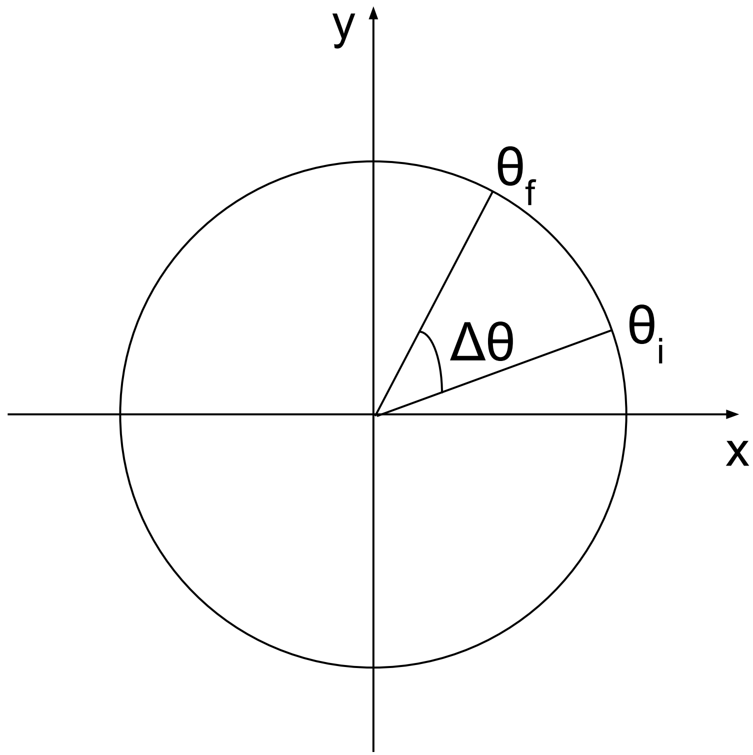
PHY 101

Circular Motion

Circular Motion

- Uniform Circular Motion
- Radial Acceleration
- Banked and Unbanked Curves
- Circular Orbits
- Nonuniform Circular Motion
- Tangential and Angular Acceleration
- Artificial Gravity

Angular Displacement



θ is the angular position.

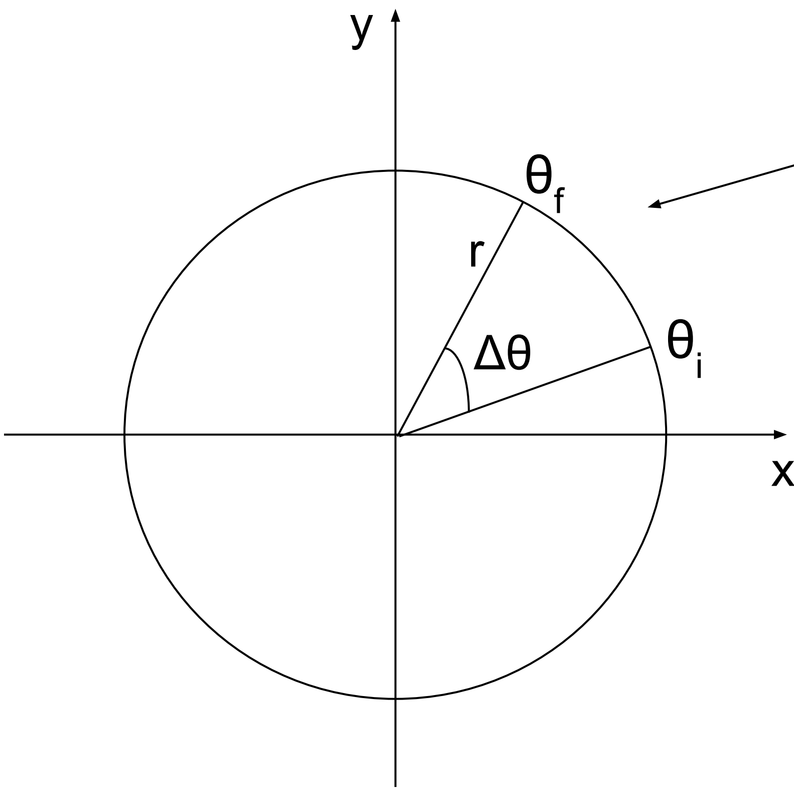
Angular displacement:

$$\Delta\theta = \theta_f - \theta_i$$

Note: angles measured CW are negative and angles measured CCW are positive. θ is measured in radians.

2π radians = 360° = 1 revolution

Arc Length



arc length = s = rΔθ

$$\Delta\theta = \frac{s}{r}$$

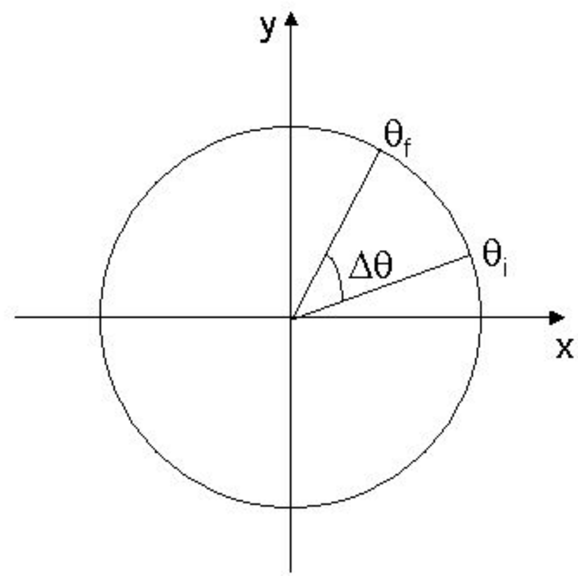
Δθ is a ratio of two lengths; it is a dimensionless ratio!

Angular Speed

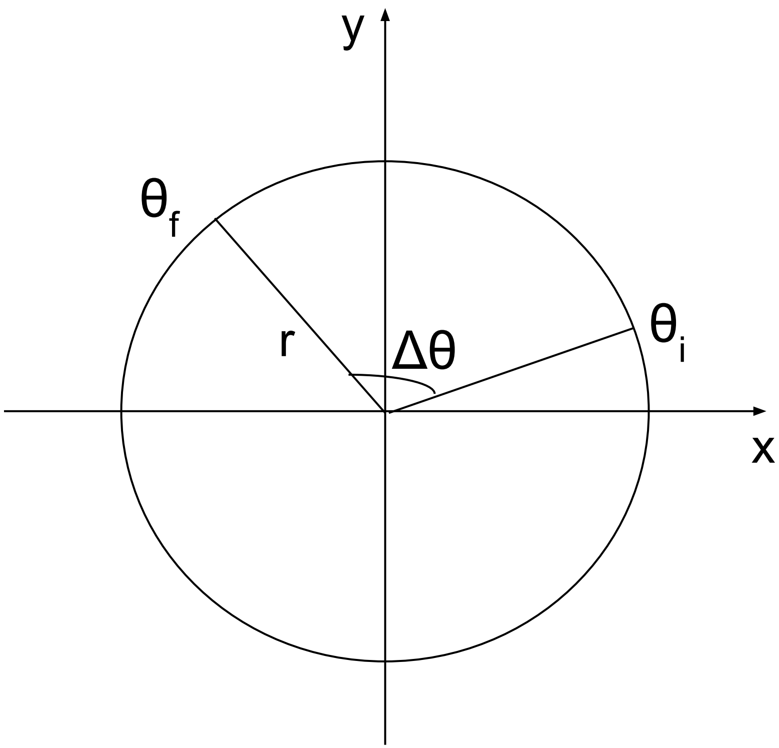
The average and instantaneous angular velocities are:

$$\omega_{\mathrm{av}} = \frac{\Delta\theta}{\Delta t} \quad \text{and} \quad \omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

ω is measured in rads/sec.



Angular Speed



An object moves along a circular path of radius r; what is its average speed?

$$v_{\mathrm{av}} = \frac{\text{total distance}}{\text{total time}} = \frac{r\Delta\theta}{\Delta t} = r\left(\frac{\Delta\theta}{\Delta t}\right) = r\omega_{\mathrm{av}}$$

Also, $v = r\omega$ (instantaneous values).

Period and Frequency

The time it takes to go one time around a closed path is called the **period** (T).

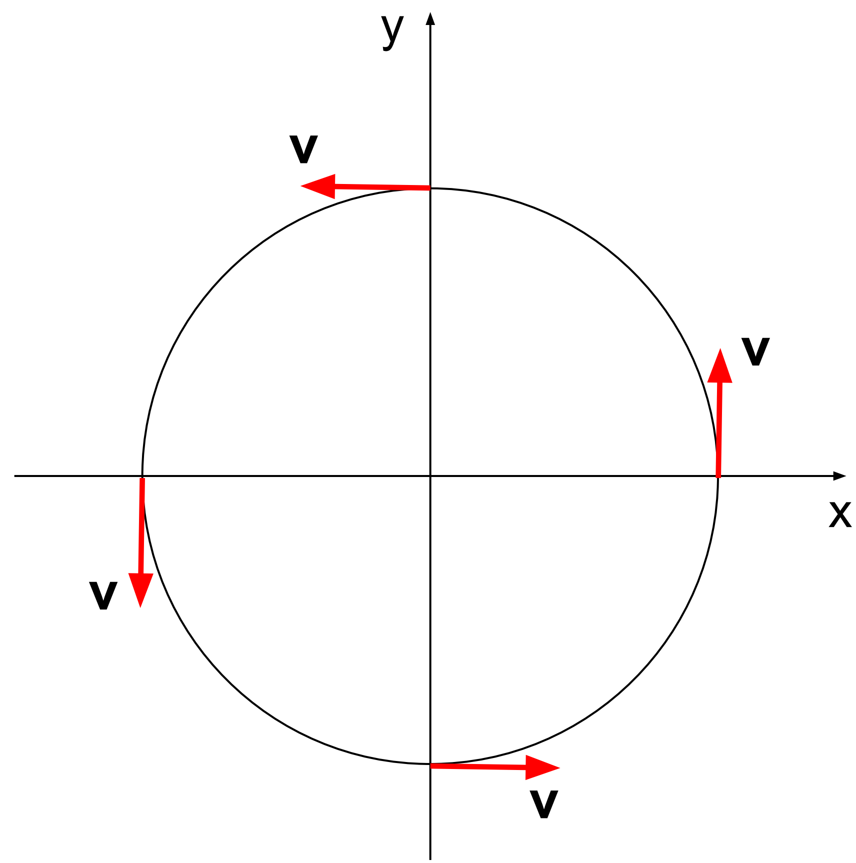
$$v_{\text{av}} = \frac{\text{total distance}}{\text{total time}} = \frac{2\pi r}{T}$$

Comparing to v = rω: $\omega = \frac{2\pi}{T} = 2\pi f$

f is called the **frequency**, the number of revolutions (or cycles) per second.

Centripetal Acceleration

Consider an object moving in a circular path of radius r at constant speed.



Here, $\Delta\mathbf{v} \neq 0$. The direction of **v** is changing.

If $\Delta\mathbf{v} \neq 0$, then $\mathbf{a} \neq 0$. Then there is a net force acting on the object.

Centripetal Acceleration

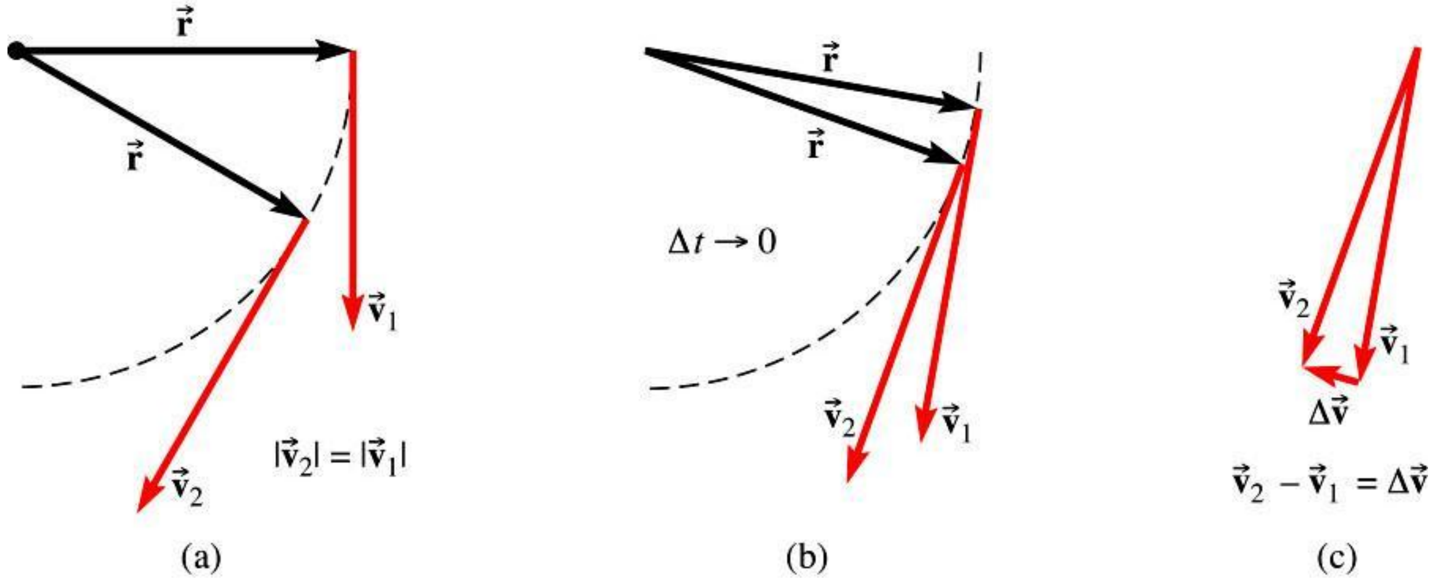
<u>**Conclusion:**</u> with no net force acting on the object it would travel in a straight line at constant speed

It is still true that $\Sigma \mathbf{F} = m\mathbf{a}$.

But what acceleration do we use?

Centripetal Acceleration

The velocity of a particle is tangent to its path.



For an object moving in uniform circular motion, the acceleration is radially inward.

Centripetal Acceleration

The magnitude of the radial acceleration is:

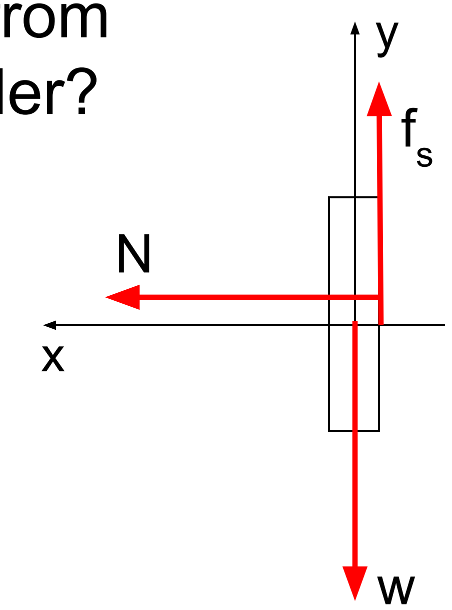
$$a_r = \frac{v^2}{r} = r\omega^2 = \omega v$$

Rotor Ride Example

The rotor is an amusement park ride where people stand against the inside of a cylinder. Once the cylinder is spinning fast enough, the floor drops out.

(a) What force keeps the people from falling out the bottom of the cylinder?

Draw an FBD for a person with their back to the wall:



It is the force of static friction.

Rotor Ride Example

(b) If $\mu_s = 0.40$ and the cylinder has r = 2.5 m, what is the minimum angular speed of the cylinder so that the people don't fall out?

Apply Newton's 2nd Law:

$$(1)\sum F_x = N = ma_r = m\omega^2 r$$

$$(2)\sum F_y = f_s - w = 0$$

From (2): $f_s = w$

From (1)

$$\mu_s N = \mu_s\left(m\omega^2 r\right) = mg$$

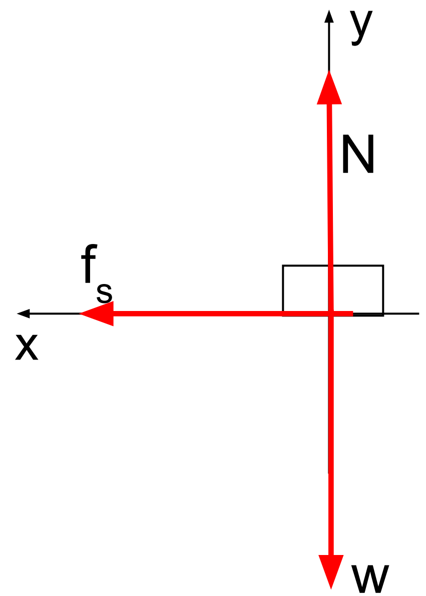
$$\omega = \sqrt{\frac{g}{\mu_s r}} = \sqrt{\frac{9.8\ \text{m/s}^2}{(0.40)(2.5\ \text{m})}} = 3.13\ \text{rad/s}$$

Unbanked Curve

A coin is placed on a record that is rotating at 33.3 rpm. If $\mu_s = 0.1$, how far from the center of the record can the coin be placed without having it slip off?

We're looking for r.

Draw an FBD for the coin:



Apply Newton's 2nd Law:

$$(1)\sum F_x = f_s = ma_r = m\omega^2 r$$

$$(2)\sum F_y = N - w = 0$$

Unbanked Curve

From (2)

$$\text{From}\left(1\right): f_s = m\omega^2 r$$

$$f_s = \mu_s N = \mu_s\left(mg\right) = m\omega^2 r$$

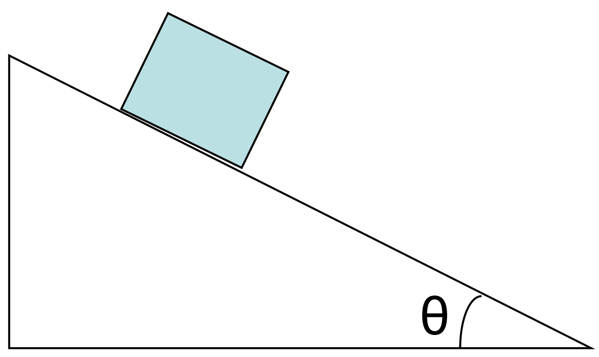
Solving for r: $r = \dfrac{\mu_s g}{\omega^2}$ What is ω?

$$\omega = 33.3\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{1\,\text{rev}}\right)\left(\frac{1\,\text{min}}{60\,\text{sec}}\right) = 3.5\ \text{rad/s}$$

$$r = \frac{\mu_s g}{\omega^2} = \frac{\left(0.1\right)\left(9.8\ \text{m/s}^2\right)}{\left(3.50\ \text{rad/s}\right)^2} = 0.08\ \text{m}$$

Banked Curves

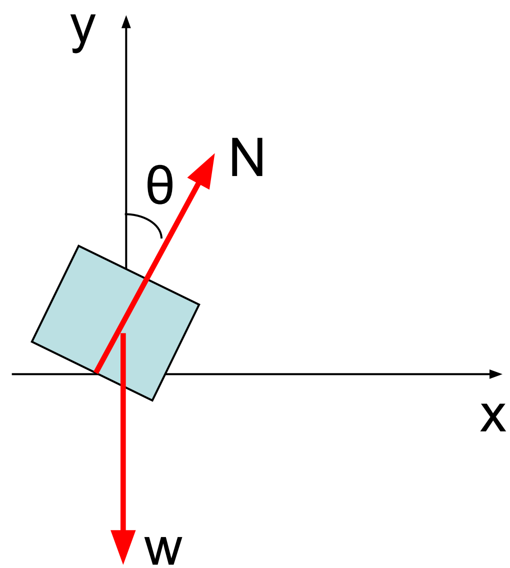
A highway curve has a radius of 825 m. At what angle should the road be banked so that a car traveling at 26.8 m/s has no tendency to skid sideways on the road? (Hint: No tendency to skid means the frictional force is zero.)



Take the car's motion to be into the page.

Banked Curves

FBD for the car:



Apply Newton's Second Law:

$$(1)\sum F_x = N\sin\theta = ma_r = m\frac{v^2}{r}$$

$$(2)\sum F_y = N\cos\theta - w = 0$$

Banked Curves

Rewrite (1) and (2):

$$(1)\ N\sin\theta = m\frac{v^2}{r}$$

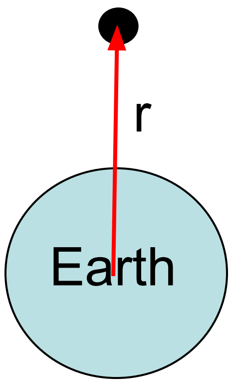
$$(2)\ N\cos\theta = mg$$

Divide (1) by (2):

$$\tan\theta = \frac{v^2}{gr} = \frac{(26.8\text{ m/s})^2}{(9.8\text{ m/s}^2)(825\text{ m})} = 0.089$$

$$\theta = 5.1°$$

Circular Orbits



Consider an object of mass m in a circular orbit about the Earth.

The only force on the satellite is the force of gravity:

$$\sum F = F_g = \frac{Gm_s M_e}{r^2} = m_s a_r = m_s \frac{v^2}{r}$$

Solve for the speed of the satellite:

$$\frac{Gm_s M_e}{r^2} = m_s \frac{v^2}{r}$$

$$v = \sqrt{\frac{GM_e}{r}}$$

Circular Orbits

Example: How high above the surface of the Earth does a satellite need to be so that it has an orbit period of 24 hours?

From previous slide: $v = \sqrt{\frac{GM_e}{r}}$ Also need, $v = \frac{2\pi r}{T}$

Combine these expressions and solve for r: $r = \left(\frac{GM_e}{4\pi^2} T^2 \right)^{1/3}$

$$r = \left(\frac{\left(6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}\right)\left(5.98 \times 10^{24}\ \mathrm{kg}\right)}{4\pi^2} \left(86400\ \mathrm{s}\right)^2 \right)^{1/3}$$

$$= 4.225 \times 10^7\ \mathrm{m}$$

$$r = R_e + h \Rightarrow h = r - R_e = 35{,}000\ \mathrm{km}$$

Circular Orbits

Kepler's Third Law

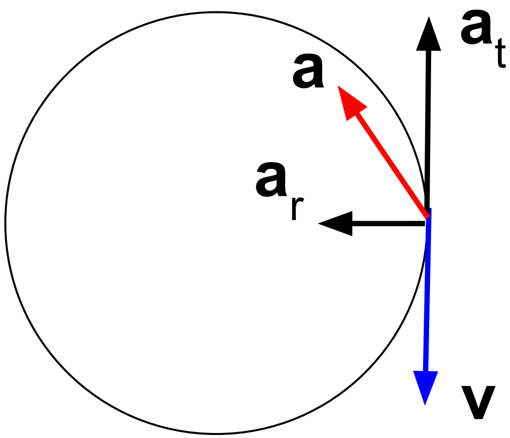
$$r = \left(\frac{GM_e}{4\pi^2} T^2 \right)^{1/3}$$

It can be generalized to: $r = \left(\frac{GM}{4\pi^2} T^2 \right)^{1/3}$

Where M is the mass of the central body. For example, it would be M_{sun} if speaking of the planets in the solar system.

Nonuniform Circular Motion

Nonuniform means the speed (magnitude of velocity) is changing.

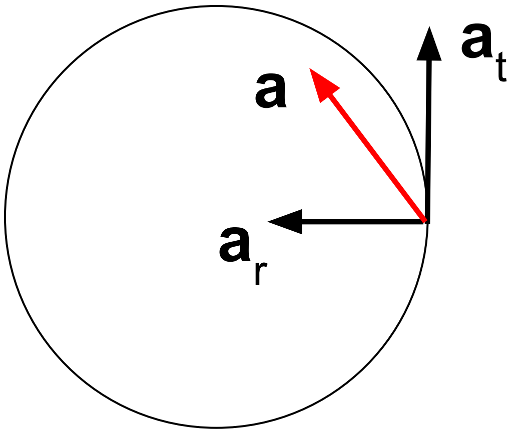


There is now an acceleration tangent to the path of the particle.

The net acceleration of the body is $a = \sqrt{a_r^{\,2} + a_t^{\,2}}$

This is true but useless!

Nonuniform Circular Motion



a_t changes the magnitude of **v**.

Changes energy - does work

a_r changes the direction of **v**.

Doesn't change energy - does NO WORK

Can write:

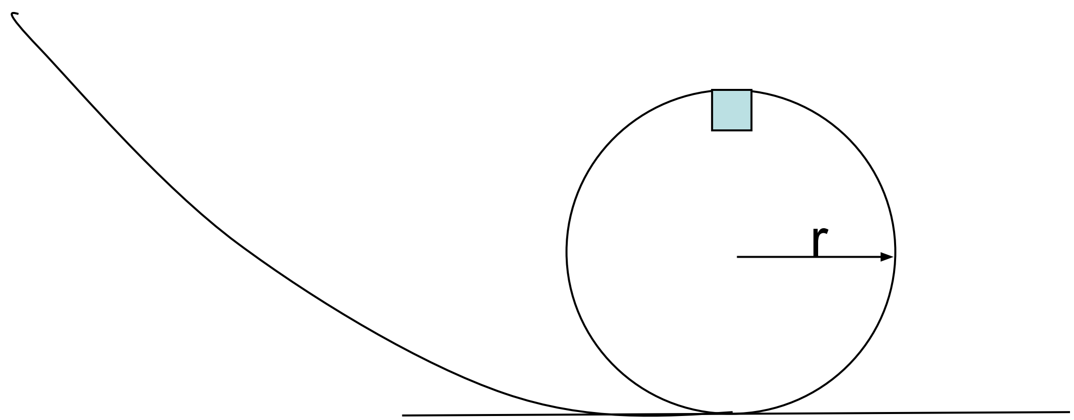
$$\sum F_r = ma_r$$

$$\sum F_t = ma_t$$

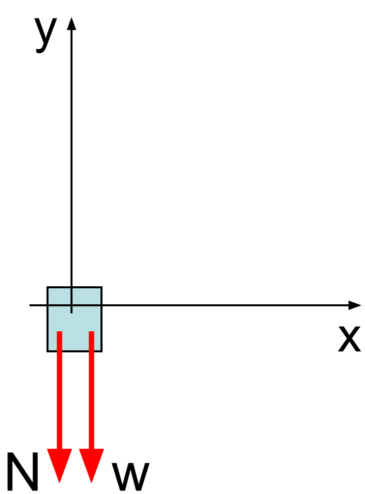
The accelerations are only useful when separated into perpendicualr and parallel components.

Loop Ride

Example: What is the minimum speed for the car so that it maintains contact with the loop when it is in the pictured position?



FBD for the car at the top of the loop:



Apply Newton's 2nd Law:

$$\sum F_y = -N - w = -ma_r = -m\frac{v^2}{r}$$

$$N + w = m\frac{v^2}{r}$$

Loop Ride

The apparent weight at the top of loop is:

$$N + w = m\frac{v^2}{r}$$

$$N = m\left(\frac{v^2}{r} - g\right)$$

N = 0 when

$$N = m\left(\frac{v^2}{r} - g\right) = 0$$

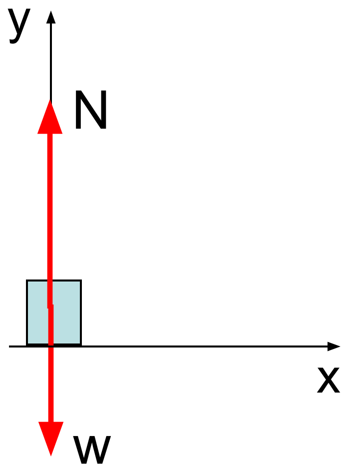
$$v = \sqrt{gr}$$

This is the minimum speed needed to make it around the loop.

Loop Ride

Consider the car at the bottom of the loop; how does the apparent weight compare to the true weight?

FBD for the car at the bottom of the loop:



Apply Newton's 2nd Law:

$$\sum F_y = N - w = ma_c = m\frac{v^2}{r}$$

$$N - w = m\frac{v^2}{r}$$

$$N = m\left(\frac{v^2}{r} + g\right)$$

Here, $N > mg$

Linear and Angular Acceleration

The average and instantaneous angular acceleration are:

$$\alpha_{\text{av}} = \frac{\Delta\omega}{\Delta t} \quad \text{and} \quad \alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

α is measured in rads/sec^2.

Linear and Angular Acceleration

Recalling that the tangential velocity is $v_t = r\omega$ means the tangential acceleration is

$$a_t = \frac{\Delta v_t}{\Delta t} = r\frac{\Delta\omega}{\Delta t} = r\alpha$$

Linear and Angular Kinematics

Linear (Tangential)

$$v = v_0 + a\Delta t$$

$$x = x_0 + v_0\Delta t + \frac{1}{2}a\Delta t^2$$

$$v^2 = v_0^2 + 2a\Delta x$$

Angular

$$\omega = \omega_0 + \alpha\Delta t$$

$$\theta = \theta_0 + \omega_0\Delta t + \frac{1}{2}\alpha\Delta t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

With $v_t = r\omega$ and $a_t = r\alpha$

"a" and "a_t" are the same thing

Dental Drill Example

A high speed dental drill is rotating at 3.14×10^4 rads/sec. Through how many degrees does the drill rotate in 1.00 sec?

Given: $\omega = 3.14\times10^4$ rads/sec; $\Delta t = 1$ sec; $\alpha = 0$

Want $\Delta\theta$.

$$\theta = \theta_0 + \omega_0 \Delta t + \frac{1}{2}\alpha \Delta t^2$$

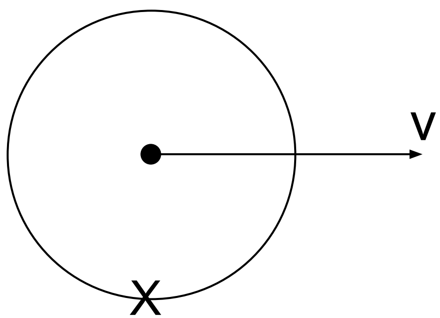
$$\theta = \theta_0 + \omega_0 \Delta t$$

$$\Delta\theta = \omega_0 \Delta t = \left(3.14\times10^4 \text{ rads/sec}\right)\left(1.0 \text{ sec}\right)$$

$$= 3.14\times10^4 \text{ rads} = 1.80\times10^6 \text{ degrees}$$

Car Example

Your car's wheels are 65 cm in diameter and are rotating at ω = 101 rads/sec. How fast in km/hour is the car traveling, assuming no slipping?

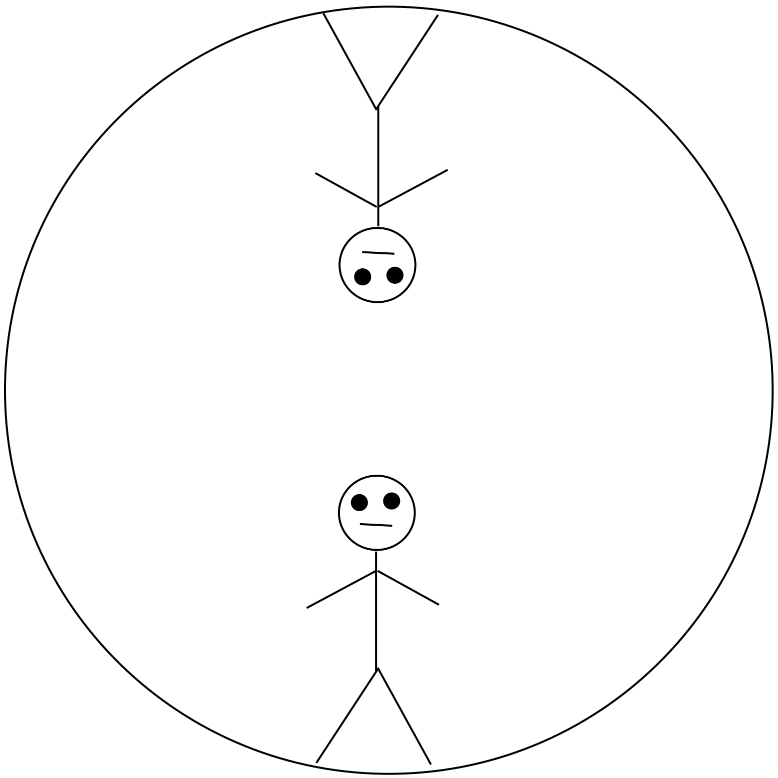


$$v = \frac{\text{total distance}}{\text{total time}} = \frac{(2\pi r)N}{(T)N} = \frac{2\pi r}{T} = \omega r$$

$$= (101\ \text{rads/sec})(32.5\ \text{cm})$$

$$= 3.28\times 10^{3}\ \text{cm/sec} = 118\ \text{km/hr}$$

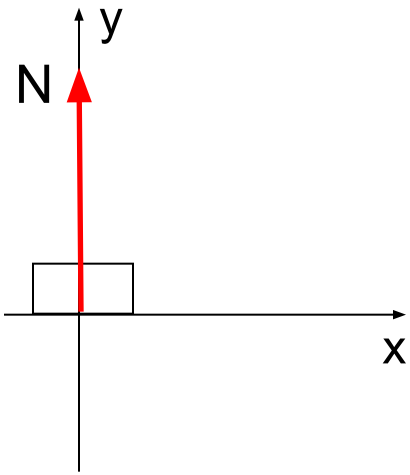
Artificial Gravity



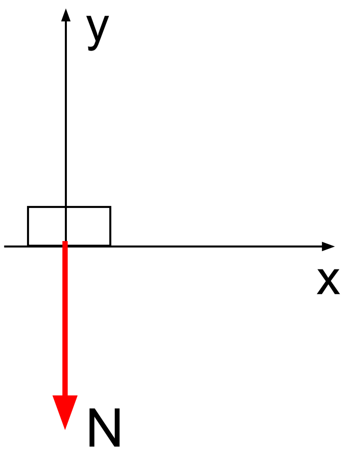
A large rotating cylinder in deep space (g≈0).

Artificial Gravity

FBD for the person



Bottom position



Top position

Apply Newton's 2nd Law to each:

$$\sum F_y = N = ma_r = m\omega^2 r \qquad \sum F_y = -N = -ma_r = -m\omega^2 r$$

Space Station Example

A space station is shaped like a ring and rotates to simulate gravity. If the radius of the space station is 120m, at what frequency must it rotate so that it simulates Earth's gravity?

Using the result from the previous slide:

$$\sum F_y = N = ma_r = m\omega^2 r$$

$$\omega = \sqrt{\frac{N}{mr}} = \sqrt{\frac{mg}{mr}} = \sqrt{\frac{g}{r}} = 0.28\ \text{rad/sec}$$

The frequency is f = ($\omega/2\pi$) = 0.045 Hz (or 2.7 rpm).

Summary

- A net force MUST act on an object that has circular motion.
- Radial Acceleration $a_r = v^2/r$
- Definition of Angular Quantities (θ, ω, and α)
- The Angular Kinematic Equations
- The Relationships Between Linear and Angular Quantities

$$v_t = r\omega \text{ and } a_t = r\alpha$$

- Uniform and Nonuniform Circular Motion