# OR 3: Chapter 9 - Finitely Repeated Games

## Recap

In the previous chapter:

- We looked at the connection between games in normal form and extensive form;
- We defined a subgame;
- We define a refinement of Nash equilibrium: subgame perfect equilibrium.

In this chapter we'll start looking at instances where games are repeated.

## Definition of a repeated game

- Definitions
- Description of strategy

## Subgame perfect Nash equilibrium in repeated games

### Theorem

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For any repeated game, any sequence of stage Nash profiles gives the outcome of a subgame perfect Nash equilibrium.

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If we consider the strategy given by:

> "Player $i$ should play strategy $\tilde{s}_i^{(k)}$ regardless of the play of any previous strategy profiles."

where $\tilde{s}_i^{(k)}$ is the strategy played by player $i$ in any stage Nash profile. The $k$ is used to indicate that all players play strategies from the same stage Nash profile.

Using backwards induction we see that this strategy is a Nash equilibrium. Furthermore it is a stage Nash profile so it is a Nash equilibria for the last stage game which is the last subgame. If we consider (in an inductive way) each subsequent subgame the result holds.

**Example**

Consider the following stage game:

$$\begin{pmatrix} (1,3) & (2,10) \\ (2,2) & (4,1) \end{pmatrix}$$

The following plot shows the various possible outcomes of the repeated game for $T = 2$:

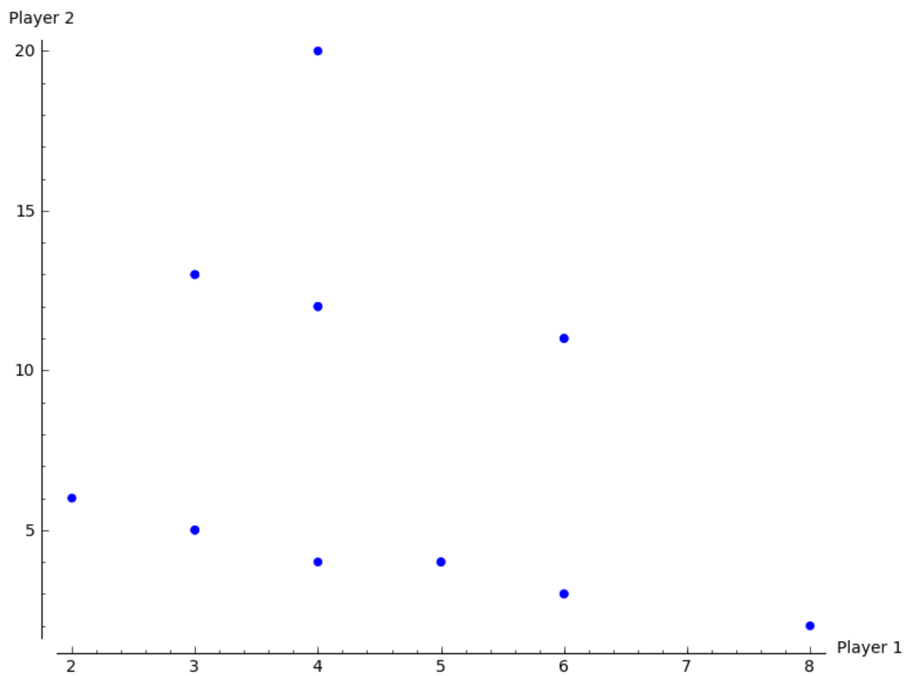


Figure 1:

If we consider the two pure equilibria $(r_1, s_2)$ and $(r_2, s_1)$, we have 4 possible outcomes that correspond to the outcome of a subgame perfect Nash equilibria:

$$(r_1 r_1, s_2 s_2) \text{ giving utility vector: } (4, 20)$$

$$(r_1 r_2, s_2 s_1) \text{ giving utility vector: } (4, 12)$$

$$(r_2 r_1, s_1 s_2) \text{ giving utility vector: } (4, 12)$$

$$(r_2 r_2, s_1 s_1) \text{ giving utility vector: } (4, 4)$$

Importantly, not all subgame Nash equilibria outcomes are of the above form.

## Reputation in repeated games

- Give example (Try above game but be ready to use a different one) of a reputation based strategy that is not a stage equilibria but is subgame perfect.