OR 3: Lecture 2 - Normal Form Games

Recap

In the previous lecture we discussed:

- Predicting rational behaviour using dominated strategies;

- The CKR;

We did discover certain games that did not have any dominated strategies.

Best response functions

Definition

In an n player normal form game. A strategy s^* for player i is a best response to some strategy profile s_{-i} if and only if $u_i(s^*, s_{-i}) \geq u_i(s, s_{-i})$ for all $s \in S_i$.

We can now start to predict rational outcomes in pure strategies by identifying all best responses to a strategy.

$$\begin{pmatrix} (1,3) & (4,2) & (2,2) \\ (4,0) & (0,3) & (4,1) \\ (2,5) & (3,4) & (5,6) \end{pmatrix}$$

We will underline the best responses for each strategy giving:

$$\begin{pmatrix} (1,\underline{3}) & (\underline{4},2) & (2,2) \\ (\underline{4},0) & (0,\underline{3}) & (4,1) \\ (2,5) & (3,4) & (\underline{5},\underline{6}) \end{pmatrix}$$

We see that (r_1, s_1) represented a pair of best responses. What can we say about the long term behaviour of this game?

Connection between best responses and dominance

Best responses against mixed strategies

We can identify best responses against mixed strategies. Let us take a look at the matching pennies game:

$$\begin{pmatrix} (1,-1) & (-1,1) \\ (-1,1) & (1,-1) \end{pmatrix}$$

If we assume that player 2 plays a mixed strategy $\sigma_2 = (x, 1-x)$ we have:

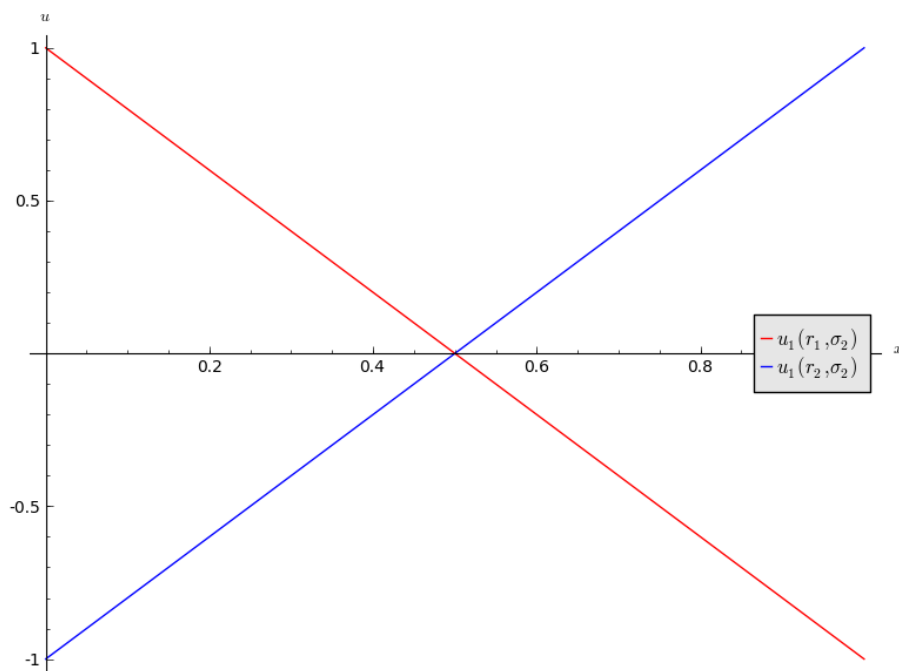$$u_1(r_1, \sigma_2) = 1 - 2x$$

and

$$u_1(r_2, \sigma_2) = 2x - 1$$

Figure 1:

1. If $x < 1/2$ then r_1 is a best response for player 1.

2. If $x > 1/2$ then r_1 is a best response for player 1.

3. If $x = 1/2$ then player 1 is indifferent.

Let us repeat this exercise for the battle of the sexes game.

$$\begin{pmatrix} (3,2) & (0,0) \\ (1,1) & (2,3) \end{pmatrix}$$

If we assume that player 2 plays a mixed strategy $\sigma_2 = (x, 1-x)$ we have:

$$u_1(r_1, \sigma_2) = 3x$$

and

$$u_1(r_2, \sigma_2) = 2 - x$$
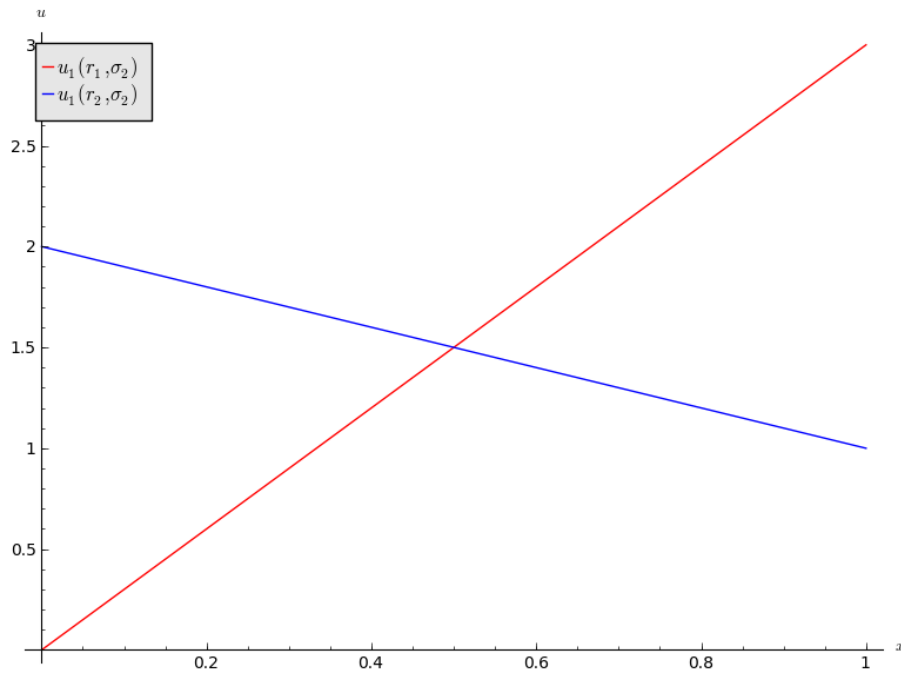

Figure 2:

1. If $x < 1/2$ then r_2 is a best response for player 1.

2. If $x > 1/2$ then r_1 is a best response for player 1.

3

3. If $x = 1/2$ then player 1 is indifferent.