ASSIGNMENT NO 2

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ROLL no: FA21-BEE-002
Subject: Multi-Cal

Q no 1

Evaluate

a) $\lim_{(x,y)\to(2,1)} \frac{x^2-2xy}{x^2-2y^2}$

Sol

$x^2 - 4y^2 = 0$

$x^2 = 4y^2$

$x = 2y$

$y = x/2$

$\lim_{(x,y)\to(2,1)} \frac{x^2 - 2x(x/2)}{x^2 - 2(x^2/2)^2}$

$\lim_{(x,y)\to(2,1)} \frac{x^2 - x^2}{x^2 - x^2}$

1 Ans.

b) $\lim_{(x,y)\to(0,0)} \frac{x-4y}{6y+7x}$

Sol

$6y = 7x$

$y = \frac{7x}{6}$

$\lim_{(x,y)\to(0,0)} \frac{x - 2(7x/6)}{6(7x/6)+7x}$

$$\lim_{(x,y)\to(0,0)} \frac{x - 14x/3}{7x + 7x}$$

$$\lim_{(x,y)\to(0,0)} \frac{3x - 14x}{14x} \Rightarrow \frac{(3-14)x}{52x}$$

$$\frac{-11}{52} \quad \text{Ans}$$

d) $\lim_{(x,y,z)\to(-1,0,4)} \frac{x^3 - 2e^z y}{6x + 2y - 3z}$

Sol

$$\frac{(-1)^3 - 0}{6(-1) + 2(0) - 3(4)}$$

$$\frac{1}{16} \quad \text{Ans}$$

Question no 2

a) $f(x,y) = \cos(x/y) \qquad v = (3,-4)$

$$f\nabla = \frac{\partial}{\partial x}(\cos(x/y))i + \frac{\partial}{\partial x}(\cos(x/y))j$$

$$= \left(\frac{-1}{y}\sin(x/y)\right)i + x(-\sin(x/y)(1/y)j$$

$$\hat{v} = \frac{3i + 4j}{\sqrt{9+16}} = \frac{3i}{\sqrt{25}} + \frac{4}{\sqrt{25}}j$$

$$\text{Duf} = -3/y \sin(x/y) + 4x/5y^2 \sin(x/y)$$

$$\text{Duf} = (1/5y \sin x/y)(4x/y - 3)$$

b) $f(x,y,z) = x^2y^3 - 4xz$ $\quad v = (-1, 2, 0)$

Sol

$\nabla f = (2y^3x\,i + (-4z)j + 3y^2x^3 j - 4x\,k$

$v = \frac{-i + 2j + 0k}{\sqrt{5}}$

$v = \frac{-i}{\sqrt{5}} + \frac{2j}{\sqrt{5}} + \frac{0k}{\sqrt{5}}$

$D_{\vec{u}} f = -\frac{1}{\sqrt{5}}(2y^3x - 4z) + \frac{2}{\sqrt{5}}(3y^2x^3) + 0$

Question no 3

$f(x,y,z) = 4x - y^2 e^{3xz}$

Sol

$\nabla f = 4 - y^2 e^{3xz}(3z)j - 2ye^{3xz} j - y^2 e^{3xz} 3x\,k$

$\nabla f_{(3,-1,0)} = 4 - (-1)^2 e^{3(3)(0)}\, 0(3)\,i - 2(-1)e^{3(3)} j - e^{3(3)0} k$

$= 4j - 2j - 9k$

$v = (-1, 4, 2)$

$\vec{v} = \frac{-1i + 4j + 2k}{\sqrt{21}}$

$v = \frac{-1(4) - 2(4) - 9(0)}{\sqrt{21}}$

$= -\frac{32}{\sqrt{21}}$

Question no 4

a) $f(x,y) = \sqrt{x^3+y^3}$ at $(-2,3)$

Sol:-

$$\nabla f = \frac{1}{2}(x^3+y^3)^{-1/2}(2x)\,i + \frac{1}{2}(x^3+y^3)^{-1/2}3y^2\,j$$

$$\nabla f_{(-2,3)} = (4+9)^{-1/2}(-2)\,i + \tfrac{1}{2}(4+9)^{-1/2}3(9)\,j$$

$$= \frac{-2}{\sqrt{13}}\,i + \frac{27}{2\sqrt{13}}\,j$$ wt)

Question no 5

a) $f = x^2y\,\hat{i} - 2(z^3-3x)\,\hat{j} + 4y^2\hat{k}$

$\nabla = 2xy\,i - 0\,j + 0\,k$

Div f $= \nabla f \cdot f$

$= (2xy\,i)\cdot(x^2y\,i - (z^3-3x)\,j + 4y^2k)$

Div f $= 2x^3y^2 + 0$

Curve $\nabla f \times f$

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & z^3-x^3 & 4y^2 \end{vmatrix}$$

$$= (8y+3z^2)\,i - 0\,j + 2x^2\,k$$

Question no 6

a) $\vec{F} = x^3 y\, i$

$$\vec{F} = \left(4x^2 + \frac{3x^2 y}{z}\right) i + \left(8xy + \frac{y^3}{z^2}\right) j + \left(11 - \frac{2x^3 y}{z^3}\right) k$$

$$\nabla \vec{F} = 8y + \frac{3x^2}{z} i - \frac{2x^3}{z^3} j - \frac{2x^3}{z^3} \hat{k}$$

$$\frac{\partial M}{\partial z} = x^3 z^{-2} = x^3 (-2) z^{-3} = -\frac{2x^3}{z^3}$$

$$\frac{\partial N}{\partial x} = 8y - \frac{3x^2}{z^3}$$

$$\frac{\partial M}{\partial z} = 4x^2 - \frac{3x^2 y}{z^2}$$

$$= 3x^2 y (-2) z^{(-3)}$$

$$= \frac{6x^2 y}{z^3}$$

as $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial x}$

Vector field is conservative

Question no 7

a) $z = \frac{x^2 - w}{y^4}$, $x = 6t + 7$

$y = \cos(2t)$ $\quad w = 4t$

Sol

$$\frac{dz}{dt} = ?$$

$$\frac{dz}{dt} = \frac{dz}{dx} \cdot \frac{dx}{dt} + \frac{dz}{y} \cdot \frac{dy}{dt} + \frac{dz}{dw}$$

$$\frac{dz}{dt} = \frac{2x}{y^4} \quad , \quad \frac{dz}{dy} = (x^2 - w)\, y^{-4}$$

$$\frac{dx}{dt} = 3t^2 \quad , \quad \frac{dz}{dy} = (x^2 - w) \cdot -4y^{-5}$$

$$= \frac{-4(x^2 - w)}{y^5}$$

$$\frac{dy}{dt} = \frac{d}{dt}\cos 2t = -2\sin 2t$$

$$\frac{dz}{dt} = \frac{2x^2}{y^4} \cdot 3t^2 + \left[\frac{-4(x^2 - w)}{y^5} - 2\sin 2t\right] + \left(-\frac{1}{y^4}\right) \cdot 4$$

$$= \frac{6x^2t^2}{y^4} + \frac{8(x^2 - w)\sin 2t}{y^5} - \frac{4w}{y^4}$$

b) $z = x^2y^4 - 2y$, $y = \sin(x^3)$

Sol $\frac{dz}{dx} = ?$

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$\frac{\partial z}{\partial y} = (4y^3x^2 - 2)$$

$$\frac{dy}{dx} = 2x\cos x^2$$

$$\frac{dz}{dx} = (4x^2y^3 - 2)(2x\cos x^2)$$

$$= 8x^3y^3 \cos x^2 - 4x\cos x^2$$

c)

$$x^3y^4 - 3 = \sin(xy)$$

diff w.r.t x

$$\frac{d}{dx}(x^3y^4 - 3) = \frac{d}{dx}\sin(xy)$$

$$3x^2y^4 + x^3 4y^3 \frac{dy}{dx} = y\cos xy.$$

$$4x^3y^3 \frac{dy}{dx} = y\cos(xy) - 3x^2y^4$$

$$\frac{dy}{dx} = \frac{y\cos(xy) - 3x^2y^4}{4x^3y^3}$$ Ans