

Cooperative Games

Game Theory

Vincent Knight

A **characteristic function game** G is given by a pair (N, v) where N is the number of players and $v : 2^N \rightarrow \mathbb{R}$ is a **characteristic function** which maps every coalition of players to a payoff.

Alice, *Bob*, Celine share a meal:

Alice, Bob, Celine share a meal:

$$v(c) = \begin{cases} 80, & \text{if } c = \{A\} \\ 56, & \text{if } c = \{B\} \\ 70, & \text{if } c = \{C\} \\ 80, & \text{if } c = \{A, B\} \\ 85, & \text{if } c = \{A, C\} \\ 72, & \text{if } c = \{B, C\} \\ 90, & \text{if } c = \{A, B, C\} \end{cases}$$

$$\phi_i(G) = \frac{1}{n!} \sum_{\pi \in \Pi_n} \Delta_{\pi}^G(i)$$

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(A, B, C)	$(80, 0, 10)$

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(A, B, C)	$(80, 0, 10)$
(A, C, B)	$(80, 5, 5)$

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(A, B, C)	$(80, 0, 10)$
(A, C, B)	$(80, 5, 5)$
(B, A, C)	$(24, 56, 10)$

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(A, B, C)	$(80, 0, 10)$
(A, C, B)	$(80, 5, 5)$
(B, A, C)	$(24, 56, 10)$
(B, C, A)	$(18, 56, 16)$

$$v(c) = \begin{cases} 80, & \text{if } c = \{A\} \\ 56, & \text{if } c = \{B\} \\ 70, & \text{if } c = \{C\} \\ 80, & \text{if } c = \{A, B\} \\ 85, & \text{if } c = \{A, C\} \\ 72, & \text{if } c = \{B, C\} \\ 90, & \text{if } c = \{A, B, C\} \end{cases}$$

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(A, B, C)	$(80, 0, 10)$
(A, C, B)	$(80, 5, 5)$
(B, A, C)	$(24, 56, 10)$
(B, C, A)	$(18, 56, 16)$
(C, A, B)	$(15, 5, 70)$

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(A, B, C)	$(80, 0, 10)$
(A, C, B)	$(80, 5, 5)$
(B, A, C)	$(24, 56, 10)$
(B, C, A)	$(18, 56, 16)$
(C, A, B)	$(15, 5, 70)$
(C, B, A)	$(18, 2, 70)$

$$v(c) = \begin{cases} 80, & \text{if } c = \{A\} \\ 56, & \text{if } c = \{B\} \\ 70, & \text{if } c = \{C\} \\ 80, & \text{if } c = \{A, B\} \\ 85, & \text{if } c = \{A, C\} \\ 72, & \text{if } c = \{B, C\} \\ 90, & \text{if } c = \{A, B, C\} \end{cases}$$

π	δ_{π}^G
(A, B, C)	$(80, 0, 10)$
(A, C, B)	$(80, 5, 5)$
(B, A, C)	$(24, 56, 10)$
(B, C, A)	$(18, 56, 16)$
(C, A, B)	$(15, 5, 70)$
(C, B, A)	$(18, 2, 70)$
ϕ	$(39.2, 20.7, 30.2)$