OR 3: Chapter 15 - Matching games

Recap

In the previous chapter:

- We defined matching games;
- We described the Gale-Shapley algorithm;
- We proved certain results regarding the Gale-Shapley algorithm.

In this Chapter we'll take a look at another type of game.

Cooperative Games

In cooperative game theory the interest lies with understanding how coalitions form in competitive situations.

Definition

A **characteristic function game** G is given by a pair (n, v) where n is the number of players and $v : 2^{[n]} \to \mathbb{R}$ is a **characteristic function** which maps every coalition of players to a payoff.

Let's consider the following game:

> "3 players must share a taxi. Here are the costs for each individual journey: - Player 1: 6 - Player 2: 12 - Player 3: 42"

This is illustrated below:

To construct the characteristic function we first obtain the power set (ie all possible coalitions) $2^{\{1,2,3\}} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \Omega\}$ where Ω denotes the set of all players ($\{1, 2, 3\}$).

The characteristic function is given below:

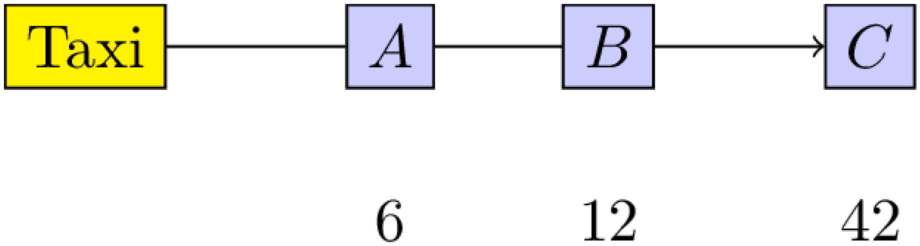

Figure 1:

$$
v(S) = \begin{cases} 6, & \text{if } S = \{1\} \\ 12, & \text{if } S = \{2\} \\ 42, & \text{if } S = \{3\} \\ 12, & \text{if } S = \{1,2\} \\ 42, & \text{if } S = \{1,3\} \\ 42, & \text{if } S = \{2,3\} \\ 42, & \text{if } S = \{1,2,3\} \end{cases}
$$

Definition

A characteristic function game $G = (n, v)$ is called **monotone** is it satisfies $v(S_2) \geq v(S_1)$ for all $S_1 \subseteq S_2$.

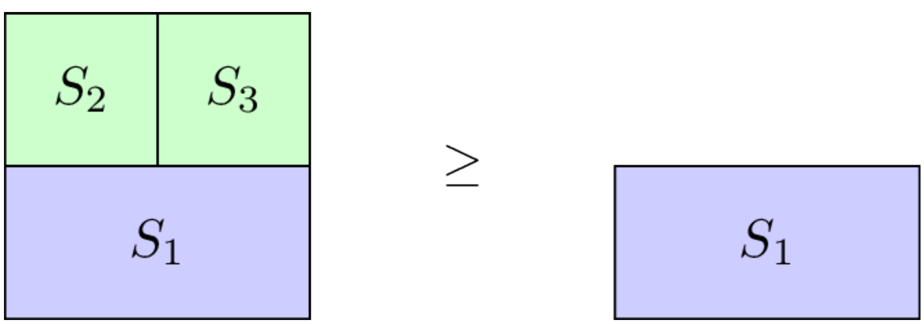

Figure 2:

Our taxi example is monotone, however the $G = (3, v_1)$ with v_1 defined as:

$$
v_1(S) = \begin{cases} 6, & \text{if } S = \{1\} \\ 12, & \text{if } S = \{2\} \\ 42, & \text{if } S = \{3\} \\ 10, & \text{if } S = \{1,2\} \\ 42, & \text{if } S = \{1,3\} \\ 42, & \text{if } S = \{2,3\} \\ 42, & \text{if } S = \{1,2,3\} \end{cases}
$$

is not.

Definition

A characteristic function game $G = (n, v)$ is called **superadditive** if it satisfies $v(S_1 \cup S_2) \geq v(S_1) + v(S_2)$.

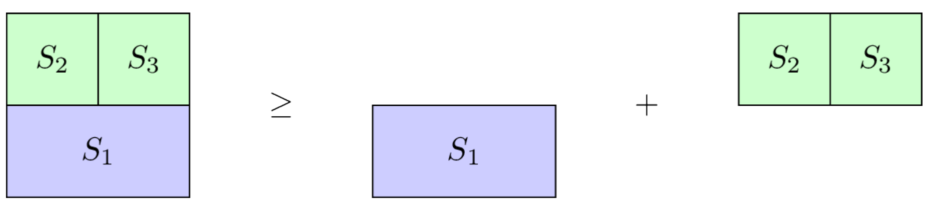

Figure 3:

Our taxi example is not superadditive, however the $G = (3, v_2)$ with v_2 defined as:

$$
v_2(S) = \begin{cases} 6, & \text{if } S = \{1\} \\ 12, & \text{if } S = \{2\} \\ 42, & \text{if } S = \{3\} \\ 18, & \text{if } S = \{1,2\} \\ 48, & \text{if } S = \{1,3\} \\ 55, & \text{if } S = \{2,3\} \\ 80, & \text{if } S = \{1,2,3\} \end{cases}
$$

is.

Shapley Value

Solution concept Required properties Shapley value