

INTRODUCTION

The Universe means totality of Space.

The physical Universe includes; the Sun and the Solar system, the stars in their galaxies, the natural Satellite (moons), the artificial Satellites, meteors and Meteorite, Space dust and Space Vacuum.

The Universe is represented mathematically as a rectangle. And any object of study in the Universe can be conveniently represented in the rectangle, while the rest of Space in the rectangle refers to the environment of the object.

Space - Time (t, x) Reference frames

The four fundamental forces in nature are:

- (i) Electromagnetic (EM) forces
- (ii) Gravitational " "
- (iii) Strong Nuclear " "
- (iv) Weak " "

A reference frame is a mathematical Coordinate System, ^{which} contain an Observer and the Object he focus at with respect to an Origin.

Examples of coordinate systems are Euclidean or Cartesian coordinate system, Spherical coordinate system, ^{and} Cylindrical ~~and~~ coordinate system.

Assignment: State one peculiar use to which each of the coordinate system mentioned above can be applied.

The two categories of reference frames are: (i) Inertia reference frame (ii) non-Inertia reference frame

An Inertia reference frame is that which is completely motionless with respect to the rotation of the earth or the Universe at large.

A System is the object of focus or study, and the rest of the Universe is its environment.

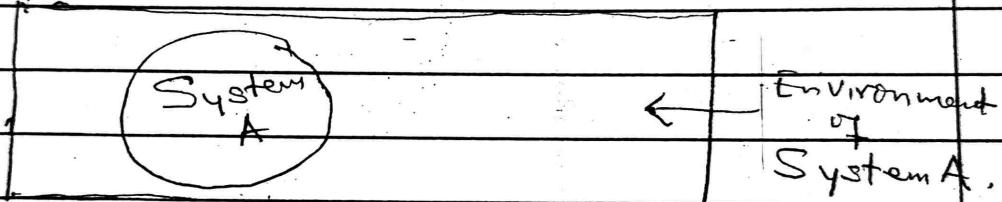


Fig 1: Showing a System Illustration

The System could be a particle, object or field of a force. There is always a form of interaction between a System and its environment - often involving motion of one type or the other.

Each interaction registers an event, and every event occurs at a time (t) and a location (x, y) or (x, y, z) , considering 1D ~~Dimension~~, 2D or 3D cases.

Consider two Observers of the same phenomenon or events A and B, which are in different reference frame X and Y.

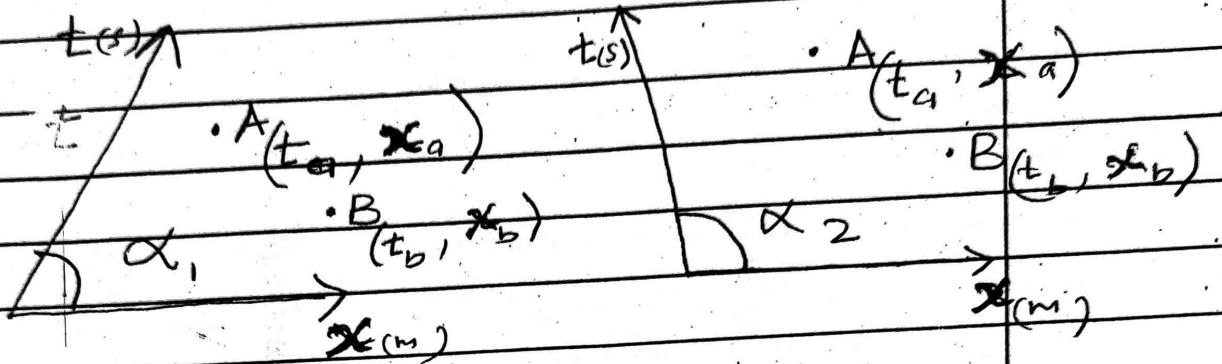


Fig 2: Showing Event A by Two Different Reference frame α_1 and α_2
[where $\alpha_1 \neq \alpha_2$]

Here the events A and B (t_a, x_a) and $B(t_b, x_b)$ have temporal (time) difference of occurrence $(t_b - t_a)$ generally defined as T, and spatial separation $(x_b - x_a)$ denoted by L or Length. Points A (t_a, x_a) and B (t_b, x_b) may be space-time coordinate of the same particle or object Q of mass M. Recall M, L and T are fundamental dimensions of physical quantity.

For application of physics laws to analyse their observations and compare their results, the two observers must use described physics terms, for example stating precisely whether their motion is linear, Circular or Spiral.

If one observer moves at 'Constant Velocity' w.r.t the other it is referred to

Galilean Transformation.

The Galilean transformation is the simplest case of observation of same event by two observers in that one observer moves at constant velocity w.r.t. the other.

Domains of Physics

The following are the four domains of physics:

- I. Non quantum and non relativistic Domain called Newtonian Mechanics
- II. Non quantum and Relativistic Domain
- III. Quantum and non relativistic Domain
- IV. Quantum and relativistic Domain

Mechanics is the study of the relationship between Net force and resulting motion or simply why motion occurs. Newtonian mechanics refers to study of ~~major~~ interaction between objects which are large relative to atomic size and have speed of interaction low relative to speed of light.

Force and Motion

When objects interact they exert forces on one another. The net force experienced by each object is their interaction determining how each of the object move.

Motion is said to occur when an object changes its position and/or orientation w.r.t time.

Representation of position along x, y and z axes is defined by

$$\mathbf{r} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$$

where i, j and k are orthogonal unit vectors along the axes x, y and z. S.I. Unit of distance (m) between any two point is metre(m) and S.I. Unit of time is seconds. Through

you should give your answers to the required units, otherwise give them in S.I. Units.

Exercise What is the difference b/w r and

- 1. Solution r is a position vector while a is just a distance between any two positions.

Types and Planes of Motion

Types of Motion

- i. Rectilinear motion
- ii. Rotational or Circular Motion
- iii. Projectile "
- iv. Helical "
- * v. SHM
- * vi. Orbital
- * vi. Spin

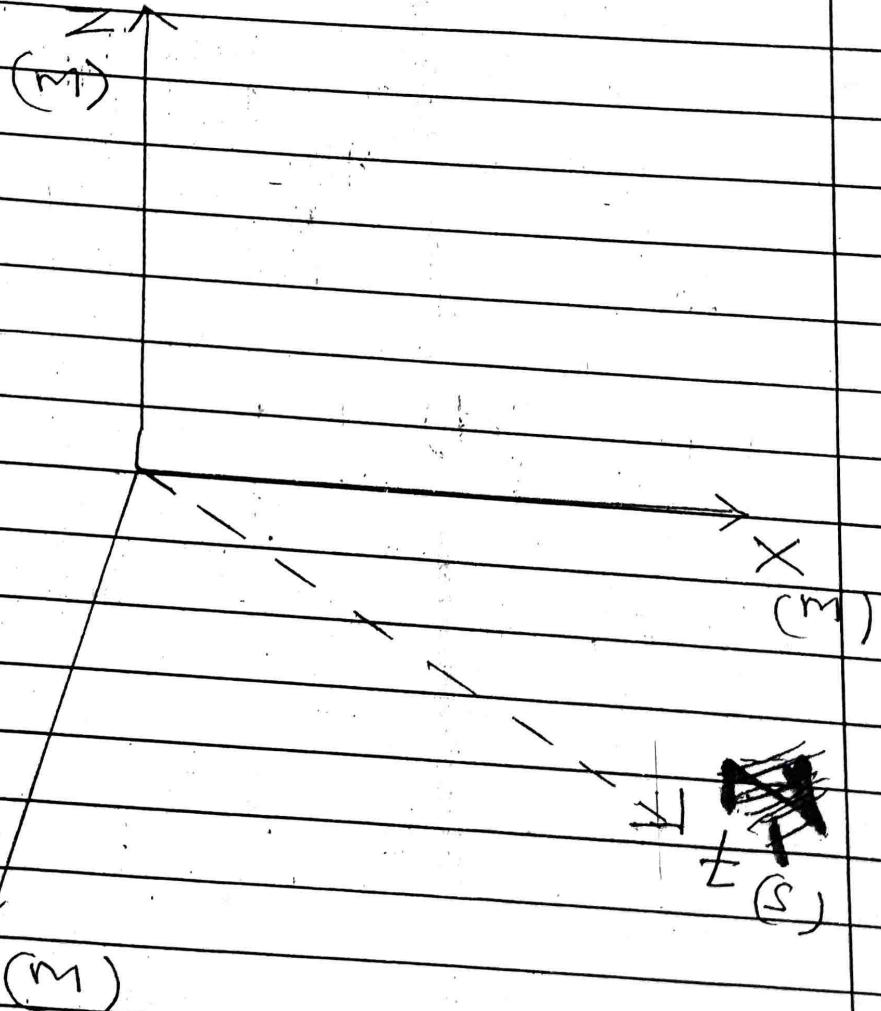


FIG 3

Rectilinear motion may be defined as the motion of an object which changes its position along ~~one~~ ^{one} defined axis (X , Y or Z) w.r.t time.

Rotational motion is the motion of an object which changes both its

① position along one or more defined axes (x, y, z) ^{planes}
(XY, XZ, YZ) ^{space}
and its ② orientation w.r.t time

Projectile motion is the motion of an object describe when it changes its position w.r.t time in one or more planes (XY, XZ, YZ) simultaneously.

* Helical motion is the motion of an object describe when it accelerates or decelerates uniformly in a rotational motion.

Simple Harmonic Motion [SHM]

is the motion an object describe when its acceleration is directly proportional to its displacement from a fixed point, such that it is always directed towards that point

$$\text{i.e } \ddot{a} \propto -r \Rightarrow \ddot{a} = -k r$$

where $r = x$ or y or z axis

The negative sign indicate that although the acceleration is larger at larger displacement, it's always in the opposite direction to the displacement

SHM is one example of Klaes or group motions in which the moving object moves about a fixed point, or position.

Spin is the motion of an object which changes ^{only} its orientation w.r.t time.

Orbital motion is the motion an object describe when it performs oscillatory motion about the position of another object. Through the object constantly move about the position of the primary object, the primary object's position may simultaneously also be changing.

Gravitational Motion

Forces generally may be classified into two, namely 'contact forces' and 'field forces'.

Gravitational force is the cause of gravitational motion, it is a field force, i.e. it acts in a field.

A field is defined as the region in which a force can be experienced.

Gravitational force F_{kj} exerted on a body of mass m_j by the Earth at distance r_{jk} from the centre of the earth is given by

$$F_{kj} = G M_k m_j \frac{1}{(r_{jk})^2} \quad \dots \quad (1)$$

Where M_k = mass of the earth

F_{kj} is the net force exerted by M_k on m_j . Hence motion occur as m_j is attracted or moved towards the earth i.e M_k , along \hat{r}_{jk} (Unit Vector).

Recall r may be written

$$\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$$

$$\text{and } \vec{r} = \hat{i} + \hat{j} + \hat{k}$$

where a, b and c are constants and \hat{i}, \hat{j} and \hat{k} are unit vectors along X, Y and Z axis respectively.

Escape Velocity

If mass m_j is escaping from the Earth mass M_k and radius r_e , then

Potential energy gained = Kinetic energy lost by the body

Work done measure the energy change. The work done (Δw) by gravity when the body m_j moved a short distance dz upwardly:

$$\Delta w = -F dz$$

$$= -G M_k m_j \frac{dz}{z^2}$$

Total work done

$$\text{while body escapes} = \int_r^\infty -G M_k m_j \frac{dz}{z^2}$$

Do not write
in either
margin

Question.....
Write on both sides of the paper

$$= -GM_{kM_j} \int_r^{\infty} \frac{1}{z^2} dz$$

$$= -GM_{kM_j} \left[-\frac{1}{z} \right]_r^{\infty}$$

$$= GM_{kM_j} \left[\frac{1}{z} \right]_r^{\infty}$$

$$= GM_{kM_j} \left(0 - \frac{1}{r} \right)$$

$$= -\frac{GM_{kM_j}}{r}$$

At escape

$$\frac{1}{2} MV^2 = GM_{kM_j} \left[\frac{1}{r} \right]$$

$$V = \sqrt{\frac{2GM}{r}}$$

$$\text{but } g = \frac{GM}{r^2}$$

$$\text{therefore } V = \sqrt{2gr}$$

$$= [2 \times 9.8 \text{ m/s}^2 \times 6.4 \times 10^6]^{\frac{1}{2}}$$

$$= 11.2 \text{ km/s}$$

Keppler's Law of Planetary motion states:

1. Each planet moves in an ellipse which has the Sun at one focus

2. The line joining the sun to the moving planet(s) sweep out equal areas in equal times

3. The Squares of the times of revolution of the planet i.e (periodic time T) about the Sun are proportional to the Cubes of the mean distance r from it, i.e.

r^3 cannot be shown on the illustration diagram, been a mean distance.

$$\frac{r^3}{T^2} = \text{Constant}$$

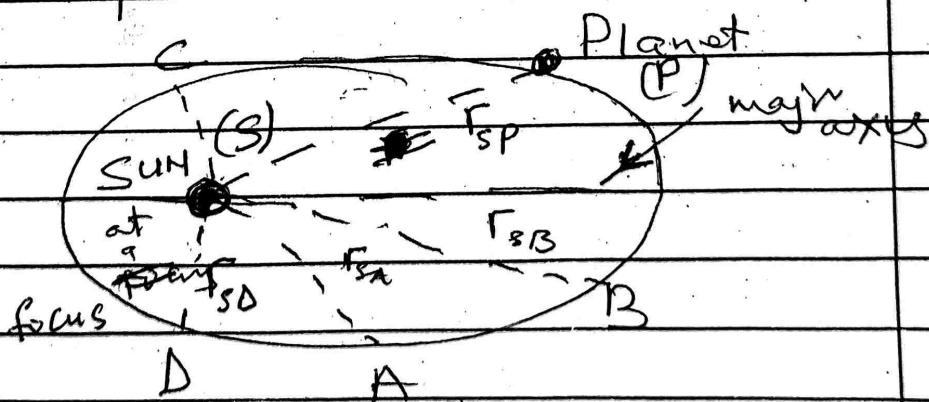


FIG 4

4. Generally in rotational motions, the method ~~is~~ commonly used in calculating magnitudes and directions of force, velocity and acceleration is calculus.

IF a spherical balloon move in a three dimensional space such that the relationship between the radial path (r) in of its motion is related to its times (t) of travel in the motion, by

$$st^2 - t + 2$$

$$\frac{dr}{dt} = 10t - 1$$

$$\frac{d^2r}{dt^2} = 10m/s^2$$

(i) Estimate the acceleration of the balloon assuming uniform motion.

$$\text{Given } r = 3t^2 - t + 4$$

$$\text{then } V \text{ (Velocity)} = \frac{dr}{dt} = 6t - 1 \text{ m/s}$$

and the constant acceleration (a) of the uniform motion is given by $a = 6m/s^2$

$$a = \frac{d^2r}{dt^2} = 6 m/s^2$$

(ii) If the balloon experience a constant net force (F_N) in the motion and its mass (m) is 0.01kg , evaluate the net force causing the motion.

$$F_N = \frac{ma}{m} = 0.01\text{kg} \times 6m/s^2$$

$$F_N = 0.06\text{N}$$

(iii) What will happen to the balloon if the set of forces acting on it suddenly

Changed and the F_H Vanished (i.e.

$H = 0$ Newton in (a) Still air

$$\rightarrow F_H = Mg \\ = 0.08 \text{ kg} \times 9.81 \text{ m/s}^2 \\ = 0.08 \text{ N}$$

(b) breeze of Uniform Velocity 0.3 m/s^2

(c) breeze of " " " $- 0.3 \text{ m/s}^2$

(iv) what will be the acceleration in the breeze 0.3 m/s^2 , 2 seconds after been in the still air.

Generally $V = u + at$

$$\therefore 0.3 \text{ m/s} = 0 + a(2s)$$

$$a = 0.15 \text{ m/s}^2$$

(v) Considering acceleration \bar{a}

(a) What is meant by Uniform motion [$\bar{a} = \text{constant}$]

(b) What is meant by Non Uniform motion [\bar{a} varies]
- for example in circular motion.

(c) What is meant by a Constant Velocity motion [$\bar{a} = 0 \text{ m/s}^2$]
note not 0.

I. MOMENTS OF INERTIA.

Consider the stone being whirled by a child in the example above. The static reference frame may be represented thus

~~Fig. 5. A stone of mass M is whirled in a horizontal circle of radius r about a vertical axis O. The angular velocity is ω. The forces acting on the stone are its weight W downwards and the reaction R of the string upwards. The angle between the string and the vertical is θ.~~

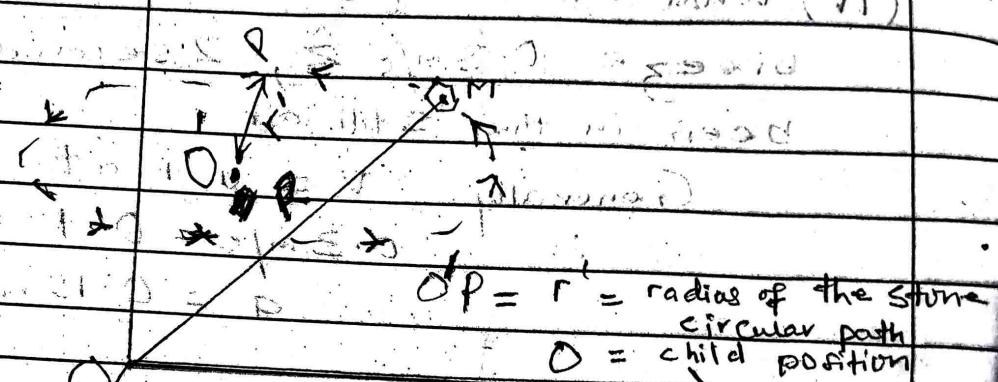
To find ~~for rotation:~~

1. Moment of Inertia (I)

2. C.G.

3. Total Angular Momentum (L)

4. Torque (T)



~~(i) If the stone has a mass M , then the angular momentum is $L = I\omega$. (ii) If the stone has a mass m , then the angular momentum is $L = mvr$. (iii) If the stone has a mass m , then the linear momentum is $p = mv$. (iv) If the stone has a mass m , then the torque is $T = Fr$.~~

~~If the stone has a mass M , then the angular momentum is $L = I\omega$. (v)~~

~~If the stone has a mass M , then the linear momentum is $p = Mr\omega$. (vi)~~

FIG 5

If the stone mass (M) is made up of several thousands of sand particles of different masses $m_1, m_2, m_3, m_4, m_5, \dots, m_n$ and in their compact state each of the sand particles are at distance $r_1, r_2, r_3, r_4, \dots, r_n$ from the origin O , then the stone's compact location vector \vec{r} is given by

$$\vec{r} = \sum_{i=1}^n m_i \cdot \vec{r}_i = \frac{\sum m_i \vec{r}_i}{M}$$

- (2)

$$\text{Where } I = \sum_{i=1}^n m_i r_i^2 \quad (3)$$

= Moment of Inertia for rotation about specified axis.

Applications of Moment of Inertia

The centre of mass of the object (stone) is called Centre of gravity (C G). The centre of gravity of an object, defined its stable location position called balance position or balance point, outside which the object will be unstable under the force of gravity.

In some circumstances a rotating body's total angular momentum (L) about a fixed origin is given by the moment of inertia (I) and angular velocity (ω) thus:

$$\text{Total angular Mmnt}(L) = I\omega \quad (4)$$

Also the net turning force called torque (T) is given by the moment of inertia and angular acceleration (α) of the object. thus:

$$\text{Torque}(T) = I\alpha \quad (5)$$

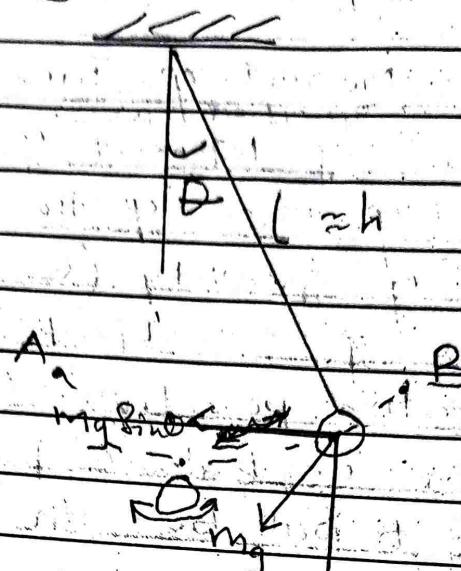
The relationship between SHM and moment of inertia can be briefly analysed using the popular pendulum bob.

In simple pendulum motion the bob is said to perform SHM if the pendulum displacement (θ) is small such that

$$\sin \theta \approx \theta \quad (6)$$

The forces basically acting on the

bob is given by figure 6.



FTG6: Simple Pendulum Mechanics

The relationship between the bob's changing position w.r.t time (t) is given by

$$\hat{V}(t) = 0.9 + 2.6 \sin(2\pi t)$$

$$= 1.8 \cos^2 x$$

$$\bar{a} \approx -0.944$$

$$t = 2$$

$$\hat{a} = -272 \text{ N}$$

$$= -2 \cdot 72 \text{ m/s}$$

423

$$= -0.515$$

$$F(t) = A \sin \omega t$$

$$V(t) = \frac{dV_0}{dt} = \frac{d}{dt} [A \sin(\omega t)]$$

$$= A d \frac{d}{dt} \sin \omega t$$

$$= Aw \cos wt, \text{ along } \hat{r}.$$

$$\bar{a}(t) = \bar{g} = \frac{dV(t)}{dt} = \frac{d}{dt} Aw \cos wt$$

$$= -A\omega^2 \sin \omega t$$

$$\Rightarrow \omega = \text{high}$$

Now by definition

$$\omega = 2\pi f$$

$$\text{and Period } (T) = \frac{1}{f} = \frac{\omega}{2\pi}$$

$$= 2\pi \sqrt{\frac{l}{mgh}}$$

$$\therefore T^2 = \frac{4\pi^2 l}{mgh}$$

$$\Rightarrow I = \frac{T^2 mgh}{4\pi^2} \quad (7)$$

(moment of inertia)
about O (center)

which estimates the turning effect of
~~force~~ of gravity (F_g) on the bob about O.

II. ROTATION OF RIGID BODIES

Another quantity angular momentum (\bar{I}) is also related to the net turning force (\bar{T}) w.r.t time (t) of the rotational motion of the object, thus:

$$\bar{T} = \frac{d\bar{I}}{dt} \quad (8)$$

$$\text{and } \bar{T} = \bar{r} \times \bar{F} \quad (9)$$

Given $\bar{F} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$ and $\bar{r} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$

then

$$\bar{T} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

- (10)

For example if

$$\bar{F} = 3\hat{i} + 4\hat{j} + 8\hat{k}$$

and $\bar{F} = 6\hat{i} + 2\hat{j} - 4\hat{k}$

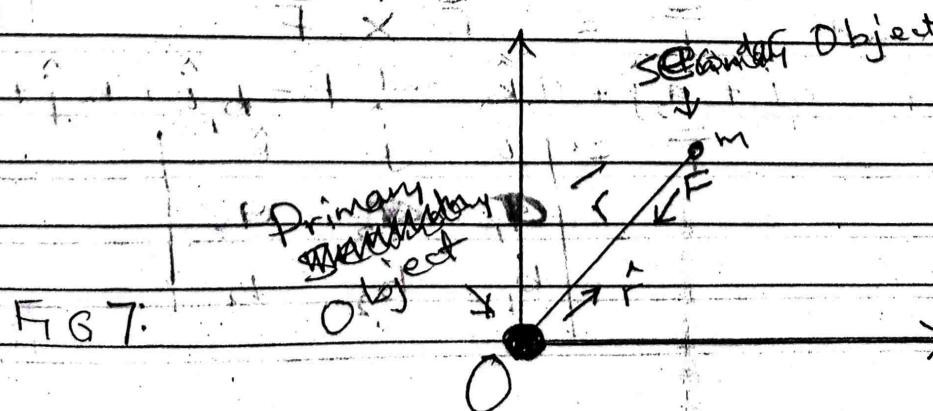
then $\bar{I} = \bar{F} \times \bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 8 \\ 6 & 2 & -4 \end{vmatrix}$

$$= 6 \begin{vmatrix} 4 & 8 \\ 2 & -4 \end{vmatrix} \hat{-j} \begin{vmatrix} 3 & 8 \\ 6 & -4 \end{vmatrix} + \hat{k} \begin{vmatrix} 3 & 4 \\ 6 & 2 \end{vmatrix}$$

$$\Rightarrow \bar{I} = -32\hat{i} + 60\hat{j} - 18\hat{k}$$

Generally the rotating objects angular momentum \bar{I} , about the origin, O of an inertial reference frame, is constant, (if not net force act on it), i.e. $\bar{I} = 0$; if the net force acting on it is always directed towards that Origin, as in SHM.

Consider orbital planetary motion in which the primary object exerts an attractive central force F ^{on the} _(mass m) secondary object in an inertia frame whose origin is at the force centre.



Question.....
Write on both sides of the Paper

i.e.

$$\bar{F} = -|F| \hat{r}$$

where $|F| = \sqrt{b_1^2 + b_2^2 + b_3^2}$; if as
in the previous example (1) $\bar{F} = 6\hat{i} + 2\hat{j} - 4\hat{k}$

$$|F| = \sqrt{6^2 + 2^2 + (-4)^2}$$

$$= 7.5 \text{ N.} \quad 7.48 \text{ N}$$

= magnitude of the

attractive central force

$$(2) \quad F = 5\hat{i} + 2\hat{j} - 4\hat{k}$$

$$|F| = 6.71 \text{ N}$$

$$-8 = -32$$

$$(-12 - 48)$$

$$x 6 \oplus$$

$$9 - 12$$

$$7 \times$$