# OR 3: Chapter 4 - Best responses

## Recap

In the [previous lecture](Chapter_03-Dominance.docx) we discussed:

* Predicting rational behaviour using dominated strategies;
* The CKR;

We did discover certain games that did not have any dominated strategies.

## Best response functions

### Definition of a best response

In an player normal form game. A strategy for player is a best response to some strategy profile if and only if for all .

We can now start to predict rational outcomes in pure strategies by identifying all best responses to a strategy.

We will underline the best responses for each strategy giving ( is underlined if it is a best response to and vice versa):

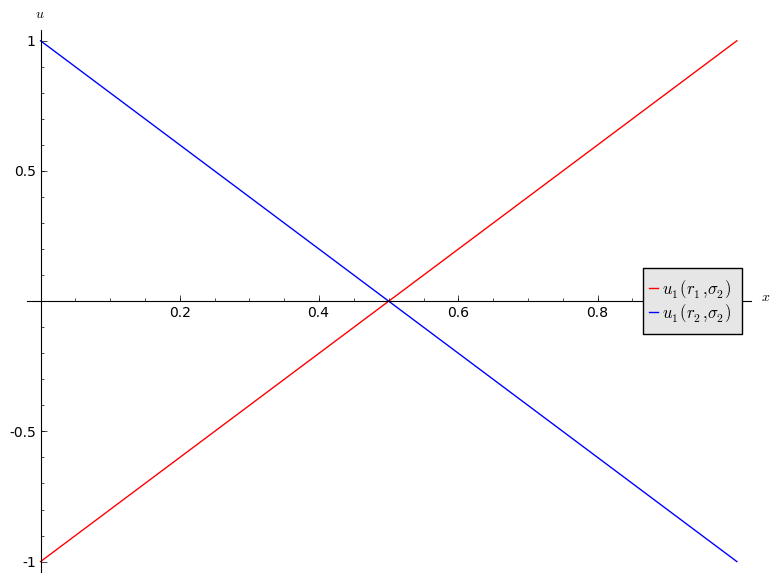
We see that represented a pair of best responses. What can we say about the long term behaviour of this game?

## Best responses against mixed strategies

We can identify best responses against mixed strategies. Let us take a look at the matching pennies game:

If we assume that player 2 plays a mixed strategy we have:

and



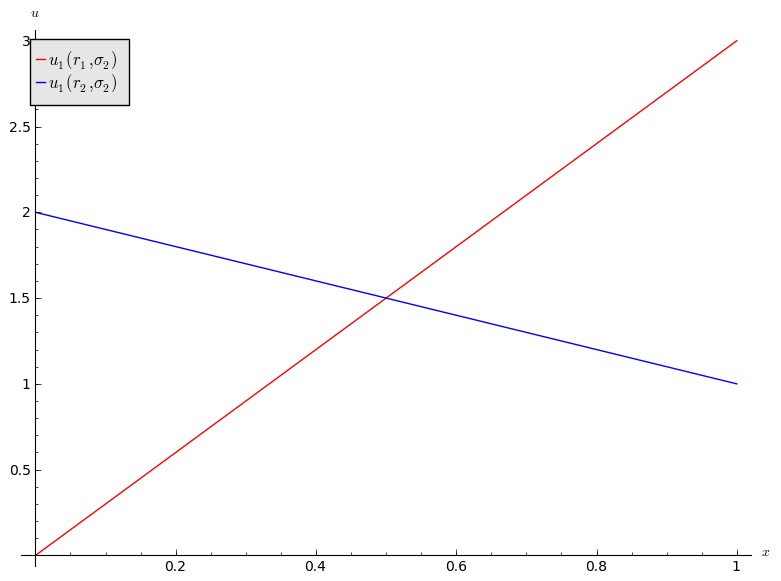
Mixed strategies fox the matching pennies game.

1. If then is a best response for player 1.
2. If then is a best response for player 1.
3. If then player 1 is indifferent.

Let us repeat this exercise for the battle of the sexes game.

If we assume that player 2 plays a mixed strategy we have:

and



Mixed strategies for the battle of the sexes game.

1. If then is a best response for player 1.
2. If then is a best response for player 1.
3. If then player 1 is indifferent.

## Connection between best responses and dominance

### Definition of the undominated strategy set

In an player normal form game, let us define the undominated strategy set :

If we consider the following game:

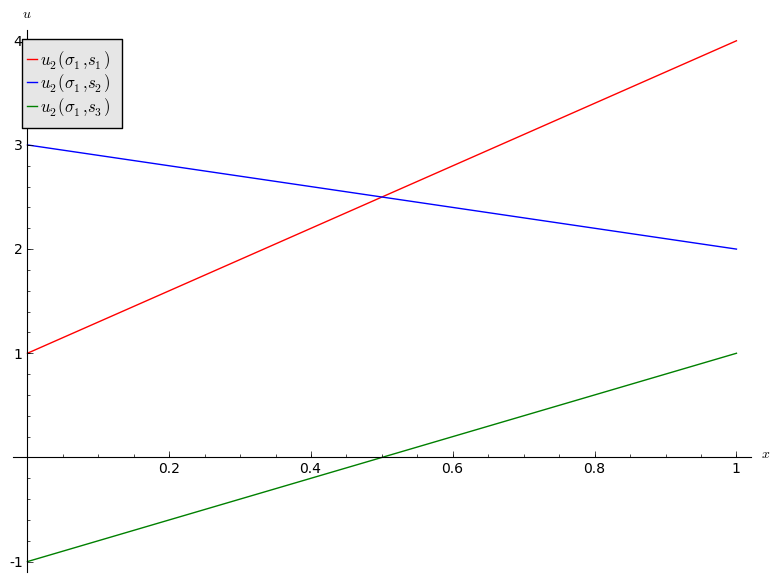
We have:

### Definition of the best responses strategy set

In an player normal form game, let us define the best responses strategy set :

In other words is the set of functions that are best responses to some strategy profile in .

Let us try to identify for the above game. Let us assume that player 1 plays . This gives:

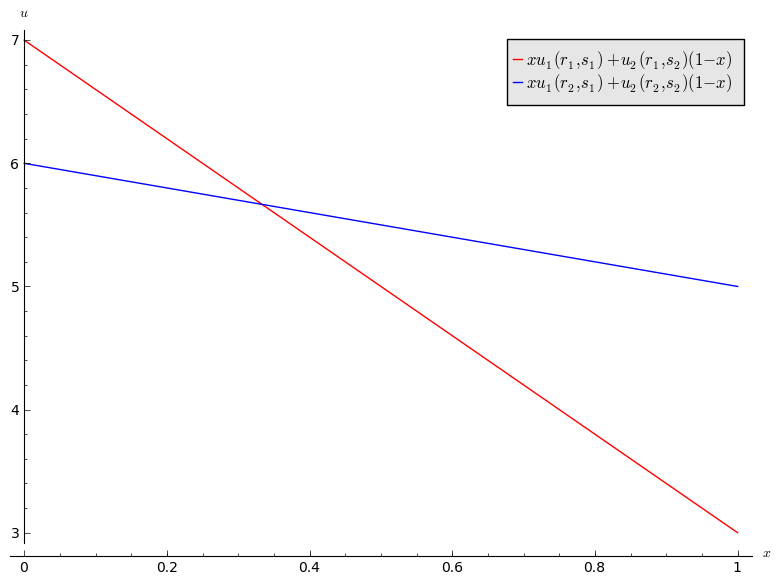


Utilities of player 2 in our example.

We see that is never a best response for player 2:

We will now attempt to identify for the above game. Let us assume that player two plays . This gives:

However as noted earlier is dominated by so:



Upper bounds on

We see that and are best responses for player 1 (however cannot be as for any value of the corresponding curves would be below the curves shown):

We have seen in our example that . This leads us to two Theorems (the proofs are omitted).

### Theorem of equality in 2 player games

In a 2 player normal form game for all .

This is however not always the case:

### Theorem of inclusion in player games

In an player normal form game for all .