21960197 向平 Assignment

Ex 2.5.2

Use the Jacobi Method to solve the sparse system within three correct decimal places (forward error in the infinity norm) for n = 100. The correct solution is [1,−1,1,−1, . . . ,1,−1]. Report the number of steps needed and the backward error. The system is

$$\begin{bmatrix} 2 & 1 & & & \\ 1 & 2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 2 & 1 \\ & & & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ -1 \end{bmatrix}$$

```
% Jacobi method
% matrix setup
n = 100;
e = ones(n,1);
A = spdiags([e,2*e,e],-1:1,n,n);
D = 2*eye(n);
L = tril(A)-D;

b = zeros(n,1);
b(1) = 1;
b(n) = -1;

x_c = ones(n,1);
x_c(2:2:end) = -1;
x = zeros(n,1);
iteration = 1;
while norm((x_c-x),"inf")>10^(-2)
    iteration = iteration +1;
    % matrix calculation instead of for-loop
    x = D\(b-(L+L')*x);
end
x
```

```
x = 100×1
    0.9997
   -0.9994
    0.9991
   -0.9988
    0.9985
   -0.9981
    0.9978
   -0.9975
    0.9972
   -0.9969
      ⋮
```

```
iteration
```

```
iteration = 10018
```

```
backward_error = norm(A*x-b,"inf")
```

```
backward_error = 9.6740e-06
```

Ex 2.5.6

Carry out the steps of Computer Problem 2 for (a) Gauss–Seidel Method and (b) SOR with ω = 1.5.

```
% Gauss-Seidel method
% matrix setup
n = 100;
e = ones(n,1);
A = spdiags([e,2*e,e],-1:1,n,n);
b = zeros(n,1);
b(1) = 1;
b(n) = -1;
x_c = ones(n,1);
x_c(2:2:end) = -1;
x = zeros(n,1);
iteration = 1;
while  norm((x_c-x),"inf")>10^(-2)
    iteration = iteration +1;
    % for-loop for simplification
    for i=1:n
        if i==1
            x(i) = (1-x(i+1))/2;
        elseif i==n
            x(i) = (-1-x(i-1))/2;
        else
            x(i) = (-x(i-1)-x(i+1))/2;
        end
    end
end

x
```

```
x = 100×1
    0.9997
   -0.9994
    0.9990
   -0.9987
    0.9984
   -0.9981
    0.9978
   -0.9975
    0.9972
   -0.9969
      ⋮
```

```
iteration
```

```
iteration = 5011
```

```
backward_error = norm((A*x-b),"inf")
```

```
backward_error = 9.6744e-06
```

```
% SOR method
% matrix setup
w = 1.5;
n = 100;
e = ones(n,1);
A = spdiags([e,2*e,e],-1:1,n,n);
b = zeros(n,1);
b(1) = 1;
b(n) = -1;
x_c = ones(n,1);
x_c(2:2:end) = -1;
x = zeros(n,1);
iteration = 1;
while  norm((x_c-x),"inf")>10^(-3)
    iteration = iteration +1;
    % for-loop for simplification
    for i=1:n
        if i==1
            x(i) = (1-w)*x(i) + w*(1-x(i+1))/2;
        elseif i==n
            x(i) = (1-w)*x(i) + w*(-1-x(i-1))/2;
        else
            x(i) = (1-w)*x(i) + w*(-x(i-1)-x(i+1))/2;
        end
    end
end
x
```

```
x = 100×1
    1.0000
   -0.9999
    0.9999
   -0.9999
    0.9998
   -0.9998
    0.9998
   -0.9997
    0.9997
   -0.9997
      ⋮
```

```
iteration
```

```
iteration = 2461
```

```
backward_error = norm((A*x-b),"inf")
```

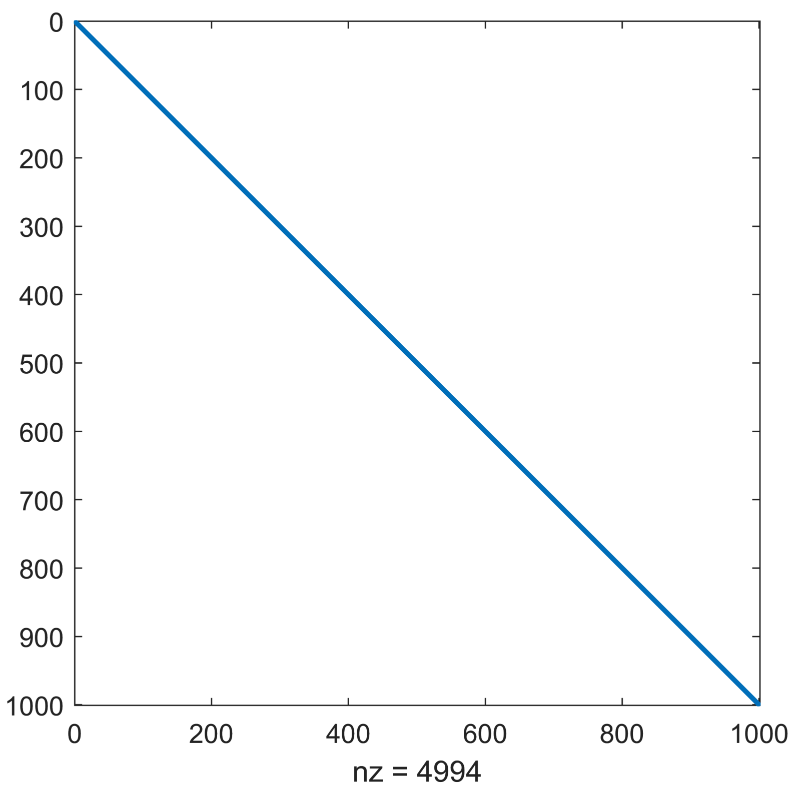
```
backward_error = 9.7363e-07
```

Ex 2.6

Let A be the n × n matrix with n = 1000 and entries $A(i,i) = i, A(i,i+1) = A(i+1,i) = \frac{1}{2}, A(i,i+2) = A(i+2,i) = \frac{1}{2}$, for all i that fit within the matrix. (a) Print the nonzero structure spy(A). (b) Let x_e be the vector of n ones. Set $b = A\,x_e$, and apply the Conjugate Gradient Method, without preconditioner, with the Jacobi preconditioner, and with the Gauss–Seidel preconditioner. Compare errors of the three runs in a plot versus step number.

```
% matrix setup

n = 1000;
e = ones(n,1);
dia = (1:1:n)';
A = spdiags([0.5*e,0.5*e,dia,0.5*e,0.5*e],-2:2,n,n);
spy(A)
```




```
% without preconditioner
xe = ones(n,1);
b = A*xe;
x = 0.5*ones(n,1);
error_1 = zeros(n,1);
d = b-A*x;
r = b-A*x;
for i=1:n
    if r<=eps
        break;
    end
    a = (d'*r)/(d'*A*d);
```

```
    x = x+a*d;
    r = b-A*x;
    beta = -1*(d'*A*r)/(d'*A*d);
    d = r+beta*d;
    error_1(i) = norm(x-xe,"inf");
end
x
```

```
x = 1000×1
    1.0000
    1.0000
    1.0000
    1.0000
    1.0000
    1.0000
    1.0000
    1.0000
    1.0000
    1.0000
      ⋮
```

```
% with Jacobi preconditioner

M = diag(1:n);
M_ = inv(M);
x = 0.5*ones(n,1);
error_2= zeros(n,1);
r = b-A*x;
%d = M_*r;  % inv(M)*r = M\r
d = M\r;
z = d;
for i=1:n
    if r<=eps
        break;
    end
    a = (r'*z)/(d'*A*d);
    x = x + a*d;
    r_old = r;
    z_old = z;
    r = r - a*A*d;
    %z = M_*r;
    z = M\r;
    beta = (r'*z)/(r_old'*z_old);
    d = z+beta*d;
    error_2(i) = norm(x-xe,"inf");
end
x
```

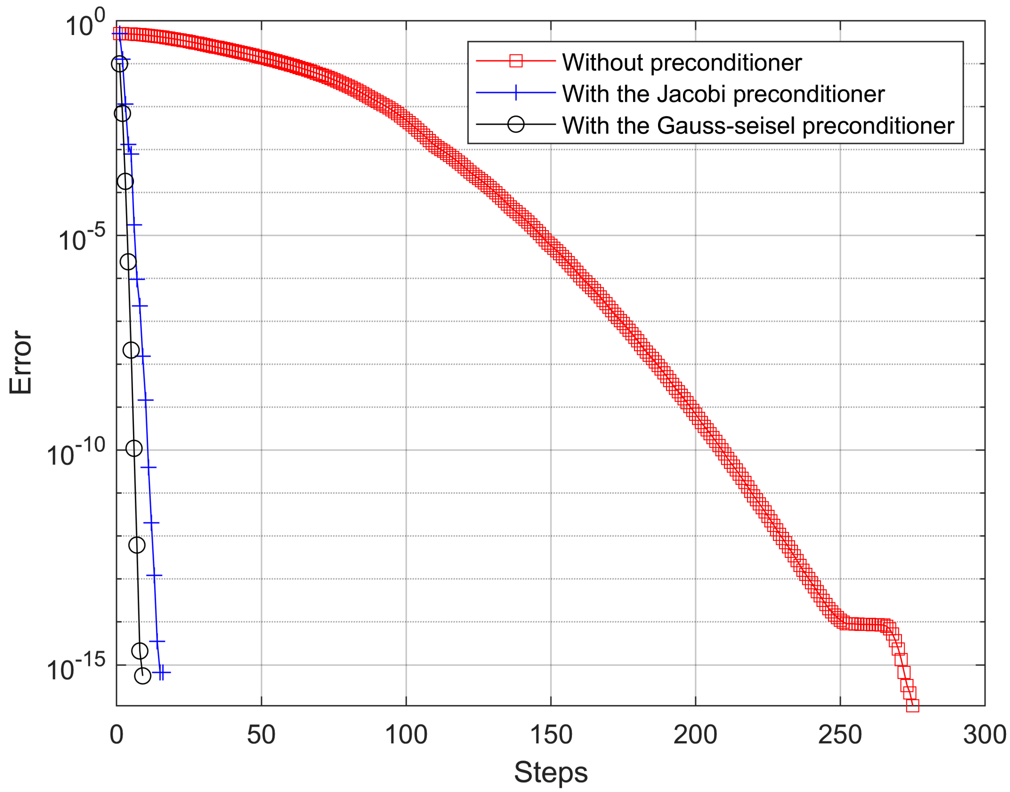
```
x = 1000×1
    1.0000
    1.0000
    1.0000
    1.0000
    1.0000
    1.0000
    1.0000
    1.0000
```

```
    1.0000
    1.0000
      ⋮
```

```
% with Gauss-Seidel preconditioner
xe = ones(n,1);
b = A*xe;
D = diag(1:n);
L = tril(A)-D;
% M = (D+L)*inv(D)*(D+L');
M = (D+L)/D*(D+L');
M_ = inv(M);
x = 0.5*ones(n,1);
error_3 = zeros(n,1);
r = b-A*x;
% d = M_*r;  % inv(M)*r = M\r
d = M\r;
z = d;
for i=1:n
    if r<=eps
        break;
    end
    a = (r'*z)/(d'*A*d);
    x = x + a*d;
    r_old = r;
    z_old = z;
    r = r - a*A*d;
    %z = M_*r;
    z = M\r;
    beta = (r'*z)/(r_old'*z_old);
    d = z+beta*d;
    error_3(i) = norm(x-xe,"inf");
end
x
```

```
x = 1000×1
    1.0000
    1.0000
    1.0000
    1.0000
    1.0000
    1.0000
    1.0000
    1.0000
    1.0000
    1.0000
      ⋮
```

```
steps = 1:n;
semilogy(steps,error_1,'r-s',steps,error_2,'b-+',steps,error_3,'k-o');
grid on;
xlabel('Steps');
ylabel('Error');
legend('Without preconditioner','With the Jacobi preconditioner','With the Gauss-seisel precond
```



Without preconditioner
With the Jacobi preconditioner
With the Gauss-seisel preconditioner
Error
Steps
10^0
10^-5
10^-10
10^-15
0
50
100
150
200
250
300