Infinitely Repeated Games

Game Theory

Vincent Knight

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- s_D: defect at every stage

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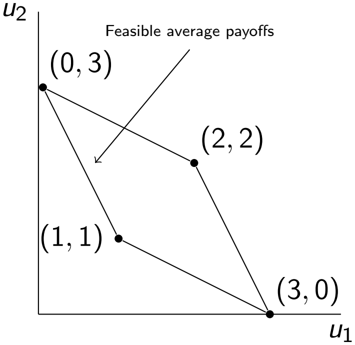
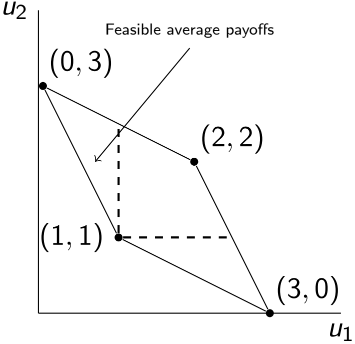
Possible interpretation of δ: probability of game ending at any stage.

$$\bar{T} = \frac{1}{1-\delta}$$

$$\frac{1}{\bar{T}} U_i(r,s) = (1-\delta) U_i(r,s)$$



u_2
Feasible average payoffs
$(0, 3)$
$(2, 2)$
$(1, 1)$
$(3, 0)$
u_1


u_2
Feasible average payoffs
$(0, 3)$
$(2, 2)$
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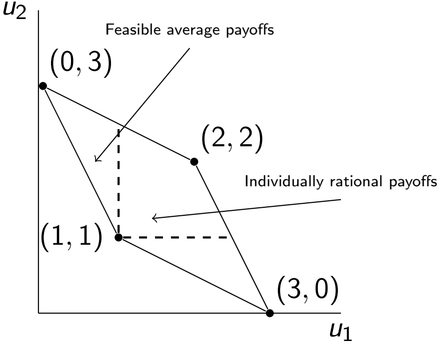
u_1


u_2
Feasible average payoffs
$(0,3)$
$(2,2)$
Individually rational payoffs
$(1,1)$
$(3,0)$
u_1

Folk Theorem

Let (u_1^*, u_2^*) be a pair of Nash equilibrium payoffs for a stage game. For every individually rational pair (v_1, v_2) there exists $\bar{\delta}$ such that for all $1 > \delta > \bar{\delta} > 0$ there is a subgame perfect Nash equilibrium with payoffs (v_1, v_2).