

"Begin by playing $\bar{\sigma}_i$...
 if any player deviates we
 σ_i^* \forall future stages"

Deviation

P_1 deviates at stage k :

$$U_1^{(k)} = \sum_{t=1}^{k-1} \delta^t v_1 + \delta^{k-1} \overbrace{u_1(\sigma_1^1, \bar{\sigma}_2)}^{\text{red checkmark}} \\ + u_1^* \left(\frac{1}{1-\delta} - \sum_{t=1}^k \delta^{t-1} \right)$$

Deviate in 1st stage } makes
 most sense. $\downarrow (k=1)$

$$U_1^{(1)} = u_1(\sigma_1^1, \bar{\sigma}_2) + u_1^* \left[\frac{\delta}{1-\delta} \right]$$

if then

$$u_1(\sigma_1', \bar{\sigma}_1) + \frac{u_1^* \delta}{1-\delta} < \frac{V_1}{1-\delta}$$

NE

\uparrow ? \uparrow ?

$$(1-\delta)u_1(\sigma_1', \bar{\sigma}_1) + u_1^* \delta \leq V_1$$

$$u_1(\sigma_1', \bar{\sigma}_1) - V_1 \leq \delta \left(\overbrace{u_1(\sigma_1', \bar{\sigma}_1)}^{V_1} - \underbrace{u_1^*}_{\geq 0} \right)$$

$$NE \Leftrightarrow \text{long run } \delta \geq \bar{\delta}$$

where

$$0 < \bar{\delta} = \frac{u_1(\sigma_1', \bar{\sigma}_1) - V_1}{u_1(\sigma_1', \bar{\sigma}_1) - u_1^*} < 1$$

NE ✓ subgame perfect?

$$\delta \rightsquigarrow \delta^k$$

