21960197 向平 Assignment 2

Ex 2.1

Put together the code fragments in this section to create a Matlab program for "naive" Gaussian elimination (meaning no row exchanges sllowed). Use it to solve the systems of Exercise 2.

$$(a)\quad \begin{aligned} 2x-2y-z&=-2\\ 4x+y-2z&=1\\ -2x+y-z&=-3 \end{aligned}$$

```
a = [[2,-2,-1];[4,1,-2];[-2,1,-1]];
b = [-2,1,-3];
n=3;
solution = Gaussian_elimination(a,b,n)
```

```
solution = 3×1
     1
     1
     2
```

$$(b)\quad \begin{aligned} x+2y-z&=2\\ 3y+z&=4\\ 2x-y+z&=2 \end{aligned}$$

```
a = [[1,2,-1];[0,3,1];[2,-1,1]];
b = [2,4,2];
n = 3;
solution = Gaussian_elimination(a,b,n)
```

```
solution = 3×1
     1
     1
     1
```

$$(c)\quad \begin{aligned} 2x+y-4z&=-7\\ x-y+z&=-2\\ -x+3y-2z&=6 \end{aligned}$$

```
a = [[2,1,-4];[1,-1,1];[-1,3,-2]];
b = [-7,-2,6];
n = 3;
solution = Gaussian_elimination(a,b,n)
```

```
solution = 3×1
    -1
     3
     2
```

Ex 2.2

Use the code fragments for Gaussian elimination in the previous section to write a Matlab script to take a matrix *A* as input and output *L* and *U*. No row exchanges are allowed——the program should be designed to shut down if it encounters a zero pivot. Check your program by factoring the matrices in Exercise 2.

(a) $\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix}$

```
a = [[3,1,2];[6,3,4];[3,1,5]];
% the resuts of LU factorization
[L,U] = LU_factorization(a)
```

```
L = 3×3
     1     0     0
     2     1     0
     1     0     1
U = 3×3
     3     1     2
     0     1     0
     0     0     3
```

```
% verification
L*U-a
```

```
ans = 3×3
     0     0     0
     0     0     0
     0     0     0
```

(b) $\begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix}$

```
a = [[4,2,0];[4,4,2];[2,2,3]];
% the resuts of LU factorization
[L,U] = LU_factorization(a)
```

```
L = 3×3
   1.000000000000000                   0                   0
   1.000000000000000   1.000000000000000                   0
   0.500000000000000   0.500000000000000   1.000000000000000
U = 3×3
     4     2     0
     0     2     2
     0     0     2
```

```
% verification
L*U-a
```

```
ans = 3×3
     0     0     0
     0     0     0
     0     0     0
```

(c) $\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 1 & 3 & 4 & 4 \\ 0 & 2 & 1 & -1 \end{bmatrix}$

```
a = [[1,-1,1,2];[0,2,1,0];[1,3,4,4];[0,2,1,-1]];
% results of LU factorization
[L,U] = LU_factorization(a)
```

```
L = 4×4
     1     0     0     0
     0     1     0     0
     1     2     1     0
     0     1     0     1
U = 4×4
     1    -1     1     2
     0     2     1     0
     0     0     1     2
     0     0     0    -1
```

```
% verification
L*U-a
```

```
ans = 4×4
     0     0     0     0
     0     0     0     0
     0     0     0     0
     0     0     0     0
```

Ex 2.3

For the $n \times n$ matrix with entries $A_{\text{ij}} = \dfrac{5}{(i+2j-1)}$, set $x = [1, \ldots, 1]^{\text{T}}$ and $b = A\,x$. Use the Matlab program from Computer Problem 2.1.1 or Matlab's backslash command to compute x_c, the double precision computed solution. Find the infinity norm of the forward error and the error magnification factor of the problem $A\,x = b$, and compare it with the condition number of A:

$(a)\; n = 6$

```
n = 6;
A = zeros(n,n);
for i=1:n
    for j=1:n
        A(i,j) = 5/(i+2*j-1);
    end
end
x = ones(n,1);
b = A*x;
x_c = A\b
```

```
x_c = 6×1
   0.999999999999995
   1.000000000000839
   0.999999999989868
```

```
   1.000000000036509
   0.999999999950057
   1.000000000022946
```

```
forward_error = norm((x_c-x),"inf");
backward_error = norm((A*x_c-b),"inf");
relative_forward_error = forward_error/norm(x,"inf");
relative_backward_error = backward_error/norm(b,"inf");
error_magnification_factor = relative_forward_error/relative_backward_error;
condition_number = norm(A)*norm(inv(A));
forward_error
```

```
forward_error =
     4.994282765125035e-11
```

```
error_magnification_factor
```

```
error_magnification_factor =
     6.888251562500000e+05
```

```
condition_number
```

```
condition_number =
     3.914178445695272e+07
```

$(b)\ n = 10$

```
n = 10;
A = zeros(n,n);
for i=1:n
    for j=1:n
        A(i,j) = 5/(i+2*j-1);
    end
end
x = ones(n,1);
b = A*x;
x_c = A\b
```

```
x_c = 10×1
   0.999999999683380
   1.000000055075762
   0.999998167391172
   1.000022878235027
   0.999861309263353
   1.000460451642242
   0.999119681351108
   1.000965693050518
   0.999435812304193
   1.000135959939740
```

```
forward_error = norm((x_c-x),"inf");
backward_error = norm((A*x_c-b),"inf");
relative_forward_error = forward_error/norm(x,"inf");
relative_backward_error = backward_error/norm(b,"inf");
error_magnification_factor = relative_forward_error/relative_backward_error;
condition_number = norm(A)*norm(inv(A));
```

```
forward_error
```

```
forward_error =
     9.656930505175243e-04
```

```
error_magnification_factor
```

```
error_magnification_factor =
     7.961475491035258e+12
```

```
condition_number
```

```
condition_number =
     6.688367107931060e+13
```

Ex 2.4

Find the PA=LU factorization (using partial pivoting) of the following matrics:

$$(a)\ \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & -1 \\ -1 & 1 & -1 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & -1 \\ -1 & 1 & -1 \end{bmatrix} \xrightarrow[\text{swap}:\ P_{213}]{} \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 0 \\ -1 & 1 & -1 \end{bmatrix} \xrightarrow[\text{elim}\ :\ L_{21}\left(-\frac{1}{2}\right), L_{31}\left(\frac{1}{2}\right)]{} \begin{bmatrix} 2 & 1 & -1 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{3}{2} & -\frac{3}{2} \end{bmatrix} \xrightarrow[\text{swap}:\ P_{132}]{} \begin{bmatrix} 2 & 1 & -1 \\ 0 & \frac{3}{2} & -\frac{3}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\xrightarrow[\text{elim}:\ L_{32}\left(-\frac{1}{3}\right)]{} \begin{bmatrix} 2 & 1 & -1 \\ 0 & \frac{3}{2} & -\frac{3}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore, the PA=LU factorization is

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix},\ A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & -1 \\ -1 & 1 & -1 \end{bmatrix},\ L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{1}{2} & \frac{1}{3} & 1 \end{bmatrix},\ U = \begin{bmatrix} 2 & 1 & -1 \\ 0 & \frac{3}{2} & -\frac{3}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$(b)\ \begin{bmatrix} 0 & 1 & 3 \\ 2 & 1 & 1 \\ -1 & -1 & 2 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 0 & 1 & 3 \\ 2 & 1 & 1 \\ -1 & -1 & 2 \end{bmatrix} \xrightarrow[\text{swap} : P_{213}]{} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 3 \\ -1 & -1 & 2 \end{bmatrix} \xrightarrow[\text{elim} : L_{31}\left(\frac{1}{2}\right)]{} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & -\frac{1}{2} & \frac{5}{2} \end{bmatrix} \xrightarrow[\text{elim} :\ L_{32}\left(\frac{1}{2}\right)]{} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 4 \end{bmatrix}$$

Therefore, the PA=LU factorization is

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ A = \begin{bmatrix} 0 & 1 & 3 \\ 2 & 1 & 1 \\ -1 & -1 & 2 \end{bmatrix},\ L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix},\ U = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 4 \end{bmatrix}$$

(c) $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & 2 \\ -1 & 0 & 3 \end{bmatrix}$

Solution:

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & 2 \\ -1 & 0 & 3 \end{bmatrix} \xrightarrow[\text{swap} : P_{213}]{} \begin{bmatrix} 2 & 4 & 2 \\ 1 & 2 & -3 \\ -1 & 0 & 3 \end{bmatrix} \xrightarrow[\text{elim} :\ L_{21}\left(-\frac{1}{2}\right), L_{31}\left(\frac{1}{2}\right)]{} \begin{bmatrix} 2 & 4 & 2 \\ 0 & 0 & -4 \\ 0 & 2 & 4 \end{bmatrix} \xrightarrow[\text{swap} :\ P_{132}]{} \begin{bmatrix} 2 & 4 & 2 \\ 0 & 2 & 4 \\ 0 & 0 & -4 \end{bmatrix}$$

Therefore, the PA=LU factorization is

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix},\ A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & 2 \\ -1 & 0 & 3 \end{bmatrix},\ L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{1}{2} & 0 & 1 \end{bmatrix},\ U = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 2 & 4 \\ 0 & 0 & -4 \end{bmatrix}$$

(d) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ -2 & 1 & 0 \end{bmatrix}$

Solution:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ -2 & 1 & 0 \end{bmatrix} \xrightarrow[\text{swap} : P_{321}]{} \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow[\text{elim} :\ L_{21}\left(\frac{1}{2}\right)]{} \begin{bmatrix} -2 & 1 & 0 \\ 0 & \frac{1}{2} & 2 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow[\text{swap} :\ P_{132}]{} \begin{bmatrix} -2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{2} & 2 \end{bmatrix} \xrightarrow[\text{elim} :\ L_{32}\left(-\frac{1}{2}\right)]{} \begin{bmatrix} -2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Therefore, the PA=LU factorization is

$$P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},\ A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ -2 & 1 & 0 \end{bmatrix},\ L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix},\ U = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$