

# **Circular and Oscillatory Motion**

**Dr. A.O. Adewale**

**Recommended Text:**

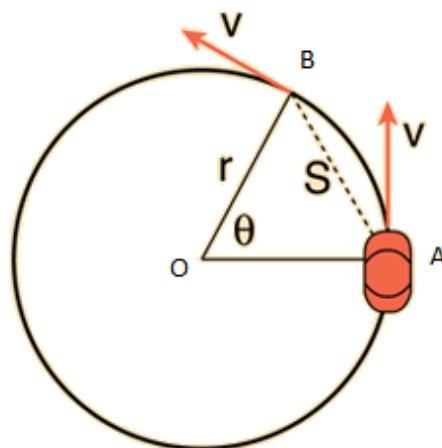
- 1. Physics for University Beginners Vol 1 by Adekola O. Adewale**
- 2. Physics for University Beginners Vol 2 by Adekola O. Adewale**

# Outline

- **Angular Displacement**
- **Angular velocity**
- **Torque**
- **Angular acceleration**
- **Centripetal acceleration**
- **Centripetal force**
- **Rotational kinetic energy**
- **Work done in rotation**
- **Conservation of angular momentum**
- **Simple harmonic motion**
- **Energy in simple harmonic motion**
- **Damped and forced oscillations**
- **Resonance**

# Angular Displacement

- The angular displacement is defined as the angle through which an object moves on a circular path. It is the angle, in radians, between the initial and final positions.



$$\theta = \frac{AB}{r}$$

# Angular Velocity

- Is the speed and velocity constant for a body moving in circular path? Explain.
- What is uniform circular motion?
- The speed  $v$  is the rate of change of the arc length  $AB$  with time. That is:

$$v = \frac{\Delta(r\theta)}{\Delta t} = \frac{r\Delta\theta}{\Delta t}$$

# Angular Velocity

- The average angular speed of the motion, denoted by the symbol  $\omega$ , is the angular displacement divided by the total time taken to travel the distance  $S$ :

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\theta - \theta_o}{t - t_o}$$

- SI unit of angular speed is radians per second (rad/s).
- In calculus notation, we can have:  $\omega = \frac{d\theta}{dt}$

# Angular Acceleration

- The angular acceleration,  $\alpha$ , can be defined as the time rate of change of angular velocity:

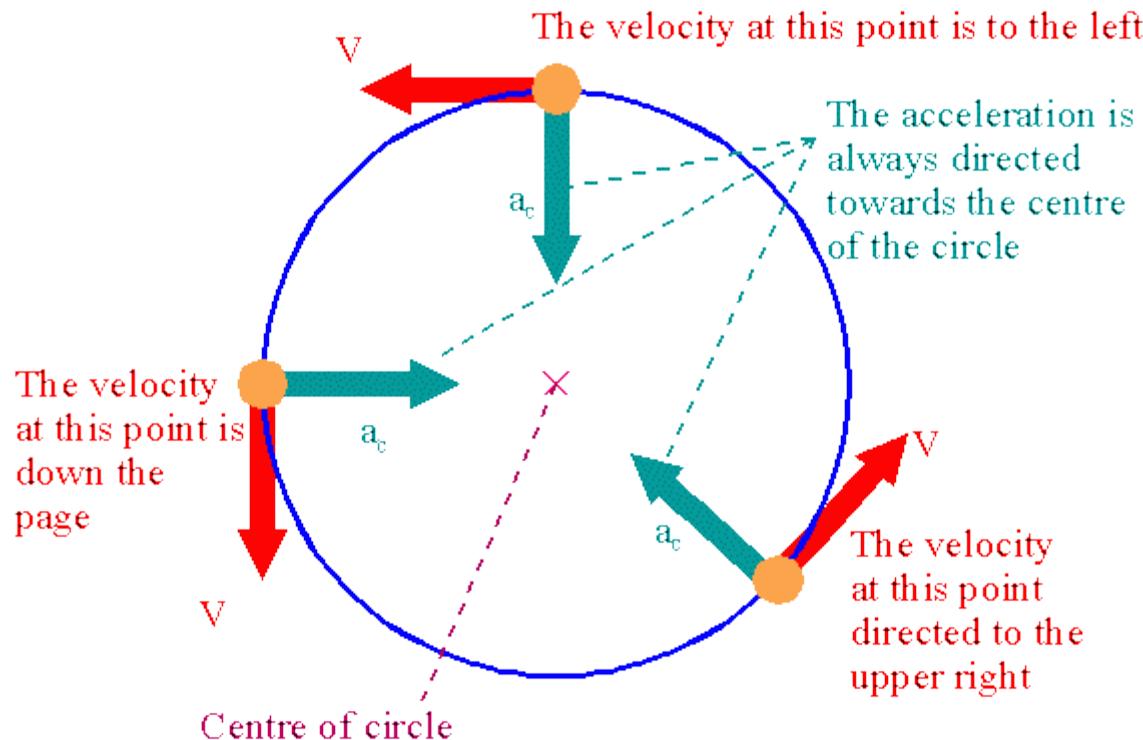
$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_o}{t - t_o} \quad \longrightarrow \quad \omega = \omega_o + \alpha t$$

- The S.I. unit of angular acceleration is radians per second per second ( $\text{rad/s}^2$ ).
- We can define angular displacement ( $\theta - \theta_o$ ) as follows:  $\theta - \theta_o = \left( \frac{\omega + \omega_o}{2} \right) \times t$
- Using the two equations above derive Eqns 5.8 and 5.9

$$\theta - \theta_o = \omega_o t + \frac{1}{2} \alpha t^2 \quad \omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

# Centripetal Acceleration

- Is an object in circular path accelerating?



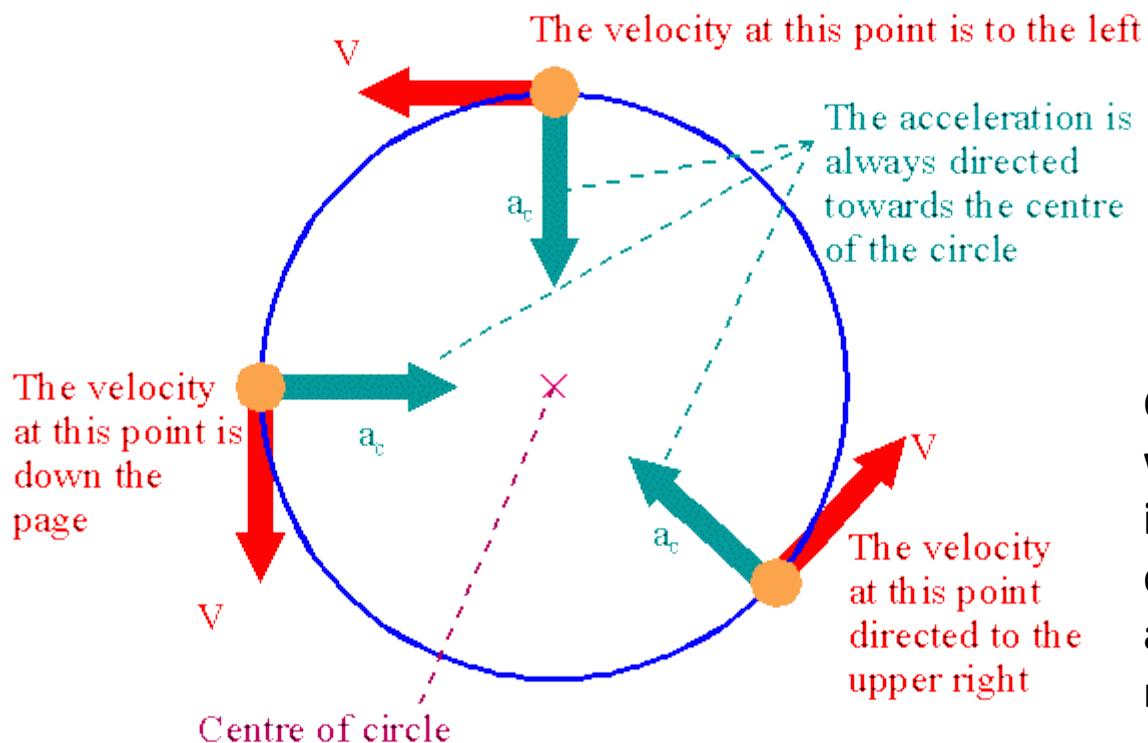
$$a = \frac{v^2}{r}$$

$$a = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

**Because the acceleration is always directed toward the centre of the circle, it is sometimes called centripetal acceleration, which means centre-seeking acceleration.**

# Centripetal Force

- From Newton's second law, the centripetal acceleration is:



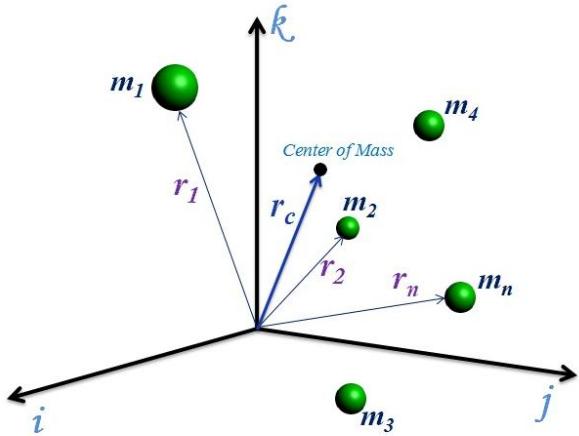
$$F = ma = \frac{mv^2}{r}$$

where  $m$  is the mass of the particle.

**Centripetal force is a force which acts on a body moving in a circular path and is directed towards the centre around which the body is moving.**

# Rotational Kinetic Energy

- Recall the following: Moment of a vector (chapter 7), Rotation of rigid bodies (chapter 7), centre of mass & centre of gravity (chapter 7), Moment of inertia (chapter 7)



The moment of inertia  $I$ , of the rigid body is defined as:

$$I = m_1 r_1^2 + m_2 r_2^2 + \dots = \sum_i m_i r_i^2$$

# Rotational Kinetic Energy

- The motion of a rigid body can always be divided into translation of the centre of mass and rotation about the centre of mass. Hence, the kinetic energy of a rigid body has both translational and rotational parts.
- The rotational kinetic energy  $K$  of a rigid body is: 
$$K = \frac{1}{2} I \omega^2$$
- The translational kinetic energy  $K$  of a rigid body is: 
$$K_{trans} = \frac{1}{2} M v_{cm}^2$$

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# Rotational Kinetic Energy

- The total KE is

$$K = \frac{1}{2} I_{cm} \omega^2 + \frac{1}{2} M v_{cm}^2$$

- where the subscript *cm* refers to the centre of mass.
- Assignment 2: Show that the acceleration of a uniform cylinder rolling down an inclined plane without slipping is

$$a = \frac{2g \sin \theta}{3}$$

# Work and Power in Rotational Motion

- **Newton's second law for rotational motion states that the total external torque  $\tau$  is equal to the product of the moment of inertia  $I$  and the angular acceleration  $\alpha$ :**  $\tau = I\alpha$
- **The angular acceleration of a rotating body is defined as:**  $\alpha = \frac{d\omega(t)}{dt}$
- **The total work done by the torque during an angular displacement from  $\theta_1$  to  $\theta_2$  is defined as:** 
$$W = \int_{\theta_1}^{\theta_2} \tau d\theta$$

# Work and Power in Rotational Motion

- If the torque remains constant while the angle changes by a finite amount, then:

$$W = \int_{\theta_1}^{\theta_2} \tau d\theta = \tau(\theta_2 - \theta_1)$$

- When a torque does work on a rotating rigid body, the change in the rotational kinetic energy of the rigid body equals the work done by forces exerted from outside the body

$$W = \int_{\theta_1}^{\theta_2} I\alpha d\theta = \int_{\theta_1}^{\theta_2} I \frac{d\omega}{dt} d\theta = \int_{\omega_1}^{\omega_2} I \frac{d\theta}{dt} d\omega = \int_{\omega_1}^{\omega_2} I\omega d\omega$$

$$W = \frac{1}{2} I\omega_2^2 - \frac{1}{2} I\omega_1^2$$

# Work and Power in Rotational Motion

- The power associated with work done by a torque acting on a rotating body is:  
$$P = \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau\omega$$
- 7.14. The flywheel of a stationary engine has a moment of inertia of  $60\text{kgm}^2$ . What is the kinetic energy if its angular acceleration is  $2\text{rads}^{-2}$ ?
- A. 10J      B. 20J   C. 30J   D. 15J   E. 60J

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# Assignment (More Questions)

- **Self-Assessment Questions (SAQs) for Chapter 7**
- **7.1.** An astronaut is being tested in a centrifuge. The centrifuge has a radius of 10m and, in starting, rotates according to  $\theta = 0.30t^2$ , where  $t$  is in seconds and  $\theta$  is in radians. When  $t = 5.0\text{s}$ , what are the magnitude of the astronaut's angular velocity and linear velocity.  
•
- **7.2.** Calculate the rotational inertial of a wheel that has a kinetic energy of 24,400J when rotating at 602 rev/min.  
A.  $24.56\text{kgm}^2$    B.  $6.14\text{kgm}^2$    C.  $3.07\text{kgm}^2$    D.  $12.3\text{kgm}^2$   
•
- **7.3.** A solid cylinder of mass 800kg and radius 6.0cm rolls without slipping down an inclined plane which makes an angle of  $30^\circ$  with the horizontal. Calculate the linear speed of the cylinder after rolling a distance of 1.5m. Assume that the moment of inertia of the cylinder about an axis through its middle, perpendicular to its plane is  $\frac{1}{2}MR^2$ , where  $M$  is its mass and  $R$  its radius.

# Assignment (More Questions)

- **Self-Assessment Questions (SAQs) for Chapter 7**
- **7.4.** A helicopter has a propeller of mass of 70kg and radius of gyration of 0.75m. Find its moment of inertia. Calculate the torque needed to produce an angular acceleration of  $4 \text{ rev/s}^2$ .
- 
- **7.5.** A 0.5kg uniform sphere of 7.0cm radius spins at 30 rev/s on an axis through its centre. Find its (a) K.E, (b) angular momentum, and (c) radius of gyration.
- 
- **7.6.** A force of 200N acts tangentially on the rim of a wheel 25cm in radius. Calculate the torque.
- 
- **7.7.** A 20kg solid disk rolls on a horizontal surface at the rate of 4.0m/s. Calculate its total kinetic energy.

# Assignment (More Questions)

- **Self-Assessment Questions (SAQs) for Chapter 7**
- **7.8.** A disc of moment of inertia  $10\text{kgm}^2$  about its centre rotates steadily about the centre with an angular velocity of  $20\text{rad/s}$ . Calculate (i) its rotational energy, (ii) its angular momentum about the centre, (iii) the number of revolutions per second of the disc.
- 
- **7.9.** A constant torque of  $200\text{Nm}$  turns a wheel about its centre. The moment of inertia about this axis is  $100\text{kgm}^2$ . Find (i) the angular velocity gained in  $4\text{s}$ , (ii) the kinetic energy gained after  $20\text{revs}$ .
- 
- **7.10.** A solid spherical ball of radius  $0.12\text{ m}$  and mass  $1.5\text{kg}$  rolls without slipping, moving in a straight line on a horizontal surface. If the velocity of the ball is  $0.2\text{m/s}$ , calculate its kinetic energy. (b) If, instead, the ball slides without rolling, at the same speed, calculate its kinetic energy.

# **Future Class**

- Read Simple Harmonic Motion**

# Centripetal Acceleration

- The angular acceleration,  $\alpha$ , can be defined as the time rate of change of angular velocity:

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