MAT 101 OER COMPILED BY SIR TEMI

Contains Extraction of Question from the Database of LASU . All questions Are likely to be Repeated in your Examination

Note : Solutions are After the Questions

1. Solve for x $2^{2x+1}.3^{2x+2}+2^x.3^{x+2}-2=0$ ans:-1

2. If $\log_9(x+3)-\log_9\frac{1}{3}=\frac{3}{2}$. Find the value of x ans:-6

3. The sum of the first ten terms of an arithmetic series is 60, the sum of the first twenty-two terms is 220.

   Find the common difference and the first term of the series. ans:$\frac{2}{3}, 3$

4. Find the sum to infinity of the geometric series $16+12+9+...$ ans:64

5. A geometric series has first term 27 and common ratio $\frac{4}{3}$. Find the least number of terms the series can have if the sum exceeds 550. ans:8

6. Solve for x $\frac{(x-2)(x-3)}{(x-1)} \leq 0$ ans:$\{x : 2 \leq x \leq 3\}$

7. Find the inverse of the function defined by $h : x \to \frac{x}{x^2-1}$. ans:$h^{-1}(x) = \frac{1-\sqrt{1+4x^2}}{2x}$

8. A polynomial $p(x)$ is divided by x^2-x and the remainder is $a+bx$. Determine the constants a and b. ans:$p(0)$ and $(p(1)-p(0))$

9. Simplify $\frac{x^{\frac{1}{3}}+x^{-\frac{2}{3}}}{x^{\frac{1}{2}}}$ ans:$\frac{x+1}{x^2}$

10. Simplify $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^{a+b}}{x^{a-b}}\right)^{\frac{a^2}{b}}$ ans:$x^{3a^2-b^2}$

11. Find the positive square root of $49-12\sqrt{5}$ ans:$2+3\sqrt{5}$

12. Give the first three terms of $(1+x)^{15}$ in ascending order ans:$1+15x+105x^2$

13. If $\log_a\left(1+\frac{1}{8}\right)=l$, $\log_a\left(1+\frac{1}{15}\right)=m$ and $\log_a\left(1+\frac{1}{24}\right)=n$. Find $l-m-n$? ans:$\log_a\left(1+\frac{1}{80}\right)$

14. Find the value of a if the function $a(x^2+2x-8)$ has a minimum value of -27. ans:3

15. If $\log_2 2a = c$, $2\log_8 a = b$ and $b-c=-4$. Find a? ans:512

16. Express $\frac{1+2x+3x^2}{(1-x)(1+x^2)}$ in partial fractions ans:$\frac{3}{1-x}-\frac{2}{1+x^2}$

17. The roots of the equation $x^2+7x+11=0$ are α and β, where $\alpha > \beta$. Find $\alpha-\beta$. ans:$\sqrt{5}$

18. Find the values of x such that $|2x-3| > |x+3|$ ans:$x > 6$

19. Solve the equation $x^{\frac{1}{3}}-3x^{-\frac{1}{3}}=2$. ans:$-1$

20. If the equation $x^2-3x+1=n(x-3)$ has equal roots, find the possible value of n. ans:5

21. Find the greatest value of $-2x^2+4x+3=0$ ans:1

22. Obtain the set of values of x for which $2x > \frac{1}{x}$ ans:$-\frac{1}{\sqrt{2}} < x < 0$

23. Write down the first five terms of the sequence specified by this recurrence relation: $U_1 = 0, U_n = \frac{1}{5-U_{n-1}}(n \geq 2)$ ans:$0, \frac{1}{5}, \frac{5}{24}, \frac{24}{115}, \frac{115}{551}$

24. The polynomial $f(x)$ is given by $f(x) = x^4 + x^3 - 7x^2 + 3x + 2$. If also, $f(x) = (x-1)(x-2)(x+3)(x+c) + Px + Q$, find the values of P, Q and c. ans:$4, -4, 1$

25. Find the inverse of matrix $Q = \begin{pmatrix} 2 & 4 & 3 \\ 1 & -2 & -2 \\ -3 & 3 & 2 \end{pmatrix}$

ans:$\frac{1}{11}\begin{pmatrix} 2 & 1 & -2 \\ 4 & 13 & 7 \\ -3 & -18 & -8 \end{pmatrix}$

26. If the minimum value of $x^2 + 2x + k$ is 3, find the value of k. ans:4

27. If α and β are the roots of the equation $3x^2 - 7x - 1$. find the values of $(\alpha - \beta)^2$. ans:$\frac{61}{9}$

28. Solve the inequality $5 - 4x > 8$ and write the solution set in interval notation ans:$\{-\infty, -\frac{3}{4}\}$

29. The sum of the first ten terms of an arithmetic series is 60, the sum of the first twenty-two terms is 220.

    Find the common difference and the first term of the series. ans:$\frac{2}{3}, 3$

30. For all x, except $x = 1$, $\frac{x^2+1}{x-1}$ equals ans:$x + 1 + \frac{2}{x-1}$

31. Functions h_1, h_2 are defined by $h_1 : x \to \log_2 x$, $h_2 : x \to \frac{1}{x}$. Find $h_1(h_2(x)) + h_1(x)$. ans:0

32. Solve completely, the equation $\sqrt{(x^2 - 3x + 6)} = 1 - \sqrt{(x^2 - 3x + 3)}$ ans:1

33. Solve $x(x+1) + \frac{12}{x(x+1)} = 8$ ans:$-3, 2, -2, 1$

34. If α and β are the roots of the expression $2x^2 + 8x + 7 = 0$, write down the value of $\sqrt{\alpha^2 + \beta^2} + 1$ ans:4

35. If $\log_4 u = \log_8 v + 1$. Find u in terms of v. ans:$u = 4v^{\frac{2}{3}}$

36. Using the remainder's theorem, find the remainder when $x^4 - 5x^3 + 6x^2 - 7$ is divided by $x^2 - 4x + 3$ ans:$-x - 4$

37. The fifth term of an A.P is 23 and the twelfth is 37. Find the first term, the common difference and the sum of the first eleven terms. ans:$15, 2, 275$

38. Find the first four terms in the expansion of $(1-x)^{-\frac{1}{2}}$ ans:$1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{15}{48}x^3$

39. The function f is given by the equation $f(x) = x^2 + 6x + 7$ for all numbers x. Which of the following statement is true? ans:The graph of $y = f(x)$ in the $x - y$ plane has a minimum point at $(-3, -2)$

40. Find the least value of $3x^2 - 4x + 2$ ans:$\frac{2}{3}$

41. Express $\frac{3x^2+2x-9}{(x^2-1)^2}$ in partial fractions ans:$\frac{3}{x^2-1} - \frac{2x-6}{(x^2-1)^2}$

Find the value of $\sum_{r=1}^{\infty} \frac{k}{10^r}$ *ans:*$\frac{k}{9}$

42. If the roots of the equation $x^2 - 2(k-2)x + 2k - 10 = 0$ are real. Find the possible values of k when the roots of the equation differ by 6. ans:5, 1

43. Find the first four terms in the expansion of $(1-x)^4(1+2x)^7$ in ascending powers of x ans:$1 + 13x + 76x^2 + 504x^4$

44. Find the 6th term of the geometric sequence $\sin 2\alpha, -\sin\alpha\cos 2\alpha, \sin 2\alpha\cos^2\alpha$ ans:$\frac{1}{32}\sin 2\alpha$

45. Find the term x^5 in the expansion of $(1+x)^{10}$ ans:$252x^5$

46. If $\sqrt[3]{x} = 3$ and $x = \sqrt{y}$, what is the value of y? ans:729

47. Find the sum to infinity of the geometric series $16 + 12 + 9 + \ldots$ ans:64

48. Solve the equation $\log_a(x^2+3) - \log_a x = \frac{2}{\log_2 a}$ ans:1

49. If $\mathcal{U}$ is the universal set $\{1, 3, 5, 7, 11\}$. $A = \{1, 3, 7, 11\}$ and $B = \{3, 5, 7, 19\}$ are the

subsets. List the elements of $(A \cap B') - (A \cup B')$. ans:$\emptyset$

50. If α and β are the roots of the expression $2x^2 + 8x + 7 = 0$, write down the value of $\sqrt{\alpha^2 + \beta^2} + 1$ ans:4

51. Using the remainder's theorem, find the remainder when $x^4 - 5x^3 + 6x^2 - 7$ is divided by $x^2 - 4x + 3$ ans:$-x - 4$

52. The fifth term of an A.P is 23 and the twelfth is 37. Find the first term, the common difference and the sum of the first eleven terms. ans:15, 2, 275

53. It is given that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are three consecutive terms of an arithmetic series. Which of the following statements is true of the series. ans:a^2, b^2, c^2 are also three consecutive terms of an series

54. Out of the 1000 undergraduate students in the faculty of science 650 are offering Algebra and 500 are offering Trigonometry. How many students are offering both Algebra and trigonometry, if only 210 students are offering neither Algebra or trigonometry. ans:360

55. Find the set of values of k for which $f(x) = x^2 + 3kx + k$ is greater than zero for real values of x. ans:$0 < k < \frac{4}{9}$

56. If $\log_4 a = \log_8 b + 1$. Find a in terms of b. ans:$a = 4b^{\frac{2}{3}}$

57. When $x^3 + 2x^2 + px - 3$ is divided by $x + 1$ the remainder is the same as when it is divided by $x - 2$. Find p. ans:-5

58. If the quadratic expression $kx^2 - px + q$ has a repeated root, obtain the value for k. ans:$\frac{p^2}{4q}$

59. Find the square root of $14 + 6\sqrt{5}$ ans:$\pm(3 + \sqrt{5})$

60. Solve the simultaneous equations $\log_2 xy = 7$, $\log_2 \frac{x^2}{y} = 5$ ans:8, 16

61. Determine the set of range for the function $f(x) = x^2 - 2x + 3$ ans:$2 \leq x \leq 6$

62. The first term of an arithmetic series is $3n + 5$ where n is a positive integer. The last term is $(17n + 17)$ and the common difference is 2. Find in terms of n the sum of the series. ans:$(7n + 7)(10n + 11)$

63. Suppose $M = \begin{pmatrix} -x & 2 \\ -x+1 & x-1 \end{pmatrix}$, find the possible value(s) of x if $|M| = 0$. ans:1 or 2

64. The co-efficient of x^5 in the expansion of $(1+5x)^8$ is equal to the co-efficient of x^4 in the expansion of $(k + 5x)^7$. Find the value of k? ans:2

65. Given that the universal set $\mathcal{U} = \{x \in z| -10 \leq x \leq 10\}$, $A = \{x \in z|x \ is \ divisible \ by \ 3\}$, $B = \{x \in z|is \ a \ prime \ number\}$, $C = \{x \in z|roots \ of \ x^4 + 2x^3 - 5x^2 - 6x = 0\}$, where A, B and C are subsets of the universal $\mathcal{U}$. Find $A - (BUC)$? ans:$\{-9, -6, 6, 9\}$

66. Solve for x if $\log_3(x + 2) = \log_9(6x + 4)$ ans:0

67. Find the co-efficient of x^3 in the expansion of $(1 - x)^{-2}(1 + x)^{-2}$ ans:0

68. The logarithm of a number to the base $\sqrt{2}$ is a, what is its logarithm to the base $2\sqrt{2}$? ans:$\frac{a}{3}$

69. Express $\frac{x^2-7x-6}{x^2(x-3)}$ in partial fraction. $\frac{2}{x^2} + \frac{3}{x} - \frac{2}{x-3}$

70. If $(\log_4 x)^2 = \log_2 x \log_n x$, find the value of n. ans:16

71. If p and q are the roots of the equation $2x^2 - x - 4 = 0$. Find $p^3 + q^3$. ans:$\frac{25}{8}$

72. Solve the equation $\frac{3^{5x+2}}{9^{1-x}} = \frac{27^{4+3x}}{729}$. ans:-3

73. If $p = \log_a(ab^2)$, $q = \log_b(a^2b)$. Which of the following statements is/are true?
I. $pq = p + q + 3$ II. $\sqrt{(p+q)^2 - pq} = 3$ III. $3pq = p^2 - q^2$ ans:I only

74. Simplify $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$ ans:1

75. Given the simultaneous equations, $2^x = 3^y$ and $x + y = 1$. Obtain an expression for x in terms of logarithm ans:$\frac{\log 2}{\log 3}$

76. A survey of 18 families in a housing estate , all of whom kept a cat or dog or both, revealed that 8 families kept a cat and 14 families kept a dog. How many kept both? ans:4

77. Given that $x^2 - 3x + 2$ is a factor of $x^4 + ax^2 + bx + 8$, find the values of a and b. ans:$-3, -6$

78. Express $\frac{5x+4}{(x-1)(x+2)^2}$ in partial fractions ans:$\frac{1}{x-1} - \frac{1}{x+2} + \frac{2}{(x+2)^2}$

79. If $n(n^2+5)$ is a multiple of 6, by inductive hypothesis which of the following is also a multiple of 6? ans:$n(n^2+5) + 3n(n+1) + 6$

80. Given that $f(x) = ax + b$ and that $f(2) = 7$ and $f(3) = 12$, find a and b. ans:$a = 5, b = -3$

81. Suggest a suitable domain and co-domain for the function $f(x) = \frac{32x+3}{4x+1}$ ans:$x : x \neq -\frac{1}{4}, y : y \neq 8$

82. If $\frac{1}{3\sqrt{5}+\sqrt{2}} \equiv a\sqrt{2} + b\sqrt{5}$, then a and b are respectively. ans:$-\frac{1}{43}, \frac{3}{43}$

83. Solve $\frac{(x-2)(x-3)}{(x-1)} \leq 0$ ans:$x < 1$ or $2 \leq x \leq 3$

84. The three real, distinct and non-zero numbers a, b, c are such that a, b, c are in arithmetic progression and a, c, b are in geometric progression. Find the numerical value of the common ratio of the geometric progression. ans:$-\frac{1}{2}$

85. Use mathematical induction to find the sum to n term of the series $\frac{1}{1.2} + \frac{1}{2.3} + ... + \frac{1}{n(n+1)}$. ans:$\frac{n}{n+1}$

86. Solve the equation $3^{2x+1} - 28(3^{x-1}) + 1 = 0$ ans:1

87. The 3rd and the 6th terms of a G.P. are 108 and -32 respectively. Find the sum of the 1st 7 terms. ans:154.3

88. Let A and B be any arbitrary sets, then $A' - B'$ is equivalent to. ans:$(A - B)'$

89. Find the first four terms of the expansion of $\left(1 - \frac{x}{2}\right)^6$ in ascending powers of x. ans:$1 - 3x + \frac{15}{4}x^2 - \frac{5}{2}x^3 + ...$

90. Find the least value of $2x^2 - x + 1$ ans:$\frac{7}{8}$

91. $f(x) \equiv 2x^3 + p + qx + 6$ where p and q are constants. When $f(x)$ is divided by $(x+1)$, the remainder is 12. When $f(x)$ is divided by $(x-1)$, the remainder is -6. Find the value of p and q ? ans:$-3, -11$

92. Find the values of m and n if the expression $mx^3 + nx^2 - 28x + 15$ is exactly divisible by $x+3$ and leaves a remainder -60 when it is divided by $x-3$. When m and n have these values, find all the values for x for which the expression is zero. ans:$-3, \frac{1}{2}, 5$

93. The values of A, B and R respectively, given that $x^2 + 9x - 3 \equiv (x+1)(Ax+B) + R$. ans:$1, 8$ *and* -11

94. Find the range of values of p for which the expression $x^2 - (x-1)p + 3 = 0$ has real roots. ans:$p \leq -2$ or $p \geq 6$

95. Solve for x $2^{2x+1}.3^{2x+2} + 2^x.3^{x+2} - 2 = 0$ ans:-1

96. Solve for x $\frac{x-2}{x^2-x+1} > 0$ ans:$x > 2$

97. Given a function $h : x \to \frac{x}{x^2-1}$, $x < 0$, $x \neq -1$. Find its inverse. ans:$\frac{1-\sqrt{1+4x^2}}{2x}$, $x \neq 0$

98. In a G.P, $U_3 = 32$ and $U_6 = 4$, Find the sum of the first eight terms of the G.P ans:255

99. Solve the $(2x^2 - x)^2 - 9(2x^2 - x) + 18 = 0$ ans:$\frac{3}{2}$

100. Find an expression in descending powers of x and with whole number co-efficient for $(2x-3)^4 - (2x+3)^4$. ans:$-192x^3$

101. Simplify $\frac{(x^{\frac{3}{2}}+x^{\frac{1}{2}})(x^{\frac{1}{2}}-x^{-\frac{1}{2}})}{(x^{\frac{3}{2}}-x^{\frac{1}{2}})^2}$ ans:$\frac{x+1}{x(x-1)}$

102. Give the first three terms of $(2+x)^5$ in ascending powers of x ans:$32 + 80x + 8x^2 + 40x^3$

103. State the range of validity of the expansion of $(1-2x)^{-\frac{3}{2}}$. ans:$\frac{1}{2} > x > -\frac{1}{2}$

104. Find the domain of the function $f : x \to \frac{x}{x^2+3x+2} + \frac{2}{x+1}$ where x is real. ans:$x : x \neq -2, x \neq -1$

105. Find the set of values of y for which $y^2 - 9x + 20$ is negative. ans:$4 < x < 5$

106. Express $\frac{1}{x^4+5x^2+6}$ in partial fractions ans:$\frac{1}{x^2+2} - \frac{1}{x^2+3}$

107. If α and β are the roots of the equation $2x^2 - x - 4 = 0$. Find the equation whose roots are $\alpha - \frac{\beta}{\alpha}$ and $\beta - \frac{\alpha}{\beta}$. ans:$2x^2 - 7x + 5 = 0$

108. Simplify $log6 + log4 + log20 - log3 - log16$ ans:1

109. Find the value of p for which the matrix $\begin{pmatrix} p+1 & 6 \\ 1 & p \end{pmatrix}$ does not have an inverse. ans:-3

110. If one root of $ax^2+bx+c=0$ is triple the other, which of the following is true ans:$3b^2=16ac$

111. The sum of the first n terms of a series is $1-(\frac{3}{4})^n$. Find the first term and the common ratio? ans:$\frac{1}{4},\frac{3}{4}$

112. $x-1, x+1$ are factors of the expression x^3+ax^2+bx+c and it leaves a remainder 12 when divided by $x-2$. Find a, b and c. ans:$1,-1,-1$

113. In proving $\sum_{r=1}^{n}=r(r+1)(r+2)=\frac{1}{4}n(n+1)(n+2)$ by induction, which of the following is true for $n=k+1$. ans:$n(n+1)\left(\frac{n^2+5n+6}{4}\right)$

114. Find the sum of the first n terms of the series $\log 3+\log 6+\log 12+...$ ans:$n\log 3+\frac{n(n-1)}{2}\log 2$

115. Obtain the set of values of x for which $2|x-1|>|x+1|$ ans:$x<\frac{1}{3}$ *or* $x>3$

116. Find the co-efficient of x^2 in $(3-x)^{10}$ ans:295245

117. Find the sum to infinity of the series $1+\frac{1}{2}+\frac{1}{4}+...$ ans:2

118. Given that the expression ax^3+8x^2+bx+6 is exactly divisible by x^2-2x-3, find the values of a and b. ans:-5, 19

119. Three consecutive terms of a geometric series have product 343 and sum $\frac{49}{2}$. Find the numbers. ans:$\frac{7}{2},7,14$

120. Expand $\frac{3x+5}{(1-x)(1+3x)}$ as a series of ascending powers of x up to and including the term in x^2 and also state its range of validity. ans:$5-7x+29x^2+...$, $-\frac{1}{3}<x<\frac{1}{3}$

121. One of the following is not a factor of $x^3-ax^2-a^2x+a^3$ ans:$x-a^2$

122. The first three terms of a geometric progression are $k-3, 2k-4, 4k-3$ in that order. Find the value of k. ans:7

123. An equilateral triangle field has sides of length l. Find an expression for the area of the field, leaving your answer in surd form. ans:$\frac{\sqrt{3}}{4}l^2$

124. If the equation $x^2-3x+1=p(x-3)$ has equal roots, find the possible value of p. ans:5

125. Find the values of n for which the expression $(2n+3)x^2-6x+4-n$ is a perfect square. ans:$3,-\frac{1}{2}$

126. For what value of m is $9x^2+mx+16$ a perfect square? ans:-24

127. The domain of $g(x)=\sqrt{x}$ is ans:$[0,\infty)$

128. Find in surd form, the square root of $4+2\sqrt{3}$ ans:$\pm(1+\sqrt{3})$

129. In a certain arithmetic progression, the sum of the first and fifth terms is 18 and the fifth term is 6 more than the third term. Find the sum of the first ten terms of the progression ans:165

130. If $a = 27^{\frac{2}{3}}$ and $b = 3^{-4}$. Find the value of ba^2 ans:1

131. Given the matrix $A = \begin{pmatrix} -1 & -4 \\ 1 & 3 \end{pmatrix}$. Find A^3 ? ans:$\begin{pmatrix} -5 & -12 \\ 3 & 7 \end{pmatrix}$

132. Find the set values of k for which $f(t) = t^2 + 3kt + k$ is greater than zero for all real values of t. ans:$k : k = 0, 0.1, 0.2, 0.3, 0.4$

133. Solve $\frac{x+4}{x+1} \leq \frac{x-2}{x-4}$ ans:$4 \leq x \leq 14$

134. Find the value of x satisfying the equation $\sqrt{3x+4} = 3 + \sqrt{x-3}$ ans:4

135. In an AP, the tenth term is 3 and the sum of the first six terms is 76.5. Find the sum of the first term and the common difference. ans:15.0

136. Solve $x^{\frac{1}{3}} - 3x^{-\frac{1}{3}} = 2$ ans:-1

137. Solve for x if $\log_3(x+2) = \log_9(6x+4)$

ans:0

138. Three consecutive terms of a geometric series have product 343 and sum $\frac{49}{2}$. Find the numbers. ans:$\frac{7}{2}, 7, 14$

139. Solve $\frac{1}{x+6} \geq \frac{2}{2-3x}$ ans:$-6 < x \leq -2$ or $x > \frac{2}{3}$

QUESTION

Solve for x:

$2^{2x+1}.3^{2x+2} + 2^x.3^{x+2} - 2 = 0$

SOLUTION

$2^1.(2^x)^2.3^2.(3^x)^2 + (2^x).(3^x).(3^2) - 2 = 0$

$18(2^x.3^x)^2 + 9(2^x.3^x) = 0$

$Let\ (2^x.3^x) = P$

$\rightarrow 18P^2 + 9P - 2 = 0$

$P = -{}^2/_3 \text{ or } P = {}^1/_6$

$Recall\ P = 2^x.3^x$

$when\ P = -{}^2/_3$

$2^x.3^x = -{}^2/_3$ ------ *impossible*

$when\ P = {}^1/_6$

$2^x.3^x = = {}^1/_6$

$2^x.3^x = 6^{-1}$

$2^x.3^x = (2.3)^{-1}$

$\therefore x = x = -1$

QUESTION

Simplify $(\frac{x^a}{x^b})^{a+b} \times (\frac{x^{a+b}}{x^{a-b}})^{a^2/b}$

SOLUTION

$$\frac{x^{a(a+b)}}{x^{b(a+b)}} \times \frac{x^{(a+b)^{a^2/b}}}{x^{(a-b)^{a^2/b}}}$$

$$\frac{x^{a^2+b}}{x^{ab+b^2}} \times \frac{x^{\frac{a^3}{b}+a2}}{x^{\frac{a^3}{b}-a^2}}$$

$$x^{a^2+ab-ab-b^2} \times x^{a\frac{3}{b}+a^2-a\frac{3}{b}+a^2}$$

$$x^{a^2-b^2} \times x^{a^2+b^2}$$

$$x^{a^2-b^2-2a^2}$$

$$= x^{3a^2-b^2}$$

QUESTION

Solve the equation

$x^{1/3} - 3x^{-1/3} = 2$

SOLUTION

$-\frac{3}{x^{1/3}} = 2$

Let $P = x^{1/3}$

$P - \frac{3}{P} = 2 \rightarrow P^2 - 2p - 3 = 0$

$P = -1$ or $P = 3$

Recall $P = x^{1/3}$

When $P = -1, \rightarrow x^{1/3} = -1$

$(x^{1/3})^3 = (-1)^3$

$x^1 = -1 \therefore x = -1$

When $P = 3$

$x^{\frac{1}{3}} = 3$

$x = 3^3 = 27$

QUESTION

$$\frac{\left(x^{\frac{3}{2}}+x^{\frac{1}{2}}\right)(x^{\frac{1}{2}}-x^{-\frac{1}{2}})}{(x^{\frac{3}{2}}-x^{\frac{1}{2}})^2}$$

SOLUTION

$$\frac{x^{\frac{3}{2}}\left(x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)+x^{\frac{1}{2}}(x^{\frac{1}{2}}-x^{-\frac{1}{2}})}{\left(x^{\frac{3}{2}}-x^{\frac{1}{2}}\right)\left(x^{\frac{3}{2}}-x^{\frac{1}{2}}\right)}$$

$$\frac{x^2-x^1+x^1-x^0}{x^3+x^1}$$

$$\frac{x^2-x^1}{x^3+x} = \frac{(x+1)\cancel{(x-1)}}{x\cancel{(x-1)}(x-1)}$$

$$\frac{(x+1)}{x(x-1)}$$

QUESTION

If $\log_9(x+3) - \log_9\frac{1}{3} = \frac{3}{2}.$

Find the value of x.

SOLUTION

$\log_9(x+3) - \log_9\frac{1}{3} = \frac{3}{2}$

$\log_9\left(\frac{x+3}{\frac{1}{3}}\right) = \frac{3}{2}$

$\cancel{\log_9}(3(x+3)) = \cancel{\log_9} 9^{3/2}$

$3(x+3) = 27$

$x + 3 = 9$

$x = 6$

QUESTION

if $\log_2 2a = C, 2\log_8 a = b$ *and* $b - c = -4$

find a

SOLUTION

$2\log_8 a = b$

$b - c = -4$

$\therefore 2\log_8 a - \log_2 2a = -4$

$\frac{2}{3}\log_2 a - \log_2 2a = -4$

$\log_2 a^{2/3} - \log_2 2a = -4$

$\log 2\left[\frac{a^{\frac{2}{3}}}{2a}\right] = \log_2 2^{-4}$

$\frac{a^{2/3}}{2a} = \frac{1}{2^4}$

$\frac{a^{2/3}}{a} = \frac{1}{8}$

$a^{\frac{2}{3}-1} = 2^{-3}$

$a^{\frac{-1}{3}} = 2^{-3}$

$(a^{-\frac{1}{3}})^3 = (2^{-3})^3$

$a = 2^9 = 512$

QUESTION

if $\log_4 U = \log_8 V + 1$. Find U in terms of V.

SOLUTION

$\log_4 U = \log_8 V + 1$

$\log_{2^2} U = \log_{2^3} V + \log_2 2$

$\frac{1}{2}\log_2 U = \frac{1}{3}\log_2 V + \log_2 2$

$\log_2 U^{\frac{1}{2}} = \log_2 2^{V1/3}$

$U^{\frac{1}{2}} = 2V^{\frac{1}{3}}$

$U = (2V^{\frac{1}{3}})^2$

$U = 4V^{\frac{2}{3}}$

QUESTION

solve the equation

$\log_a(x^2+3) - \log_a x = \frac{2}{\log_2 a}$

SOLUTION

$\log_a(x^2+3) - \log_a x = \frac{2}{\log_2 a}$

$\log_a(\frac{x^2+3}{x}) = \frac{2}{\log_2 a}$

$\log_a(\frac{x^2+3}{x}) \times \frac{2}{\log_2 a} = 2$

$\frac{\log_2(\frac{x^2+3}{x})}{\log_2 a} \times \log_2 a = 2$

$\log_2(\frac{x^2+3}{x}) = \log_2 2^2$

$\frac{x^2+3}{x} = 4$

$x^2 + 3 = 4x, x^2 - 4x + 3 = 0$

$\therefore x = 1$ or $x = 3$

QUESTION

find the positive square root of $49 - 12\sqrt{5}$

SOLUTION

$49 - 12\sqrt{5} = (\sqrt{P} - \sqrt{q})^2$

$49 - 12\sqrt{5} = P - 2\sqrt{pq} + q$

$\therefore p + q = 49 ------ (I)$

$-12\sqrt{5} = -2\sqrt{pq}$

$6\sqrt{5} = \sqrt{pq}$ -------- square both sides

$36.5 = pq$

$pq = 180$ ---------- (II)

from equ (I) $p = 49 - q$ ------- (*)

put equ (*)in equ (II)

$q(49 - q) = 180$

$49q - q^2 = 180$

$q^2 - 49q + 180 = 0$

$\therefore q = 4 \;\; or \; q = 45$

$put \; q = 4 \; or \; q = 45 \; in \; equ \; (*)$

$\therefore p = 49 - 4 = 45$

$p = 49 - 45 = 4$

$\therefore 49 - {}^{12}\sqrt{5} = \sqrt{4} - \sqrt{45}$

$= 2 - 3\sqrt{4}$

QUESTION

solve completely, the equation

$\sqrt{(x^2 - 3x + 6)} = 1 - \sqrt{(x^2 - 3x + 3)}$

SOLUTION

$x^2 - 3x + 6 = (1 - \sqrt{(x^2 - 3x + 3)})^2$

$\cancel{x^2} - 3\cancel{x} + 6 = 1 - 2\sqrt{(x^2 - 3x + 3)} + x^2 - 3x$
$+ 3$

$\cancel{2} = -\cancel{2}\sqrt{x^2 - 3x + 3}$

$1 = -1\sqrt{x^2 - 3x + 3}$

$1 = x^2 - 3x + 3$

$x^2 - 3x + 3 = 0$

$\therefore x = 1 \quad or \quad \cancel{x} = 2$

QUESTION

$solve \;\; x\,(x + 1) + \dfrac{12}{x(x + 1)} = 8$

SOLUTION

$let \; P = x(x + 1)$

$\Rightarrow P + \frac{12}{P} = 8$

$p^2 - 8p + 12 = 0$

$\therefore p = 6, \;\; p = 2$

$Recall \; P = x(x + 1)$

$when \; p = 6,$

$\rightarrow 6 = x(x + 1)$

$x^2 + x - 6 = 0$

$x = -3 \;\; or \; x = 2$

$when \; p = 2,$

$\rightarrow 2 = x^2 + x$

$x^2 + x - 2 = 0$

$\therefore x = -2 \;\; or \; x = 1$

$\therefore answer \;\rightarrow x = -3, -2, -1, 2$

QUESTION

find the square root of $14 + 6\sqrt{5}$

SOLUTION

$14 + 6\sqrt{5} \rightarrow (\sqrt{P} + \sqrt{q})^2$

$14 + 6\sqrt{5} \rightarrow p + q + 2\sqrt{Pq}$

$p + q = 14 - - - - (I)$

$6\sqrt{5} = 2\sqrt{pq}$

$3\sqrt{5} = \sqrt{pq}$

$45 = pq - - - - (II)$

$from \; equ \; (I) P = 14 - q - - - - (*)$

$put \; (*) in \; equ \; (II)$

$45 = (14 - q)q$

$q^2 - 14q + 45 = 0$

$q = 9 \;\; or \; q = 5$

$put \; q \; in \; equ (*)$

$P = 5 \;\; or \;\; p = 9$

$\therefore 14 + 6\sqrt{5} = \pm(\sqrt{5} + \sqrt{9})$

$= \pm(3 + \sqrt{5})$

QUESTION

solve the simultaneous equation

$log_2 xy = 7, \;\; log_2 \frac{x^2}{y} = 5$

SOLUTION

$log_2 xy = 7, - - - - (I)$

$log_2\frac{x^2}{y} = 5, ----(II)$

$from\ equ\ (I) \quad xy = 2^7$

$xy = 128 ----(*)$

From equ (II) $\frac{x^2}{y} = 2^5$

$\frac{x^2}{y} = 32 ---(**)$

$from\ equ * x = \frac{128}{y} ----(x)$

$put\ equ\ (x) into\ equ\ (**)$

$\frac{(\frac{128}{y})^2}{y} = 32$

$\frac{16384}{y^3} = 32$

$32y^3 = 16384$

$y^3 = 512$

$y = \sqrt{512} = 8$

put $y = 8$ in equ (x)

$\therefore x = \frac{128}{8} = 16$

$\therefore x = 16, y = 8$

QUESTION

solve the equation

$\frac{3^{5x+2}}{9^{1-x}} = \frac{27^{4+3x}}{729}$

SOLUTION

$\frac{3^{5x+2}}{9^{1-x}} = \frac{27^{4+3x}}{729}$

$\frac{3^{5x+2}}{3^{2(1-x)}} = \frac{3^{3(4+3x)}}{3^6}$

$\frac{3^{5x+2}}{3^{2-2x}} = \frac{3^{12+9x}}{3^6}$

$3^{5x+2+2x} = 3^{12+9x}$

$7x = 6 + 9x$

$2x = -6$

$x = -3$

QUESTION

Given the simultaneous equation

$2^x = 3^y$ and $x + y = 1$

obtain an expression for x in terms of logarit

SOLUTION

$2^x = 3^y ----(I)$

$x + y = 1 ------(II)$

from (I) $x\log_2 = y\log_3$

$\therefore y = \frac{x\log_2}{\log_3}$

put y in equ(II)

$x + \frac{x\log_2}{\log_3} = 1$

$x\log_3 + x\log 2 = \log 3$

$x(\log 3 + \log 2) = \log 3$

$x = \frac{\log 3}{\log 3 + \log 2}$

QUESTION

If $\frac{1}{3\sqrt{5}+\sqrt{2}} \equiv a\sqrt{2} +$ $b\sqrt{5}$, then a and b are respectively?

SOLUTION

$\frac{1}{3\sqrt{5}+\sqrt{2}} ----$Rationalize

$\frac{1}{3\sqrt{5}+\sqrt{2}} \text{ x } \frac{3\sqrt{5}+\sqrt{2}}{3\sqrt{5}+\sqrt{2}}$

$= \frac{3\sqrt{5}-\sqrt{2}}{45-2} = \frac{3\sqrt{5}-\sqrt{2}}{43}$

$= \frac{3\sqrt{5}}{43} - \frac{\sqrt{2}}{43}$

$= \frac{-\sqrt{2}}{43} + \frac{3\sqrt{5}}{43}$

compare to $a\sqrt{2} + b\sqrt{5}$

$\therefore a = \frac{-1}{43}, b = \frac{3}{43}$

QUESTION

solve the equation

$3^{2x+1} - 28(3^{x-1}) + 1 = 0$

SOLUTION

$3^{2x+1} - 28(3^{x-1}) + 1 = 0$

$(3^x).3^1 - 28\frac{(3^x)}{3}. +1 = 0$

let $P = 3^x$

$\rightarrow 3P^2 - \frac{28P}{3} + 1 = 0$

$\rightarrow 9P^2 - 28P + 3 = 0$

$9P^2 - 27P - P + 3 = 0$

$9P(P-3) - 1(P-3) = 0$

$9P - 1 = 0$ or $P - 3 = 0$

$P = 1/9$ or $P = 3$

Recall $P = 3^x$

when $P = 1/9$

$\therefore 3^x = 1/9 \rightarrow 3^x = 3^{-2} \therefore x = -2$

when $P = 3$

$\therefore 3^x = 3 \rightarrow 3^x = 3^1 \therefore x = 1$

QUESTION

solve the equation

$(2x^2 - 2)^2 - 9(2x^2 - 2) + 18 = 0$

SOLUTION

Let $P = 2x^2 - 2$

$\therefore P^2 - 6P - 3P + 18 = 0$

$P(p-6) - 3(P-6) = 0$

$p - 3 = 0$ or $p - 6 = 0$

$p = 3$ or $p = 6$

Recall $P = 2x^2 - 2$

when $p = 3, \rightarrow 2x^2 - 2 - 3 = 0$

$x = -1, x = 3/2$

when $p = 6, \rightarrow 2x^2 - 2 - 6 = 0$

$x = -\frac{3}{2}$ or $x = 2$

QUESTION

find in surd form, the square root of

$4 + 2\sqrt{3}$

SOLUTION

$4 + 2\sqrt{3} \equiv (\sqrt{P} + \sqrt{Q})^2$

$\equiv p + q + 2\sqrt{pq}$

$P + q = 4$ ---------- (I)

$\sqrt{3} = \sqrt{pq}$

$3 = pq$ ------------ (II)

from (I) $- - - p = 4 - q$ - - - - - - - (*)

put equ (*) in equ (II)

$3 = (4 - q)q$

$q^2 - 4q + 3 = 0$

$q^2 - 3q - q + 3 = 0$

$q(q-3) - 1(q-3) = 0$

$q - 1 = 0$ or $q - 3 = 0$

$q = 1$ or $q = 3$

put $q = 1$ or 3 in equ (*)

i.e. $p = 3$ when $q = 1$

$p = 1$ when $q = 3$

$\therefore 4 + 2\sqrt{3} = \pm(1 + \sqrt{3}$

QUESTION

if $a = 27^{\frac{2}{3}}$ and $b = 3^{-4}$, find the value of ba^2

SOLUTION

$ba^2 \implies 3^{-4} X \left(27^{\frac{2}{3}}\right)^2$

$3^{-4} X \left(27^{\frac{4}{3}}\right)$

$3^{-4} X (3^3)4/3$

$3^{-4} X 3^4 = 1$

QUESTION

find the value of x satisfying the equation

$\sqrt{3x+4} = 3 + \sqrt{x-3}$

SOLUTION

$\sqrt{3x+4} = 3 + \sqrt{x-3}$ *square both sides*

$(\sqrt{3x+4})^2 = (3 + \sqrt{3x+4})^2$

$3x + 4 = 9 + 6\sqrt{x-3} + x - 3$

$2x - 2 = 6\sqrt{x-3}$

$x - 1 = 3\sqrt{x-3}$ *square both sides*

$x^2 - 2x + 1 = 9(x-3)$

$x^2 - 2x + 1 = 9x - 27$

$x^2 - 11x + 28 = 0$

$x = 4, x = 7$

QUESTION

find the three inverse of the function defined by

$h: x \to \frac{x}{x^2-1}$

Solution

$y = \frac{x}{x^2-1}$

$yx^2 - y = x$

$yx^2 - x - y = 0$

from $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$X = \frac{1\pm\sqrt{1+4y^2}}{2y}$

$\therefore\ h^{-1} = \frac{1\pm\sqrt{1+4x^2}}{2x}$

QUESTION

function h_1, h_2 *are defined by*

$h_1: x \to \log_2 x, h_2: x \to \frac{1}{x},$

find $h_1(h_2(x)) + h_1(x)$

Solution

$h_1: x \to \log_2 x$

$h_2: x \to \frac{1}{x}$

$\therefore h_1(h_2(x)) + h_1(x)$

$\to \log_2 \frac{1}{x} + \log_2 x$

$-\log_2 x + \log_2 x = 0$

QUESTION

Given that $f(x) = ax + b$ *and that* $f(2) = 7$ *and* $f(3) = 12,$ *find a and b*

Solution

$f(x) = ax + b$

$f(2) \Rightarrow 2a + b = 7 - - - (I)$

$f(3) \Rightarrow 3a + b = 12 - - - (II)$

$\therefore a = 5 \quad b = -3$

QUESTION

Suggest a suitable domain and codoma

for the function $f(x) = \frac{32x+3}{4x+1}$

Solution

$f(x) = \frac{32x+3}{4x+1}$

$\therefore 4x + 1 \neq 0$

$4x \neq -1$

$x \neq -\frac{1}{4}$

$\therefore$ Domain: $\{x : x \neq -\frac{1}{4}\}$

$\frac{32x+3}{4x+1} = y$

$4xy + y = 32x + 3$

$x = \frac{3-y}{4y-32}$

$f^{-1}(x) = \frac{3-y}{4y-32}$

$4y - 32 \neq 0$

$4y \neq 32$

$y \neq 8$

$\therefore codomain: \{y: y \neq 8\}$

$\therefore \left\{x: x \neq -\frac{1}{4}\right\}, \{y: y \neq 8\}$

QUESTION

Find the domain of the function

$F: x \rightarrow \frac{x}{x^2+3x+2} + \frac{2}{x+1}$ *where x is real*

Solution

$F: x \rightarrow \frac{x}{x^2+3x+2} + \frac{2}{x+1}$

$F: x \rightarrow \frac{x(x+1)+2(x^2+3x+2)}{(x^2+3x+2)(x+1)}$

$\therefore (x^2+3x+2)(x+1) \neq 0$

$(x+1)(x+2)(x+1) \neq 0$

$\therefore x \neq -2$ or $x \neq -1$

QUESTION

The domain of $g(x) = \sqrt{x}$ *is?*

Answer

The domain of $g(x) = \sqrt{x}$ *is* $[0, \alpha]$

QUESTION

the roofs of teh equation $x^2 + 7x + 11 = 0$ *are* α *and* β. *Where* $\alpha > \beta$. *Find* $\alpha - \beta$.

Solution

$x^2 + 7x + 11 = 0$

$\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$\alpha + \beta = -\frac{b}{a} = -7, \ \alpha\beta = \frac{c}{a} = 11$

$\alpha - \beta = \sqrt{(-7)^2 - 4(11)}$

$= \sqrt{49 - 44} = \sqrt{5}$

QUESTION

If the equation $x^2 - 3x + 1 = n(x-3)$ *has equal roots, find the possible value of* n

Solution

$x^2 - 3x + 1 = n(x-3)$

$x^2 - 3x + 1 = nx - 3n$

$x^2 - (3+n)x + 1 + 3n = 0$

for equal root $b^2 - 4ac = 0$

$\therefore (-(3+n)^2 - (4 \times 1 \times (1+3n)) = 0$

$9 + 6n + n^2 - (4 + 12n) = 0$

$9 + 6n + n^2 - 4 - 12n = 0$

$n^2 - 6n + 5 = 0$

$\therefore n = 5$ *or* $n = 1$

QUESTION

Find the greatest value of $-2x^2 + 4x + 3 = 0$

Solution

$-2x^2 + 4x + 3 = 0$

maximum or greatest value

$x = -\frac{b}{2a}$

$x = -\frac{4}{2(-2)} = -\frac{4}{-4} = 1$

$x = 1$

QUESTION

if the minimum value of $x^2 + 2x + K$ *is 3. Find the value of K.*

Solution

for minimum value,

$\Rightarrow C - \frac{b^2}{4a}$

$K - \frac{4}{4} = 3$

$K - 1 = 3$

$K = 4$

QUESTION

if α *and* β *are the roots of the equation* $3x^2 - 7x - 1$, Find the value of $(\alpha - \beta)^2$

Solution

$\alpha-\beta=\sqrt{(\alpha+\beta)^2-4\alpha\beta}$

$\therefore(\alpha-\beta)^2=(\alpha\mp)^2-4\alpha\beta$

$(\frac{-7}{3})^2-4\left(-\frac{1}{3}\right)$

$=\frac{49}{9}+\frac{4}{3}=\frac{49+12}{9}=\frac{61}{9}$

QUESTION

if α and β are the roots of the expression $2x^2+8x+7=0$, write down the value of $\sqrt{\alpha^2\beta^2}+1$

Solution

$2x^2+8x+7=0$

$\sqrt{\alpha^2\beta^2}+1$

$\Rightarrow\sqrt{(\alpha+\beta)^2-2\alpha\beta}+1$

$\alpha+\beta=-\frac{b}{a}=-\frac{8}{2}=-4$

$\alpha\beta=\frac{c}{a}=\frac{7}{2}$

$\rightarrow\sqrt{(-4)^2-2\left(\frac{7}{2}\right)}+1$

$\rightarrow\sqrt{16-7}+1$

$\rightarrow 3+1=4$

QUESTION

Find the least value of $3x^2-4x+2$

Solution

least value $\rightarrow c-\frac{b^2}{4a}$

$\rightarrow 2-\frac{(-4)^2}{4(3)}$

$\rightarrow 2-\frac{16}{12}=\frac{8}{12}=\frac{2}{3}$

QUESTION

if the quadratic expression kx^2-px+q has a repeated root, obtain the value of K.

Solution

for the root to be repeated

$b^2=4ac$

$\therefore p^2=4kq$

$\therefore K=\frac{p^2}{4q}$

QUESTION

if p and q are the roots of the equation $2x^2-x-4=0$. Find p^3+q^3

Solution

$2x^2-x-4=0$

$p+q=-\frac{b}{a}=\frac{1}{2}$

$pq=\frac{c}{a}=-\frac{4}{2}=-2$

$\therefore p^3+q^3=(p+q)^3-3pq(p+q)$

$=\left(\frac{1}{2}\right)^3-3(-2)\left(\frac{1}{2}\right)$

$\frac{1}{8}+\frac{3}{1}=\frac{25}{8}$

QUESTION

find the least value of $2x^2-x+1$

Solution

least value $=c-\frac{b^2}{4a}$

$=c-\frac{(-1)^2}{4(2)}=1-\frac{1}{8}=\frac{7}{8}$

QUESTION

Find the range of values of P for which the expression $x^2-(x-1)p+3=0$ real roots

Solution

$x^2-(x-1)p+3=0$

for Real roots

$b^2-4ac\geq 0$

$x^2-xp+p+3=0$

$(-p)^2 - 4 \times 1 \times (p+3) \geq 0$

$p^2 - 4p - 12 \geq 0$

$(p-6)(p+2) \geq 0$

$p - 6 \geq 0 \quad p + 2 \geq 0$

$p \geq 6 \text{ or } p \geq -2$

QUESTION

Find the value of n for which the expression

$(2n+3)x^2 - 6x + 4 - n$ *is a perfect square*

Solution

$(2n+3)x^2 - 6x + 4 - n$

$b^2 = 4ac$

$(-6)^2 = 4(2n+3)(4-n)$

$36 = 32n + 8n^2 + 48 - 12n$

$8n^2 - 20n - 12 = 0$

$2n^2 - 5n - 3 = 0$

$\therefore n = -\frac{1}{2}, n = 3$

QUESTION

for what value of m is $9x^2 + mx + 16$ *a perfect square?*

Solution

$9x^2 + mx + 16$

$b^2 = 4ac$

$m^2 = 4 \times 9 \times 16$

$m^2 = 576$

$m = \sqrt{576}$

$m = 24$

QUESTION

if one root of $ax^2 + bx + c = 0$ *is triple the other*

Solution

$\alpha \quad \& \quad \beta$

$\alpha \quad \& \quad \beta = 3\alpha$

$\alpha + \beta \rightarrow \alpha + 3\alpha = -\frac{b}{a}$

$4\alpha = -\frac{b}{a}$

$\alpha = -\frac{b}{4a} - - - - (I)$

$\alpha\beta \rightarrow \alpha 3\alpha = 3\alpha^2 = \frac{c}{a} - - - (II)$

$3\left(\frac{-b}{4a}\right)^2 = \frac{c}{a}$

$\frac{3b^2}{16\ ^2} = 16ac$

QUESTION

the polynomial $f(x)$ *is given by*

$f(x) = x^4 + x^3 - 7x^2 + 3x + 2$

if also, $f(x) = (x-1)(x-2)(x+3)(x+c) + px + q$

find the values of P, Q *and* C

SOLUTION

$f(x) = x^4 + x^3 - 7x^2 + 3x + 2$

$f(x) = (x-1)(x-2)(x+3)(x+c) + px + q$

$= (x^2 - 3x + 2)(x^2 + 3x + 3c + cx) + px + q$

$= x^4 + 3x^3 + Cx^3 + 3Cx^2 - 3x^3 - 9x^2 - 3cx^2 - 9cx + 2x^2 + 6x + 2cx + 6c + px + c$

$= x^4 + cx^3 - 7x^2 - 7cx + 6x + Px + 6c + q$

$= x^4 + cx^3 - 7x^2 + (-7c + 6 + p)x + 6c + q$

compare

$x^4 + x^3 - 7x^2 + 3x + 2$

$C = 1$

$-7c + 6 + P = 3$

$-7 + 6 + P = 3$

$P = 4$

$6c + q = 2$

$6 + q = 2$

$q = -4$

$\therefore P = 4, Q = -4, C = 1$

QUESTION

For all except $x = 1, \frac{x^2+1}{x-1}$ *equals*

SOLUTION

$\frac{x^2+1}{x-1}$

using long division method

$$\begin{array}{r} x+1 \\ x-1\overline{)x^2+1} \\ \underline{x^2-x} \\ x+1 \\ \underline{x-1} \\ 2 \end{array}$$

$\therefore \frac{x^2+1}{x-1} = x + 1 + \frac{2}{x-1}$

QUESTION

when $x^3 + 2x^2 + px - 3$ *is divided by* $x + 1$ *the remainder is the same as when it is divided by* $x - 2$. *Find* p

SOLUTION

let $f(x) = x^3 + 2x^2 + px - 3$

since the remainder when $f(x)$ *is divided by* $x + 1$ *is*

the same as when it is divided by $x - 2$

$\therefore f(-1) = f(2)$

$\Rightarrow (-1)^3 + 2(-1)^2 + p(-1) - 3 = (2)^3 + 2(2)^2 + p(2) - 3 - 1 + 2 - p - \cancel{3} = 8 + 8 + 2p - \cancel{3}$

$1 - p = 16 + 2p$

$-15 = 3p$

P = -5

QUESTION

Given that $x^2 - 3x + 2$ *is a factor of* $x^4 + ax^2 + bx + 8$, *find the values of a and b*

SOLUTION

since $x^2 - 3x + 2$ *is a factor, then* $(x-1)(x-2)$ *are factors*

Let $f(x) = x^4 + ax^2 + bx + 8$

using factor theorem

for $x - 1$,

$f(1) = (1)^4 + a(1)^2 + b(1) + 8 = 0$

$a + b = -9 ----(I)$

for $x - 2$,

$f(2) = (2)^4 + a(2)^2 + b(2) + 8 = 0$

$16 + 4a + 2b + 8 = 0$

$4a + 2b = -24 ----(II)$

solve equ(I)& (II)simultaneously

$a + b = -9 \text{ x } 2$

$2a + 2b = -18 ----(I)$

$4a + 2b = -24$

subtract (I)from (II)

$4a - 2a + 2b - 2b = -24 + 18$

$2a = -6$

$\therefore a = -3$

put $a = -3$ *in equ (I)*

$-3 + b = -9$

$b = -9 + 3$

$b = -6$

$a = -3, b = -6$

QUESTION

$f(x) = 2x^3 + p + qx + 6$ *where p and q are constants. When* $f(x)$ *is divided by* $(x + 1)$, *the remainder is 12*

When $f(x)$ *is divided* $(x - 1)$, *the remainder is* -6.

Find the value of p and q

SOLUTION

$f(x) = 2x^3 + p + qx + 6$

$x + 1 : f(-1) = 2(-1)^3 + p + q(-1) + 6 = 12$

$-2 + p - q + 6 = 12$

$p - q = 8$ ---------(I)

$x - 1 : f(1) = 2(1)^3 + p + q(1) + 6 = -6$

$2 + p + q + 6 = -6$

$p + q = -14$-------(II)

Solve $P - q = 8$

$P + q = -14$

solve simultaneously (I)& (II)

$2p = -6$

$p = -3$ *put* $p = -3$ *in equ(I)*

$-3 - q = 8$

$-q = 11$

$q = -11$

$\therefore p = -3$ & $q = -11$

QUESTION

The values ofd A, B and R respectively, given that

$x^2 + 9x - 3 \equiv (x + 1)(Ax + B) + R$

SOLUTION

$x^2 + 9x - 3 \equiv (x + 1)(Ax + B) + R$

$\equiv Ax^2 + Bx + Ax + B + R$

$\equiv Ax^2 + (A + B)x + B + R$

$\therefore A = 1, \ A + B = 9$

$B + R = -3$

$B = 8$

$R = -3 - 8 = -11$

$\therefore$ A, B, R = 1,8, −11 respectively

QUESTION

$x - 1, x + 1$ are factors of teh expression

$x^3 + ax^2 + bx + c$ *and it leaves a remainder* 12

when divided by $x - 2$. *Find* a, b, c.

SOLUTION

$F: x^3 + ax^2 + bx + c$

$x = 1: F \Rightarrow 1 + a + b + c = 0$

$a + b + c = -1$ − − − −(I)

$x = -1: F \Rightarrow -1 + a - b + c = 0$

$a - b + c = 1$ − − − −(II)

$x = 2: F \Rightarrow 8 + 4a + 2b + c = 12$

$4a + 2b + c = 4$ − − − −(III)

solve (I), (II)& (III)simultaneously

subtract equ(I)from (II)and (III)

Equ (II) − (I)

$\rightarrow \cancel{a} - \cancel{a} - b - b + \cancel{c} - \cancel{c} = 1 - -1$

$-2b = 2$

$b = -1$

Equ (III) − (I)

$\rightarrow 4a - a + 2b - b + c - c = 4 - -1$

$3a + b = 5$

$3a - 1 = 5$

$3a = 6, a = 2$

put $a = 2, b = -1$ in equ (I)

$2 + (-1) + C = -1$

1+C = -1, c=-2

$\therefore a, b, c = 2, -1, -2$ respetively

QUESTION

Given the expression $ax^3 + 8x^2 + bx + 6$ is exactly divisible by $x^2 - 2x - 3$, find the values of a and b.

SOLUTION

$x^2 - 2x - 3 \to (x+1)(x-3)$

$f(x) = ax^3 + 8x^2 + bx + 6$

$x = -1: F \Rightarrow a(-1)^3 + 8(-1)^2 + b(-1) + 6 = 0$

$\Rightarrow -a + 8 - b + 6 = 0$

$A + b = 14 - - - (I)$

$x = 3: F = a(3)^3 + 8(3)^2 + b(3) + 6 = 0$

$27a + 72 + 3b + 6 = 0$

$27a + 3b = -78$

$9a + b = -26 - - - - (II)$

solve (I)& (II)simultaneously

Add (I) & (II)

$8a = -40, a = -5$

put $a = -5$ in equ(I)

$-5 + b = 14$

$b = 19$

$\therefore a = -5, \; b = 19$

QUESTION

The sum of the first ten terms of an arithmetic series is 60, the sum of teh first twenty-two terms is 220.

Solution

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{10} = 5(2a + 9d) = 60$

$10a + 45d = 60 \dots\dots(I)$

$S_{22} = 11(2a + 21d) = 220$

$22a + 231d = 220 \dots\dots(II)$

solve (I)&(II)simultaneosuly

$a = 3 \quad d = 2/3$

first term $= 3$

common difference $= 2/3$

QUESTION

find the sum to infinity of teh geometry series

$16 + 12 + 9 + \cdots$

Solution

$S\propto = \frac{a}{1-r}$

$a = 16, r = 3/4$

$S\propto = \frac{16}{1-\frac{3}{4}} = \frac{16}{\frac{1}{4}} = 64$

QUESTION

Write down the first five terms of the sequence specified by this recurrence relation

$U_1 = 0, \;\; U_n = \frac{1}{5-U_{n-1}} \; (n \geq 2)$

Solution

$U_1 = 0$

$U_n = \frac{1}{5-U_{n-1}} \; (n \geq 2)$

$U_2 = \frac{1}{5-U_1} = \frac{1}{5-0} = \frac{1}{5}$

$U_3 = \frac{1}{5-U_2} = \frac{1}{5-\frac{1}{5}} = \frac{5}{24}$

$U_4 = \frac{1}{5-U_3} = \frac{1}{5-\frac{5}{24}} = \frac{24}{115}$

$U_5 = \frac{1}{5-U_4} = \frac{1}{5-\frac{24}{115}} = \frac{115}{551}$

First five terms of the sequence is $0, \frac{1}{5}, \frac{5}{24}, \frac{24}{115}, \frac{115}{551}$

QUESTION

The fifth term of an A.P is 23 and the twelfth is 37. Find the first term, the common differrence and the sum of the first eleven terms.

Solution

$T_n = a + (n-1)d$

$T_5 = a + 4d = 23 \dots\dots\dots (I)$

$T_{12} = a + 11d = 37 \dots\dots\dots (II)$

Solve (I) & (II) simultaneously

$a = 15, d = 2$

First term 15, common difference 2

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{11} = \frac{11}{2}(2(15) + (10 x 2))2)$

$= 275$

QUESTION

The 3rd and the 6th term of a G.P are 108 and −32 respectively. Find the sum of the 1st 7terms.

Solution

$T_n = ar^{n-1}$

$T_3 = ar^2 = 108 \dots\dots (I)$

$T_6 = ar^5 = -32 \dots\dots (II)$

Solve (I) and (II)

$r = -\frac{2}{3}, a = 243$

$\therefore S_7 = \frac{a(1-r^2)}{1-r}$

$= \frac{243\,(1-(-\frac{2}{3})^7)}{1-(-\frac{2}{3})}$

$= \frac{243\,(1+0.0585)}{\frac{5}{3}}$

$= \frac{243\,(1.0585)}{1.6667}$

$= 154.3$

QUESTION

In a G.P, $U_3 = 32$ and $U_6 = 4$, Find the sum of the first eight terms of the G.P

Solution

$U_3 = ar^2 = 32 \dots\dots (I)$

$U_6 = ar^5 = 4 \dots\dots (II)$

Solve (I) &(II)

$a = 128, r = 1/2$

$\therefore S_n = \frac{a(1-r^n)}{1-r}$

$= \frac{128\left(1-\left(\frac{1}{2}\right)^8\right)}{1-\frac{1}{2}}$

$= 255$

QUESTION

Find the sum to infinity of the seies $1 + \frac{1}{2} + \frac{1}{4} + \cdots$

Solution

$1 + \frac{1}{2} + \frac{1}{4} + \cdots$

$r = 1/2$

$S\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = 2$

QUESTION

The first three terms of a geometric progression are $k-3, 2k-4, 4k-3$ in that order. Find the value of K.

Solution

$k-3, 2k-4, 4k-3$

$r \Rightarrow \frac{4k-3}{2k-4} = \frac{2k-4}{k-3}$

$(4k-3)(k-3) = (2k-4)(2k-4)$

$16k^2 - 24 + 9 = 4k^2 - 16k + 16$

$12k^2 - 8k - 7$

$K = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$K = -(-8) \pm \frac{\sqrt{(-8)^2-4(12)(-7)}}{2x12}$

$K = -\frac{1}{2}$ or $K = \frac{7}{6}$

QUESTION

in a certain arithmetic progression, the sum of the first and fifth term is 18 and the term is 6 more than the third term. Find the sum of the first term of the progresion

Solution

$\therefore S_n = \frac{a(1-r^n)}{1-r}$

$T_1 + T_5 = 18$

$a + a + 4d = 18$

$2a + 4d = 18 \ldots\ldots (I)$

$T_5 = 6 + T_3$

$T_5 - T_3 = 6$

$a + 4d - a - 2d = 6$

$2d = 6$

$d = 3$

$put\ d = 3\ in\ equ(I)$

$2a + 12 = 18$

$2a = 6$

$a = 3$

$S_{10} = \frac{10}{2}(2(3) + 9(3))$

$= 5(6 + 27)$

$= 5(33)$

$= 165$

QUESTION

In an A.P the tenth term and the sum of the six term is 76.5. Find the sum of the first term and the common difference

Solution

$T_{10} = a + 9d = 3 \ldots\ldots (I)$

$S_6 = \frac{6}{2}(2a + 5d) = 76.5$

$6a + 15d = 76.5 \ldots\ldots (II)$

$solve\ (I)\&(II)$

$a = 16.5, d = -1.5$

The sum of the first term and the common difference.

$a + d = 16.5 + -1.5$

$= 15$

QUESTION

Give the first thre terms of $(1 + x)^{15}$ in ascending order

SOLUTION

$(1 + x)^{15} \rightarrow using\ binomial\ theorem$

$15C_0(1)^{15}(x^0) + 15C_1(1)^{14}(x^1) + 15C_2(1)^{13}(x^2)$

$= 1 + 15x + 105x^2$

QUESTION

Find the first four terms in the expansion of $(1 - x)^{\frac{-1}{2}}$

SOLUTION

$(1 + x)^n = 1 + nx + n\frac{(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3$

$(1 + (-x))^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)(-x) + \left[\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2}\right](-x)^2$

$+ \left[\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)-\left(\frac{1}{2}-2\right)}{6}\right](-x)^3 + \ldots\ldots$

$= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{15}{48}x^3 + \cdots$

QUESTION

find the term x^5 in the expansion of $(1 + x)^{10}$

SOLUTION

$(x + y)^n \rightarrow {}^nCr.x^{n-r}y^r$

$\therefore (1 + x)^{10}, x = 1, y = x, n = 10$

$nCr.x^{n-r}y^r$

$\rightarrow {}^{10}C_r 1^{10-r} x^r$

$x^r = x^5$

$r = 5$

${}^{10}C_5 x^5 = 252x^5$

The coefficient is 252

QUESTION

The coefficient of x^5 in the expansion of $(1+5x)^8$ is equal to the coefficient of x^4 in the expansion of $(k+5x)^7$. Find the value of K.

SOLUTION

From $(1+5x)^8$

${}^nC_r x^{n-r} y^r \rightarrow {}^8C_r (1)^{8-r}(5x)^r$

coefficient of x^5

$\rightarrow x^r = x^5 \therefore r = 5$

${}^8C_5(5x)^5$

$\rightarrow 56 \times 3125x^5$

$\rightarrow 175000$

$(k+5x)^7$

${}^7C_r K^{7-r}(5x)^r = x^4$

$x^r = x^4$

$\therefore r = 4$

${}^7C_4 K^3 (5x)^4 = 175000$

$35K^3 X 625 = 175000$

$21875K^3 = 175000$

$K^3 = \frac{175000}{21875}$

$K^3 = 8, K = 2$

QUESTION

Find the coeficient of x^3 in the expansion $(1-x)^{-2}(1+x)^{-2}$

SOLUTION

$(1-x)^{-2} \rightarrow 1 - 2x + \frac{2x^2}{2!} \ldots\ldots$

$\rightarrow 1 - 2x + x^2$

$(1+x)^{-2} \rightarrow 1 + 2x + x^2$

$(1-x)^{-2}(1+x)^{-2} \rightarrow (1-2x+x^2)(1+2x+x^2)$

$1 + 2x + x^2 - 2x - 4x^2 - 2x^3 + x^2 + 2x^3 + x^4$

$= 1 + 2x^2 - 4x^2 + x^4$

$\therefore coefficient\ of\ x^3 = 0$

QUESTION

find the first four terms of the expansion of $(1-\frac{x}{2})^6$ in ascending power of x.

SOLUTION

$\left(1-\frac{x}{2}\right)^6 = {}^6C_0 . 1^6 . \left(-\frac{x}{2}\right)^0 + {}^6C_1 . 1^5 . \left(-\frac{x}{2}\right)^1 + {}^6C_2 . 1^4 . \left(-\frac{x}{2}\right)^2 + {}^6C_3 . 1^3 . \left(-\frac{x}{2}\right)^3 + {}^6C_4 . 1^2 . \left(-\frac{x}{2}\right)^4 + {}^6C_5 . 1^1 . \left(-\frac{x}{2}\right)^5 + {}^6C_6 . 1^0 . \left(-\frac{x}{2}\right)^6$

$\rightarrow 1 - 3x + \frac{15}{4}x^2 - \frac{5}{2}x^3 + \cdots$

QUESTION

Give the first three terms of $(2+x)^5$ in ascending powers of x.

SOLUTION

$(2+x)^5 \rightarrow {}^5C_0 2^5 x^0 + {}^5C_1 2^4 x^1 + {}^5C_2 2^3 x^2 + {}^5C_3 2^4 x^3 + \cdots$

$= 32 + 80x + 80x^2 + 40^3$

QUESTION

find the coefficient of x^2 in $(3-x)^{10}$

SOLUTION

$x^2\ in\ (3-x)^{10}$

$^{n}C_r 3^{n-r}(-x)^r$

$but\ r = 2$

$10C_2 3^{10-}\ (-x)^2$

$10C_2 3^8 x^2$

$295242x^2$

Question

Express $\frac{1+2x+3x^2}{(1-x)(1+x^2)}$ in partial fractions.

Solution

$\frac{1+2x+3x^2}{(1-x)(1+x^2)} = \frac{A}{(1-x)} + \frac{Bx+c}{(1+x^2)}$

$\frac{1+2x+3x^2}{(1-x)(1+x^2)} = \frac{A(1+x^2)+(Bx+c)(1-x)}{(1-x)(1+x^2)}$

$1 + 2x + 3x^2 = A + Ax^2 + Bx - Bx^2 + C - Cx$

x^2: $3 = A - B \cdots\cdots (i)$

x^1 : $2 = B - C \cdots\cdots (ii)$

x^0: $1 = A + C \cdots\cdots (iii)$

from (i) $A = 1 - c \cdots\cdots (*)$

put $(*)$ into (ii)

$3 = (1 - C) - B$

$\therefore C + B = -2 \cdots\cdots (iv)$

Solve (ii) & (iv)

$B - C = 2 \cdots\cdots (ii)$

$B + C = -2 \cdots\cdots (iv)$

Add (ii) & (iv)

$2B = 0, B = 0$

Put $B = 0$ in (iv)

$\therefore C = -2,$ put C in $(*)$

$A = 1 - (-2) = 1 + 2 = 3$

$\therefore A = 3, B = 0, C = -2$

$\frac{1+2x+3x^2}{(1-x)(1+x^2)} = \frac{3}{(1-x)} - \frac{2}{(1+x^2)}$

Question

Express $\frac{3x^2+2x-9}{(x^2-1)^2}$ in partial fraction

Solution

$\frac{3x^2+2x-9}{(x^2-1)^2} = \frac{Ax+B}{x^2-1} + \frac{Cx+D}{(x^2-1)^2}$

$\frac{3x^2+2x-9}{(x^2-1)^2} = \frac{(Ax+B)(x^2-1)+Cx+D}{(x^2-1)^2}$

$3x^2 + 2x - 9 = Ax^3 - Ax + Bx^2 - B + Cx$

$Ax^3 = 0 \qquad \therefore A = 0$

$Bx^2 = 3x^2 \qquad \therefore B = 3\ -Ax + Cx = 2x$

$\therefore -A + C = 2 \cdots (i)$

$-B + D = -9 \cdots (ii)$

$C = 2, \quad -3 + D = -9$

$D = -9 + 3 = -6$

$\therefore A = 0, B = 3, C = 2, D = -6$

$\frac{3x^2+2x-9}{(x^2-1)^2} = \frac{3}{x^2-1} + \frac{2x-6}{(x^2-1)^2}$

Question

Express $\frac{x^2-7x-6}{x^2(x-3)}$ in partial fraction.

Solution

$\frac{x^2-7x-6}{x^2(x-3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3}$

$\frac{x^2-7x-6}{x^2(x-3)}$

$= \frac{Ax(x-3)+B(x-3)+Cx^2}{x^2(x-3)}$

$x^2 - 7x - 6 = Ax^2 - 3Ax + Bx - 3B + Cx^2$

$A + c = 1$

$-3A + B = -7$

$-3B = -6, \ B = 2$

put B in (ii)

$-3A + 2 = -7, \ A = 3$

Put $A = 3$ in (i)

$3 + C = 1, \ C = -2$

$\therefore A = 3, B = 2, C = -2$

$\frac{x^2-7x-6}{x^2(x-3)} = \frac{3}{x} + \frac{2}{x^2} - \frac{2}{x-3}$

Question

Express $\frac{5x+4}{(x-1)(x+2)^2}$ in partial fractions

Solution

$$\frac{5x+4}{(x-1)(x+2)^2}$$

$$= \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$5x + 4 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

$A + B = 0 \cdots\cdots (i)$

$4A + B + C = 5 \cdots (ii)$

$4A - 2B - C = 4 \cdots\cdots (iii)$

Form (i) $\quad A = -B \cdots (*)$

Put $(*)$ in (ii) and (iii)

$\to 4(-B) + B + C = 5$

$-3B + C = 5 \cdots (iv)$

$\to 4(-B) - 2B - C = 4$

$-6B - C = 4 \cdots (v)$

Add (iv) and (v)

$-9B = 9, \; B = -1$

Put $B = -1$ in (iv)

$-3(-1) + C = 5, \quad C = 2$

From (i) $A = -B, \; A = 1$

$\therefore A = 1, B = -1, C = 2$

$$\frac{5x+4}{(x-1)(x+2)^2} = \frac{1}{x-1} - \frac{1}{x+2} + \frac{2}{(x+2)^2}$$

Question

Express $\frac{1}{x^4+5x^2+6}$ in partial fraction

Solution

$$\frac{1}{x^4+5x^2+6} = \frac{1}{(x^2+2)(x^2+3)}$$

$$= \frac{Ax+B}{(x^2+2)} + \frac{Cx+D}{(x^2+3)}$$

$$1 = (Ax+B)(x^2+3) + (Cx+D)(x^2+2)$$

$x^3: A + c = 0$

$x^2: B + D = 0$

$x^1: 3A + 2C = 0$

$x^0: 3B + 2D = 1$

$A = 0, B = 1, C = 0$

$D = -1$

$$\frac{1}{x^4+5x^2+6} = \frac{1}{(x^2+2)} - \frac{1}{(x^2+3)}$$

QUESTION

find the inverse of matrix

$$Q = \begin{pmatrix} 2 & 4 & 3 \\ 1 & -2 & -2 \\ -3 & 3 & 2 \end{pmatrix}$$

Solution

$$Q^{-1} = \frac{1}{|Q|}.AdjQ$$

Determinant of Q

$$|Q| = 2\begin{pmatrix} -2 & -2 \\ 3 & 2 \end{pmatrix} - 4\begin{pmatrix} 1 & -2 \\ -3 & 2 \end{pmatrix} + 3\begin{pmatrix} 1 & -2 \\ -3 & 3 \end{pmatrix}$$

$$= 2(-4+6) - 4(2-6) + 3(3-6)$$

$$= 11$$

cofactor

$$\begin{pmatrix} (+)2 & (-)4 & (+)3 \\ (-)1 & (+)-2 & (-)-2 \\ (+)-3 & (-)3 & (+)2 \end{pmatrix}$$

$$\begin{pmatrix} \begin{pmatrix} -2 & -2 \\ 3 & 2 \end{pmatrix} & -\begin{pmatrix} 1 & -2 \\ -3 & 2 \end{pmatrix} & +\begin{pmatrix} 1 & -2 \\ -3 & 3 \end{pmatrix} \\ -\begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix} & +\begin{pmatrix} 2 & 3 \\ -3 & 2 \end{pmatrix} & -\begin{pmatrix} 2 & 4 \\ -3 & 3 \end{pmatrix} \\ +\begin{pmatrix} 4 & 3 \\ -2 & -2 \end{pmatrix} & -\begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix} & +\begin{pmatrix} 2 & 4 \\ 1 & -2 \end{pmatrix} \end{pmatrix}$$

$$\therefore cofactor = \begin{pmatrix} 2 & 4 & -3 \\ 1 & 13 & -18 \\ -2 & 7 & -8 \end{pmatrix}$$

Transpose of cofactor

$$= \begin{pmatrix} 2 & 1 & -2 \\ 4 & 13 & 7 \\ -3 & -18 & -8 \end{pmatrix}$$

$\therefore Q^{-1} = \frac{Transpose\ of\ cofactor}{Determinan}$

$$\therefore \frac{1}{11}\begin{bmatrix} 2 & 1 & -2 \\ 4 & 13 & 7 \\ -3 & -18 & -8 \end{bmatrix}$$

QUESTION

$suppose\ M = \begin{pmatrix} -x & 2 \\ -x+1 & x-1 \end{pmatrix},$

$find\ the\ possible\ value of\ x\ if\ /m/ = 0$

Solution

$Determinant\ of\ M = 0$

$\therefore\ -x(x-1) - 2(x-2+1) = 0$

$-x^2 + x + 2x - 2 = 0$

$x^2 - 3x + 2 = 0$

$\therefore x = 1\ \ or\ x = 2$

QUESTION

$find\ the\ value\ of\ P\ for\ which\ the\ matrix$

$\begin{pmatrix} P+1 & 6 \\ 1 & P \end{pmatrix} does\ not\ have\ an\ inverse$

SOLUTION

$if\ a\ matrix\ does\ not\ have\ an\ inverse\ then\ the$

$determinant\ of\ the\ matrix\ is\ zero$

$\therefore \begin{pmatrix} P+1 & 6 \\ 1 & P \end{pmatrix} = 0$

$P(P+1) - 6 = 0$

$P^2 + P - 6 = 0$

$\therefore P = -3, P = 2$

QUESTION

$Given\ that\ matrix\ A = \begin{pmatrix} -1 & -4 \\ 1 & 3 \end{pmatrix} find\ A^3$

Solution

$A^2 = \begin{pmatrix} -1 & -4 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} -1 & -4 \\ 1 & 3 \end{pmatrix}$

$= \begin{pmatrix} 1-4 & 4-12 \\ -1+3 & -4+9 \end{pmatrix} = \begin{pmatrix} -3 & -8 \\ 2 & 5 \end{pmatrix}$

$\therefore A^3 = A^2 x\ A$

$= \begin{pmatrix} -3 & -8 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} -1 & -4 \\ 1 & 3 \end{pmatrix}$

$= \begin{pmatrix} 3-8 & 12-24 \\ -2+5 & -8+15 \end{pmatrix}$

$= \begin{pmatrix} -5 & -12 \\ 3 & 7 \end{pmatrix}$

TRIGONOMETRY [redacted]

QUESTION

If circumference of a circle is divided into 360 congruent parts, angle subtended by a part at cen[tre] of circle is called (a) angle (b) radian (c) degree (d) minute

QUESTION

$cos^2\left(\frac{\theta}{2}\right)$ =? (a)$1 - cos^2(\theta)$(b)$csc^2 2\theta$ (c)$1 - sin^2\theta$ (d) $sec^2 2\theta$

SOLUTION

$COS^2\left(\frac{\theta}{2}\right)$ =?

$Since\ sin^2\theta + cos^2\theta = 1$

$\therefore sin^2\left(\frac{\theta}{2}\right) + cos^2\left(\frac{\theta}{2}\right) = 1$

$\rightarrow cos^2\left(\frac{\theta}{2}\right) = 1 - sin^2\frac{\theta}{2} \rightarrow C$

QUESTION

If $\sin x = 3/4$, then $\cos x$ =? (a) $2/3$ (b) $\sqrt{3}/2$ (c) $\sqrt{7}/4$ (d) ½

SOLUTION

$If\ sinx = \frac{3}{4} = \frac{opp}{hyp}$

$from\ pythagoras$

$adj = \sqrt{hyp^2 - opp^2}$

$adj = \sqrt{16-9} = \sqrt{7}$

$\therefore \cos x = \frac{adj}{hyp} = \frac{\sqrt{7}}{4}$

QUESTION

$\sin\left(\frac{12\pi}{6}\right) =?$ (a) $1/\sqrt{2}$(b) $1/\sqrt{3}$ (c)½ (d) $\sqrt{3}$

SOLUTION

$\sin\left(\frac{12\pi}{6}\right) =?$

$\rightarrow Sin(2\pi) = Sin360^0 = 0$

QUESTION

$\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$ (a) $sec^2\theta$(b) $csc^2\theta$ (c)$2sec^2\theta$

(d) $2csc^2\theta$

SOLUTION

$\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$

$\frac{1-\sin\theta+1+\sin}{(1+\sin\ )(1-\sin\theta)} = \frac{2}{1-\sin^2\theta}$

$= \frac{2}{\cos^2\theta} = 2.\frac{1}{\cos^2\theta} = 2sec^2\theta --- C$

QUESTION

If the roots of the quadratic equation $x^2 + Ax + B = 0$ are $\tan(\pi/6)$ and $\tan(\pi/12)$ then the value of $A - B$ is (a) 1 (b)2 (c) -1 (d) 3

SOLUTION

$Roots\ are\ x = \tan\frac{x}{6}, x = tan\frac{\pi}{12},$
$x = \frac{1}{\sqrt{3}},\ \ x = 2-\sqrt{3}$

$\left(x - \frac{1}{\sqrt{3}}\right)\left(x - (2-\sqrt{3})\right) = 0$

$x^2 - (2-\sqrt{3})x - \frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}}(2-\sqrt{3}) = 0$

$x^2 - \left(2-\sqrt{3}+\frac{1}{\sqrt{3}}\right)x + \frac{1}{\sqrt{3}}(2-\sqrt{3}) = 0$

$Compare\ to\ x^2 + Ax + B = 0$

$\therefore$ A=-(2-$\sqrt{3} + \frac{1}{\sqrt{3}}$) $B = \frac{1}{\sqrt{3}}(2-\sqrt{3})$

$A - B = -2+\sqrt{3} - \frac{1}{\sqrt{3}} - \frac{2}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{3}}$

$= -2+\sqrt{3} - \frac{1}{\sqrt{3}} - \frac{2}{\sqrt{3}} + 1$

$=-1 --- C$

QUESTION 1

$\sqrt{\sqrt{\sqrt{2+2\cos(4\pi/3)}}}$ = (a) $1/\sqrt{2}$ (b) 1 (c)$\sqrt{3}$ (d) ½

SOLUTION

$\sqrt{\sqrt{\sqrt{2+2\cos(4\pi/3)}}}$

$\sqrt{\sqrt{\sqrt{2+2Cos240}}}$

$\sqrt{\sqrt{\sqrt{2+2Cos120}}}$

$\sqrt{\sqrt{\sqrt{2+2(-\frac{1}{2})}}}$

$\sqrt{\sqrt{\sqrt{2-1}}} = \sqrt{\sqrt{\sqrt{1}}} = 1 --- B$

QUESTION

If $K(\sin(18°)+\cos(36°)) = 5,$

then $K =$ (a) 5 (b) 4 (c) $2\sqrt{5}$ (d) $\frac{\sqrt{5}}{2}$

SOLUTION

$K(\sin(18°)+\cos(36°))$

$\therefore k = \frac{5}{sin18+cos3} = \frac{\frac{5}{\sqrt{5}}}{2}$

$= \frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5} --- C$

QUESTION

Evaluate $sin^2\left(\frac{4\pi}{3}\right) + sin^2(\pi/6)$

(a)3/4 (b) 5/2 (c) 5/4 (d)4/5

SOLUTION

$sin^2\left(\frac{4\pi}{3}\right) + sin^2(\pi/6)$

$sin^2(240) + sin^2(30)$

$(-\frac{\sqrt{3}}{2})^2 + (1/2)^2$

$\frac{3}{4} + \frac{1}{4} = 1$

$sin^2\left(\frac{4\pi}{3}\right) + sin^2\left(\frac{\pi}{6}\right)$

$(\frac{-\sqrt{3}}{2})^2 + \frac{1}{2} = \frac{5}{4} - - - C$

QUESTION

If $x = cos^4\left(\frac{\pi}{24}\right) - sin^4\left(\frac{\pi}{24}\right)$, then $x =$

(a)$(\sqrt{5}-1)/2\sqrt{2}$ (b)$(\sqrt{5}-1)/4$

(c) $(\sqrt{3}+1)/2\sqrt{2}$ (d) $(\sqrt{2+\sqrt{2}})/4$

SOLUTION

$x = cos^4\left(\frac{\pi}{24}\right) - sin^4\left(\frac{\pi}{24}\right)$

$x = cos^2\left(\frac{\pi}{24}\right) - sin^2\left(\frac{\pi}{24}\right)$

$x = cos^2\left(\frac{15}{2}\right) - sin^2\left(\frac{15}{2}\right)$ *multiple formula*

$x = cos2\left(\frac{15}{2}\right) = cos15^o$

$= \frac{1+\sqrt{3}}{2\sqrt{2}} - - - C$

QUESTION

If $\sin\alpha - \sin\beta = m$ and $\cos\alpha - \cos\beta = n$ then $\cos(\alpha-\beta) =$ (a)$\frac{2+m^2+n^2}{2}$ (b)$\frac{m^2+n^2}{2}$ (c)$\frac{2-m^2-n^2}{2}$ (d)$-(\frac{m^2+n^2}{2})$

SOLUTION

$\sin\alpha - \sin\beta = m$

$\cos\alpha - \cos\beta = n$

$\cos(\alpha-\beta) = ?$

$\cos(\alpha-\beta) = cos\alpha cos\beta$

$m + n = \sin\alpha - \sin\beta + sin$

$m^2 = (sin\alpha - sin\beta)^2$

$= sin^2\alpha - 2sin\alpha sin\beta + sin^2\beta$

$n^2 = (cos\alpha - cos\beta)^2$

$= cos^2\alpha - 2cos\alpha cos^2\beta + cos^2\beta$

$\therefore m^2+n^2 = sin^2\alpha - 2sin\alpha sin\beta + sin^2\beta +$

$cos^2\alpha - 2cos\alpha cos\beta + cos^2\beta$ $m^2 + n^2 = 2 -$

$2sin\alpha sin\beta - 2cos\alpha cos\beta$

$m^2 + n^2 = 2 - 2[cos\alpha cos\beta + sin\alpha sin\beta$

$m^2 + n^2 - 2 = -2[cos\alpha cos\beta + sin\alpha sin\beta$

$but \cos(\alpha-\beta) = cos\alpha cos\beta + sin\alpha sin\beta$

$\therefore\ m^2 + n^2 - 2 = -2\cos(\alpha-\beta)$

$cos(\alpha-\beta) = \frac{m^2+n^2-2}{-2} = -\frac{(2-m^2-n^2}{-2}$

$\therefore cos(\alpha-\beta) = \frac{(2-m^2-n^2}{2} - - - C$

QUESTION

If $\sin(x-y) - \cos(x-y) = 0$, find $\tan y$

(a)$\sin(x-\pi/4)$ (b) $cos(x-\pi/4)$ (c) $\tan(x -$

(d) 0

SOLUTION

$\frac{\sin(x-y)}{\cos(x-y)} = \frac{\cos(x-y)}{\cos(x-y)}$

$\tan(x-y) = 1$

$x - y = 45^0$

$y = x - 45$

$\therefore tany = \tan(x - \frac{\pi}{4}x - - - - C$

QUESTION

If $\tan(x+y) = 4/3$ and $\tan x = 1/2$, evaluate $\tan y$ (a) 5/6 (b) 11/6 (c) 1/2 (d)3/4

SOLUTION

$\tan(x+y) = \frac{tanx+tany}{1-tanxtany} - \frac{4}{3}$

$\frac{\frac{1}{2}+tany}{1-\frac{1}{2}tany} = \frac{4}{3}$

$\frac{1+\frac{2tany}{2}}{2-\frac{tany}{2}} = \frac{4}{3}$

$\frac{1+2tany}{2-tany} = \frac{4}{3}$

$3 + 6tany = 8 - 4tany$

$10tany = 5$

$tany = \frac{5}{10} = 12 - - - C$

QUESTION

Simplify $\cos(A+B)\cos(A-B)$ (a) $cos^2A + cos^2B$ (b) $\cos 2A$ (c) $cos^2A + sin^2B$ (d) $\sin 2B$

SOLUTION

$(cosAcosB - SinAsinB)(cosAcosB + sinAsinB)$

$\cos^2 A\cos^2 B - \sin^2 A\sin^2 B$

$\cos^2 A(1-\sin^2 B) - (1-\cos^2 A)\sin^2 B$

$\cos^2 A - \cos^2 A\sin^2 B + \cos^2 A\sin^2 B$

$\cos^2 A - \sin^2 B - - - C$

QUESTION

Given that in a triangle with sides a, b, and c and $\cos A = \frac{b^2+c^2-a^2}{2bc}$, then $sin^2\left(\frac{A}{2}\right)$ is (a) $\frac{a^2-c^2}{bc}$ (b) $\frac{a^2+c^2}{2bc}$ (c) $\frac{-(b-c)^2-a^2}{4bc}$ (d) $\frac{c^2-a^2}{b^2}$

SOLUTION

$\cos A = \frac{b^2+c^2-a^2}{2bc}$

From multiple formula

$cos2A = 1 - 2sin^2A$

$\therefore cosA = 1 - 2sin^2A/2$

$2sin^2A/2 = 1 - cosA$

$sin^2A/2 = \frac{1}{2}[1 - cosA]$

but $cosA = \frac{b^2+c^2-a^2}{2bc}$

$\therefore sin^2A/2 = \frac{1}{2}[1 - \frac{b^2-c^2-a^2}{2bc}]$

$= \frac{1}{2}[\frac{2bc-(b^2-c^2-a^2)}{2bc}]$

$= \frac{-b^2-c^2+a^2+2bc}{4bc}$

$= \frac{-(b-c)^2+a^2}{4bc} - - - - C$

QUESTION

Simplify $cos^2(\alpha+\beta) + cos^2(\alpha-\beta)$ (a) $1 - \cos\alpha\cos\beta$ (b) $\cos\alpha\cos\beta - 1$ (c) $1+\cos 2\alpha\cos 2\beta$ (d) $\cos 2\alpha\cos 2\beta - \sin\alpha\cos\beta$

QUESTION

Given that $t = \tan\frac{x}{2}$, express $\cos x$ in terms of t (a) $\frac{2t}{1+t^2}$ (b) $\frac{1+t^2}{t^2-1}$ (c) $\frac{1-t^2}{t^2+1}$ (d) $\frac{2t}{1-t^2}$

SOLUTION

$t = tanx/2$

$cosx = ?$

$from\ cos2x = cos^2x - sin^2x$

$\therefore cosx = \frac{cos^2x/2 - sin^2x/2}{1}$

$= \frac{cos^2x/2 - sin^2x/2}{sin^2x/2 + c\ \ ^2x/2}$

$\frac{\frac{cos^2x/2}{cos^2x/2} - \frac{sin^2x/2}{cos^2x/2}}{\frac{sin^2x/2}{cos^2x/2} + \frac{cos^2x/2}{cos^2x/2}}$

$\rightarrow cosx = \frac{1-tan^2x/2}{tan^2x/2+1} = \frac{1-t^2}{t^2+1}$

QUESTION

If $\sin\theta = 1/\sqrt{3}$ and $0 \le \theta \le \pi/2$, then $\cot\theta$ is (a) $\sqrt{3}$ (b) $1/\sqrt{2}$ (c) $\sqrt{2}$ (d) $\sqrt{3/2}$

SOLUTION

$\sin\theta = 1/\sqrt{3} = \frac{opp}{hyp}$

$\therefore adj = \sqrt{\sqrt{5^2 - 1^2}}$

$= \sqrt{3-1}$

$= \sqrt{2}$

$cot\theta = \frac{1}{tan\theta} = \frac{\frac{1}{opp}}{adj} = \frac{adj}{opp} = \frac{\sqrt{2}}{1} = \sqrt{2}$

QUESTION

Simplify $\left(\frac{1+\sin x}{1+co}\right)\left(\frac{1+\sec x}{1+cosec\, x}\right)$ (a) $1+\tan x$ (b) sin^2x (c)$\tan x$ (d) $1+\cos x$

SOLUTION

$\left(\frac{1+\sin x}{1+\cos}\right)\left(\frac{1+\sec}{1+cosec\, x}\right)$

$\frac{1+sinx}{1+cosx} \cdot \frac{1+\frac{1}{cosx}}{1+\frac{1}{sinx}} = \frac{1+sinx}{1+cosx} \cdot \frac{\frac{cos+1}{cosx}}{\frac{sinx+1}{sinx}}$

$\frac{1+sinx}{1+cosx} \cdot \frac{cosx+1}{cosx} \cdot \frac{sinx}{sinx+1} = \frac{sinx}{cosx} = tanx - -C$

QUESTION

Given that the side of a triangle are of length $a = 3.57m, b = 2.61m\ and\ c = 4.72m$, then the area of the triangle is (a) $5.45m^2$ (b)$43.98m^2$ (c)$4.609m^2$ (d) $10.9m^2$

SOLUTION

$Area = \sqrt{s(s-a)(s-b)(s-c)}$

$where\ s = \frac{a+b+c}{2}$

$s = \frac{3.57+2.61+4.72}{2}$

$= \frac{10.9}{2} = 5.45$

$\therefore Area =$

$\sqrt{\begin{matrix}5.45(5.45-3.57)\\(5.45-2.61)(5.45-4.72)\end{matrix}}$

$= \sqrt{(5.45)(1.88)(2.84)}$

$= \sqrt{29.09864}$

$= 5.3943155$

$= 5.4m^2 - - - -A$

QUESTION

Simplify $\sin^2 2\varphi + 2\cos^2\varphi cos2\varphi$ (a) $2\cos\varphi$ (b) $ccos^2\varphi$ (c) $2\cos^2\varphi$ (d) 1

SOLUTION

$\sin^2 2\varphi + 2\cos^2\varphi cos2\varphi$

$(2sin\varphi cos\varphi)^2 + 2cos^2\varphi[1 - 2sin^2\varphi]$

$\cancel{4sin^2\varphi\ cos^2\varphi} + 2\cos^2\varphi \cancel{-4sin^2\varphi cos^2\varphi}$

$\rightarrow 2cos^2\varphi$

QUESTION

Given that $t = \tan\frac{x}{2}$, express $\sec x - \tan x$ in terms of t (a) $\frac{2(1+t)}{1+t^2}$ (b) $\frac{(1+t)^2}{1+t^2}$ (c) $\frac{(1-t)^2}{1-t^2}$ (d) $\frac{2(1+t)}{1-t}$

SOLUTION

$t = \tan\frac{x}{2}$

$\therefore secx - tanx$

$\rightarrow \frac{1}{cosx} - \frac{sinx}{cosx}$

$\rightarrow \frac{cosx-sinx}{cosx}$

$but\ sinx = \frac{2t}{1+t^2}$

$cosx = \frac{1-t^2}{1+t^2}$

$\therefore secx - tanx = \frac{cosx-sinx}{cosx}$

$\frac{\frac{1-t^2}{1+t^2} - \frac{2t}{1+t^2}}{\frac{1-t^2}{1+t^2}}$

$\frac{\frac{1-t^2-2t}{1+t^2}}{\frac{1-t^2}{1+t^2}}$

$\frac{1-t^2-2t}{1-t^2}$

QUESTION

Simplify $\sin 2A\cos 4A + \sin 3A\cos 9A$ (a)$\sin 5A\cos 13A$ (b)$2\sin A\cos A$ (c)$\frac{1}{2}(\sin 12A$ $\sin 2A)$ (d) $\tan 3A$

SOLUTION

$\sin 2A\cos 4A + \sin 3A\cos 9A$

from factor formular

$sinAcosB = \frac{1}{2}[\sin(A+B) + Sin(A-B)]$

$\therefore Sin2Acos4A =$

$\frac{1}{2}[\sin(2A+4A) + Sin(2A-4A)]$

$+\frac{1}{2}[Sin(3A+9A) + Sin(3A-9A)]$

$= \frac{1}{2}[Sin6A - Sin2A + Sin12A - Sin6A]$

$= \frac{1}{2}[Sin12A - Sin2A] - - - C$

QUESTION

1. $\sin\theta + \sin\varphi = a$and $\cos\theta + \cos\varphi = b$, the value of $\cos^2\frac{1}{2}(\theta-\varphi)$ is

(a)a^2+b^2(b) $a-b$ (c)$\frac{1}{4}(a^2+b^2)$ (d)$\frac{1}{4}(a^2-b^2)$

SOLUTION

from factor formula

$Sin\theta + Sin\theta = a \equiv 2sin\frac{1}{2}(\theta+\theta)cos\frac{1}{2}(\theta+\theta) - - - (I)$

$cos\theta + cos\theta = b \equiv 2cos\frac{1}{2}(\theta+\theta)cos\frac{1}{2}(\theta+\theta) - - - (II)$ *squaring and adding* $(I)\&(II)$

$4cos^2 1/2\ (\theta+\theta)[sin^2\frac{1}{2}(\theta+\theta) + cos^2\frac{1}{2}(\theta+\theta)] = a^2+b^2$

$4cos^2\frac{1}{2}(\theta-\theta) = a^2+b^2$

$cos^2\frac{1}{2}(\theta-\theta) = \frac{a^2+b^2}{4}$

QUESTION

Which of these is not correct?

(a)$\cos 70° + \cos 20° = \sqrt{2}\sin 65°$

(b) $\cos 70° + \cos 20° = \sqrt{2}\cos 25°$

(c) $\sin 50° + \sin 40° = \sqrt{2}\cos 5°$

(d) $\cos 70° + \cos 20° = \sqrt{2}\sin 25°$

SOLUTION

$cos45^0 - 6315$

$= -2Sin\left(\frac{75+15}{2}\right)Sin\left(\frac{75-15}{2}\right)$

$= -2Sin45Sin30$

$= -2.\frac{1}{\sqrt{2}}.\frac{1}{2} = -\frac{1}{\sqrt{2}}$

QUESTION

$(\sin\theta + \sin\varphi)/(\cos\theta + \cos\varphi)$is equivalent to (a) $\tan(\theta+\varphi)$ (b) $\tan\frac{1}{2}(\theta-\varphi)$ (c) $\tan\frac{1}{2}(\theta+\varphi)$(d) $\tan(\theta-\varphi)$

Solution

$\frac{sin\theta - sin\theta}{cos\theta + cos\theta}$

$\rightarrow \frac{2cos(\frac{\theta+\theta}{2})sin(\frac{\theta-\theta}{2})}{2cos(\frac{\theta+\theta}{2})cos(\frac{\theta-\theta}{2})}$

$= \frac{\frac{sin(\theta-\theta)}{2}}{\frac{cos(\theta-\theta)}{2}}$

$= \frac{tan(\theta-\theta)}{2} - - - - B$

QUESTION 51

If $\sin\left(x+\frac{\pi}{4}\right) = y$, then x is (a) $\arcsin y$ (b) $\frac{y}{\sin -\frac{\pi}{4}}$ (c) $\arcsin y - \frac{\pi}{4}$ (d) $\arcsin(y - \frac{\pi}{4})$

Solution

$\sin\left(x+\frac{\pi}{4}\right) = y$

$x + \frac{\pi}{4} = arcsiny$

$x = arcsiny - \frac{\pi}{4} - - - - C$

QUESTION

Is it true that $\tan^2\theta - \sin^2\theta = \tan^2\theta sin^2\theta$(a) No (b) Sometimes (c) Yes (d) I don't know

QUESTION

Evaluate $\tan\left(\frac{\pi}{4}+x\right)\tan\left(\frac{\pi}{4}-x\right)$ (a)0 (b) $\frac{\pi}{2}$ (c) 1 (d) $\pi^2/(16-x^2)$

Solution

$\tan\left(\frac{\pi}{4}+x\right)\tan\left(\frac{\pi}{4}-x\right)$

$\left(\frac{\tan45+\tan x}{1-\tan45\tan}\right)\left(\frac{\tan45-\tan x}{1+\tan45\tan}\right)$

$\left(\frac{1+\tan x}{1-\tan x}\right)\left(\frac{1-\mathrm{ta}}{1+\tan}\right)=1---\mathrm{C}$

QUESTION

Given that $p=\sin 2\alpha+\sin 2\beta$ and $q=\cos 2\alpha+\cos 2\beta$, then p/q is (a) 1 (b) $\tan(2\alpha+2\beta)$ (c) $\tan(\alpha+\beta)$ (d) $\tan(2\alpha-2\beta)$

Solution

$p=\sin 2\alpha+\sin 2\alpha$

$q=\cos 2\alpha+\cos 2\beta$

$\frac{p}{q}=\frac{\sin2\alpha+\sin2}{\cos 2\alpha+\cos 2}$

from factor formula

$=\frac{2Sin\left(\frac{2\alpha+2\beta}{2}\right)Co\left(\frac{2\alpha-2\beta}{2}\right)}{2\cos\left(\frac{2\alpha+2\beta}{2}\right)\cos\left(\frac{2\alpha-2\beta}{2}\right)}$

$=\tan(\alpha+\beta)---C$

QUESTION

Which of the following is not correct?

(a) $\tan\theta+\cot\theta=\sec\theta\csc\theta$

(b) $4-3\cos^2\theta=1+3\sin^2\theta$

(c) $4+3\cos^2\theta=1+3\sin^2\theta$

(d) $\sin^2\theta+\cos^2\theta=1$

Solution

Answer

(c) $4+3\cos^2\theta=1+3\sin^2\theta$

QUESTION

If $\tan\theta=\frac{a}{b}$ and $0\le\theta\le\frac{\pi}{2}$, then $\frac{b\cos\theta-a\sin}{b\cos\theta+a\sin}$ is

$\frac{b-a}{b^2+a^2}$ (b) $\frac{b^2-a^2}{b^2+a^2}$ (c) $\frac{b^2-a^2}{b^2-a^2}$ (d) $\frac{b+a}{b-a}$

Solution

$\frac{bCos\theta-aSin\theta}{bcos\theta+aSin}$

divide through by cosθ

$\frac{b-a\tan\theta}{b+ata}$

$\frac{\frac{b}{1}-a\left(\frac{a}{b}\right)}{b+a\left(\frac{a}{b}\right)}$

$\frac{\frac{b^2-a^2}{b}}{\frac{b^2+a^2}{b}}$

$=\frac{b^2-a^2}{b^2+a^2}---B$

QUESTION

Evaluate $\sin 15°$ if $\cos 30°=\frac{\sqrt{3}}{2}$ (a) $\frac{1}{2}\sqrt{(3-\sqrt{}}$

(b) $\frac{1}{2}\sqrt{(3+\sqrt{3})}$ (c) $\frac{1}{2}\sqrt{(2-\sqrt{3})}$ (d) $\frac{1}{2}\sqrt{(2-\sqrt{}}$

Solution

$sin15=?$

$Cos30=\sqrt{\frac{3}{2}}$ *from multiple formula*

$cos2\theta=1-2sin^2\theta$

$cos30=cos2(15)=1-2sin^215=\frac{\sqrt{3}}{2}$

$1-2sin^215=\frac{\sqrt{3}}{2}$

$1-\frac{\sqrt{3}}{2}=2sin^215$

$\frac{2-\sqrt{3}}{2}=2sin^215$

$\frac{2-\sqrt{3}}{4}=2sin^215$

$sin15=\sqrt{\frac{2-\sqrt{3}}{4}}=\frac{1}{2}\sqrt{2-\sqrt{3}}$

QUESTION

Simplify $(\cos x - \sin x)^2$ (a) $1 - \cos 2x$ (b) $1 + \cos 2x$ (c) $1 - \sin 2x$ (d) $1 + \sin 2x$

Solution

$(cosx - sinx)^2$

$(cosx - sinx)(cosx - sinx)$

$cos^2x - 2sinxcosx + sin^2x$

$1 - 2sinxcosx$

$1 - sin2x ----C$

QUESTION

Find $\tan^2 x$ if $5\cos^2 x + 30\sin^2 x = 21$ (a) 21/5 (b) 5/21 (c) 16/25 (d) 25/16

Solution

$5\cos^2 x + 30\sin^2 x = 21$

$5\cos^2 x + 30\sin^2 x = 21\,(sin^2x + cos^2x)$

$5\cos^2 x + 30\sin^2 x = 21\,sin^2x + 21cos^2x$

$sin^2x = 16cos^2x$

$\frac{sin^2x}{cos^2x} = \frac{16}{9}$

$\therefore tan^2x = \frac{16}{9}$ *No answer*

QUESTION

If $\sin x + \sin y = 3$and $\cos x + \cos y = 4$, the $\cos^2(\frac{x-y}{2})$ is (a)$\pm(2/5)$ (b) $\pm(3/4)$ (c) $\pm(5/4)$ (d) $\pm(4/3)$

Solution

Relation it to question 43

$\cos^2\left(\frac{x-y}{2}\right) = \frac{3^2+4^2}{4}$

$= \frac{9+16}{4} = \frac{25}{4}$

$= \pm\left(\frac{25}{4}\right)$ — *No answer*

$cos\left(\frac{x-4}{2}\right) = \pm(\frac{5}{2})$

QUESTION

Evaluate $\tan^2(\frac{\pi}{4} - \frac{x}{2})$ (a) $\frac{1+sin\,x}{1-\sin x}$ (b) $\frac{1+tan\ x}{1-\tan\ x}$ (c) $\frac{1-sin\,x}{1+\sin x}$ (d) $\frac{1-tan\ x}{1+\tan\ x}$

QUESTION

If, then $\tan^2 x$ is (a) $\frac{2}{5}$ (b) $\frac{5}{6}$ (c) $\frac{1}{3}$ (d) $\frac{3}{5}$

Solution

$6\cos^2 x + 2\sin^2 x = 5$

$6\cos^2 x + 2\sin^2 x = 5\,(sin^2x+cos^2x)$

$6\cos^2 x + 2\sin^2 x = 5\,sin^2x+5cos^2x$

$-3\sin^2 x = -cos^2x$

$tan^2x = \frac{1}{3} ---C$

QUESTION

If $\cot^2 x + 3\csc^2 x = 7$, then $\tan x$ is (a) ±3 (b) ±7 (c) ±1 (d) $\pm\frac{3}{7}$

Solution

$\cot^2 x + 3\csc^2 x = 7$

$\cot^2 x + 3\,(1 + \cot^2 x) = 7$

$\cot^2 x + 3 + 3\cot^2 x = 7$

$4\cot^2 x = 4$

$\cot^2 x = 1$

$\tan^2 x = 1$

$tan\,x = \pm1 ---C$

QUESTION

If $3\cos^2\theta + 28\sin^2\theta = 19$, then $\tan\theta$ is (a) $\pm\frac{3}{19}$ (b) $\pm\frac{5}{4}$ (c) $\pm\frac{4}{5}$ (d) $\pm\frac{3}{28}$

Solution

$3\cos^2\theta + 28\sin^2\theta = 19$

$3\cos^2\theta + 28\sin^2\theta = 19sin^2\theta + 19cos^2\theta$

$9sin^2\theta = 16cos^2\theta$

$tan^2\theta = \frac{16}{9}$

$tan\theta = \pm\frac{4}{3} --- No\ answer$

QUESTION

Evaluate $\sin\theta + \sin\left(\theta + \frac{2\pi}{3}\right) + \sin\left(\theta + \frac{4\pi}{3}\right)$ (a) $\frac{4\pi}{3}$

(b) $\frac{2\pi}{3}$ (c) 0 (d) 1

Solution

$\sin\theta + \sin\left(\theta + \frac{2\pi}{3}\right) + \sin\left(\theta + \frac{4\pi}{3}\right)$

$\frac{2\pi}{3} = 120, \frac{4\pi}{3} = 420$

Additional formula

$Sin\theta + [Sin\theta cos120\theta + sin120cos\theta]$

$+[sin\theta cos240 + cos\theta sin240]$

$\rightarrow sin\theta - \frac{1}{2}sin\theta + \frac{\sqrt{3}}{2}cos\theta - \frac{1}{2}sin\theta - \frac{\sqrt{3}}{2}cos\theta$

$\rightarrow sin\theta - \frac{1}{2}sin\theta - \frac{1}{2}sin\theta = 0 --- C$

QUESTION

If $\tan\alpha = x$, then $\frac{\sin\theta + x\cos\theta}{\cos\theta - x\sin\theta}$ is (a) $\sin(\theta+\alpha)$ (b) $\cos(\theta+\alpha)$ (c) $\tan(\theta+\alpha)$ (d) $sec(\theta+\alpha)$

Solution

$\frac{\sin\theta + x\cos\theta}{\cos\theta - x\sin\theta}$

$x = tan\alpha$

$\therefore \frac{sin\theta + tan\alpha cos\theta}{cosx - tan\alpha sin\theta}$

$\frac{sin\theta + \frac{sin\alpha cos\theta}{cos\alpha}}{cos\theta - \frac{sin\alpha sin\theta}{cos\alpha}}$

$\frac{cos\alpha sin\theta + sin\alpha cos\theta}{cos\theta sin\alpha - sin\alpha sin\theta}$

$= \frac{sin(\theta+\alpha)}{cos(\theta+\alpha)}$

$= tan(\theta + \alpha) --- C$

QUESTION

Find $\tan\alpha$ if $\sin(\theta - \alpha) = \cos(\theta - \alpha)$ (a)1 (b)

(c) $tan(\theta - \frac{\pi}{4})(d)\sin(\theta - \frac{\pi}{4})$

Solution

Since $\sin(\theta - \alpha) = cos(\theta - \alpha)$

$\therefore \theta - \alpha = 45°$

$\alpha = \theta - 45°$

$\therefore tan\alpha = \tan(\theta - \frac{\pi}{4})$

QUESTION

Simplify $\frac{1+\tan^2 x}{(1+\tan\ \ )^2}$ (a) $\frac{1}{1+ta}$ (b) $\frac{1}{1+2\tan x}$ (c) $\frac{1}{1+\sin 2x}$ (d) $\frac{1}{1+2\ \ \ \ x}$

Solution

$\frac{1+\tan^2 x}{(1+\tan\ .)^2}$

$\frac{sec^2x}{1+2tanx+sec^2x} = \frac{sec^2x}{2tanx+se\ ^2x} = \frac{\frac{1}{cos^2x}}{\frac{2sinx}{cosx}+\frac{1}{cos^2x}}$

$\frac{\frac{1}{cos^2x}}{\frac{2cosxsin}{cos^2x}+1} = \frac{1}{1+sin2x} --- C$

QUESTION

If $\tan^2 x = 1/3$, then $2\sin^2 x + 6\cos^2 x$ is (a)

(b) 2/5 (c) 5 (d) 3

Solution

$\tan^2 x = 1/3$

$\frac{\sin^2 x}{\cos^2 x} = \frac{1}{3}$

$3^2x = cos^2x$

$2sin^2x - 5sin^2x = 5cos^2x - 6\cos_3 2x$

$6cos^2x + 2sin^2x = 5sin^2x + 5cos^2x$

$6cos^2x + 2sin^2x = 5[sin^2x + cos^2x]$

$\therefore 6cos^2x + 2sin^2x = 5 --- C$

1. Solve the inequality

$\frac{x-2}{x-3} > \frac{x-3}{x+4}$

(a) -4<x<17/8, x>3 (b)x>3 (c) -4<x>17/8 (d) 17/8<x<3,x<-4

Solution

Multiply through by $(x-3)^2(x+4)^2$

$$\frac{x-2}{x-3}(x-3)^2(x+4)^2 > \frac{x-3}{x+4}(x-3)^2(x+4)^2$$

$(x-2)(x-3)(x+4)^2 > (x-3)(x-3)^2(x+4) > 0$

$(x-3)(x+4)\,[(x-2)(x+4)-(x-3)^2] > 0$

$(x-3)(x+4)[x^2+4x-2x-8x-x^2+6x-9] > 0$

$$(x-3)(x+4)(8n-17) > 10$$

Using sign test

	$x < -4$	$-4 < x < {}^{17}/_{8}$	${}^{17}/_{8} < x < 3$	$x > 3$
$x-3$	-	-	-	+
$x+4$	-	+	+	+
$8x-17$	-	-	+	+
$(x-3)(x+4)(8x-17)$	-	+	-	+

:- Thus the inequality is true if $-4x < x < {}^{17}/_{8}$ or $x > 3.$

2. Find the value of x

$\frac{x^2+x-2}{x^2+4} > {}^{1}/_{2}$

Solution

$\frac{x^2+x-2}{x^2+4} > {}^{1}/_{2}$

Multiply through by x^2+4

$$x^2+x-2 > {}^{1}/_{2}(x^2+4)$$

$$2x^2+2x-4 > x^2+4$$

$$x^2+2x-8 > 0$$

$$(x+4)(x-2) > 0$$

$x - 2 > 0$ or $x + 4 < 0$

$x > 2$ or $x < -4$.

3. The inequality

$\frac{2x^2-3x-5}{x^2+2x+6} < \frac{1}{2}$ has solution

Solution

$2x^2 - 3x - 5 < \frac{1}{2}(x^2 + 2x + 6)$

$4x^2 - 6x - 10 < x^2 + 2x + 6$

$4x^2 - 6x - 10 - x^2 - 2x - 6 < 0$

$3x^2 - 8x - 1 < 0$

$3x(x - 4) + 4(x - 4) < 0.$

$(3x + 4)(x - 4) < 0$

:- $x - 4 < 0, 3x + 4 > 0$,

$x < 4$ or $x > -\frac{4}{3}$

$x \in (-\frac{4}{3}, 4)$

4. The function is defined by

$g: x \rightarrow 4 - 2x - x^2$ where $x \in R$

state the range of g.

(a) $-\infty < x < \infty$ (b) $0 \leq x \leq 1$

(c) $-20 \leq x \leq 20$ (d) $-\infty < x \leq 5$

Solution

Since the function g is a prolynomial function, then the range is a set of real number.

:- $Range\ (-\infty, \infty)$

5. Find the sum to infinity of the series

$$1 + \frac{3}{7} + \frac{9}{49} + \frac{27}{343} + \frac{81}{2401} + \dots$$

Solution

$a = 1$

$r = \frac{3}{7}$

$S_\infty = \frac{a}{1-r} = \frac{1}{1-3/7}$

$= 7/4$

Questions 6

What is the first to last term of 1 + 3 + 5 + 7 + …

Solution

$T_n = a + (n-1)d$

$a = 1 \quad d = 2$

$T_n = 1+2(n-1)$

$= 1 + 2n - 2$

$T_n = 2n - 1$

The first term to last term is $2n - 1$

Question 7

Describe the set [6, 12, 18, 24, 30, 36]

[a] 6z [b] [6x: x∈Z] [c] [6x: 1 < x < 6] [d] [6x: ≤ 1 x ≤ 6] zEZ

Answer D

6x: 1 ≤ x 6 - mean that the set is 6x. and the x is defined to be positive integer that falls from 1 to 6

Question 8

Let s = [a, b, c, d, e], then the cardinality of subsets obtainable from the set s is given by [a] 16 [b] 21 [c] 32 [d] 64

Solution

Set s is a set of 5 elements, . 1

∴ the subsets 2^5 = = 32 subsets including itself and the empty set.

∴ The cardinality of subset of s = 32

Question 9

If A = {1, 2, 3, 4, 5} B = {3, 4, 5, 6} and C = {4, 5, 6, 7}

Then (A U B) - C is

AUB = {1, 2, 5, 4, 5, 6}

(A U B) - C = {1, 2, 3}

Question 10

Let P = {rEZ : /x – 2/ < 4}

Q = {rER : / $x^2 - 4x = 0$}

Find P – Q

Solution

P = {rEZ : /x – 2/ < 4}

x – 2 < 4 or x – 2 > - 4

x < 6 or x > - 2

:- $P\{x \in \ -2 < x < 6\}$

$P\{-1,0,1,2,3,4,5\}$

$Q: \{x \in I\mathbb{R}\ x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0\ or\ x = 4$

:- $P - Q = \{-1,1,2,3,5)$

10. Given that the universal set

$\{u = \{x \in \mathbb{Z}/-10 \leq x \leq 10$

$A = \left\{x \in \frac{\mathbb{Z}}{x} is\ divisible\ by\ 3\right\}$

$B = \{x \in \mathbb{Z}\ x\ is\ a\ prime\ number\}$

$C = \{x \in \mathbb{Z}\ roots\ of\ x^4 + 2x^3 - 5x^2 - x = 0$

A,B and C are subsets of the universal Set. Find $A - (B \cup C)$?

<u>Solution</u>

$A = \{-10, -9, -8, -7, -6, -5, -4, -3, -2, -1,0,1,2,3,4,5,6,7,8,9,10\}$

$A = \{-9, -6, -3, 3, , 9$

$B = \{2, 3, 5, 7\}$

$C = \{-3, -1, 0, 2\}$

$B \cup C = \{-3, -1, 0, 2, 3, 5, 7\}$

$A - (B \cup C) = \{-9, -6, 6, 9\}$