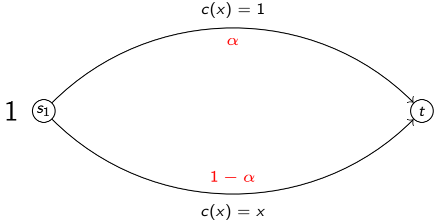
Connection between Nash flows an optimal flows

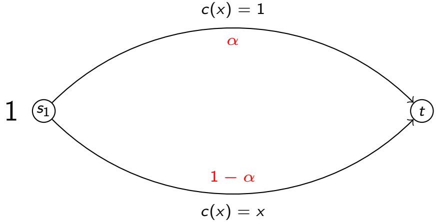
Game Theory

Vincent Knight

$$(G, r, c)$$

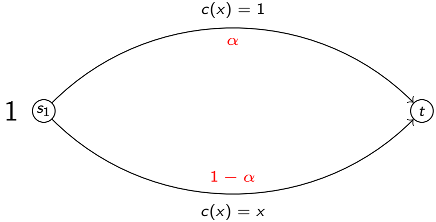
- $G = (V, E)$, with a defined set of sources s_i and sinks t_i;
- A commodity r_i;
- A set of latencies: c_e.


$c(x) = 1$
α
1
s_1
t
$1 - \alpha$
$c(x) = x$



Define the potential function:

$$\Phi(f) = \sum_{e \in E} \int_0^{f_e} c_e(x)\,dx$$

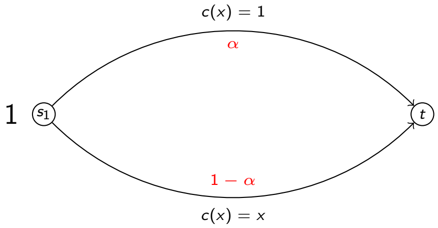


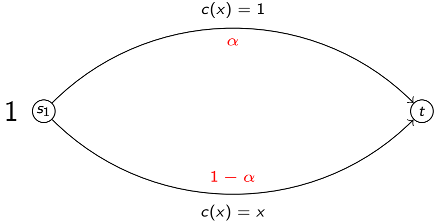
Define the potential function:

$$\Phi(f) = \sum_{e \in E} \int_0^{f_e} c_e(x) dx$$

$$\Phi(f) = \alpha + \frac{(1-\alpha)^2}{2} = \frac{1}{2} - \frac{\alpha^2}{2}$$

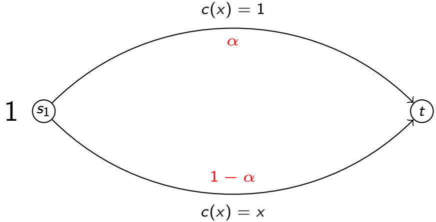
$$f^* = (0, 1) \text{ minimises } \Phi(f)$$


$c(x) = 1$
α
1
s_1
t
$1 - \alpha$
$c(x) = x$



Define marginal costs:

$$c^* = \frac{d}{dx}(xc(x))$$



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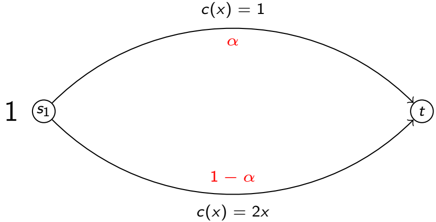
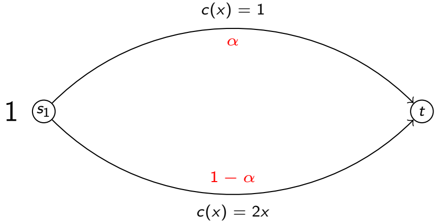
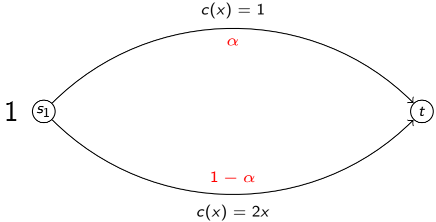



$c(x) = 1$
α
1
s_1
t
$1 - \alpha$
$c(x) = 2x$



$\tilde{f} = (.5, .5)$ is a Nash flow

Theorem: A feasible flow $\tilde{f}$ is a Nash flow for (G, r, c) if and only if $\tilde{f}$ minimises $\Phi(f)$.

Theorem: A feasible flow f^* is an optimal flow for (G, r, c) if and only if f^* is a Nash flow for (G, r, c^*).