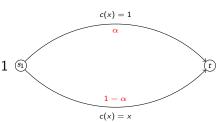
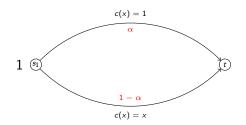
## Connection between Nash flows an optimal flows Game Theory

Vincent Knight

## (G, r, c)

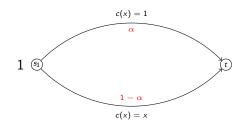
- ▶ G = (V, E), with a defined set of sources  $s_i$  and sinks  $t_i$ ;
- ► A commodity *r<sub>i</sub>*;
- ▶ A set of latencies: c<sub>e</sub>.





Define the potential function:

$$\Phi(f) = \sum_{e \in E} \int_0^{f_e} c_e(x) dx$$

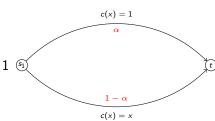


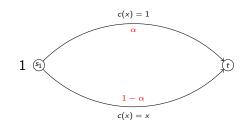
Define the potential function:

$$\Phi(f) = \sum_{e \in E} \int_0^{f_e} c_e(x) dx$$

$$\Phi(f) = \alpha + \frac{(1 - \alpha)^2}{2} = \frac{1}{2} - \frac{\alpha^2}{2}$$

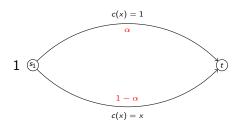
$$f^* = (0, 1) \text{ minimises } \Phi(f)$$



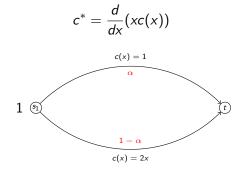


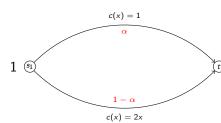
Define marginal costs:

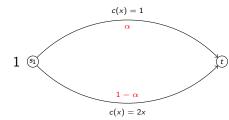
$$c^* = \frac{d}{dx}(xc(x))$$



## Define marginal costs:







 $\tilde{f}=(.5,.5)$  is a Nash flow

**Theorem:** A feasible flow  $\tilde{f}$  is a Nash flow for (G, r, c) if and only if  $\tilde{f}$  minimises  $\Phi(f)$ .

**Theorem:** A feasible flow  $f^*$  is an optimal flow for (G, r, c) if and only if  $f^*$  is a Nash flow for  $(G, r, c^*)$ .