1. $P(\grave{A}) = 1 - P(A)$

2. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

3. if A, B are event mutually exclusive
$$P(A \cap B) = 0$$

4. $P(A - B) = P(A) - P(A \cap B)$

5. $P(A \cap \grave{B}) = P(A) - P(A \cap B)$

6. $P(A \cup \grave{B}) = 1 + P(A \cap B) - P(B)$

7. $P(A \cap B)^{\grave{}} = 1 - P(A \cap B)$

8. $P(A \cup B)^{\grave{}} = 1 - P(A \cup B)$

9. $P(\grave{A} \cap \grave{B}) = 1 - P(A \cup B)$

10. $P(A' \cup B') = 1 - P(A \cap B)$

11. $P(\bar{A} \cup \bar{B} \cup \grave{C}) = 1 - P(A \cap B \cap C)$

12. $P(\bar{A} \cap \grave{B} \cap \grave{C}) = 1 - P(A \cup B \cup C)$

13. $P(A|B) = \dfrac{P(A \cap B)}{P(A)}$

Notes

1) $P(A) = P(A \cap B) + P(A \cap B^c)$

2) $P(B) = P(A \cap B) + P(A^c \cap B)$

3) $P(A \cap B^c) = P(A) - P(A \cap B)$

4) $P(A^c \cap B) = P(B) - P(A \cap B)$

5) $P(A^c \cap B^c) = 1 - P(A \cup B)$

6) $P(A \cup B) = P(A) + P(A^c \cap B)$

the first is defective (D)

Theorem

Two events A and B are independent if and only if

$$P(A \cap B) = P(A)\,P(B)$$

(Multiplicative Rule for independent events)

Note:

Two events A and B are independent if one of the following conditions is satisfied:

(i) $P(A|B) = P(A)$

$\Leftrightarrow$ (ii) $P(B|A) = P(B)$

$\Leftrightarrow$ (iii) $P(A \cap B) = P(A)\,P(B)$

Solution

X	$f(x)$	$X f(x)$	X^2	$X^2 f(x)$
1	0.1	0.1	1	0.1
2	0.3	0.6	4	1.2
3	0.2	0.6	9	1.8
4	0.3	1.2	16	4.8
5	0.1	0.5	25	2.5
Total		$\Sigma X f(x) = 3.0$		$\Sigma X^2 f(x) = 10.4$

$$E(X) = \Sigma X f(x) = 3$$

$$E(X^2) = \Sigma X^2 f(x) = 10.4$$

$$Var(X) = E(X^2) - [E(X)]^2 = 10.4 - (3)^2 = 1.4$$

$$std(X) = \sqrt{Var(X)} = \sqrt{1.4} = 1.18$$

Continuous Distributions

Probability density function	$f(x) = F'(x)$ $P\{a < X < b\} = \int_a^b f(x)dx$

and any experiment with a binary outcome is called a Bernoulli trial.

Bernoulli distribution

Bernoulli distribution

$$
\begin{aligned}
p &= \text{probability of success} \\
P(x) &= \begin{cases} q = 1 - p & \text{if } x = 0 \\ p & \text{if } x = 1 \end{cases} \\
\mathrm{E}(X) &= p \\
\mathrm{Var}(X) &= pq
\end{aligned}
$$

Binomial distribution

Now consider a sequence of independent Bernoulli trials and count the number of successes in it. This may be the number of defective computers in a shipment, the number of updated files in a folder, the number of girls in a family, the number of e-mails with attachments, etc.

Binomial distribution

Binomial probability mass function is

$$P(x) = P\{X = x\} = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, \ldots, n,$$

$P(2) = \binom{10}{2} 0.2^2 0.8^{10-2} = 45 \times 0.2^2 \times 0.8^8 =$

$P(3) = \binom{10}{3} 0.2^3 0.8^{10-3} = 120 \times 0.2^3 \times 0.8^7 =$
0.20133

$F(3) = 0.10737 + 0.26844 + 0.30199 + 0.20133 =$
0.8791

$P(X \geq 4) = 1 - F(3) = 1 - 0.8791 = 0.1209$

Binomial distribution

Binomial distribution	n = number of trials p = probability of success $P(x) = \binom{n}{x} p^x q^{n-x}$ $E(X) = np$ $Var(X) = npq$

Binomial distribution

An interactive system consists of 8terminals that are connected to the central computer.
At any time, each terminal is ready to

Poisson distribution

DEFINITION

The number of rare events occurring within a fixed period of time has Poisson distribution.

Poisson distribution	$\lambda = $ frequency, average number of events $P(x) = e^{-\lambda}\frac{\lambda^x}{x!},\ x = 0, 1, 2, \ldots$ $\mathrm{E}(X) = \lambda$ $\mathrm{Var}(X) = \lambda$

Families of continuous distributions

Uniform distribution

$$
\begin{aligned}
(a, b) &= \text{range of values} \\
f(x) &= \frac{1}{b-a}, \quad a < x < b \\
\mathbf{E}(X) &= \frac{a+b}{2} \\
\mathrm{Var}(X) &= \frac{(b-a)^2}{12}
\end{aligned}
$$

Families of continuous distributions

Exponential distribution	λ	$=$	frequency parameter, the number of events per time unit
	$f(x)$	$=$	$\lambda e^{-\lambda x}, \quad x > 0$
	$\mathbf{E}(X)$	$=$	$\frac{1}{\lambda}$
	$\mathrm{Var}(X)$	$=$	$\frac{1}{\lambda^2}$

The quantity λ is a parameter of Exponential distribution,

Normal distribution

Normal distribution

$$\begin{aligned}
\mu &= \text{expectation, location parameter} \\
\sigma &= \text{standard deviation, scale parameter} \\
f(x) &= \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{\frac{-(x-\mu)^2}{2\sigma^2}\right\}, \ -\infty < x < \infty \\
\mathrm{E}(X) &= \mu \\
\mathrm{Var}(X) &= \sigma^2
\end{aligned}$$