

# 1 Homework sheet 4 - Evolutionary games, games with incomplete information and stochastic games

1. Consider the pairwise contest games with the following associated two player games:

$$\begin{pmatrix} (2, 3) & (4, 5) \\ (4, 5) & (1, 2) \end{pmatrix}$$

$$\begin{pmatrix} (1, -1) & (-1, 1) \\ (-1, 1) & (1, -1) \end{pmatrix}$$

$$\begin{pmatrix} (\alpha, 3) & (1, \beta) \\ (1, \beta) & (0, 0) \end{pmatrix}$$

Identify all evolutionary stable strategies.

2. Consider the following game:

In a mathematics department, researchers can choose to use one of two systems for typesetting their research papers: LaTeX or Word. We will refer to these two strategies as  $L$  and  $W$  respectively. A user of  $W$  receives a basic utility of 1 and as  $L$  is more widely used by mathematicians out of the department and is in general considered to be a better system a user of  $L$  gets a basic utility of  $\alpha$ . Members of the mathematics department often collaborate and as such it is beneficial for the researchers to use the same typesetting system. If we let  $\mu$  represent the proportion of users of  $L$  we let:

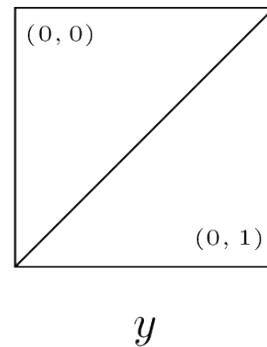
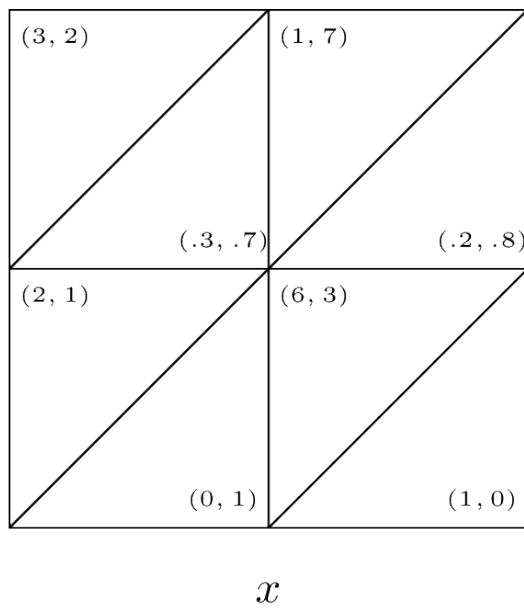
$$u(L, \chi) = 2 + 2(1 - x)$$

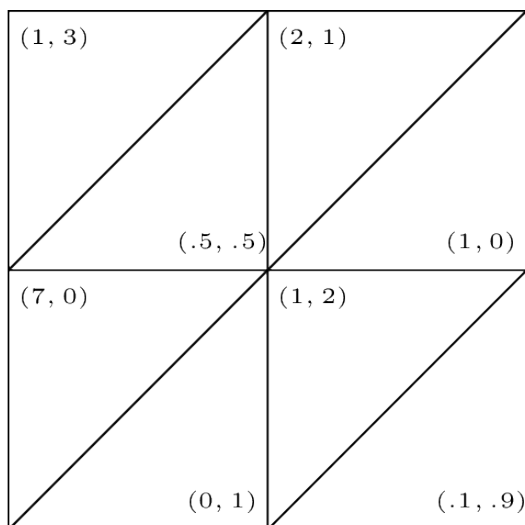
$$u(W, \chi) = 1 + 2(1 - x)$$

What are the evolutionary stable strategies?

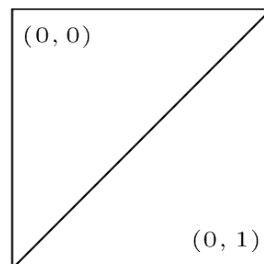
3. Consider the simple game with two players: an insurer and a driver. The insurer sets a premium price  $K \geq 0$ , once that is done the driver can choose to buy insurance or not. It is assumed that the driver will have an accident with probability  $p$ , if the driver has an accident the financial cost is  $A$ . Represent this game in normal form and obtain the Nash equilibrium for the game as a function of the parameters. Modify your analysis assuming that the utility function to the driver is given by  $u(x) = x^{1/\alpha}$  and the utility to the insurer is given by  $u(x) = x^{1/\beta}$ .

4. Repeat the analysis of the principal agent game assuming that  $p$  is the probability of the project being successful in case of a high level of effort by the employee.
  - i. What are the expected utilities to the employer and the employee?
  - ii. Obtain a condition for which the employer should offer a bonus.
5. Obtain the Markov Nash equilibrium for the following games assuming  $\delta = 1/4$ .

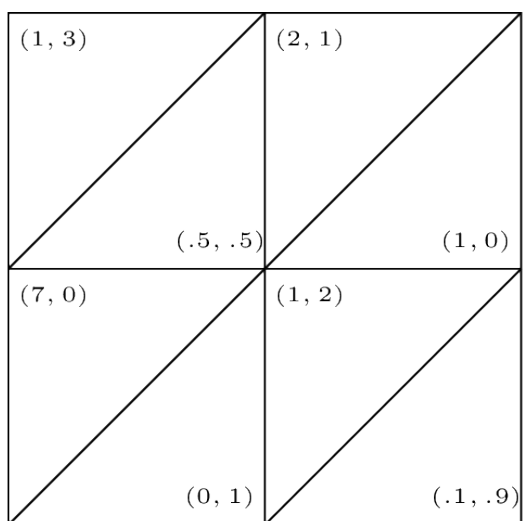




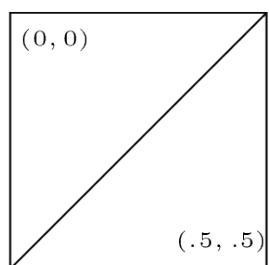
$x$



$y$



$x$



$y$

6. Construct a two state stochastic game corresponding to an infinitely repeated game with the following stage game:

$$\begin{pmatrix} (2, 2) & (0, 3) \\ (3, 0) & (1, 1) \end{pmatrix}$$

Show that the strategy  $s_g$  (“player the first strategy until either player plays

the second strategy”) can be represented as a Markov strategy. For what values of  $\delta$  is both players playing this strategy a Markov Nash equilibrium?