المحاضرة الثامنة

3.2 Determinants & elementry row operations:

Ex(1) a) $A = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix} \Rightarrow |A| = 11$

$R_1 \rightleftharpoons R_2$ interchange R_1 & R_2

$B = \begin{bmatrix} 1 & 4 \\ 2 & -3 \end{bmatrix} \Rightarrow |B| = -11$

$|A| = -|B|$

b. $A = \begin{bmatrix} 1 & -3 \\ 2 & -4 \end{bmatrix} \Rightarrow |A| = 2$

$[-2 \quad 6]$

$R_2 - 2R_1 \Rightarrow B = \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix} \Rightarrow |B| = 2$

$|A| = |B|$

c. $A = \begin{bmatrix} 2 & -8 \\ -2 & 9 \end{bmatrix} \Rightarrow |A| = 2$

$\frac{1}{2}R_1 \Rightarrow B = \begin{bmatrix} 1 & -4 \\ -2 & -9 \end{bmatrix} \Rightarrow |B| = 1$

$|B| = \frac{1}{2}|A|$

Ex(2) Find determinant using elementry row operations

$$A = \begin{bmatrix} 0 & -7 & 14 \\ 1 & 2 & -2 \\ 0 & 3 & -8 \end{bmatrix}$$

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$$R_1 \rightleftharpoons R_2 \begin{vmatrix} \boxed{0}^{1} & -7 & 14 \\ \boxed{1}^{0} & 2 & -2 \\ \boxed{0}^{0} & 3 & -8 \end{vmatrix}$$

$$-7 \text{ out of } R_2 \quad -\begin{vmatrix} 1 & 2 & -2 \\ 0 & \boxed{-7}^{1} & 14 \\ 0 & 3 & -8 \end{vmatrix}$$

$$-(-7)\begin{vmatrix} 1 & 2 & -2 \\ 0 & 1 & -2 \\ 0 & 3 & -8 \end{vmatrix} \quad [0 \quad -3 \quad 6]$$

$R_3 - 3R_2$

$$7\begin{vmatrix} 1 & 2 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & -2 \end{vmatrix} = (7)(1)(1)(-2) = -14$$

* Determinants & elementary Column operations

$\boxed{1}$

$$\begin{vmatrix} 2 & 1 & -3 \\ 4 & 0 & 1 \\ 0 & 0 & 2 \end{vmatrix} = -\begin{vmatrix} 1 & 2 & -3 \\ 0 & 4 & 1 \\ 0 & 0 & 2 \end{vmatrix}$$

$C_1 \rightleftharpoons C_2$

$\boxed{2}$

$$\begin{vmatrix} 2 & 3 & -5 \\ 4 & 1 & 0 \\ -2 & 4 & -3 \end{vmatrix} = 2\begin{vmatrix} 1 & 3 & -5 \\ 2 & 1 & 0 \\ -1 & 4 & 3 \end{vmatrix}$$

2 out of C_1

Ex(3) use Column operations to find

$$\begin{vmatrix} -1 & 2 & 2 \\ 3 & -6 & 4 \\ 5 & -10 & -3 \end{vmatrix}$$

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$$\begin{bmatrix} -2 \\ -6 \\ 10 \end{bmatrix} \quad \begin{vmatrix} -1 & 2 & 2 \\ 3 & -6 & 4 \\ 5 & -10 & -3 \end{vmatrix} = \begin{vmatrix} -1 & 0 & 2 \\ 3 & 0 & 4 \\ 5 & 0 & -3 \end{vmatrix} = 0$$

$C_2 + 2C_1$

Notes: If all entries of any row or any column of A are zeros then $|A| = 0$

Theorem: If A is a square matrix and any one of the condition below is true then $|A| = 0$

1. An entire row (column) consists of zeros.
2. Two rows (columns) are equal
3. one row (column) is a multiple of another row (column)

Ex(4) Find $|A|$ if $A = \begin{bmatrix} 1 & 4 & 1 \\ 2 & -1 & 0 \\ 0 & 18 & 4 \end{bmatrix}$

sol^n

$R_2 - 2R_1$ $\begin{vmatrix} 1 & 4 & 1 \\ \boxed{2}^{0} & -1 & 0 \\ 0 & 18 & 4 \end{vmatrix}$ $[-2 \quad -8 \quad -2]$

$R_3 + 2R_2$ $\begin{vmatrix} 1 & 4 & 1 \\ 0 & -9 & -2 \\ 0 & 18 & 4 \end{vmatrix}$ $[0 \quad -18 \quad -4]$

$\begin{vmatrix} 1 & 4 & 1 \\ 0 & -9 & -2 \\ 0 & 0 & 0 \end{vmatrix} = 0$

Ex(B) Find $|A|$ if $A = \begin{bmatrix} -3 & 5 & 2 \\ 2 & 4 & -1 \\ -3 & 0 & 6 \end{bmatrix}$

$$\begin{bmatrix} -6 \\ 4 \\ -6 \end{bmatrix} \quad \begin{vmatrix} -3 & 5 & 2 \\ 2 & 4 & -1 \\ -3 & 0 & \boxed{6}^{0} \end{vmatrix} = \begin{vmatrix} -3 & 5 & -4 \\ 2 & 4 & 3 \\ -3 & 0 & 0 \end{vmatrix}$$

$C_3 + 2C_1$

$$-3(-1)^{3+1} \begin{vmatrix} 5 & -4 \\ 4 & 3 \end{vmatrix} = -3(+15+16)$$
$$= -3(-1) = 3$$

Exercise P.124 (1–20, 21, 22, 25–30)

Ex(1) Find

$$\begin{array}{r} \\ R_1 + R_2 \\ R_3 - 2R_1 \end{array} \begin{vmatrix} +1 & 0 & 2 \\ \boxed{-1}^{0} & 1 & 4 \\ \boxed{2}^{0} & 0 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 2 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{vmatrix} = (1)(1)(-1) = -1$$

$[-2 \quad 0 \quad -4]$

22. Find

$$\begin{array}{r} \\ \\ R_3 + R_1 \end{array} \begin{vmatrix} -1 & 3 & 2 \\ 0 & 2 & 0 \\ \boxed{1}^{0} & 1 & -1 \end{vmatrix} = \begin{vmatrix} -1 & 3 & 2 \\ 0 & 2 & 0 \\ 0 & 4 & 1 \end{vmatrix}$$

$$= (-1)(-1)^2 \begin{vmatrix} 2 & 0 \\ 4 & 1 \end{vmatrix}$$
$$= -1(2-0) = -2$$

(25–30) Use elementry row or column to find det.

25) $\begin{vmatrix} 1 & 7 & -3 \\ 1 & 3 & 1 \\ 4 & 8 & 1 \end{vmatrix}$

Solⁿ

$$\begin{array}{r} \\ R_2 - R_1 \\ R_2 - 4R_1 \end{array} \begin{vmatrix} 1 & 7 & -3 \\ \boxed{1}^{0} & 3 & 1 \\ \boxed{4}^{0} & 8 & 1 \end{vmatrix} \begin{array}{l} \\ [-1 \quad -7 \quad 3] \\ [-4 \quad -28 \quad 12] \end{array}$$

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(75)

$$-4 \text{ out of } R_2 \begin{vmatrix} 1 & 7 & -3 \\ 0 & -4 & 4 \\ 0 & -20 & 13 \end{vmatrix}$$

$$(-4)\begin{vmatrix} 1 & 7 & -3 \\ 0 & 1 & -1 \\ 0 & \boxed{-20}^{0} & 13 \end{vmatrix} \quad R_3 + 20R_2 \quad [0 \quad 20 \quad -20]$$

$$(-4)\begin{vmatrix} 1 & 7 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & -7 \end{vmatrix} = (-4)(1)(1)(-7) = 28$$

(26)

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & -2 \\ 1 & -2 & -1 \end{vmatrix} \qquad \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix} \begin{vmatrix} 1 & \boxed{1}^{0} & \boxed{1}^{0} \\ 2 & -1 & -2 \\ 1 & -2 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & -3 & -4 \\ 1 & -3 & -2 \end{vmatrix} = (-3)\begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & \boxed{-4}^{0} \\ 1 & 1 & -2 \end{vmatrix}$$

$C_2 - C_1 \quad C_3 - C_1 \qquad -3 \text{ out of } C_2 \qquad 4C_2 + C_1$

$$= -3\begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 2 \end{vmatrix} = -3(1)(1)(2) = -6$$

(27)

$$R_1 \rightleftarrows R_2 \begin{vmatrix} \boxed{2}^{1} & -1 & -1 \\ 1 & 3 & 2 \\ -6 & 3 & 3 \end{vmatrix}$$

$$\begin{matrix} \\ R_2 - 2R_1 \\ R_3 + 6R_1 \end{matrix} \; -\begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & -1 \\ -6 & 3 & 3 \end{vmatrix} \quad \begin{matrix} \\ [-2 \quad -6 \quad -4] \\ [6 \quad 18 \quad 12] \end{matrix}$$

$$R_3 + 3R_2 \; -\begin{vmatrix} 1 & 3 & 2 \\ 0 & -7 & -5 \\ 0 & 21 & 15 \end{vmatrix} = -\begin{vmatrix} 1 & 3 & 2 \\ 0 & -7 & -5 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$[0 \quad -21 \quad -15]$

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(28) $\begin{vmatrix} 3 & 0 & 6 \\ 2 & -3 & 4 \\ 1 & -2 & 2 \end{vmatrix}$ $\begin{bmatrix} -3 \\ -2 \\ -1 \end{bmatrix}$

$$\begin{vmatrix} 3 & 0 & \boxed{6}^{0} \\ 2 & -3 & 4 \\ 1 & -2 & 2 \end{vmatrix} = 2\begin{vmatrix} 3 & 0 & 3 \\ 2 & -3 & 2 \\ 1 & -2 & 1 \end{vmatrix} = 2\begin{vmatrix} 3 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & -2 & 0 \end{vmatrix}$$

2 out of C_3 $\quad$ $C_3 - C_1$

$= 2(0) = 0$

29) $\begin{vmatrix} 3 & 2 & -3 \\ 7 & 5 & 1 \\ -1 & 2 & 6 \end{vmatrix}$

$R_1 \rightleftarrows R_3$ $\begin{vmatrix} 3 & 2 & -3 \\ 7 & 5 & 1 \\ -1 & 2 & 6 \end{vmatrix}$

$R_2 + 7R_1$
$R_3 + 3R_1$ $\quad - \begin{vmatrix} -1 & 2 & 6 \\ 7 & 5 & 1 \\ 3 & 2 & -3 \end{vmatrix}$ $\begin{matrix} [-7 & 14 & 42] \\ [-3 & 6 & 18] \end{matrix}$

$$- \begin{vmatrix} -1 & 2 & 6 \\ 0 & 19 & 43 \\ 0 & 8 & 15 \end{vmatrix} = (-1)(-1)(-1)^2 \begin{vmatrix} 19 & 43 \\ 8 & 15 \end{vmatrix}$$

$= (1)(19)(15) - (43)(8)$

$= 285 - 344 = -59$

30) $\begin{vmatrix} 3 & 8 & -7 \\ 0 & -5 & 4 \\ 6 & 1 & 6 \end{vmatrix}$

$R_3 - 2R_1$ $\begin{vmatrix} 3 & 8 & -7 \\ 0 & -5 & 4 \\ 6 & 1 & 6 \end{vmatrix}$ $[-6 \quad -16 \quad 14]$

$R_3 - 3R_2$ $\begin{vmatrix} 3 & 8 & -7 \\ 0 & -5 & 4 \\ 0 & -15 & 20 \end{vmatrix}$ $[0 \quad 15 \quad -12]$

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(7 7)

$$\begin{bmatrix} 3 & 8 & -7 \\ 0 & -5 & 4 \\ 0 & 0 & 8 \end{bmatrix} = (3)(-5)(8)$$
$$= -120$$

[1-14] Determine which property of det the eq. illustrates.

1. $\begin{vmatrix} 2 & -6 \\ 1 & -3 \end{vmatrix} = 0$ $R_1 = 2R_2$
$C_2 = -3C_1$

2. $\begin{vmatrix} -4 & 5 \\ 12 & -15 \end{vmatrix} = 0$ $R_2 = -3R_1$

3. $\begin{vmatrix} 1 & 4 & 2 \\ 0 & 0 & 0 \\ 5 & 6 & -7 \end{vmatrix} = 0$ all entries of R_2 are zeros.

4 $\begin{vmatrix} -3 & 2 & 1 \\ 6 & 0 & 0 \\ -3 & 2 & 1 \end{vmatrix} = 0$ $C_2 = 2C_1$

5. $\begin{vmatrix} 1 & 3 & 4 \\ -7 & 2 & -5 \\ 6 & 1 & 2 \end{vmatrix} = -\begin{vmatrix} 1 & 4 & 3 \\ -7 & -5 & 2 \\ 6 & 2 & 1 \end{vmatrix}$

$C_2 \rightleftharpoons C_3$

6. $R_1 \rightleftharpoons R_3$ $\begin{vmatrix} 1 & 3 & 4 & -5 \\ -2 & 2 & 0 & 1 \\ 1 & 6 & 2 & -7 \\ 0 & 5 & 3 & 8 \end{vmatrix} = -\begin{vmatrix} 1 & 6 & 2 & -7 \\ -2 & 2 & 0 & 1 \\ 1 & 3 & 4 & -5 \\ 0 & 5 & 3 & 8 \end{vmatrix}$.

7. $\begin{vmatrix} 5 & 10 \\ 2 & -7 \end{vmatrix} = 5\begin{vmatrix} 1 & 2 \\ 2 & -7 \end{vmatrix}$.

5 out of R_1

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8. $\begin{vmatrix} 9 & 1 \\ 3 & 12 \end{vmatrix} = 3 \begin{vmatrix} 3 & 1 \\ 1 & 12 \end{vmatrix}$

3 out of C_1

9. $\begin{vmatrix} 1 & 8 & -3 \\ 3 & -12 & 6 \\ 7 & 4 & 9 \end{vmatrix} = 12 \begin{vmatrix} 1 & 2 & -1 \\ 3 & -3 & 2 \\ 7 & 1 & 3 \end{vmatrix}$

4 out of C_2
3 out of C_3

10. $\begin{vmatrix} 1 & 2 & 3 \\ 4 & -8 & 6 \\ 5 & 4 & 12 \end{vmatrix} = 6 \begin{vmatrix} 1 & 1 & 1 \\ 4 & -4 & 2 \\ 5 & 2 & 4 \end{vmatrix}$

2 out of C_2
3 out of C_3

11. $\begin{vmatrix} -10 & 5 & 5 \\ 35 & -20 & 25 \\ 0 & 15 & 30 \end{vmatrix} = 5^3 \begin{vmatrix} -2 & 1 & 1 \\ 7 & -4 & 5 \\ 0 & 3 & 6 \end{vmatrix}$

5 out of R_1 & R_2 & R_3

12. $\begin{vmatrix} 6 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 \end{vmatrix} = 6 \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$

6 out of R_1 & R_2 & R_3 & R_4

13. $\begin{vmatrix} 2 & -3 \\ 8 & 7 \end{vmatrix} = \begin{vmatrix} 2 & -3 \\ 0 & 19 \end{vmatrix}$

$-4R_1 + R_2$

14. $\begin{vmatrix} 2 & 1 \\ 0 & -1 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix}$

$-2R_1 + R_2$

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