OR 3: Lecture 2 - Normal Form Games

Recap

In the previous lecture we discussed:

- Interactive decision making;
- Normal form games;
- Normal form games and representing information sets.

We did this looking at a game called "the battle of the sexes":

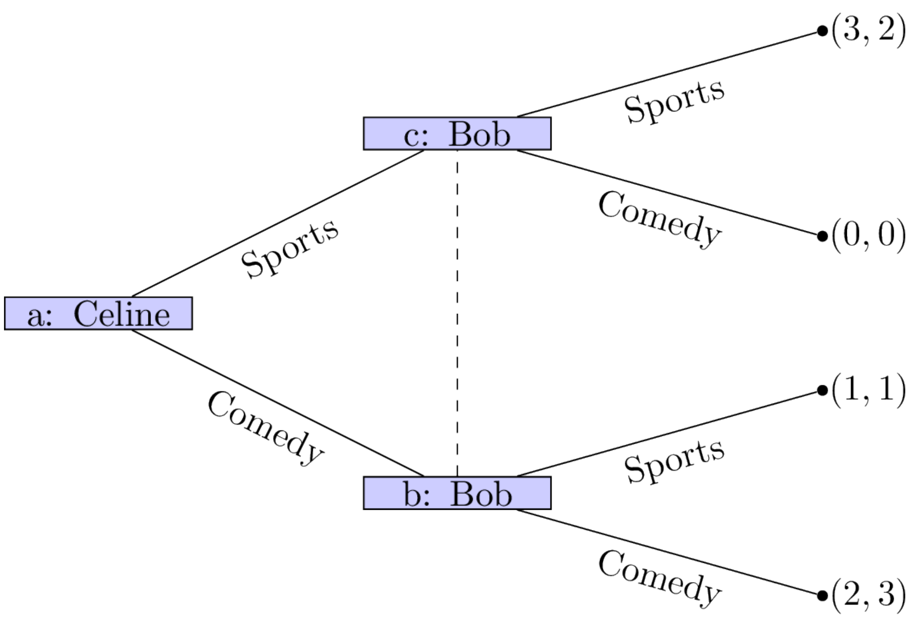

Figure 1: Celine and Bob with Information Set

Can we think of a better way of representing this game?

Normal form games

One other representation for a game is called the **normal form**.

Definition

A n player **normal form game** consists of:

1. A finite set of n players;
2. Strategy spaces for the players: $S_1, S_2, S_3, \dots S_n$;
3. Payoff functions for the players: $u_i : S_1 \times S_2 \dots \times S_n \to \mathbb{R}$

A natural way of representing a two player normal form game is using a **bi-matrix**. If we assume that $S_1 = \{r_i \mid 1 \leq i \leq m\}$ and $S_2 = \{s_j \mid 1 \leq j \leq n\}$ then the following is a **bi-matrix** representation of the game considered:

$$
\begin{array}{cc}
 & \text{Column strategies (player 2)} \\
\text{Row strategies (player 1)} & \begin{pmatrix}
(u_1(r_1, s_1), u_2(r_1, s_1)) & (u_1(r_1, s_2), u_2(r_1, s_2)) & \dots & (u_1(r_1, s_2), u_2(r_1, s_n)) \\
(u_1(r_2, s_1), u_2(r_2, s_1)) & (u_1(r_2, s_2), u_2(r_2, s_2)) & \dots & (u_1(r_2, s_2), u_2(r_2, s_n)) \\
\vdots & \dots & \dots & \vdots \\
(u_1(r_m, s_1), u_2(r_m, s_1)) & (u_1(r_m, s_2), u_2(r_m, s_2)) & \dots & (u_1(r_m, s_2), u_2(r_m, s_n))
\end{pmatrix}
\end{array}
$$

Figure 2: A bi matrix

Some examples

The battle of the sexes

This is the game we've been looking at between Bob and Celine:

$$
\begin{pmatrix}
(3, 2) & (0, 0) \\
(1, 1) & (2, 3)
\end{pmatrix}
$$

Prisoners' Dilemma

Suppose ...

$$
\begin{pmatrix}
(2, 2) & (0, 3) \\
(3, 0) & (1, 1)
\end{pmatrix}
$$

Hawk-Dove/Chicken

Suppose...

$$\begin{pmatrix} (0,0) & (3,1) \\ (1,3) & (2,2) \end{pmatrix}$$

Coordination

Suppose...

$$\begin{pmatrix} (1,1) & (0,0) \\ (0,0) & (1,1) \end{pmatrix}$$

Pareto Coordination

Suppose...

$$\begin{pmatrix} (2,2) & (0,0) \\ (0,0) & (1,1) \end{pmatrix}$$

Pigs

Suppose...

$$\begin{pmatrix} (4,2) & (2,3) \\ (6,-1) & (0,0) \end{pmatrix}$$