1 Homework sheet 4 - Evolutionary games, games with incomplete information and stochastic games

1. Consider the pairwise contest games with the following associated two player games:

$$\begin{pmatrix} (2,2) & (4,5) \\ (5,4) & (1,1) \end{pmatrix}$$

$$\begin{pmatrix} (1,1) & (0,0) \\ (0,0) & (1,1) \end{pmatrix}$$

   Identify all evolutionary stable strategies.

2. Consider the following game:

   > In a mathematics department, researchers can choose to use one of two systems for typesetting their research papers: LaTeX or Word. We will refer to these two strategies as L and W respectively. A user of W receives a basic utility of 1 and as L is more widely used by mathematicians out of the department and is in general considered to be a better system a user of L gets a basic utility of $\alpha > 1$. Members of the mathematics department often collaborate and as such it is beneficial for the researchers to use the same typesetting system. If we let μ represent the proportion of users of L we let:

   $$u(L, \chi) = \alpha + 2\mu$$
   $$u(W, \chi) = 1 + 2(1 - \mu)$$

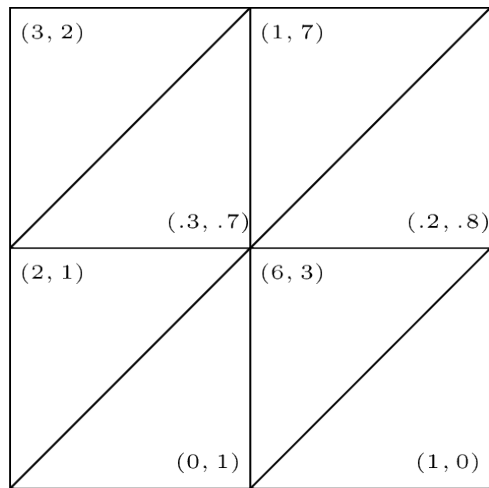   What are the evolutionary stable strategies?

3. Consider the following two normal form games:

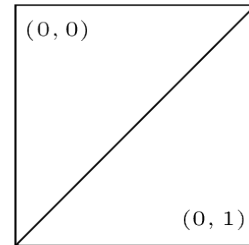 $$A = \begin{pmatrix} (3,0) & (-1,-1) & (1,2) \\ (1,0) & (-1,1) & (2,0) \end{pmatrix}$$

   $$B = \begin{pmatrix} (2,2) & (1,1) & (1,3) \\ (1,3) & (-2,-3) & (4,2) \end{pmatrix}$$

   Assume both players play either game A or game B with probability $1/2$, neither player knows which game is played. Obtain the Nash equilibrium for this game.
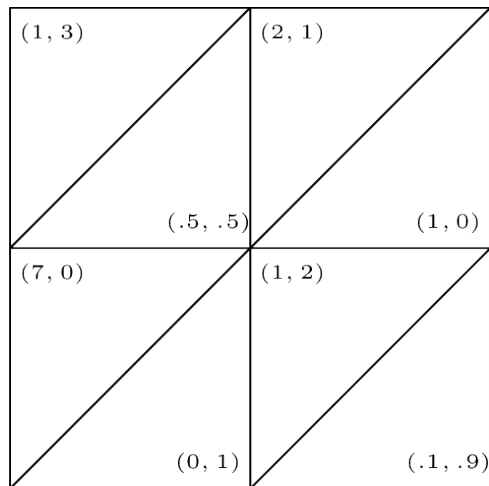
4. Repeat the analysis of the principal agent game assuming that p is the probability of the project being successful in case of a high level of effort by the employee.

    i. What are the expected utilities to the employer and the employee?

    ii. Obtain a condition for which the employer should offer a bonus.

5. Obtain the Markov Nash equilibrium (in pure strategies if it exists) for the following games assuming $\delta = 1/4$.
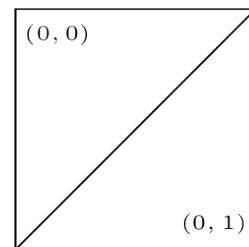
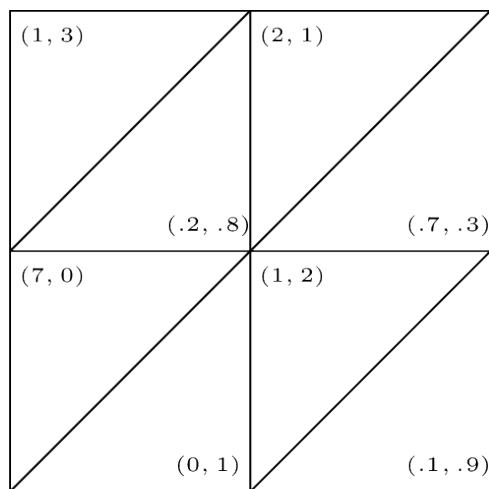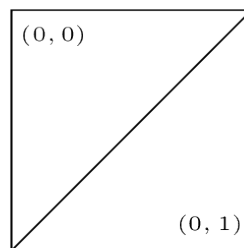|	
(3, 2) (.3, .7)	(1, 7) (.2, .8)
(2, 1) (0, 1)	(6, 3) (1, 0)

$$x$$

(0, 0)   (0, 1)

$$y$$

(1, 3) (.5, .5)	(2, 1) (1, 0)
(7, 0) (0, 1)	(1, 2) (.1, .9)

$$x$$

(0, 0)   (0, 1)

$$y$$

$$x \qquad\qquad\qquad\qquad y$$