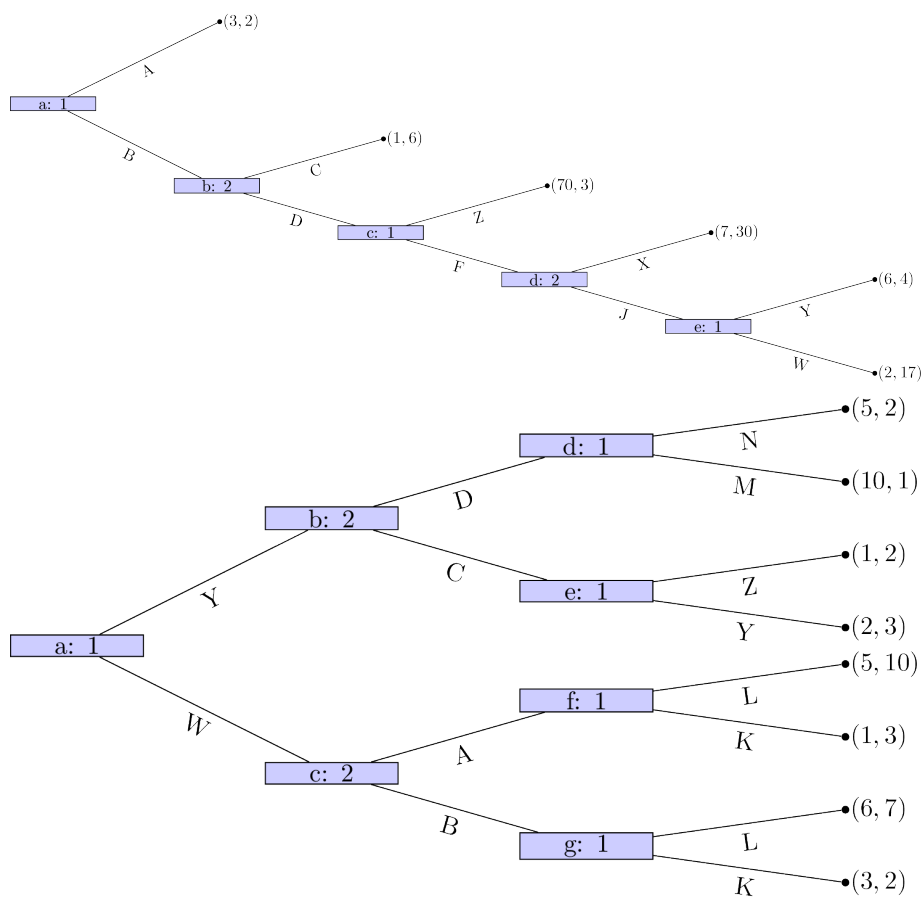
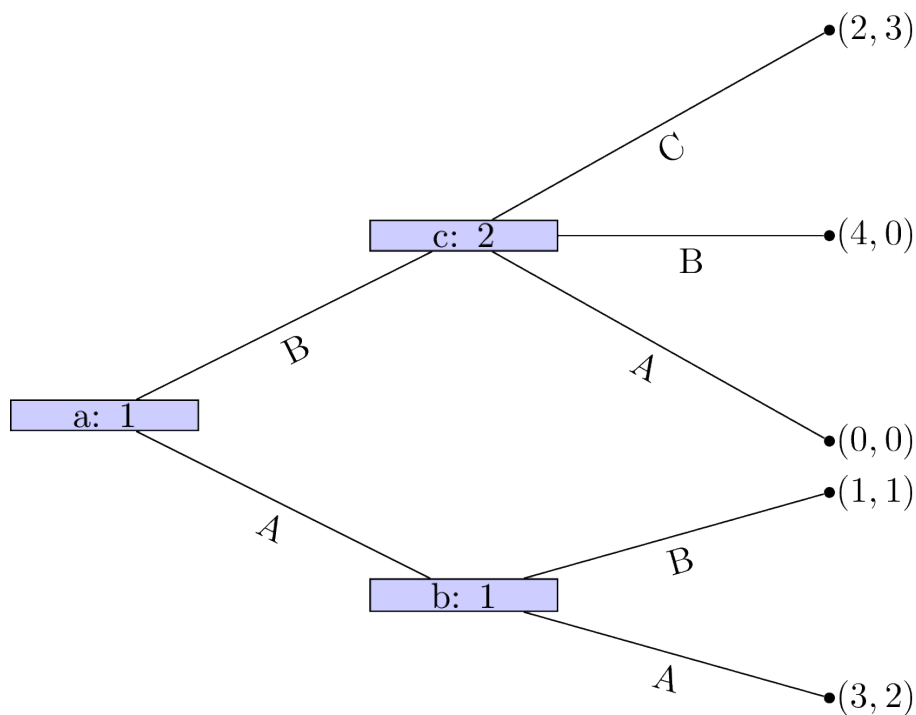
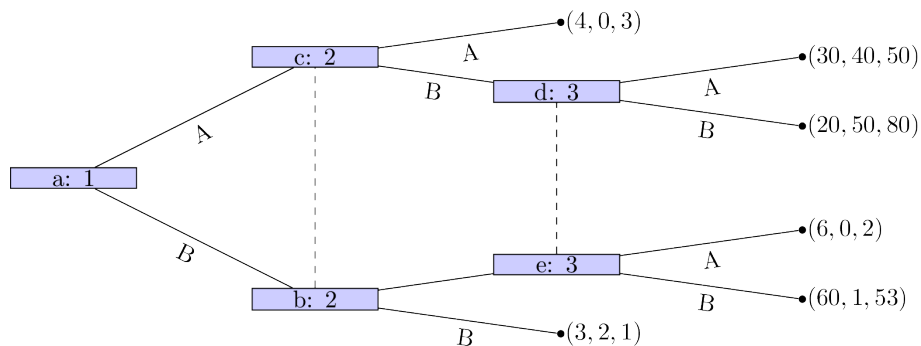


1 Homework sheet 3 - Extensive form games, subgame perfect equilibrium and repeated games

1. Obtain the Nash equilibrium for the following games using backward induction:





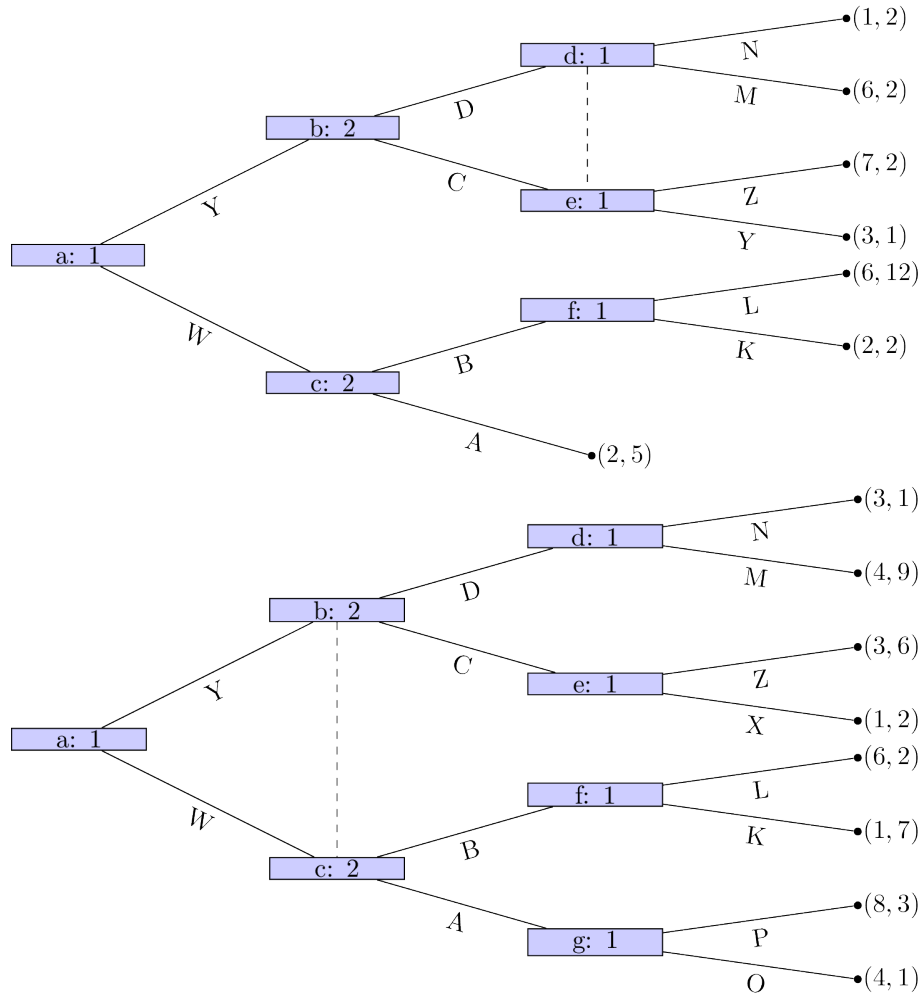
2. Obtain the Nash equilibrium for the following game:

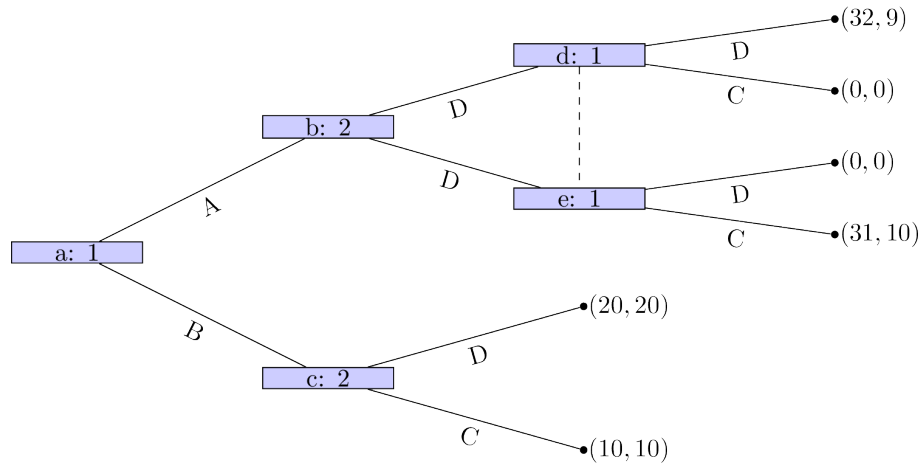
Player 1 chooses a number $x \geq 0$, which player 2 observes. After this simultaneously and independatly player 1 and player 2 choose $y_2, y_1 \in \mathbb{R}$ respectively. The utility to player 1 is given by $2y_2y_1 + xy_1 - y_1^2 - x^3/3$ and the utility to player 2 is given by $-(y_1 - 2y_2)^2$.

3. For each of the following games:

i. Identify all subgames.

- ii. Identify the corresponding normal form representations and hence obtain all Nash equilibrium.
- iii. Identify which Nash equilibrium are also subgame perfect Nash equilibrium.





4. Consider the game in exercise 3 of homework sheet 2. Assume that the vendors now position themselves sequentially. Model the game in extensive form and find the subgame perfect Nash equilibrium.
5. For the following stage games:
 - i. Plot all possible utility pairs for $T = 2$;
 - ii. Recalling that subgame perfect equilibrium for the repeated game must play a stage Nash equilibrium in the final stage attempt to identify a Nash equilibrium for the repeated game that is not a sequence of stage Nash profiles.

$$\begin{pmatrix} (4, 3) & (7, 6) \\ (1, 1) & (4, 3) \end{pmatrix}$$

$$\begin{pmatrix} (-1, 1) & (3, -7) \\ (-2, 6) & (2, 2) \end{pmatrix}$$

$$\begin{pmatrix} (5, 2) & (2, 0) & (6, 3) \\ (5, 2) & (1, 3) & (7, 1) \end{pmatrix}$$

6. Consider the following stage game:

$$\begin{pmatrix} (-1, 1) & (3, -7) \\ (-2, 6) & (2, 2) \end{pmatrix}$$

- i. For $\delta = 1/3$ obtain the utilities for the infinitely repeated game for the strategies S_D : “play the first strategy throughout” and S_C : “play the second strategy throughout”.
- ii. Plot the space of feasible average payoffs and the space of individually rational payoffs.
- iii. Obtain δ that ensures that a strategy profile exists that would give a subgame perfect Nash equilibrium with average payoffs: $(3/2, 3/2)$, $(0, 3)$, $(2, 6)$ and $(2, 0)$.