

# 1 Homework sheet 2 - Nash equilibrium in normal form games

1. Compute the Nash equilibrium (if they exist) in pure strategies for the following games:

$$\begin{pmatrix} (5, 3) & (70, -1) & (4, 2) \\ (6, 7) & (71, 2) & (2, 1) \end{pmatrix}$$

$$\begin{pmatrix} (6, 7) & (2, 1) & (4, 6) \\ (0, 4) & (3, 8) & (2, 3) \\ (1, 2) & (1, 5) & (1, 1) \end{pmatrix}$$

$$\begin{pmatrix} (\pi, e) & (1 - \pi, \sqrt{e}) \\ (\sqrt{2}, 1/e) & (2, 1) \end{pmatrix}$$

2. For what values of  $\alpha$  does a Nash equilibrium exist in pure strategies for the following game:

$$\begin{pmatrix} (3, 5) & (2 - \alpha, \alpha) \\ (4\alpha, 6) & (\alpha, \alpha^2) \end{pmatrix}$$

3. Consider the following game:

Suppose two vendors (of an identical product) must choose their location along a busy street. It is anticipated that their profit is directly related to their position on the street.

If we allow their positions to be represented by a points  $x_1, x_2$  on the  $[0, 1]_{\mathbb{R}}$  line segment then we have:

$$u_1(x_1, x_2) = \begin{cases} x_1 + (x_2 - x_1)/2, & \text{if } x_1 \leq x_2 \\ 1 - x_1 + (x_2 - x_1)/2, & \text{otherwise} \end{cases}$$

and

$$u_2(x_1, x_2) = \begin{cases} x_2 + (x_2 - x_1)/2, & \text{if } x_2 \leq x_1 \\ 1 - x_2 + (x_2 - x_1)/2, & \text{otherwise} \end{cases}$$

By considering best responses of each player, identify the Nash equilibrium for the game.

4. Consider the following game:

$$\begin{pmatrix} (3, 2) & (6, 5) \\ (1, 4) & (2, 3) \end{pmatrix}$$

Plot the expected utilities for each player against mixed strategies and use this to obtain the Nash Equilibria.

5. Assume a soccer player (player 1) is taking a penalty kick and has the option of shooting left or right:  $S_1 = \{SL, SR\}$ . A goalie (player 2) can either dive left or right:  $S_2 = \{DL, DR\}$ . The chances of a goal being scored are given below:

$$\begin{pmatrix} .8 & .15 \\ .2 & .95 \end{pmatrix}$$

- i. Assume the utility to player 1 if the probability of scoring and the utility to player 2 the probability of a goal not being scored. What is the Nash equilibrium for this game?
- ii. Assume that player 1 now has a further strategy available: to shoot in the middle:  $S_1 = \{SL, SM, SR\}$  the probabilities of a goal being scored are now given:

$$\begin{pmatrix} .8 & .15 \\ .5 & .5 \\ .2 & .95 \end{pmatrix}$$

Obtain the new Nash equilibrium for the game.

6. In the notes the following theorem is given:

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Every normal form game with a finite number of pure strategies for each player, has at least one Nash equilibrium.

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Prove the theorem for 2 player games with  $|S_1| = |S_2| = 2$ . I.e. prove the above result in the special case of  $2 \times 2$  games.