ASSIGNMENT No 1

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ROLL NO: FA21-BEE-002.
Subject: Multi variable calculus.

Question no 1

A,B,C position vector
$4i+4j+k$, $-4i+3j-4k$, $4i-j-2k$

a) Equation of plane ABC.
Sol:-

$A = (4, -4, 1)$
$B = (-4, 3, -4)$
$C = (4, -1, -2)$

$\vec{AB} = \vec{OB} - \vec{OA}$

$$= \begin{bmatrix} -4 \\ 3 \\ -4 \end{bmatrix} - \begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} -8 \\ 7 \\ -5 \end{bmatrix}$$

$$AC = \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix}$$

$n = \vec{AB} \times \vec{AC}$

$$= \begin{bmatrix} i & j & k \\ -8 & 7 & -5 \\ 0 & 3 & -3 \end{bmatrix}$$

$n = -6i - 24j - 24k$

$= i + 4j + 4k$

$d = a \cdot n = (4i + 4j + 1k) \cdot (i + 4j + 4k)$

$= -8$

Equation of plane

$r \cdot n = d$

$r (i + 4j + 4k) = -8$

$(xi + yj + zk) \cdot (i + 4j + 4k) = -8$

$x + 4y + 4z = -8$

$\boxed{x + 4y + 4z + 8 = 0}$

b) Perpendicular distance from O to ABC.

Sol:-

$p = \frac{d}{|n|}$

$= \frac{8}{\sqrt{(1)^2 + (4)^2 + (4)^2}}$

$\boxed{p = 1.39}$

c) line OD $r = a \times \lambda b$

$r = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}$

$$r = \begin{pmatrix} 2\lambda \\ 3\lambda \\ -3\lambda \end{pmatrix} \cdot \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix}$$

$$d = r \cdot n$$

$$-8 = 2\lambda + 12\lambda - 12\lambda$$

$$\lambda = -4$$

$$r = \begin{bmatrix} 2(-4) \\ 2(-4) \\ -3 -(-4) \end{bmatrix}$$

$$\boxed{r = (-8, -12, 12)}$$

Question no 2

Points A, B, C, D

$7i + 4j - k$, $11i + 3j$, $2i + 6j + 3k$, $2i + 7j + \lambda k$

a) Shortest distance b/w AB and CD

Sol:-

$$r_1 = OA - OB$$

$$= \begin{bmatrix} 11 \\ 3 \\ 0 \end{bmatrix} - \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}$$

$$r_2 = \begin{bmatrix} 2 \\ 7 \\ \lambda \end{bmatrix} - \begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ \lambda - 3 \end{pmatrix}$$

$b_1 e$

$$r_1 \times r_2 = \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ 0 & 1 & \pi-3 \end{vmatrix}$$

$$r_1 \times r_2 = \left(-(\pi-3)-1\right)\hat{i} - 4(\pi-3)j + 4k$$

$$= (2-\pi)i - (4\pi-12)j + 4k$$

$$a_2 - a_1 = \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix} - \begin{bmatrix} 7 \\ 4 \\ 7 \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \\ 4 \end{bmatrix}$$

$$|b_1 \times b_2| = \sqrt{(2-\pi)^2 + (4\pi-12)^2 + (4)^2}$$

$$= \sqrt{17\pi^2 - 100\pi + 164}$$

$$(b_1 \times b_2)\cdot(a_1 - a_2) = -5(2-\pi) - 2(4\pi-12) + 16$$

$$= -10 + 5\pi - 8\pi + 24 + 16$$

$$= 30 - 3\pi$$

$$d = \frac{(b_1 \times b_2)\cdot(a_1 - a_2)}{|b_1 \times b_2|}$$

$$d = \frac{(30-3\pi)^2}{(17\pi^2 - 100\pi + 164)^2}$$

$144\lambda^2 - 720\lambda + 576 = 0$

$4(9\lambda^2 - 45\lambda + 36) = 0$

$\lambda^2 - 4\lambda - \lambda + 4 = 0$

$\lambda - 4 = 0 \quad \lambda - 1 = 0$

$\lambda = 4 \quad \lambda = 1$

Putting $\lambda = 4$ and 1 in eq (1)

for $\lambda = 4$

$(4)^2 - 5(4) + 4 = 0$

$0 = 0$

for $\lambda = 1$

$(1)^2 - 5(1) + 4 = 0$

$0 = 0$

b) when $\lambda = 1$

$\lambda = 4$

equation of π_1

equation of π_2

Sol:

Plane ABD when $\lambda = 1$

So $D = 2i + 7j + 1k$

$$AB \times AD = \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ -5 & 3 & 2 \end{vmatrix}$$

$= i(-2-3) - j(8+5) + k(12-5)$

$= -5i - 13j + 7k$

$Eq = -5(2) - 13(7) + 7(1)$

$= \boxed{-10x - 91y + 7z}$

for $\lambda = 4$

$AB \times AD =$ ~~$48(2) + 3(7) + 7(4)$~~

$$\begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ 5 & 3 & 5 \end{vmatrix}$$

$= -8j + 15j + 17k$

$Eq = -8(x-11) - 15(y-3) + 17(z-0)$

$= -8x + 88 - 15y + 45 + 17z$

$= \boxed{8x + 15y - 17z - 133}$

c) Angle b/w π_1 and π_2

$$\emptyset = \cos^{-1}\left(\frac{(n_1 - n_2)}{|n_1||n_2|}\right)$$

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$n_1 \cdot n_2 = 94.1952\ 19$

$\theta = \cos^{-1} \dfrac{28}{\sqrt{24+16+4+9+4+4+9+28}}$

$\boxed{\theta = 31.84^{\circ}}$

Question no 2

a) Find value of t.

$l_1 : ti + j \quad -2i - j$

$l_2 : j + tk \quad -2j + k$

Sol

distance b/w lines

$\sqrt{21}$

$r_1 = OA + \lambda AB$

$r_2 = OA + \mu AB$

$$l_1 = r_1 = \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

$$l_2 = r_2 = \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} + \mu \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

$$D = \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{|b_1 \times b_2|}$$

$|b_1 \times b_2| = i + 2j + 4k$

$a_2 - a_1 = -it + kt$

$$\sqrt{21} = \frac{(i + j + 4k)\cdot(-ti + j + kt)}{\sqrt{21}}$$

$21 = -t + 4t$

$\boxed{t = 7}$

b)

sol:-

$\pi_1 = OA + \lambda AB + \mu AC$

$= 7i + j + \lambda(2i - j) + \mu(-2j + k)$

c) $5x - 6y + 7z = 0$

$\theta = ?$ b/w l_2 and π_2

n of $l_2 = (0, -2, 1)$

$\pi_2 = (5, -6, 7)$

$$\theta = \cos^{-1} \frac{|n_1 \cdot n_2|}{|n_1| \cdot |n_2|}$$

$$\theta = \cos^{-1} \frac{19}{\sqrt{4+1}\sqrt{25+36+49}}$$

$\boxed{\theta = 35.8}$

d) π_1 $n = \begin{bmatrix} 7 \\ 1 \\ 0 \end{bmatrix}$ $\pi_2 = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$

Sol

$\theta = \cos^{-1}\left(\frac{35-6+0}{\sqrt{49+1}\sqrt{25+36+49}}\right)$

$\boxed{\theta = 67.09^\circ}$

j + k + 1)

Question no 5

a) Equation of circle.

Sol

u (-2j + k)

P (-2, -1)

Q (-6, -3)

$M.P = \frac{-6-2}{2}, \frac{-3-1}{2}$

$= -4, 2$

$(x-h)^2 + (y-k)^2 = r^2$

$r^2 = 5$

$\boxed{(x+4)^2 + (y+2)^2 = 5}$

b) Radius of circle.

Let y be centre at point (4, 0)

$(4)^2 + (0-b)^2 = r^2$

$16 + b^2 = r^2$

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at point (0, 2)

$(0)^2 + (2-b)^2 = r^2$

$4 - b^2 - 4b = r^2$

$12 = -4b$

$b = -3$

$16 + 9 = r^2$

$r = 5$

c) equation of directrix

$y = 100x$

$y^2 = 4ax$

$4ax = 100x$

$a = 25$

$x = -a$

$\boxed{x = -25}$

d) eq of axis of parabola.

$x^2 = 24y$

$x^2 = 4ay$

$4ay = 24y$

$a = 6$

$\boxed{x = -6}$

e) $\left(\frac{x}{25}\right)^2 + \left(\frac{y}{16}\right)^2 = 1$

sol:-

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a = 5 \quad b = 4$

$c = a^2 - b^2$

$c = \sqrt{25-16} = \pm 3$

$F_1 = (-3, 0) \; ; \; F_2 \, (3, 0)$

lenght $2a \Rightarrow 2(5)$

$= 10.$