

1 Homework sheet 4 - Evolutionary games, games with incomplete information and stochastic games

1. Consider the pairwise contest games with the following associated two player games:

$$\begin{pmatrix} (2, 3) & (4, 5) \\ (4, 5) & (1, 2) \end{pmatrix}$$

$$\begin{pmatrix} (1, -1) & (-1, 1) \\ (-1, 1) & (1, -1) \end{pmatrix}$$

$$\begin{pmatrix} (\alpha, 3) & (1, \beta) \\ (1, \beta) & (0, 0) \end{pmatrix}$$

Identify all evolutionary stable strategies.

2. Consider the following game:

In a mathematics department, researchers can choose to use one of two systems for typesetting their research papers: LaTeX or Word. We will refer to these two strategies as L and W respectively. A user of W receives a basic utility of 1 and as L is more widely used by mathematicians out of the department and is in general considered to be a better system a user of L gets a basic utility of $\alpha > 1$. Members of the mathematics department often collaborate and as such it is beneficial for the researchers to use the same typesetting system. If we let μ represent the proportion of users of L we let:

$$\begin{aligned} u(L, \chi) &= \alpha + 2\mu \\ u(W, \chi) &= 1 + 2(1 - \mu) \end{aligned}$$

What are the evolutionary stable strategies?

Solutions

Using the theorem for necessity of stability we have the following candidate ESS:

1. σ_L : everyone uses L , thus $\mu = 1$ (we have $u(L, \chi) > u(W, \chi)$).
2. σ_W : everyone uses W , thus $\mu = 0$ (we have $u(L, \chi) < u(W, \chi)$).

3. σ_m : some use L and some use W , by the theorem we have $u(L, \chi) = u(W, \chi)$ which implies $\alpha + 2\mu = 1 + 2(1 - \mu)$ giving $\mu = \frac{3-\alpha}{4}$.

Now we consider the post entry population $\chi_\epsilon = (1-\epsilon)\sigma^* + \epsilon\sigma$ (where σ^* is the base strategy and σ is the entry population). We denote $\sigma = (\mu, 1-\mu)$ and $\sigma^* = (\mu^*, 1-\mu^*)$ and $\delta = u(\sigma^*, \chi_\epsilon) - u(\sigma, \chi_\epsilon)$. We have:

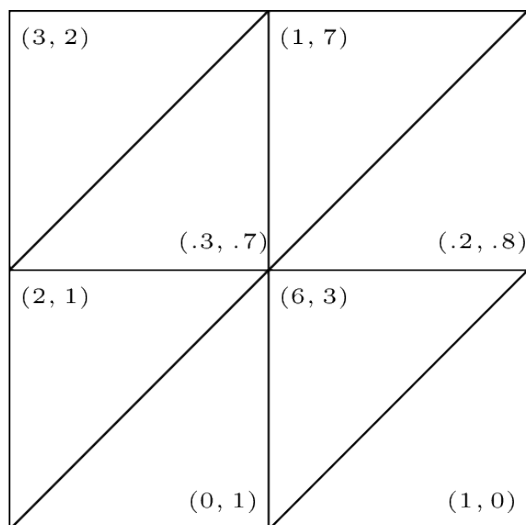
$$\delta = \mu^*u(L, \chi_\epsilon) + (1-\mu^*)u(W, \chi_\epsilon) - \mu u(L, \chi_\epsilon) + (1-\mu)u(W, \chi_\epsilon) = (\mu^* - \mu)(u(L, \chi_\epsilon) - u(W, \chi_\epsilon))$$

which gives:

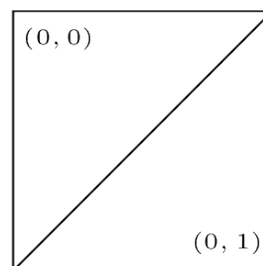
$$\delta = (\mu^* - \mu)(\alpha - 3 + 4((1-\epsilon)\mu^* + \epsilon\mu)) = (\mu^* - \mu)(4\mu^* + \alpha - 3 + 4\epsilon(\mu - \mu^*))$$

We now consider each potential ESS in turn, if $\delta > 0$ for all $\epsilon < \bar{\epsilon}$ for some $\bar{\epsilon}$ then we have an ESS (this is by definition):

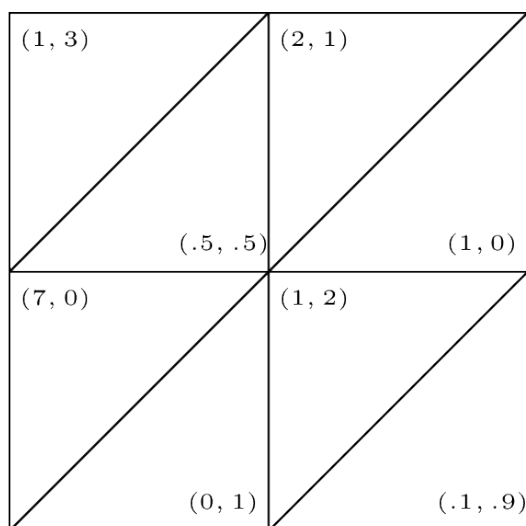
1. $\mu^* = 1$: $\delta = (1-\mu)(1 + \alpha + 4\epsilon(\mu - 1)) > (1-\mu)(1 + \alpha - 4\epsilon) > 0$ for all $\mu \neq 1$ and $\epsilon < \bar{\epsilon} = \frac{1+\alpha}{4}$. Thus σ_L is an ESS.
 2. $\mu^* = 0$: $\delta = -\mu(\alpha - 3 + 4\epsilon\mu)$. If $\alpha \geq 3$ then $\delta \leq 0$ for all values of μ, ϵ , thus if L is 3 times better than W σ_W is not an ESS. If $\alpha < 3$ $\delta > 0 \Leftrightarrow \alpha - 3 + 4\epsilon\mu < 0 \Rightarrow \alpha - 3 + 4\epsilon\mu < \alpha - 3 + 4\epsilon < 0$ for all $\mu \neq 0$ $\epsilon < \bar{\epsilon} = \frac{3-\alpha}{4}$. Thus σ_W is an ESS for $\alpha < 3$.
 3. $\mu^* = \frac{3-\alpha}{4}$: $\delta = -4\epsilon\left(\frac{3-\alpha}{4} - \mu\right)^2 < 0$ for all $\mu \neq \frac{3-\alpha}{4}$ and for all $\epsilon > 0$ so σ_m is not an ESS.
3. Consider the simple game with two players: an insurer and a driver. The insurer sets a premium price $K \geq 0$, once that is done the driver can choose to buy insurance or not. It is assumed that the driver will have an accident with probability p , if the driver has an accident the financial cost is A . Represent this game in normal form and obtain the Nash equilibrium for the game as a function of the parameters. Modify your analysis assuming that the utility function to the driver is given by $u(x) = x^{1/\alpha}$ and the utility to the insurer is given by $u(x) = x^{1/\beta}$.
4. Repeat the analysis of the principal agent game assuming that p is the probability of the project being successful in case of a high level of effort by the employee.
- i. What are the expected utilities to the employer and the employee?
 - ii. Obtain a condition for which the employer should offer a bonus.
5. Obtain the Markov Nash equilibrium for the following games assuming $\delta = 1/4$.



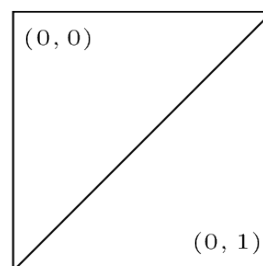
x



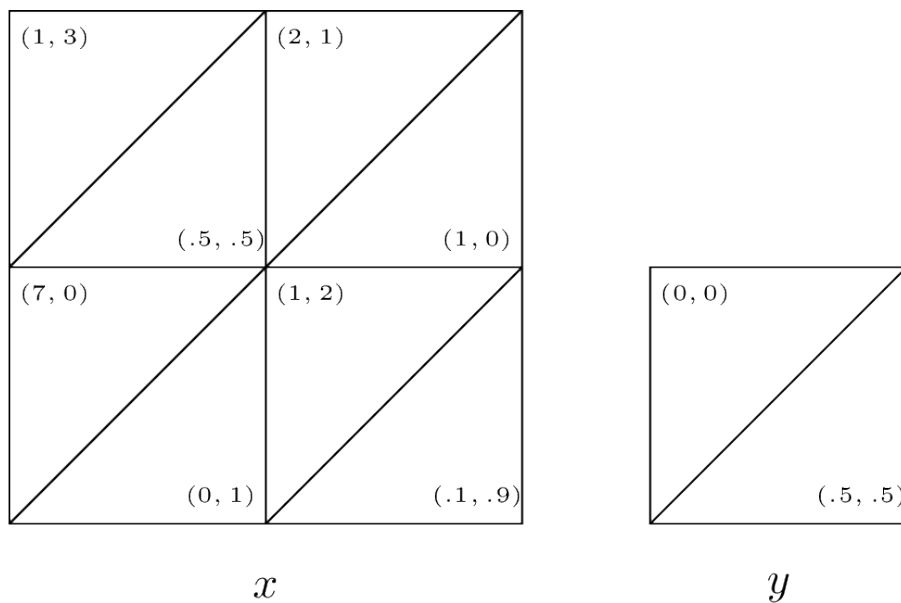
y



x



y



6. Construct a two state stochastic game corresponding to an infinitely repeated game with the following stage game:

$$\begin{pmatrix} (2, 2) & (0, 3) \\ (3, 0) & (1, 1) \end{pmatrix}$$

Show that the strategy s_g (“player the first strategy until either player plays the second strategy”) can be represented as a Markov strategy. For what values of δ is both players playing this strategy a Markov Nash equilibrium?