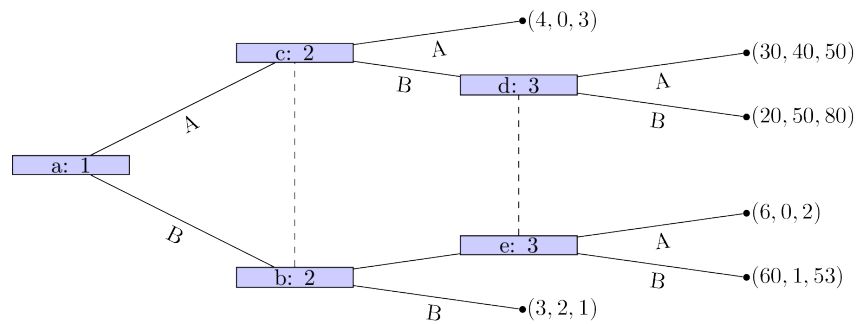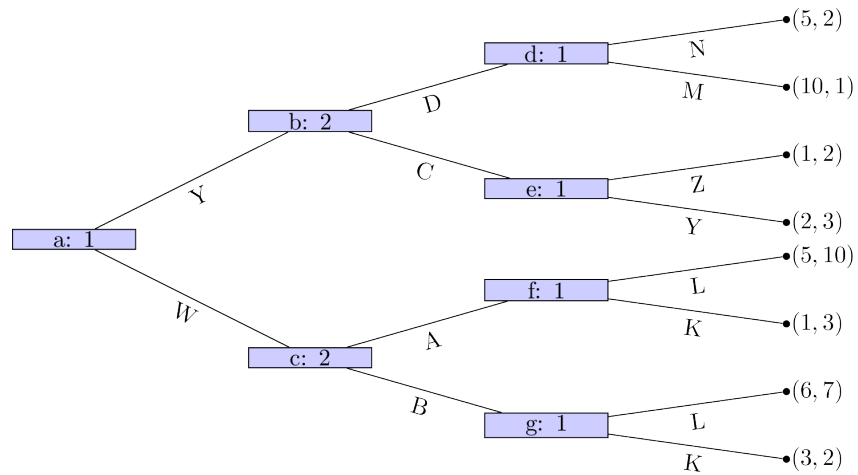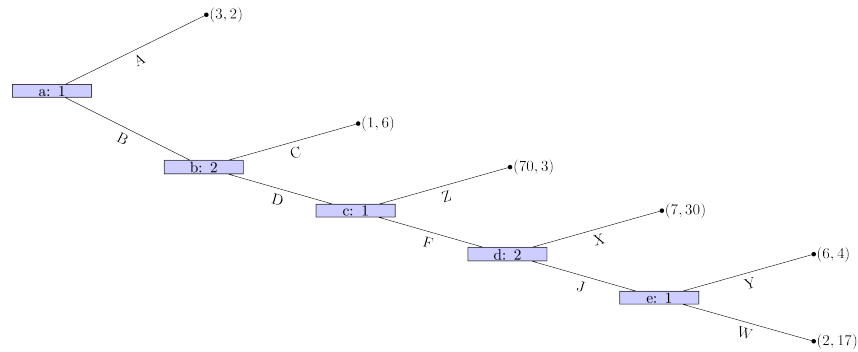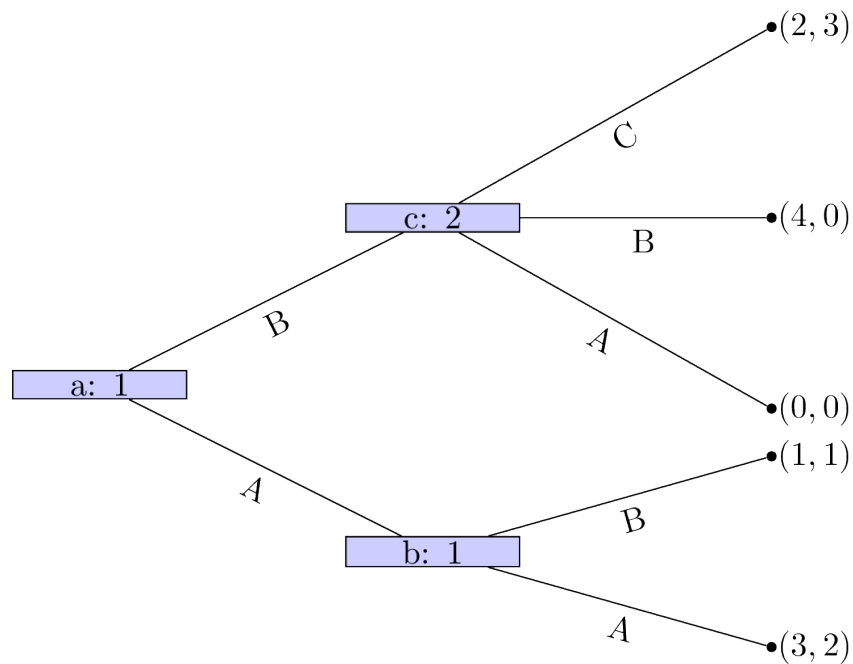# 1 Homework sheet 3 - Extensive form games, subgame perfect equilibrium and repeated games

1. Obtain the Nash equilibrium for the following games using backward induction:

2. Obtain the Nash equilibrium for the following game:

   Player 1 chooses a number $x \geq 0$, which player 2 observes. After this simulataneously and independatly player 1 and player 2 choose $y_2, y_1 \in \mathbb{R}$ respectively. The utility to player 1 is given by $2y_2y_1 + xy_1 - y_1^2 - x^3/3$ and the utility to player 2 is given by $-(y_1 - 2y_2)^2$.

3. For each of the following games:

   i. Identify all subgames.
   ii. Identify the corresponding normal form representations and hence obtain all Nash equilibrium.
   iii. Identify which Nash equilibrium are also subgame perfect Nash equilibrium.

Tree 1

- a: 1
  - Y → b: 2
    - D → d: 1
      - N → (1, 2)
      - M → (6, 2)
    - C → e: 1
      - Z → (7, 2)
      - Y → (3, 1)
  - W → c: 2
    - B → f: 1
      - L → (6, 12)
      - K → (2, 2)
    - A → (2, 5)

(d: 1 and e: 1 connected by dashed line)

Tree 2

- a: 1
  - Y → b: 2
    - D → d: 1
      - N → (3, 1)
      - M → (4, 9)
    - C → e: 1
      - Z → (3, 6)
      - X → (1, 2)
  - W → c: 2
    - B → f: 1
      - L → (6, 2)
      - K → (1, 7)
    - A → g: 1
      - P → (8, 3)
      - O → (4, 1)

(b: 2 and c: 2 connected by dashed line)

Tree 3

- a: 1
  - A → b: 2
    - D → d: 1
      - D → (32, 9)
      - C → (0, 0)
    - D → e: 1
      - D → (0, 0)
      - C → (31, 10)
  - B → c: 2
    - D → (20, 20)
    - C → (10, 10)

(d: 1 and e: 1 connected by dashed line)
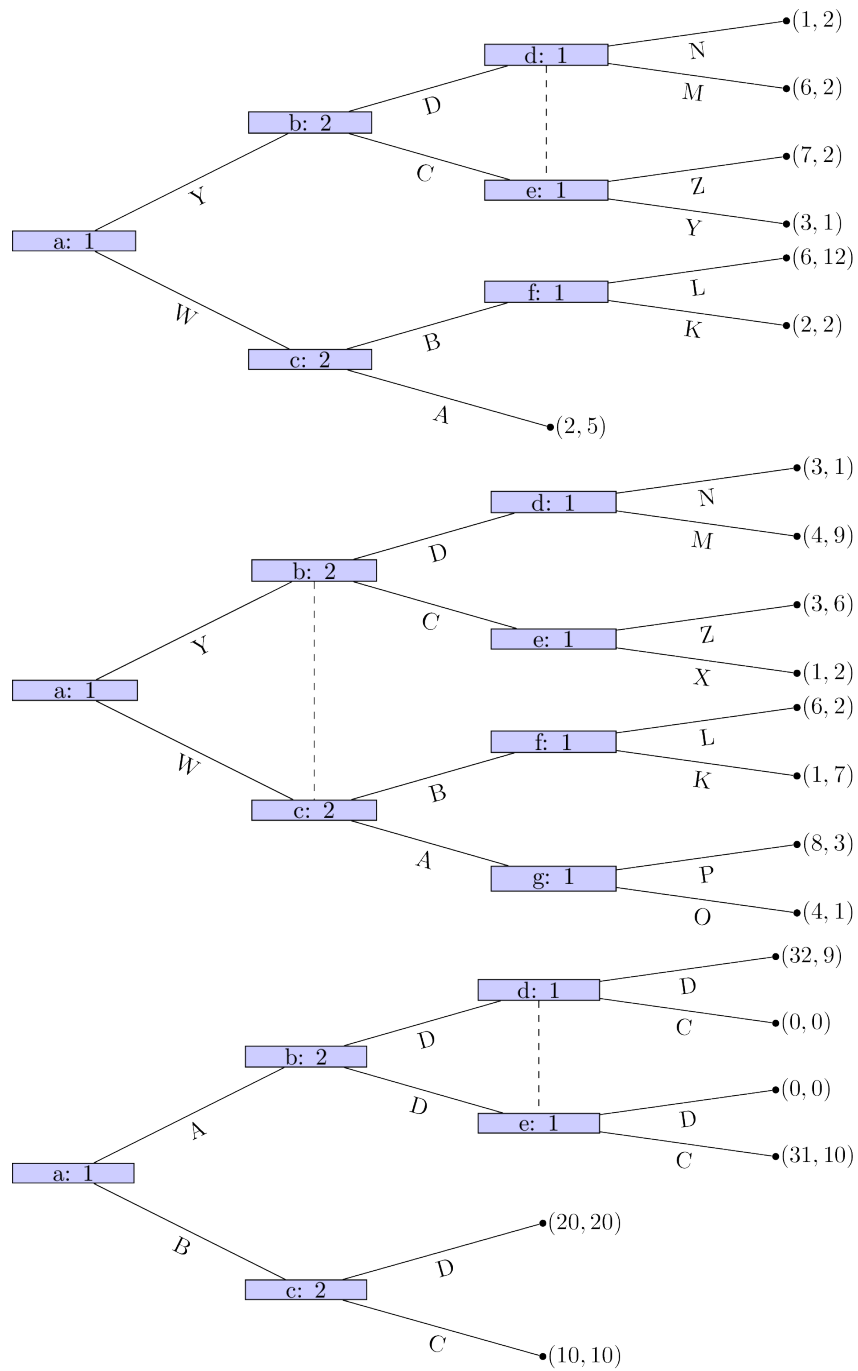
4. Consider the game in exercise 3 of homework sheet 2. Assume that the

3

vendors now position themselves sequentially. Model the game in extensive form and find the subgame perfect Nash equilibrium.

5. For the following stage games:

    i. Plot all possible utility pairs for $T = 2$;

    ii. Recalling that subgame perfect equilibrium for the repeated game must play a stage Nash equilibrium in the final stage attempt to identify a Nash equilibrium for the repeated game that is not a sequence of stage Nash profiles.

$$\begin{pmatrix} (4,3) & (7,6) \\ (1,1) & (4,3) \end{pmatrix}$$

$$\begin{pmatrix} (-1,1) & (3,-7) \\ (-2,6) & (2,2) \end{pmatrix}$$

$$\begin{pmatrix} (5,2) & (2,0) & (6,3) \\ (5,2) & (1,3) & (7,1) \end{pmatrix}$$

6. Consider the following stage game:

$$\begin{pmatrix} (-1,1) & (3,-7) \\ (-2,6) & (2,2) \end{pmatrix}$$

    i. For $\delta = 1/3$ obtain the utilities for the infinitely repeated game for the strategies S_D: "play the first strategy throughout" and S_C: "play the second strategy throughout".

    ii. Plot the space of feasible average payoffs and the space of individually rational payoffs.

    iii. Obtain δ that ensures that a strategy profile exists that would give a subgame perfect Nash equilibrium with average payoffs: $(3/2, 3/2)$, $(0, 3)$, $(2, 6)$ and $(2, 0)$.