}, \text{SR}\}$. A goalie (player 2) can either dive left or right: $S_2 = \{\text{DL}, \text{DR}\}$. The chances of a goal being scored are given below:

$$\begin{pmatrix} .8 & .15 \\ .2 & .95 \end{pmatrix}$$

i. Assume the utility to player 1 if the probability of scoring and the utility to player 2 the probability of a goal not being scored. What is the Nash equilibrium for this game?

ii. Assume that player 1 now has a further strategy available: to shoot in the middle: $S_1 = \{\text{SL}, \text{SM}, \text{SR}\}$ the probabilities of a goal being scored are now given:

$$\begin{pmatrix} .8 & .15 \\ .5 & .5 \\ .2 & .95 \end{pmatrix}$$

Obtain the new Nash equilibrium for the game.

6. In the notes the following theorem is given:

Every normal form game with a finite number of pure strategies for each player, has at least one Nash equilibrium.

Prove the theorem for 2 player games with $|S_1| = |S_2| = 2$. I.e. prove the above result in the special case of 2×2 games.