0000 0001 0010 0110 1111 1001 1101 0100

Q. value of c = ? to conclude Mean value theorem

$f(x) = \log_e x$, interval $\rightarrow [1, 3]$.

$a = 1, b = 3.$

we know that

$$f'(c) = \frac{f(b) - f(a)}{b - a} \quad \text{, so}$$

$\because f(x) = \log_e x = \ln x.$

$f'(x) = \frac{1}{x} \quad \because \frac{d}{dx}(\ln x) = x.$

$$c = \frac{\ln b - \ln a}{b - a}$$

$$c = \frac{\ln 3 - \ln 1}{3 - 1}$$

$$c = \frac{\ln 3}{2} \quad \because \ln 1 = 0.$$

$$c = \frac{1}{2} \ln 3$$

$$c = \frac{1}{2} \log_e 3.$$

Q2

$$\int \frac{\cos 4x+1}{\cot x-\tan x}\,dx = A f(x)+B.$$

calculate integral.

since, we know $2\cos^2 2x = 1+\cos 4x$

so

$$\int \frac{2\cos^2 2x}{\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}}\,dx = 2\int \frac{\cos^2 2x}{\frac{\cos^2 x-\sin^2 x}{\sin x\cos x}}\,dx$$

$$= 2\int \frac{\cos^{\cancel{2}} 2x\cdot \sin x\cos x}{\cancel{\cos 2x}}\,dx$$

since $\cos^2\theta-\sin^2\theta=\cos 2\theta$

$$= \int \cos 2x\,\sin 2x\,dx$$

$2\sin x\cos x = \sin 2x$.

$$= \frac{1}{2}\int \sin 4x - \sin 0\,dx$$

$2\cos\alpha\sin\beta = \sin(\alpha+\beta)-\sin(\alpha-\beta)$

$$= \frac{1}{2}\int \sin 4x\,dx$$

$$= -\frac{\cos 4x}{8} + C.$$

$$= -\frac{1}{8}(\cos 4x) + C$$

$$= A(f(x)) + B.$$

$\boxed{\text{so. } c \text{ can be } 1/2.}$

period of $\cos 4x \Rightarrow \frac{2\pi}{4} = \frac{\pi}{2}.$

cos is even function.

c can be a constant, the value can be found based on some condition, but now it can be anything!

Q3

Given.
$$\begin{vmatrix} f(x) & g(x) & h(x) \\ a & b & c \\ p & q & r \end{vmatrix} = mx^4 + nx^3 + rx^2 + sx + t$$

So
$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ a & b & c \\ p & q & r \end{vmatrix} = 4xm^3 + 3nx^2 + 2rx + s \quad -(1)$$

$$\begin{vmatrix} f''(x) & g''(x) & h''(x) \\ a & b & c \\ p & q & r \end{vmatrix} = 12xm^2 + 6nx + 2r \quad -(2)$$

$$\begin{vmatrix} f'''(x) & g'''(x) & h'''(x) \\ a & b & c \\ p & q & r \end{vmatrix} = 24mx + 6n \quad -(3).$$

So. (3) − (2)

$$\begin{vmatrix} f'''(x) & g'''(x) & h'''(x) \\ a & b & c \\ p & q & r \end{vmatrix} - \begin{vmatrix} f''(x) & g''(x) & h''(x) \\ a & b & c \\ p & q & r \end{vmatrix}$$

$$= 24mx + 6n - 12mx^2 - 6nx - 2r \quad -(4)$$

Now we are reqiured
$$\begin{vmatrix} f'''(0) - f''(0) & g'''(0) - g''(0) & h'''(0) - h''(0) \\ a & b & c \\ p & q & r \end{vmatrix}$$

So put zero in (4) ; $x = 0$

$\boxed{= 6n - 2r}$ Ans $\quad \boxed{2(3n - r)}$

Q4

(1) $e^{x}\cos x = 1.$

let $g(x) \Rightarrow \tan x - 1. = 0$

$\cos x = e^{-x}.$

$\cos x - e^{-x} = 0$

By rolls theorem, there is 'c' at which derivatives becomes zero. , so

$f'(x) = -\sin x - e^{-x}(-1)$

$= -\sin x + e^{-x}$

~~finding c~~ , let a, b be roots of equation

so $f(a) = 0$, $f(b) = 0$.

using rolls theorem.

$f(a) = f(b)$

$f(a) = 0$

$f(b) = 0$

$f'(c) = 0$

$-\sin c - e^{-c} = 0$

$e^{-c} = \sin c.$

$\frac{\sin c}{e^{-c}} = 1$

$e^{c}\sin c = 1$ ← ⓐ

$e^{c}\sin c - 1 = 0$

$e^{c}\sin c = 1.$

$\boxed{e^{c}\tan c = \cos c\ e^{c}.}$ ~~[illegible]~~ 1.

So it is ~~[illegible]~~

$e^{x}\tan x = \cos x \cdot e^{+c}.$ → given

$e^{c}\tan c = 1.$

$\tan c = 1/e^{c}.$ $\boxed{\text{false}}$

(2)

$e^x \sin x = 1.$

let a, b be roots

$f(x) = e^x \sin - 1. = 0$

$\sin x = e^{-x}.$

$\sin x - e^{-x} = 0.$

$f'(x) = \cos x + e^x$

$f'(c) = 0$ By rolles theorem.

$\cos x + e^x = 0$

$f(a) = f(b)$

$f(a) = 0$

$f(b) = 0$

$\cos x = -e^x.$

$-\frac{\cos x}{e^x} = 1.$ — (b)

$-\frac{\cos x}{\sin x\, e^x} = \frac{1}{\sin x}$

$\frac{e^x \sin x}{\cos x} = -\frac{1}{\sin x}$

$\tan x = -\frac{1}{\sin x e^x}$

$\boxed{\tan x = -1}$ True $\because \sin x e^x = 1.$

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(3) from first part, eq (a).

$e^c \sin c = 1.$

$e^x \sin x = 1$, $\boxed{\text{True}}$

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(4) part 2 b=).

$-\frac{\cos x}{e^x} = 1$

$e^x \cos x = -e^x \cdot e^x$

$\boxed{e^x \cos x \neq 1}$

False.

Q5

$$y = c_1 e^{c_2 x}$$

$$y' = c_1 e^{c_2 x} . c_2 . = c_1 c_2 e^{c_2 x}.$$

$$y'' = c_1 c_2^2 e^{c_2 x}.$$

$$y'' = y c_2^2.$$

Now

$$y . y'' = c_1 e^{c_2 x} \, y \, c_2^2. \longrightarrow c_1 e^{c_2 x}.$$

$$= c_1 c_2 e^{c_2 x} . c_2 c_1 e^{c_2 x}$$

$$= y' . y'$$

$$\boxed{y . y'' = (y')^2}$$