Assignment 3

[1] Simplify the following Boolean functions, using three-variable maps:

(a) $F(x,y,z) = \sum(0,2,6,7)$

xy \ z	0	1
00	1	
01	1	
11	1	1
10		

=x’z’+xy

(b) $F(x,y,z) = \sum(0,2,3,4,6)$

xy \ z	0	1
00	1	
01	1	1
11	1	
10	1	

=z’+x’y

(c) $F(x,y,z) = \sum(0,1,2,3,7)$

xy \ z	0	1
00	1	1
01	1	1
11		1
10		

=x’+yz

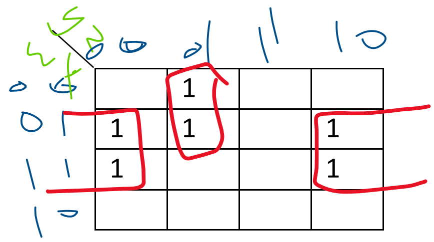
(d) $F(x,y,z) = \sum(3,5,6,7)$

xy \ z	0	1
00		
01		1
11	1	1
10		1

=xy+yz+xz

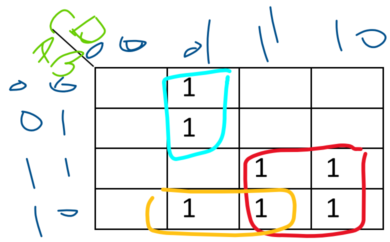
[2] Simplify the following Boolean functions, using four-variable maps:

(a) $F(w,x,y,z) = \sum(1,4,5,6,12,14,15)$



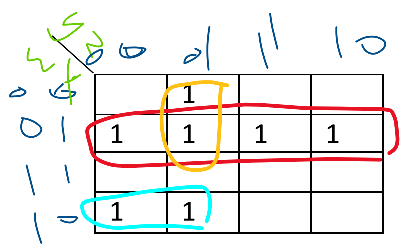
=xz'+w'y'z

(b) $F(A,B,C,D) = \sum(1,5,9,10,11,14,15)$



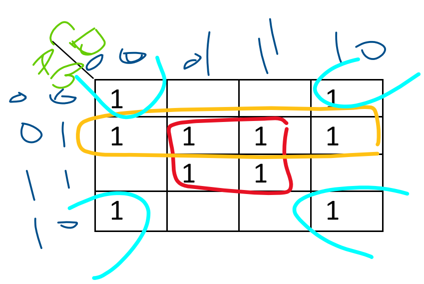
=AC+AB'D+A'C'D

(c) $F(w,x,y,z) = \sum(1,4,5,6,7,8,9)$



=w'x+w'y'z+wx'y'

(d) $F(A,B,C,D) = \sum(0,2,4,5,6,7,8,10,13,15)$



=BD+A'B+B'D'

[3] Simplify the following expressions to (1) sum-of-products and (2) products-of-sums:

(a) $\bar{x}\,\bar{z} + \bar{y}\,\bar{z} + y\bar{z} + xy$

$=\bar{z}\,(\bar{x} + \bar{y} + y)$+xy

$=\bar{z}$+xy (SOP)

$=\bar{z}$.(x+y) (POS)

(b) $AC\bar{D} + \bar{C}D + A\bar{B} + ABCD$

=AC($\bar{D}$+BD) + $\bar{C}D + A\bar{B}$

= AC($\bar{D}$+B) + $\bar{C}D + A\bar{B}$

= AC$\bar{D}$+ACB+ $\bar{C}D + A\bar{B}$

=A(C$\bar{D}$+CB + $\bar{B}$) + $\bar{C}D$

=AC+ $A\bar{B} + \bar{C}D$ (SOP)

=(A+C).($A +$ $\bar{B}$).($\bar{C}$+D) (POS)

(c) $(A + \bar{C} + \bar{D})(\bar{A} + \bar{B} + \bar{D})(\bar{A} + B + \bar{D})(\bar{A} + B + \bar{C})$

=$\bar{C}\,\bar{D}$+ $\bar{A}\,\bar{C}$ + $\bar{A}\,\bar{D}$+ $B\,\bar{D}$ **(SOP)**

=$(\bar{C} + \bar{D}).\ (\bar{A} + \bar{C}).\ (\bar{A} + \bar{D}).\ (B + \bar{D})$ **(POS)**

(d) $AB\bar{C} + A\bar{B}D + BCD$

=AD+AB$\bar{C}$+BCD (SOP)

=(A+D).(A+B+$\bar{C}$).(B+C+D) (POS)

[4] Using K-map derive the minimum SOP expression for each of the following functions:

(a) F = ab'c' + ab'c + abc' +abc

ab \ c	0	1
00		
01		
11	1	1
10	1	1

=a

=∑(4,5,6,7)

(b) Q = c' + cd + ac + a'bc'd + abc

ab\cd	00	01	11	10
00	1	1	1	
01	1	1	1	
11	1	1	1	1
10	1	1	1	1

=a+c’+d

F=∑(0,1,3,4,5,7,8,9,10,11,12,13,14,15)

[5] F(w,x,y,z) = Σ (2, 3, 6, 8, 11, 12, 14)

(a) Show F on a K-map.

wx\yz	00	01	11	10
00			1	1
01				1
11	1			1
10	1		1	

(b) Obtain a minimal sum of products expression for F.

F= =wy’z’+x’yz+xyz’+w’yz’

(c) If the minterms: m4, m10, m13 and m15 will never occur determine a new minimal sum of products expression for F.

wx\yz	00	01	11	10
00	1	1	1	1
01		1	1	1
11	1			1
10	1	1	1	

=w’y+w’z+wxz’+x’y’+x’z

[6] Convert the following Boolean function from a sum-of-products form to a simplified product-of-sums form.

$F(w,x,y,z) = \sum(0,1,2,5,8,10,13)$

wx \ yz	00	01	11	10
00	1	1		1
01		1		
11		1		
10	1			1

=x'z'+xy'z+w'y'z (SOP)

=(x'+z').(x+y'+z).(w'+y'+z) (POS)

[7] Simplify the following Boolean function F , together with the don't-care conditions d , and then express the simplified function in sum-of-minterms form:

(a) $F(x,y,z) = \sum(2,3,4,6,7)$ *and* $d(x,y,z) = \sum(0,1,5)$

xy \ z	0	1
00	d	d
01	1	1
11	1	1
10	1	d

=1

(b) $F(A,B,C,D) = \sum(0,6,8,13,14)$ *and* $d(A,B,C,D) = \sum(2,4,10)$

AB \ CD	00	01	11	10
00	1			d
01	d			1
11		1		1
10	1			d

=CD'+B'D'+ABC'

(c) $F(A,B,C,D) = \sum(4,5,7,12,13,14)$ *and* $d(A,B,C,D) = \sum(1,9,11,15)$

AB \ CD	00	01	11	10
00		d		
01	1	1	1	
11	1	1	d	1
10		d	d	

=AB+C’D+A’BC’+A’BD

(d) $F(A,B,C,D) = \sum(1,3,8,10,15)$ *and* $d(A,B,C,D) = \sum(0,2,9)$

AB \ CD	00	01	11	10
00	d	1	1	d
01				
11			1	
10	1	d		1

=B’D’+A’B’+ABCD

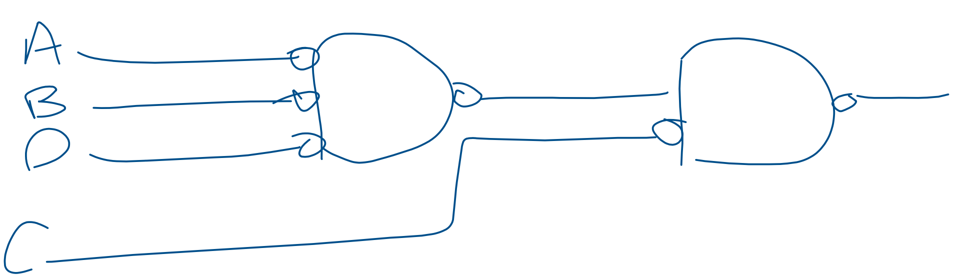
[8] Simplify the following functions, and implement them with two-level NAND gate circuits:

(a) $F(A,B,C,D) = \overline{A}\overline{B}C + \overline{A}C + ACD + AC\overline{D} + \overline{A}\overline{B}\overline{D}$

AB \ CD	00	01	11	10
00	1		1	1
01			1	1
11			1	1
10			1	1

$=C+\overline{A}\overline{B}\overline{D}$

$=C+\overline{A}\overline{B}\overline{D} = C+\overline{\overline{\overline{A}\overline{B}\overline{D}}} = \overline{\overline{C}.(A+B+D)} = \overline{\overline{C}\ \overline{\overline{(A+B+D)}}} = \overline{\overline{C}.(\overline{\overline{A}.\overline{B}.\overline{D}})}$

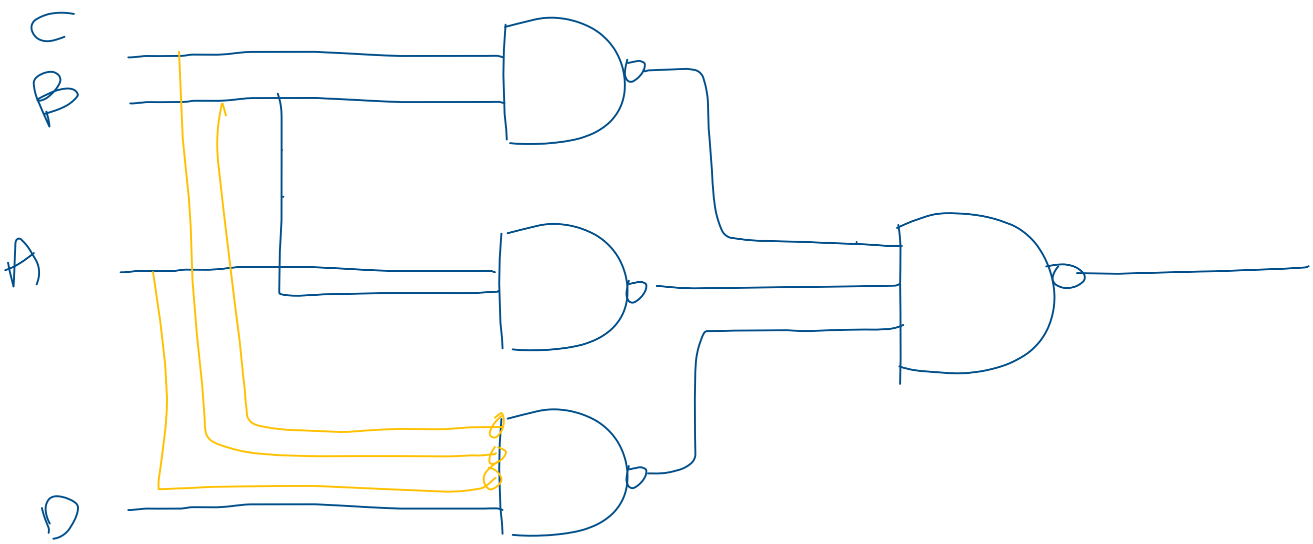


(b) $F(A,B,C,D) = AB + \bar{A}BC + \bar{A}\bar{B}CD$

AB \ CD	00	01	11	10
00		1		
01			1	1
11	1	1	1	1
10				

$=\bar{A}\bar{B}\bar{C}D+AB+BC$

$=\bar{A}\bar{B}\bar{C}D+AB+BC= \overline{\overline{\bar{A}\,\bar{B}\,\bar{C}\,D + AB + BC}} = \overline{(\overline{BC}).(\overline{AB}).(\overline{\bar{A}\bar{B}\bar{C}D})}$

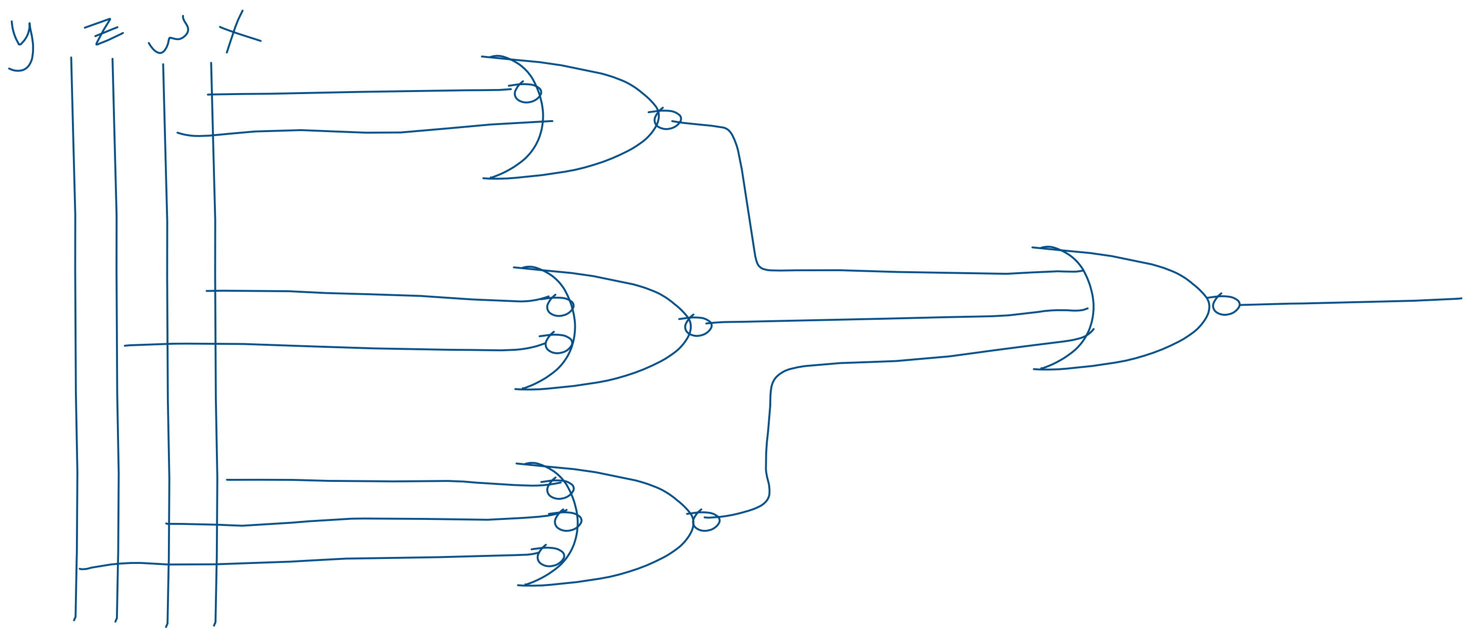


[9] Simplify the following functions, and implement them with two-level NOR gate circuits:

(a) $F = w\bar{x} + \bar{y}\,\bar{z} + \bar{w}y\bar{z}$

wx \ yz	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	0
10	1	1	1	1

$= \overline{\overline{(\bar{z} + w)(\bar{x} + \bar{z})(\bar{w} + \bar{x} + \bar{y})}} = \overline{\overline{(\bar{z} + w)} + \overline{(\bar{x} + \bar{z})} + \overline{(\bar{w} + \bar{x} + \bar{y})}}$

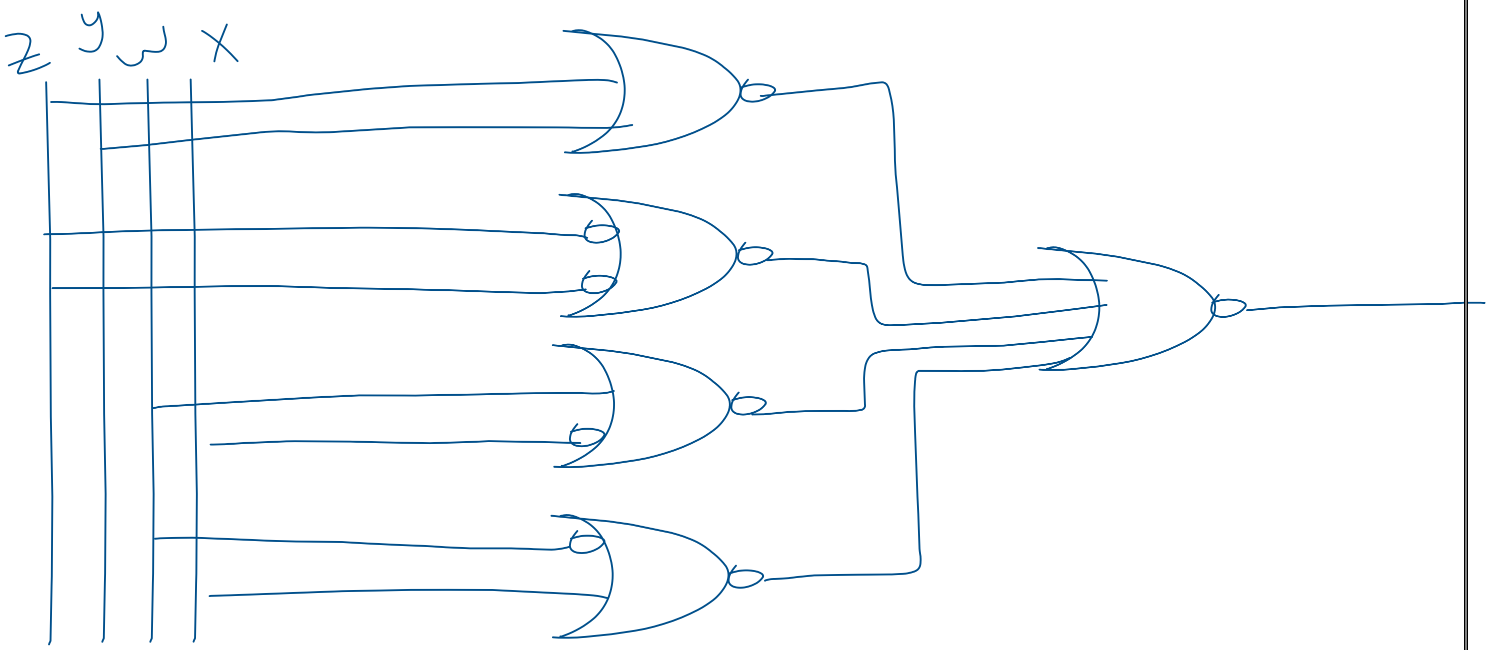


(b) $F(w,x,y,z) = \sum(1,2,13,14)$

$wx \backslash yz$	00	01	11	10
00	0	1	0	1
01	0	0	0	0
11	0	1	0	1
10	0	0	0	0

$=(y + x)(\bar{y} + \bar{z}) + (w + \bar{x})(\bar{w} + x) = \overline{\overline{(y + x)(\bar{y} + \bar{z}) + (w + \bar{x})(\bar{w} + x)}}$

$= \overline{\overline{(y + x)}\,\overline{(\bar{y} + \bar{z})} + \overline{(w + \bar{x})}\,\overline{(\bar{w} + x)}}$



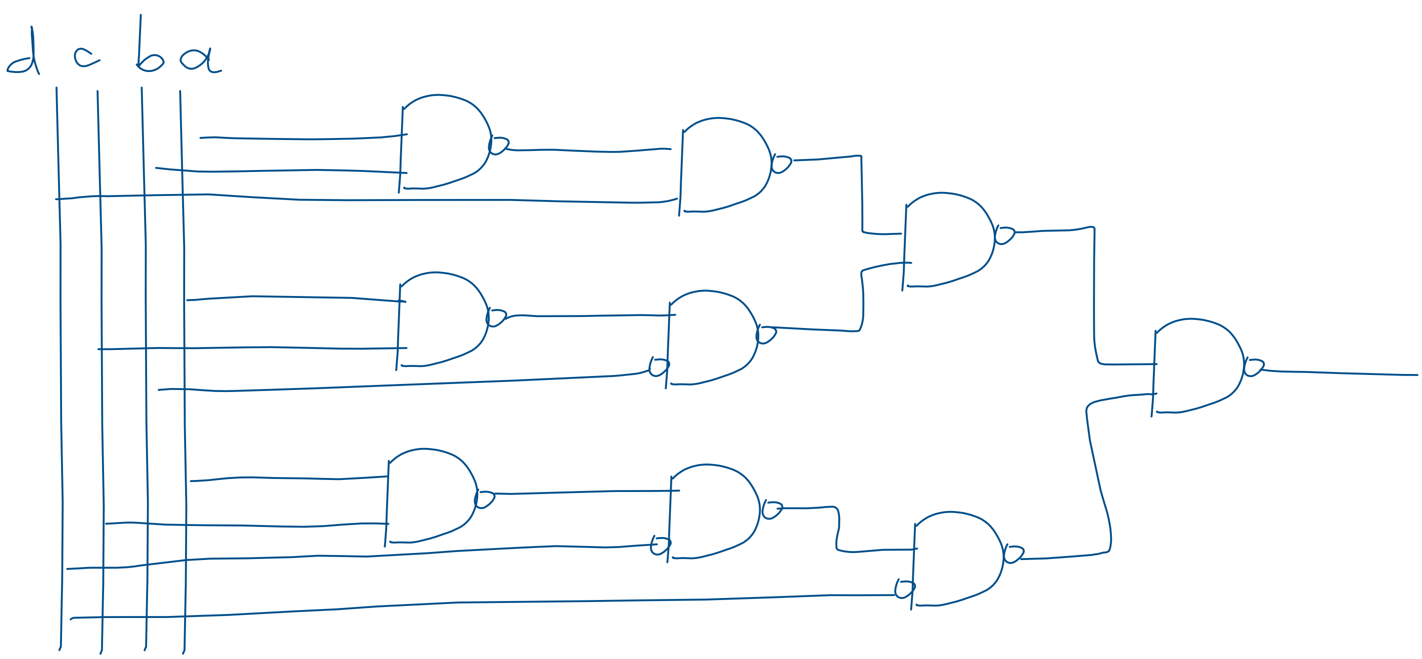
[10] (a) Implement the following function using NAND gates with a fan in of 2.

F = (ab + d')(ac + b) + (ac +b)d

$$= \overline{\overline{(ab + \bar{d})(ac + b) + (ac + b)d}}$$

$$= \overline{\overline{(ab + \bar{d})(ac + b)} + \overline{(ac + b)d}}$$

$$= \overline{\overline{\overline{(\overline{ab} + d)(\overline{ac} + \bar{b})}} + \overline{(\overline{\overline{ac} + \bar{b}})d}}$$



(b) Simplify the above function and implement using NAND gates with a fan in of 2.

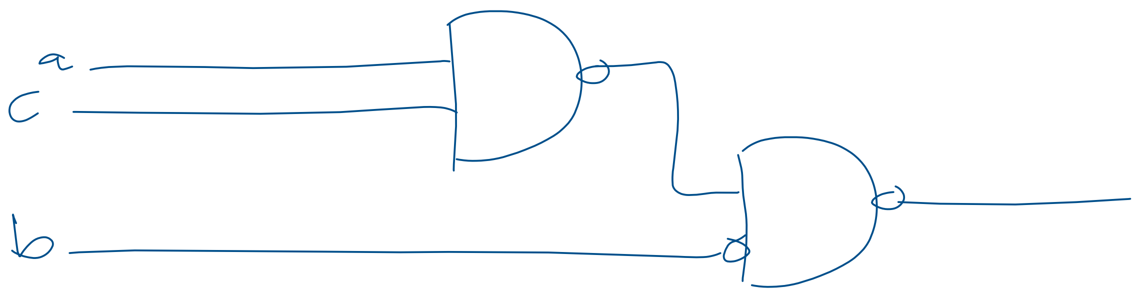
= abc + ab +ac$\overline{d}$ +$\overline{d}b$ + acd + db

= ab(c + 1) +ac($\overline{d}$ + d) +b($\overline{d}$ + d)

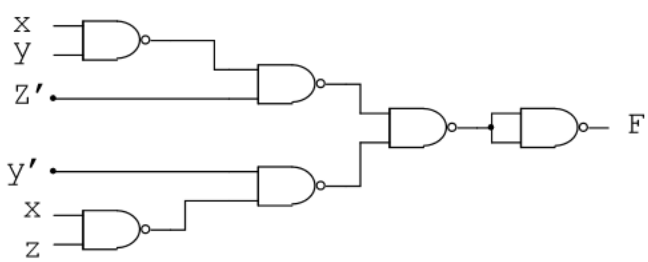
=ab + ac + b

=b(a+1) +ac

= $b + ac$ =$\overline{\overline{b + ac}}$ = $\overline{\overline{b}.\overline{ac}}$



[11] Determine the value of F in a standard form:



=$\left((xy).\overline{z}\right).\left((xz).\overline{y}\right) = \left(\overline{(xy)}.\overline{z}\right).\left(\overline{(xz)}.\overline{y}\right) = xyz + \overline{y}\overline{z}$