

الاسم: وعاء من الرصاص

الرقم الجامعي: ١٢٣٤٥٦٧٨٩

1-Evaluate the limit, if it exists

$$a-\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2}$$

$$b-\lim_{t \rightarrow -3} \frac{t^2 - 9}{2t^2 + 7t + 3}$$

$$a) \lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2} = \frac{2^2 + 2 - 6}{2 - 2} = \frac{0}{0}$$

$$\lim_{x \rightarrow 2} \frac{(x-2)(x+3)}{(x-2)} = \lim_{x \rightarrow 2} (x+3)$$

$$= 2 + 3 = 5$$

$$b) \lim_{T \rightarrow -3} \frac{T^2 - 9}{2T^2 + 7T + 3} = \frac{(-3)^2 - 9}{2(-3)^2 + 7(-3) + 3} = \frac{0}{0}$$

$$= \lim_{T \rightarrow -3} \frac{(T-3)(T+3)}{(2T+1)(T+3)}$$

$$= \lim_{T \rightarrow -3} \frac{T-3}{2T+1} = \frac{(-3)-3}{2(-3)+1} = \frac{-6}{-6+1} = \frac{-6}{-5} = \frac{6}{5}$$

2-Use the definition of continuity and the properties of limits to show that the function is continuous at the given number a :

$$f(x) = (x + 2x^3)^4, a = -1$$

$$f(a) = f(-1) = (-1 + 2(-1)^3)^4 = (-1 - 2)^4 = (-3)^4 = 81$$

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} (x + 2x^3)^4 = (a + 2(a)^3)^4$$

$$= (-1 + 2(-1)^3)^4 = 81$$

$$f(a) = \lim_{x \rightarrow a} f(x)$$

Function is continuous

3- Explain why the function is discontinuous at the given number a :

$$a) f(x) = \begin{cases} \frac{1}{x} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}, a = 1$$

$$b) f(x) = \begin{cases} e^x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}, a = 0$$

$$a) f(1) = 2$$

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{1}{x} = \frac{1}{1} = 1$$

$$f(1) \neq \lim_{x \rightarrow 1} f(x)$$

Dis continuous

$$b) \textcircled{1} f(0) = x^2 = 0^2 = 0$$

$$\textcircled{2} \lim_{x \rightarrow 0^+} x^2 = 0^2 = 0$$

$$\textcircled{3} \lim_{x \rightarrow 0^-} e^x = e^0 = 1$$

$$\lim_{x \rightarrow 0^+} f(x) \neq \lim_{x \rightarrow 0^-} f(x) \Rightarrow \text{Dis continuous}$$