PHY 101: Mechanics and Properties of Matter

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(Lecture : C19-1)

GRAVITATION

Learning Goals:

- **Understanding the laws that governs the relationship between heavenly bodies**

- **How to calculate the gravitational forces that any two bodies exert on each other.**

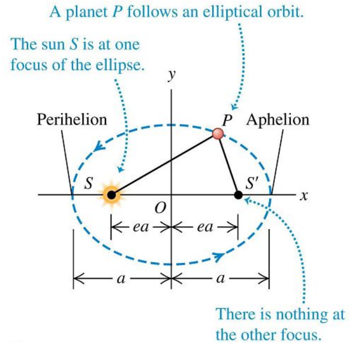
- **How to relate the weight of an object to the general expression for gravitational force (Calculating acceleration due to gravity on, above and inside the earth crust).**

- **How to calculate the speed, orbital period, and mechanical energy of a satellite in a circular orbit.**

- **How to apply and interpret Kepler’s three laws that describe the motion of planets.**

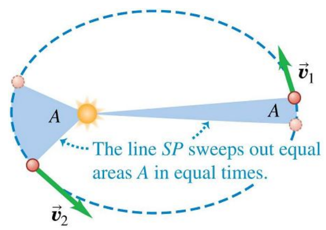
Keplers's Law of Planetory Motion

1. **The planets describes motion in elliptical orbit about the sun as a focus**



1st Law

2. **The line joining the sun and each planet sweeps out equal areas in equal times (i.e constant period) due to conservation of angular momentum**



2nd Law

3. **The square of the period T is proportional to the cube of the radius of the ellipse of each planet i.e $T^2 \alpha r^{3.}$**

$$T = \frac{2\pi r^{3/2}}{\sqrt{GM_e}}$$

3rd Law

Newton's Law of Universal Gravitation

States that:

- **Every particle of matter attracts every other particle with a force (F_g) that is directly proportional to the product of their masses (m_1 & m_2) and inversely proportional to the square of the distance (r) between them. That is**

$$F_g = \frac{Gm_1\,m_2}{r^2}$$

Where **G** is the gravitational constant.
The gravitational constant G = 6.67×10^{-11} N·m²/kg² is a fundamental physical constant that has the same value for any two particles

- **Consider the sun of mass, M and a planet of mass, m, the gravitational force which provide the centripetal force is given by:**

$$F = \frac{GM\cancel{m}}{r^2} = \cancel{m}\omega^2 r$$

$$\omega^2 r = \frac{GM}{r^2} \quad \text{or} \quad \frac{4\pi^2}{T^2} = \frac{GM}{r^3}$$

$$\therefore \boldsymbol{T^2 = \left(\frac{4\pi^2}{GM}\right) r^3} \quad \text{i.e } \mathbf{T^2 \ \alpha \ r^3}.$$

- **This is Kepler's 3rd law and it implies 2nd law since for a particular planet moving in an orbit, the radius r is constant**

Gravitational Attraction

- Our solar system is part of a spiral galaxy like this one, which contains roughly 10^{11} stars as well as gas, dust, and other matter (mostly dark matter).

- The entire assemblage is held together by the mutual gravitational attraction of all the matter in the galaxy

Weight and Acceleration Due to Gravity

- **The weight of an object on the earth is the gravitational force that the earth exrts on the object. This force gives the object an acceleration of g towards the centre of the earth**.

Hence $F = \frac{GM_E\cancel{m}}{R_E^2} = \cancel{m}g$

Therefore, the acceleration due to gravity at the earth's surface is:

Gravitational constant

Mass of the earth

Acceleration due to gravity at the earth's surface

$$g = \frac{Gm_E}{R_E^2}$$

Radius of the earth

$$\therefore \quad g \propto \frac{1}{r^2}$$

This explains why g is not constant all over the world (It is more at the pole than at the equator**. The average value of g is 9.8 ms^{-2}**

- The acceleration due to gravity, **g'** of an object at an height **h above the earth is given by**

$$\mathbf{g'} = \frac{R_E^2}{(R_E + h)^2}\,\mathbf{g}$$

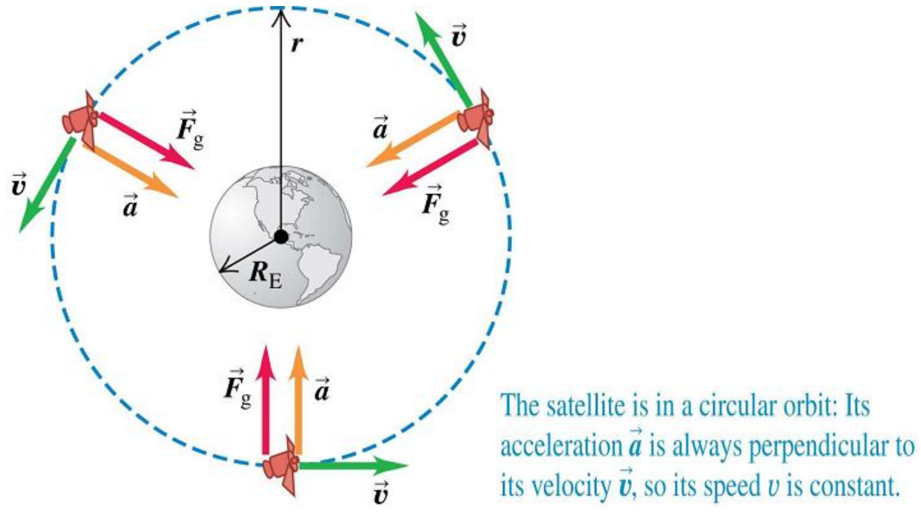
- The acceleration due to gravity, **g''** of an object at an height **h** beneath **the earth surface is given by**

$$\mathbf{g''} = \left(\frac{R_E - h}{R_E}\right)\mathbf{g}$$

Orbits Round the Earth

- **Satellites are objects that revolve other objects. For example, the moon around the earth is a natural satellite.**

- The force due to the earth's gravitational attraction provides the centripetal acceleration/ force that keeps a satellite in orbit.



- **Using the Newton's law of gravitation, we have:**

$$F = \frac{GM_E \cancel{m}}{r^2} = \cancel{m}\frac{v^2}{r}$$

$$\therefore \; \boldsymbol{V = \sqrt{GM_E/r}}$$

- For a circular orbit of radius **r** , the Speed, **V** of a satellite is just right to keep its distance from the center of the earth constant.

For one revolution of a satellite, the period is given by: $\boldsymbol{T = {}^{2\pi r}/_{V}}$

Substituting $\boldsymbol{V = \sqrt{GM_E/r}}$ **we will have**

$$\boldsymbol{T = \frac{2\pi r^{3/2}}{\sqrt{GM_e}}}$$

This confirms **Kepler's 3rd Law**

- **A satellite is said to be parked if its period is 24 Hrs (the time taken by earth to complete one rotation**

Earth's Gravitational Potential Energy

- The earth's gravitational potential energy at any point in the earth gravitational field can be defined as the total work done per unit mass on a body to move it from the point which is at a distance r from the center of the earth to infinity (where $F_g = 0$)

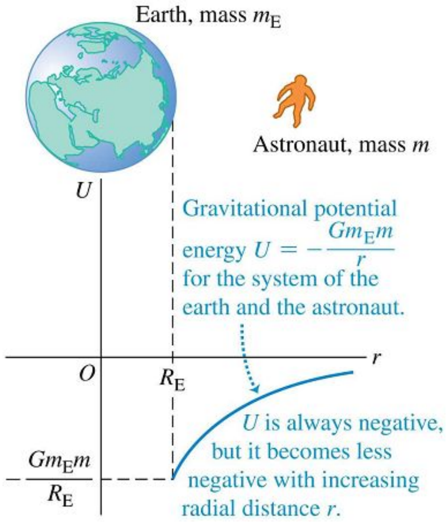
Gravitational constant · Mass of the earth

Gravitational potential energy (general expression)

$$U = -\frac{G m_E m}{r}$$

Mass of body

Distance of body from the earth's center



- Hence, the Gravitational potential, φ is thus given by

$$\varphi = \frac{work}{mass} = \frac{-GM_E}{r}$$

U depend on distance, r

Escape Velocity

- If a body is given an energy which is equal to the gravitational potential energy on the earth surface, it will go to infinity. In other words, if a body leaves the earth surface with a **velocity, $\boldsymbol{v}$** such that its kinetic is equal to the gravitational potential energy, it will never return. Thus, It is said to be escaped. That is :

$$\frac{1}{2}\mathbf{m}\boldsymbol{v}^2 = \frac{GM_Em}{r} \quad \text{hence,}$$

$$\boldsymbol{v} = \sqrt{2GM_E/r} = \sqrt{2gr} \quad \text{is called the escape velocity}$$

Example:

The mass of the moon is about one eighty-first, and its radius on-fourth, that of the earth. Calculate the acceleration due to gravity on the surface of the moon.

Solution

$$mg = \frac{GmM}{R^2} \longrightarrow g = \frac{GM}{R^2}$$

fer earth : $9.8 = \frac{GM_E}{R_E^2}$

for moon : $g = \frac{G(M_E/81)}{(R_E/4)^2}$

$$\therefore \quad g = 9.8 \times \frac{16}{81} = 1.94 \ ms^{-2}$$