21960197 向平 Assignment 6

Ex 5.1.1

Make a table of the error of the three-point centered-difference formula for $f'(0)$, where $f(x) = \sin x - \cos x$, with $h = 10^{-1}, \ldots, 10^{-12}$, as in the table in Section 5.1.2. Draw a plot of the results. Does the minimum error correspond to the theoretical expectation?

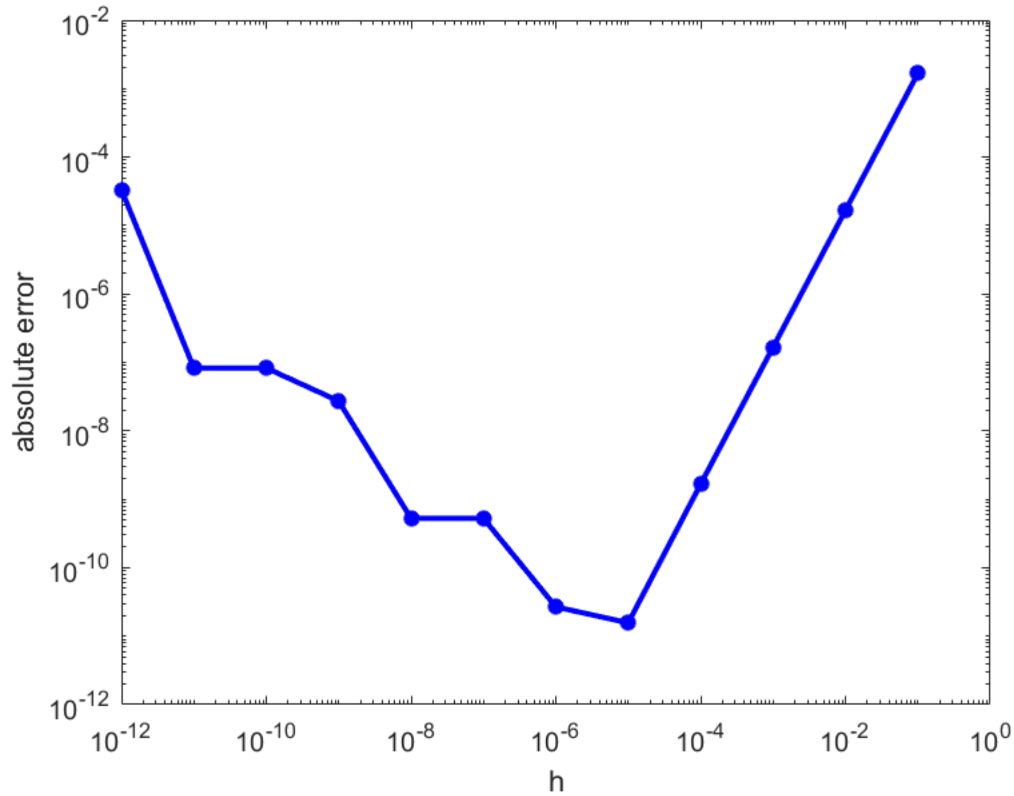
```
f = @(x)sin(x)-cos(x);
f_prime =@(h)(f(h)-f(-h))./(2*h);
f_correct =@(x)cos(x)+sin(x);
correct_ans = f_correct(0);
h = ones(12,1);
for i=1:12
    h(i) = h(i)/(10^i);
end
error = (correct_ans-f_prime(h));
format shortE;
table(h,f_prime(h),error,'VariableNames',{'h','Three_point_centered_difference_formula','error'
```

ans = 12×3 table

	h	Three_point_centered_difference_for...	error
1	1.0000e-01	9.9833e-01	1.6658e-03
2	1.0000e-02	9.9998e-01	1.6667e-05
3	1.0000e-03	1.0000e+00	1.6667e-07
4	1.0000e-04	1.0000e+00	1.6671e-09
5	1.0000e-05	1.0000e+00	1.5653e-11
6	1.0000e-06	1.0000e+00	2.6755e-11
7	1.0000e-07	1.0000e+00	5.2636e-10
8	1.0000e-08	1.0000e+00	5.2636e-10
9	1.0000e-09	1.0000e+00	-2.7229e-08
10	1.0000e-10	1.0000e+00	-8.2740e-08

⋮

```
loglog(h,abs(error),'b-*','LineWidth',2);hold on;
xlabel('h');
ylabel('absolute error');
```



From the above table and plot, we can observe that the minimum absolute error occurs about at $h = 10^{-5}$. In double precision, the theoretical smallest error occurs at $h = (3\epsilon_{\text{mach}}/M)^{1/3} \approx 10^{-5}$, which is consistent with the table and plot results.

Ex 5.2.1

Use the composite Trapezoid Rule and Simpson's Rule with m = 16 and 32 panels to approximate the definite integral. Compare with the correct integral and report the two errors.

(a) $\int_0^4 \frac{xdx}{\sqrt{x^2+9}}$

```
% composite Trapezoid rule
% m = 16
m = 16;
h = 4/m;
x = 0:h:4;
y = @(x)x./sqrt(x.^2+9);
format long;
int_result = h/2*(y(x(1))+y(x(m+1))+2*sum(y(x(2:m))))
```

```
int_result =
   1.998638181470279
```

```
int_correct = integral(y,0,4)
```

```
int_correct =
   2.000000000000000
```

```
error = abs(int_result-int_correct)
```

```
error =
   0.001361818529722
```

```
% m = 32
m = 32;
h = 4/m;
x = 0:h:4;
% y = @(x)x./sqrt(x.^2+9);
int_result = h/2*(y(x(1))+y(x(m+1))+2*sum(y(x(2:m))))
```

```
int_result =
   1.999659678077911
```

```
error = abs(int_result-int_correct)
```

```
error =
     3.403219220892151e-04
```

```
% Composite Simpson's rule
% m = 16
m = 16;
h = 4/(2*m);
x = 0:h:4;
% y = @(x)x./sqrt(x.^2+9);
int_result = h/3*(y(x(1))+y(x(end))+4*sum(y(x(2:2:2*m)))+2*sum(y(x(3:2:2*m-1))))
```

```
int_result =
   2.000000176947122
```

```
error = abs(int_result-int_correct)
```

```
error =
     1.769471218437957e-07
```

```
% m = 32
m = 32;
h = 4/(2*m);
x = 0:h:4;
int_result = h/3*(y(x(1))+y(x(end))+4*sum(y(x(2:2:2*m)))+2*sum(y(x(3:2:2*m-1))))
```

```
int_result =
   2.000000011037514
```

```
error = abs(int_result-int_correct)
```

```
error =
     1.103751356978933e-08
```

(c) $\int_0^1 xe^x dx$

```
% composite Trapezoid rule
% m = 16
m = 16;
h = 1/m;
x = 0:h:1;
y = @(x)x.*exp(x);
format long;
int_result = h/2*(y(x(1))+y(x(m+1))+2*sum(y(x(2:m))))
```

```
int_result =
   1.001444027067708
```

```
int_correct = integral(y,0,1)
```

```
int_correct =
     1
```

```
error = abs(int_result-int_correct)
```

```
error =
   0.001444027067708
```

```
% m = 32
m = 32;
h = 1/m;
x = 0:h:1;
int_result = h/2*(y(x(1))+y(x(m+1))+2*sum(y(x(2:m))))
```

```
int_result =
   1.000361038046700
```

```
error = abs(int_result-int_correct)
```

```
error =
     3.610380466998464e-04
```

```
% Composite Simpson's rule
% m = 16
m = 16;
h = 1/(2*m);
x = 0:h:1;
int_result = h/3*(y(x(1))+y(x(end))+4*sum(y(x(2:2:2*m)))+2*sum(y(x(3:2:2*m-1))))
```

```
int_result =
   1.000000041706364
```

```
error = abs(int_result-int_correct)
```

```
error =
     4.170636414002615e-08
```

```
% m = 32
m = 32;
h = 1/(2*m);
x = 0:h:1;
int_result = h/3*(y(x(1))+y(x(end))+4*sum(y(x(2:2:2*m)))+2*sum(y(x(3:2:2*m-1))))
```

```
int_result =
```

```
   1.000000002606974
```

```
error = abs(int_result-int_correct)
```

```
error =
    2.606973970031845e-09
```

Ex 5.2.7

Apply the Composite Midpoint Rule to the following improper integrals with $m = 16 \text{ and } 32$.

(c) $\int_0^1 \frac{\arctan x}{x} dx$

```
% Composite Midpoint Rule
% m = 16
m = 16;
h = 1/m;
x = 0+h/2:h:1-h/2;
y = @(x) atan(x)./x;
int_result = h*sum(y(x(:)))
```

```
int_result =
   0.916012051029311
```

```
int_correct = integral(y,0,1)
```

```
int_correct =
   0.915965594177219
```

```
error = abs(int_result-int_correct)
```

```
error =
    4.645685209170303e-05
```

```
% m = 32
m = 32;
h = 1/m;
x = 0+h/2:h:1-h/2;
int_result = h*sum(y(x(:)))
```

```
int_result =
   0.915977207391470
```

```
error = abs(int_result-int_correct)
```

```
error =
    1.161321425058315e-05
```

From the above resuts, we can observe that when the *m* increases, the calculation precision increases.

Ex 5.5.3

Approximate the integrals in Exercise 1, using n = 4 Gaussian Quadrature, and give the error.

$(a) \int_{-1}^{1} (x^3 + 2x)d\,x$

Solution:

roots x_i: $x_1 = -\sqrt{\frac{15+2\sqrt{30}}{35}}, x_2 = -\sqrt{\frac{15-2\sqrt{30}}{35}}, x_3 = \sqrt{\frac{15-2\sqrt{30}}{35}}, x_4 = \sqrt{\frac{15+2\sqrt{30}}{35}}$

coefficients c_i :

$c_1 = \frac{90-5\sqrt{30}}{180}, c_2 = \frac{90+5\sqrt{30}}{180}, c_3 = \frac{90+5\sqrt{30}}{180}, c_4 = \frac{90-5\sqrt{30}}{180}$

The $n = 2$ Gaussian Quadrature approximation is

$$\int_{-1}^{1} (x^3 + 2x)d\,x \approx c_1 f(x_1) + c_2 f(x_2) + c_3 f(x_3) + c_4 f(x_4)$$

≈

```
f = @(x)x.^3 + 2*x;
x = zeros(4,1);
x(1) = -1*sqrt((15+2*sqrt(30))/35);
x(2) = -1*sqrt((15-2*sqrt(30))/35);
x(3) = sqrt((15-2*sqrt(30))/35);
x(4) = sqrt((15+2*sqrt(30))/35);
c = zeros(4,1);
c(1) = (90-5*sqrt(30))/180;
c(2) = (90+5*sqrt(30))/180;
c(3) = (90+5*sqrt(30))/180;
c(4) = (90-5*sqrt(30))/180;
format long;
int_gaussian = sum(c.*f(x))
```

```
int_gaussian =
     0
```

```
int_correct = integral(f, -1, 1)
```

```
int_correct =
     0
```

```
error = abs(int_gaussian-int_correct)
```

```
error =
     0
```