(55)

إنشاء مبرهنة درجة

2.3 Inverses of Matrices :-

An $n\times n$ matrix is invertible (or nonsingular) when there exists an $n\times n$ matrix B s.t

$$AB = BA = I_n$$

where I_n is the identity matrix of order n

The matrix B is the inverse of A. The matrix does not have an inverse is noninvertible or singular

If A is invertible matrix then its inverse is unique

the inverse of A is denoted by A^{-1}

Ex (1) show that B is the inverse of A where

$$A = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$$

sol

$$AB = \begin{bmatrix} -1+2 & 2-2 \\ -1+1 & 2-1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$$BA = \begin{bmatrix} -1+2 & 2-2 \\ -1+1 & 2-1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$\therefore$ B inverse to A.

To Find inverse of $n\times n$ matrix A

$$[A \;\; I_n] \leftrightarrow [I_n \;\; A^{-1}]$$

Ex(2) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix}$$

$$R_1 + R_2 \quad \left[\begin{array}{cc|cc} \boxed{1} & 4 & 1 & 0 \\ \boxed{-1} & -3 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} 1 & \boxed{4} & 1 & 0 \\ 0 & \boxed{1} & 1 & 1 \end{array}\right] \quad [0\ -4 : -4\ -4]$$

$$\quad A \qquad\qquad I$$

$$R_1 - 4R_2 \Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & -3 & -4 \\ 0 & 1 & 1 & 1 \end{array}\right] \quad \therefore A^{-1} = \begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix}$$

$$\quad I \qquad\qquad A^{-1}$$

Ex(3) Find the inverse of $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$

$$\begin{array}{r} \\ R_2 - R_1 \\ R_3 + 6R_1 \end{array} \left[\begin{array}{ccc|ccc} \boxed{1} & -1 & 0 & 1 & 0 & 0 \\ \boxed{1} & 0 & -1 & 0 & 1 & 0 \\ \boxed{-6} & 2 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ [-1\ +1\ 0 : -1\ 0\ 0] \\ [6\ -6\ 0 ; 6\ 0\ 0] \end{array}$$

$$\quad A \qquad\qquad I$$

$$R_3 + 4R_2 \quad \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & \boxed{1} & -1 & -1 & 1 & 0 \\ 0 & \boxed{-4} & 3 & 6 & 0 & 1 \end{array}\right] [0\ 4\ -4 : -4\ 4\ 0]$$

$$-R_3 \quad \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & \boxed{-1} & 2 & 4 & 1 \end{array}\right] = \qquad A^{-1} = \begin{bmatrix} -2 & -3 & -1 \\ -3 & -3 & -1 \\ -2 & -4 & -1 \end{bmatrix}$$

$$R_1 + R_2 \quad \left[\begin{array}{ccc|ccc} 1 & \boxed{-1} & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & -4 & -1 \end{array}\right] \qquad I \qquad A^{-1}$$

$$\begin{array}{r} R_1 + R_3 \\ R_2 + R_3 \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & \boxed{-1} & 0 & 1 & 0 \\ 0 & 1 & \boxed{-1} & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & -4 & -1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -3 & -1 \\ 0 & 1 & 0 & -3 & -3 & -1 \\ 0 & 0 & 1 & -2 & -4 & -1 \end{array}\right]$$

Ex(7) Show that the matrix has no inverse

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 2 \\ -2 & 3 & -2 \end{bmatrix}$$

$$\begin{array}{l} \\ R_2 - 3R_1 \\ R_3 + 2R_1 \end{array} \left[\begin{array}{ccc|ccc} \boxed{1} & 2 & 0 & 1 & 0 & 0 \\ \boxed{3} & -1 & 2 & 0 & 1 & 0 \\ \boxed{-2} & 3 & -2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ [-3 \ -6 \ 0 : -3 \ 0 \ 0] \\ [+2 \ +4 \ 0 : +2 \ 0 \ 0] \end{array}$$

$$\frac{R_2}{-7} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & \boxed{-7} & 2 & -3 & 1 & 0 \\ 0 & \boxed{7} & -2 & 2 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} \\ \\ R_3 - 7R_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & -\frac{2}{7} & \frac{3}{7} & -\frac{1}{7} & 0 \\ 0 & \boxed{7} & -2 & 2 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ [0 \ -7 \ 2 : -3 \ 1 \ 0] \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & -\frac{2}{7} & \frac{3}{7} & -\frac{1}{7} & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \end{array}\right]$$

$\neq I \Rightarrow$ no inverse.

~~If A is 2x2 matrix s.t~~ $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible

$\Leftrightarrow ad - cb \neq 0$ & $A^{-1} = \dfrac{1}{ad-cb} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Ex 5 Find the inverse if possible for

(i) $A = \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$ $\quad (3)(2) - (-1)(-2) = 6 - 2 = 4 \neq 0$

there is an A^{-1} s.t

$$A^{-1} = \begin{bmatrix} \frac{2}{4} & \frac{1}{4} \\ \frac{2}{4} & \frac{3}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{3}{4} \end{bmatrix}$$

(b) $B = \begin{bmatrix} 3 & -1 \\ -6 & 2 \end{bmatrix}$ $3\times 2 - (-1)(-6) = 6 - 6 = 0$

there is no B^{-1}

Properties of inverse matrices :-

If A is invertible matrix, k is a +ve integer $c \neq 0$, $c \in \mathbb{R}$ then A^{-1}, A^{k}, cA, A^{T} are invertible &

1. $(A^{-1})^{-1} = A$ 2. $(A^{k})^{-1} = \underbrace{A^{-1} \cdot A^{-1} \cdots A^{-1}}_{k \text{ factors}} = (A^{-1})^{k}$

3. $(cA)^{-1} = \frac{1}{c} A^{-1}$ 4. $(A^{T})^{-1} = (A^{-1})^{T}$

Ex 6 Compute A^{-1} by two different way where

$A = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}$ $(1)(4) - (1)(2) = 4 - 2 = 2 \neq 0$

$\Rightarrow A^{-1} = \begin{bmatrix} \frac{4}{2} & -\frac{1}{2} \\ -\frac{2}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 & -\frac{1}{2} \\ -1 & \frac{1}{2} \end{bmatrix}$

(i) $(A^{-1})^{2} = \begin{bmatrix} 2 & -\frac{1}{2} \\ -1 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 2 & -\frac{1}{2} \\ -1 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 4+\frac{1}{2} & -1-\frac{1}{4} \\ -2-\frac{1}{2} & \frac{1}{2}+\frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{9}{2} & -\frac{5}{4} \\ -\frac{5}{2} & \frac{3}{4} \end{bmatrix}$

(ii) $(A^{2})^{-1} = \begin{bmatrix} 1+2 & 1+4 \\ 2+8 & 2+16 \end{bmatrix}^{-1} = \begin{bmatrix} 3 & 5 \\ 10 & 18 \end{bmatrix}^{-1}$ $3\times 18 - 5\times 10$

$54 - 50 = 4 \neq 0$

$= \begin{bmatrix} \frac{18}{4} & -\frac{5}{4} \\ -\frac{10}{4} & \frac{3}{4} \end{bmatrix} = \begin{bmatrix} \frac{9}{2} & -\frac{5}{4} \\ -\frac{5}{2} & \frac{3}{4} \end{bmatrix}$

$\therefore (A^{-1})^{2} = (A^{2})^{-1}$

If A & B are invertible matrices of order n then AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$

Ex 7 Find $(AB)^{-1}$ for

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \quad B^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 1 & 0 \\ \frac{2}{3} & 0 & -\frac{1}{3} \end{bmatrix}$$

$$\therefore (AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 1 & 0 \\ \frac{2}{3} & 0 & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7+2-1 & -3-2+0 & -3+0+1 \\ -7-1+0 & 3+1+0 & 3+0+0 \\ \frac{14}{3}+0+\frac{1}{3} & -2+0+0 & -2+0-\frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -5 & -2 \\ -8 & 4 & 3 \\ 5 & -2 & -\frac{7}{3} \end{bmatrix}$$

If C is invertible matrix then

1) if $AC = BC \Rightarrow A = B$

2) $CA = CB \Rightarrow A = B$

If A is invertible matrix, then the system of linear eqs $Ax=b$ has a unique sol^n $x=A^{-1}b$

Ex 8 use inverse matrix to solve

a. $2x+3y+z=-1$
$3x+3y+z=1$
$2x+4y+z=-2$

b. $2x+3y+z=4$
$3x+3y+z=8$
$2x+4y+z=5$

c. $2x+3y+z=0$
$3x+3y+z=0$
$2x+4y+z=0$

sol

all systems have $A=\begin{bmatrix} 2 & 3 & 1 \\ 3 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}$

to find A^{-1}:

$$\begin{array}{c} \\ \frac{R_1}{2} \\ \\ \\ \end{array} \begin{array}{c} \quad A \qquad\qquad\qquad I \\ \left[\begin{array}{ccc|ccc} \boxed{2}^{1} & 3 & 1 & 1 & 0 & 0 \\ 3 & 3 & 1 & 0 & 1 & 0 \\ 2 & 4 & 1 & 0 & 0 & 1 \end{array}\right] \end{array}$$

$$\begin{array}{c} \\ R_2-3R_1 \\ R_3-2R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \boxed{3}^{0} & 3 & 1 & 0 & 1 & 0 \\ \boxed{2}^{0} & 4 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} [-3 \;\; -\frac{9}{2} \;\; -\frac{3}{2} \;\; \vdots \;\; -\frac{3}{2} \;\; 0 \;\; 0] \\ [-2 \;\; -3 \;\; -1 \;\; \vdots \;\; -1 \;\; 0 \;\; 0] \end{array}$$

$$\begin{array}{c} \\ -\frac{2}{3}R_2 \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \boxed{-\frac{3}{2}}^{1} & -\frac{1}{2} & -\frac{3}{2} & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \end{array}\right]$$

$$\begin{array}{c} \\ \\ R_3-R_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{3} & 1 & -\frac{2}{3} & 0 \\ 0 & \boxed{1}^{0} & 0 & -1 & 0 & 1 \end{array}\right] [0 \;\; -1 \;\; -\frac{1}{3} \;\; \vdots \;\; -1 \;\; \frac{2}{3} \;\; 0]$$

$$\begin{array}{c} \\ \\ -3R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{3} & 1 & -\frac{2}{3} & 0 \\ 0 & 0 & -\frac{1}{3} & -2 & \frac{2}{3} & 1 \end{array}\right]$$

$$R_1 - \tfrac{3}{2} R_2 \quad \begin{bmatrix} 1 & \boxed{\tfrac{3}{2}}^{\,0} & \tfrac{1}{2} & \vdots & \tfrac{1}{2} & 0 & 0 \\ 0 & 1 & \tfrac{1}{3} & \vdots & 1 & -\tfrac{2}{3} & 0 \\ 0 & 0 & 1 & \vdots & 6 & -2 & -3 \end{bmatrix} \quad [\,0 \;\; -\tfrac{3}{2} \;\; -\tfrac{1}{2} \;\vdots\; -\tfrac{3}{2} \;\; 1 \;\; 0\,]$$

$$R_2 - \tfrac{1}{3} R_3 \quad \begin{bmatrix} 1 & 0 & 0 & \vdots & -1 & 1 & 0 \\ 0 & 1 & \boxed{\tfrac{1}{3}}^{\,0} & \vdots & 1 & -\tfrac{2}{3} & 0 \\ 0 & 0 & 1 & \vdots & 6 & -2 & -3 \end{bmatrix} \quad [\,0 \;\; 0 \;\; -\tfrac{1}{3} \;\vdots\; -2 \;\; \tfrac{2}{3} \;\; 1\,]$$

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & -1 & 1 & 0 \\ 0 & 1 & 0 & \vdots & -1 & 0 & 1 \\ 0 & 0 & 1 & \vdots & 6 & -2 & -3 \end{bmatrix}$$

$$\qquad I \qquad\qquad\qquad A^{-1}$$

Sol^n of a sy. in a. is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \overset{A^{-1}}{\begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 6 & -2 & -3 \end{bmatrix}} \overset{b}{\begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}} = \begin{bmatrix} 1+1+0 \\ 1+0-2 \\ -6-2+6 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$$

in b.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 6 & -2 & -3 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \\ 5 \end{bmatrix} = \begin{bmatrix} -4+8+0 \\ -4+0+5 \\ 24-16-15 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ -7 \end{bmatrix}$$

in C.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 6 & -2 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

6. show that B is inverse to A where

$$A = \begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2-34+33 & 2-68+66 & 4+51-55 \\ -1+22-21 & -1+44-42 & -2-33+35 \\ 0+6-6 & 0+12-12 & 0-9+10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \therefore \text{ A \& B are inverse to eachother}$$

Find the inverse of the matrix if exist

11) $\begin{bmatrix} -7 & 33 \\ 4 & -19 \end{bmatrix}$ $\quad (-7)(-19) - (33)(4)$

$= 133 - 132 = 1 \neq 0 \Rightarrow$ inverse exist

the inverse is $\begin{bmatrix} \frac{-19}{1} & \frac{-33}{1} \\ \frac{-4}{1} & \frac{-7}{1} \end{bmatrix} = \begin{bmatrix} -19 & -33 \\ -4 & -7 \end{bmatrix}$

12) $\begin{bmatrix} -1 & 1 \\ 3 & -3 \end{bmatrix} \Rightarrow (-1)(-3) - (1)(3) = 3 - 3 = 0$

no inverse

(13) $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{bmatrix}$

$$\begin{array}{l} \\ R_2 - 3R_1 \\ R_3 - 3R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ \boxed{3}^{0} & 5 & 4 & 0 & 1 & 0 \\ \boxed{3}^{0} & 6 & 5 & 0 & 0 & 1 \end{array}\right] \left[-3 \;\; -3 \;\; -3 \; \vdots \; -3 \;\; 0 \;\; 0\right]$$

$$\frac{R_2}{2} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & \boxed{2}^{1} & 1 & -3 & 1 & 0 \\ 0 & 3 & 2 & -3 & 0 & 1 \end{array}\right]$$

$$R_3 - 3R_2 \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & \boxed{3}^{0} & 2 & -3 & 0 & 1 \end{array}\right] \left[0 \;\; -3 \;\; -\frac{3}{2} \; \vdots \; \frac{9}{2} \;\; -\frac{3}{2} \;\; 0\right]$$

$$2R_3 \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{3}{2} & \frac{1}{2} & 1 \\ 0 & 0 & \boxed{\frac{1}{2}}^{1} & \frac{3}{2} & -\frac{3}{2} & 1 \end{array}\right]$$

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$R_1 - R_2$ $\left[\begin{array}{ccc|ccc} 1 & \boxed{1}^{0} & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{3}{2} & \frac{1}{2} & 1 \\ 0 & 0 & 1 & 3 & -3 & 2 \end{array}\right]$ $[0 \;\; -1 \;\; -\frac{1}{2} \;\vdots\; \frac{3}{2} \;\; -\frac{1}{2} \;\; -1]$

$R_1 - \frac{1}{2}R_3$
$R_2 - \frac{1}{2}R_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & \boxed{\frac{1}{2}} & \frac{5}{2} & -\frac{1}{2} & -1 \\ 0 & 1 & \boxed{\frac{1}{2}} & -\frac{3}{2} & \frac{1}{2} & 1 \\ 0 & 0 & 1 & 3 & -3 & 2 \end{array}\right]$ $[0 \;\; 0 \;\; -\frac{1}{2} \;\vdots\; -\frac{3}{2} \;\; \frac{3}{2} \;\; -1]$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -2 \\ 0 & 1 & 0 & -3 & 2 & 0 \\ 0 & 0 & 1 & 3 & -3 & 2 \end{array}\right]$$

$I \qquad A^{-1}$

(44) if $A^{-1} = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 1 & 3 \\ 4 & 0 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 6 & 5 & -3 \\ -2 & 4 & -1 \\ 1 & 3 & 4 \end{bmatrix}$

Find

(a) $(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 6+0-12 & -24+5+0 & 12+15-3 \\ -2+0-4 & 8+4+0 & -4+12-1 \\ 1+0+16 & -4+3+0 & 2+9+4 \end{bmatrix}$

$= \begin{bmatrix} -6 & -19 & 24 \\ -6 & 12 & 7 \\ 17 & -1 & 15 \end{bmatrix}$

(b) $(A^T)^{-1} = (A^{-1})^T$

$= \begin{bmatrix} 1 & 0 & 4 \\ -4 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}$

(c) $(2A)^{-1} = \frac{1}{2}A^{-1} = \begin{bmatrix} \frac{1}{2} & -2 & 1 \\ 0 & \frac{1}{2} & \frac{3}{2} \\ 2 & 0 & \frac{1}{2} \end{bmatrix}$

Use an inverse to solve the following system

46 (a) $2x - y = -3$ $\quad$ (b) $2x - y = -1$

$\quad\quad 3x + y = 7$ $\quad\quad\quad 3x + y = -3$

For both $A = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} \Rightarrow 2 - (-3) = 5 \neq 0$

$\Rightarrow A^{-1}$ exist

$\Rightarrow$ one solution

$$\Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ -\frac{3}{5} & \frac{2}{5} \end{bmatrix}$$

(a) $\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}b = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ -\frac{3}{5} & \frac{2}{5} \end{bmatrix} \begin{bmatrix} -3 \\ 7 \end{bmatrix} = \begin{bmatrix} -\frac{3}{5} + \frac{7}{5} \\ \frac{9}{5} + \frac{14}{5} \end{bmatrix} = \begin{bmatrix} \frac{4}{5} \\ \frac{23}{5} \end{bmatrix}$

(b) $\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}b = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ -\frac{3}{5} & \frac{2}{5} \end{bmatrix} \begin{bmatrix} -1 \\ -3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{5} - \frac{3}{5} \\ \frac{3}{5} - \frac{6}{5} \end{bmatrix} = \begin{bmatrix} -\frac{4}{5} \\ -\frac{3}{5} \end{bmatrix}$