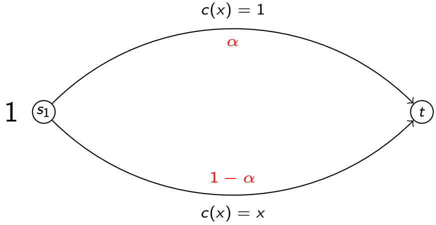
Routing Games

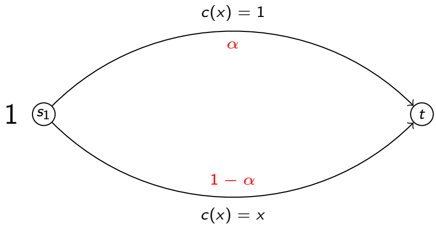
Game Theory

Vincent Knight

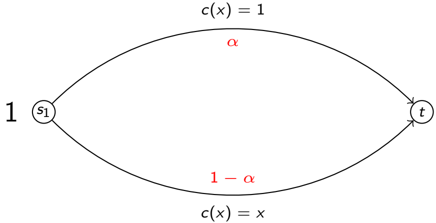
$$(G, r, c)$$

- $G = (V, E)$, with a defined set of sources s_i and sinks t_i;
- A commodity r_i;
- A set of latencies: c_e.


$c(x) = 1$
α
1
s_1
t
$1 - \alpha$
$c(x) = x$

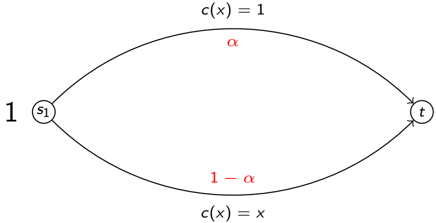


$$C(f) = \sum_{P \in \mathcal{P}} c_P(f_P) f_P = \sum_{e \in E} c_e(f_e) f_e$$



$$C(f) = \sum_{P \in \mathcal{P}} c_P(f_P) f_P = \sum_{e \in E} c_e(f_e) f_e$$

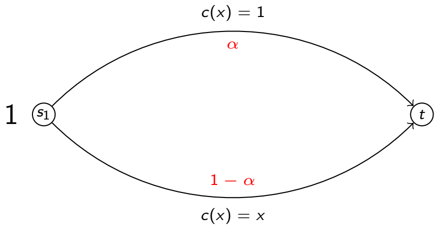
$$\alpha = .75 \Rightarrow f = (.75, .25)$$



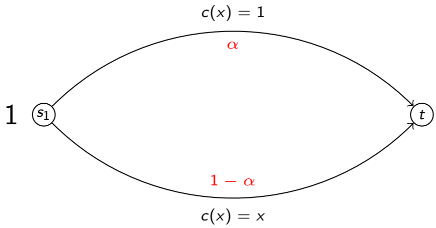
$$C(f) = \sum_{P \in \mathcal{P}} c_P(f_P) f_P = \sum_{e \in E} c_e(f_e) f_e$$

$$\alpha = .75 \Rightarrow f = (.75, .25)$$

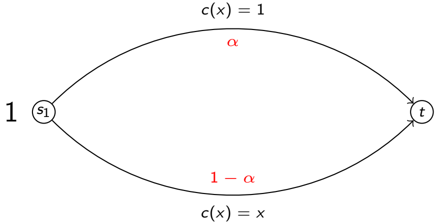
$$C(f) = 1 \times .75 + c(.25) \times .25 = 1 \times .75 + .25 \times .25 = 1.125$$



$c(x) = 1$
α
1
s_1
t
$1 - \alpha$
$c(x) = x$

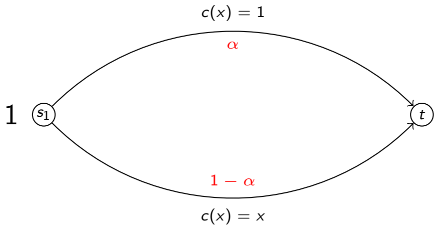


An optimal flow f^* minimizes $C(f)$.



An optimal flow f^* minimizes $C(f)$.

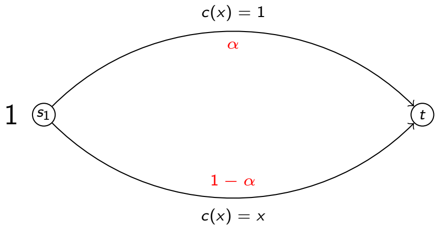
$$C(\alpha) = \alpha + (1 - \alpha)^2 = 1 - \alpha + \alpha^2$$

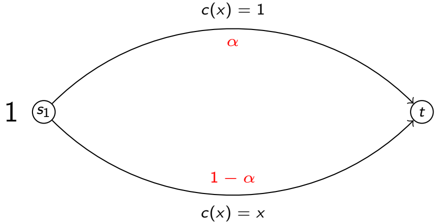


An optimal flow f^* minimizes $C(f)$.

$$C(\alpha) = \alpha + (1 - \alpha)^2 = 1 - \alpha + \alpha^2$$

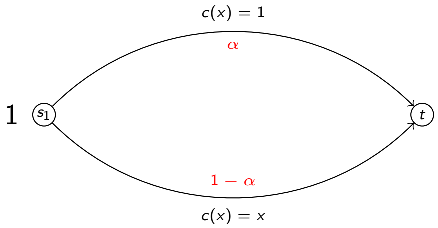
$$f^* = (.5, .5)$$


$c(x) = 1$
α
1
s_1
t
$1 - \alpha$
$c(x) = x$



A Nash flow: $\tilde{f}$ iff for every commodity i and any two paths $P_1, P_2 \in \mathcal{P}_i$ such that $f_{P_1} > 0$ then:

$$c_{P_1}(f) \leq c_{P_2}(f)$$



A Nash flow: $\tilde{f}$ iff for every commodity i and any two paths $P_1, P_2 \in \mathcal{P}_i$ such that $f_{P_1} > 0$ then:

$$c_{P_1}(f) \leq c_{P_2}(f)$$

$$\tilde{f} = (0, 1)$$