

1 Homework sheet 4 - Evolutionary games, games with incomplete information and stochastic games

1. Consider the pairwise contest games with the following associated two player games:

$$\begin{pmatrix} (2, 2) & (4, 5) \\ (5, 4) & (1, 1) \end{pmatrix}$$

$$\begin{pmatrix} (1, 1) & (0, 0) \\ (0, 0) & (1, 1) \end{pmatrix}$$

$$\begin{pmatrix} (\alpha, \alpha) & (1, \beta) \\ (\beta, 1) & (0, 0) \end{pmatrix}$$

(Assume $\alpha, \beta > 0$ and $\alpha \neq \beta$)

Identify all evolutionary stable strategies.

2. Consider the following game:

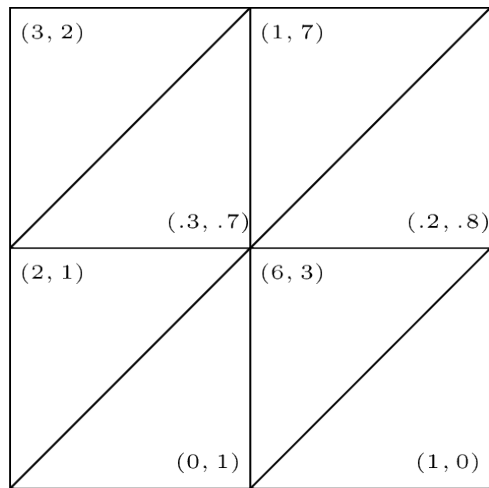
In a mathematics department, researchers can choose to use one of two systems for typesetting their research papers: LaTeX or Word. We will refer to these two strategies as L and W respectively. A user of W receives a basic utility of 1 and as L is more widely used by mathematicians out of the department and is in general considered to be a better system a user of L gets a basic utility of $\alpha > 1$. Members of the mathematics department often collaborate and as such it is beneficial for the researchers to use the same typesetting system. If we let μ represent the proportion of users of L we let:

$$\begin{aligned} u(L, \chi) &= \alpha + 2\mu \\ u(W, \chi) &= 1 + 2(1 - \mu) \end{aligned}$$

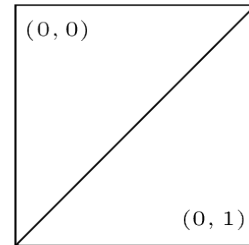
What are the evolutionary stable strategies?

3. Consider the simple game with two players: an insurer and a driver. The insurer sets a premium price $K \geq 0$, once that is done the driver can choose to buy insurance or not. It is assumed that the driver will have an accident with probability p , if the driver has an accident the financial cost is A . Represent this game in extensive form and obtain the Nash equilibrium for the game as a function of the parameters. Modify your analysis assuming that the utility function to the driver is given by $u(x) = x^{1/\alpha}$ and the utility to the insurer is given by $u(x) = x^{1/\beta}$.

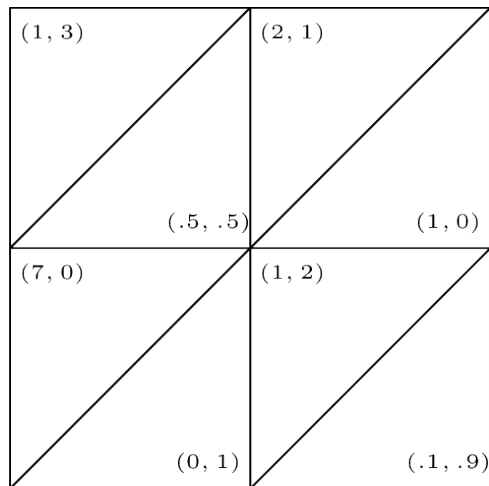
4. Repeat the analysis of the principal agent game assuming that p is the probability of the project being successful in case of a high level of effort by the employee.
- What are the expected utilities to the employer and the employee?
 - Obtain a condition for which the employer should offer a bonus.
5. Obtain the Markov Nash equilibrium for the following games assuming $\delta = 1/4$.



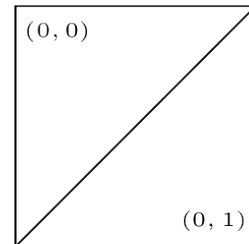
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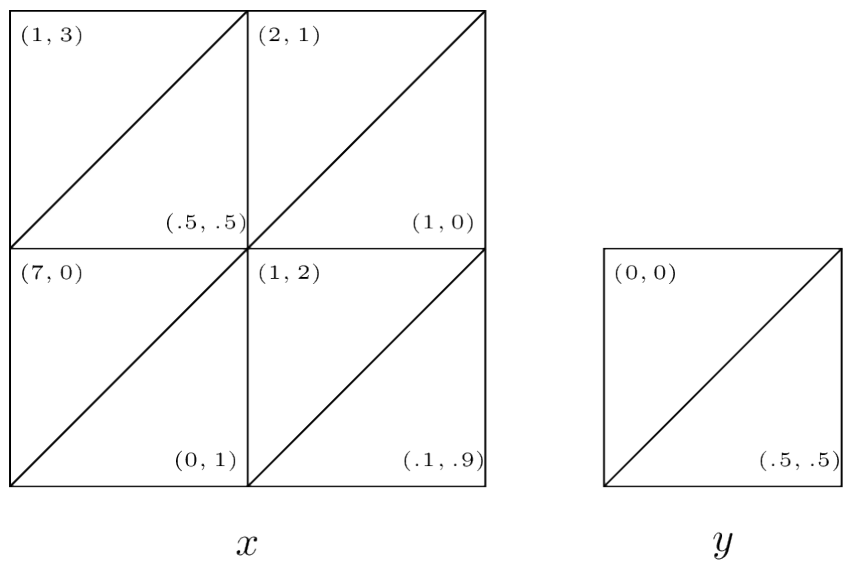
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y



6. Construct a two state stochastic game corresponding to an infinitely repeated game with the following stage game:

$$\begin{pmatrix} (2, 2) & (0, 3) \\ (3, 0) & (1, 1) \end{pmatrix}$$

Show that the strategy s_g (“player the first strategy until either player plays the second strategy”) can be represented as a Markov strategy. For what values of δ is both players playing this strategy a Markov Nash equilibrium?