Assignment 2

[1] Consider a Boolean Algebra <B,+,.,0,1> Show that

(a) x.y + x'.z + yz = x.y + x'.z

$xy+x'z+yz=xy+x'z+(1)yz$

$=xy+x'z+(x+x')yz$

$=xy+x'z+(xyz+x'yz)$

$=(xy+xyz)+(x'z+x'yz)$

$=xy(1+z)+x'z(1+y)$

$=xy(1)+x'z(1)$

$=xy+x'z$

(b) (a + b)'(a' + b)' = 0

(a + b)'(a' + b)'

=(a' .b') (a.b')

= a'.a.b'

=0. b'

= 0

[2] Algebraically simply the following expressions:

(a) F = abc + a'bc + a'b'c +abc' + a'b'c'

= bc(a'+a) + a ' c ' (b '+ b) + abc'

= bc+ a'c'+ abc'

(b) F = y(wz' + wz) + xy

=yw (z'+z)+xy

=yw + xy

=y(w+x)

[3] Simplify the following Boolean expressions to a minimum number of literals :

(a) $xy + x\bar{y}$

$= x(y + \bar{y})$

$= x(1) = x$

(b) $(x+y)(x+\bar{y})$

$= x+xy+x\bar{y}+y\bar{y}$

$=x(1+y+\bar{y})+0$

$=x$

(c) $xyz+\bar{x}y+xy\bar{z}$

$=xy(z+\bar{z})+\bar{x}y$

$=xy(1)+\bar{x}y$

$=y(x+\bar{x})$

$=y$

(d) $(\overline{A+B})(\bar{A}+\bar{B})$

$=\overline{AB}\ (\bar{A}+\bar{B})$

$=\overline{AAB}+\overline{AB}\,\bar{B}$

$=\overline{AB}+\overline{AB}$

$=\overline{AB}$

[4] Reduce the following Boolean expressions to the indicated number of literals :

(a) $\bar{A}\bar{C}+ABC+A\bar{C}$ to three literals.

$=\bar{C}(\bar{A}+A)+ABC$

$=\bar{C}+ABC$

$=\bar{C}+AB$

(b) $\overline{(\bar{x}\bar{y}+z)}+z+xy+wx$ to three literals.

$=(\overline{\bar{x}\bar{y}})\,\bar{z}+z+xy+wz$

$=[(x+y)\,\bar{z}+z]+xy+wz$

$=(z+\bar{z})(z+x+y)+xy+wz$

$=z+wz+x+xy+y$

$=z(1+w)+x(1+y)+y$

$=x+y+z$

(c) $\bar{A}B(\bar{D}+\bar{C}D)+B(A+\bar{A}CD)$ to one literals.

$=B(AD+\overline{AC}\ D+A+\bar{A}CD)$

$=B(\overline{AD}+A+\bar{A}D(C+\bar{C}))$

$=B(A+\bar{A}(\bar{D}+D))$

$=B(A+\bar{A})$

$=B$

(d) $(\bar{A}+C)(\bar{A}+\bar{C})(A+B+\bar{C}D)$ to four literals.

$=(\bar{A}\bar{A} + \bar{A}\bar{C} + \bar{A}C + C\bar{C})(A + B + \bar{C}D)$

$=(\bar{A} + \bar{A}\bar{C} + \bar{A}C)(A + B + \bar{C}D)$

$= \bar{A}(\bar{A} + \bar{C} + C)(A + B + \bar{C}D)$

$=\bar{A}(\bar{A}+1)(A + B + \bar{C}D)$

$=(\bar{A}A + \bar{A}B + \overline{A\bar{C}}D)$

$= \bar{A}B+\bar{A}\bar{C}D$

$=\bar{A}(B + \bar{C}D)$

[5] Find the complement of the following expressions :

(a) $x\bar{y}+\bar{x}y$

$F=(x\bar{y} + \bar{x}y)'$

=(x'+y).(x+y')

=x.x'+ x'.y' +y.x +y.y'

= 0+x'y' + x.y +0

=x'y' + xy

(b) $(a+c)(a+\bar{b})(\bar{a}+b+\bar{c})$

$=\overline{[(a+c)(a+\bar{b})(\bar{a}+b+\bar{c})]}$

$=\overline{(a+c)}+\overline{(a+\bar{b})}+\overline{(\bar{a}+b+\bar{c})}$

$=(\bar{a}.\bar{c}) + (\bar{a}.b) + (a.\bar{b}.c)$

(c) $z+\bar{z}(\bar{v}w+xy)$

$=\overline{z + \bar{z}(\bar{v}w + xy)}$

$=\bar{z}.[z + \overline{(\bar{v}w + xy)}]$

$=\bar{z}.[z + ((v + \bar{w}).(\bar{x} + \bar{y}))]$

$= z\bar{z} + \bar{z}.(v + \bar{w}).(\bar{x} + \bar{y})$

$=0+\bar{z}.(v + \bar{w}).(\bar{x} + \bar{y})$

$=\bar{z}.(v + \bar{w}).(\bar{x} + \bar{y})$

(d) $(w+\bar{y}z)(x+\bar{w}z)$

$=\overline{(w + \bar{y}z)(x + \bar{w}z)}$

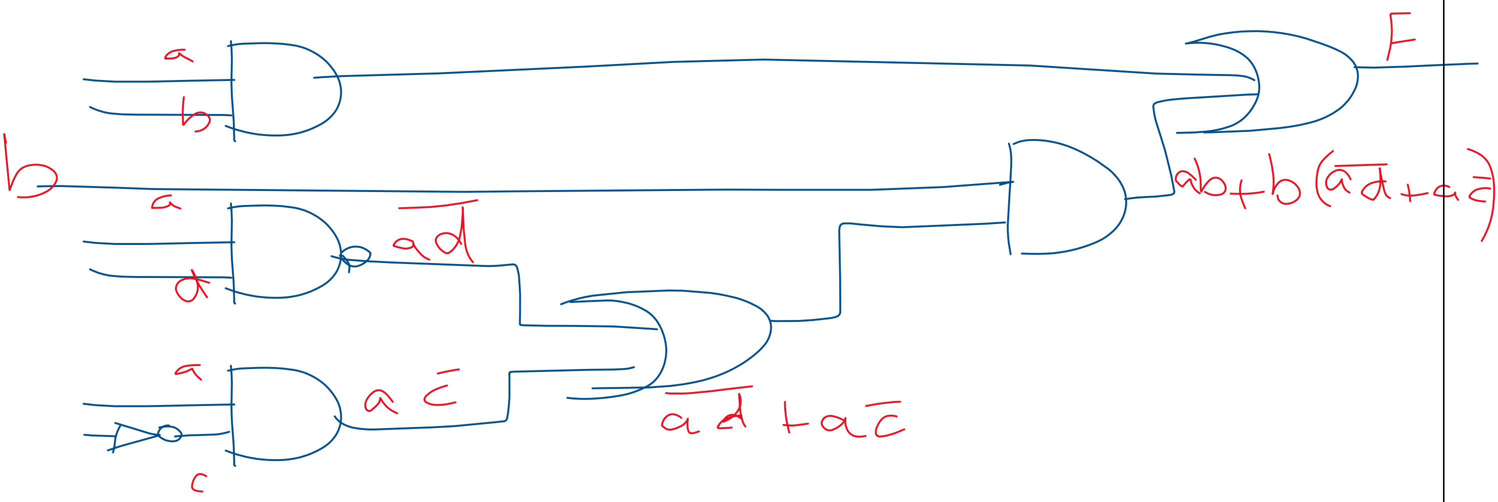
$=\overline{(w + \bar{y}z)} + \overline{(x + \bar{w}z)}$

$=(\bar{w}.(\overline{(\bar{y})z})) + (\bar{x}.\overline{(\bar{w}z)})$

$=(\bar{w}.(y + \bar{z})) + (\bar{x}.(w + \bar{z}))$

$=\overline{wy} + \overline{wz} + \bar{x}w + \overline{xz}$

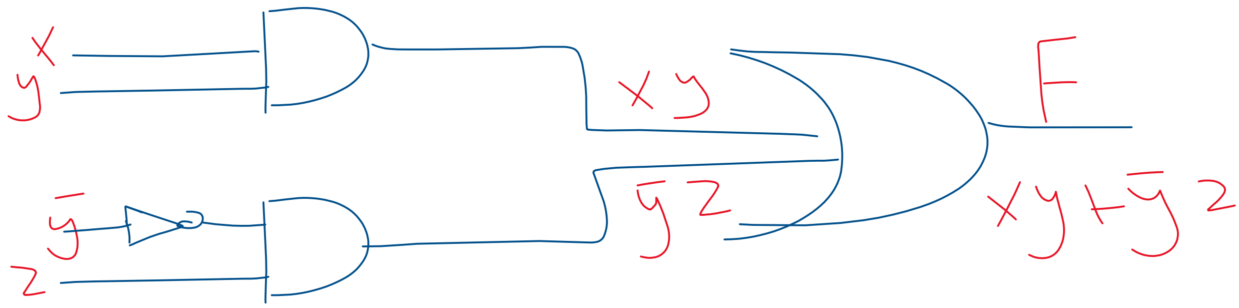
[6] Given F = ab + b($\overline{ad}$+ a$\bar{c}$) implement it with AND, OR and inverter gates.



[7] Implement the Boolean function

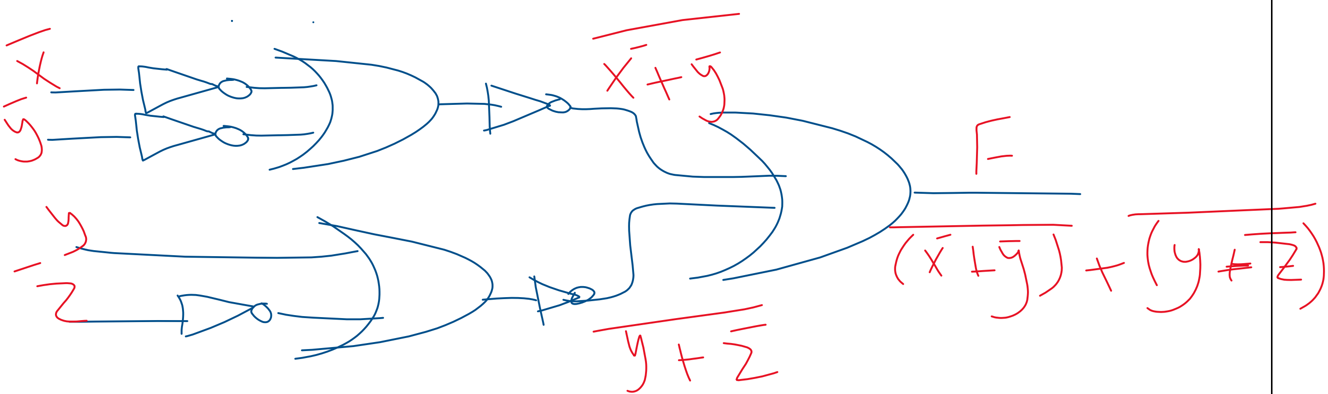
$$F = xy + \bar{y}z$$

(a) With AND, OR, and inverter gates



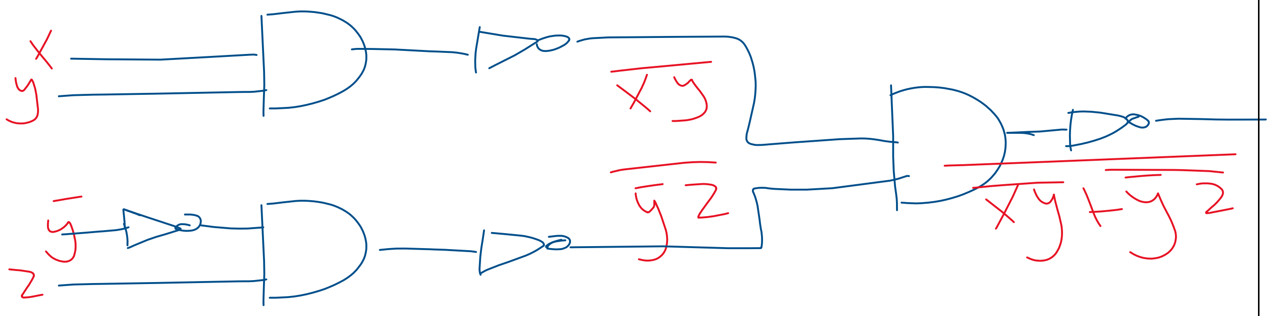
(b) With OR and inverter gates

= $\overline{(\bar{x} + \bar{y})} + \overline{(y + \bar{z})}$



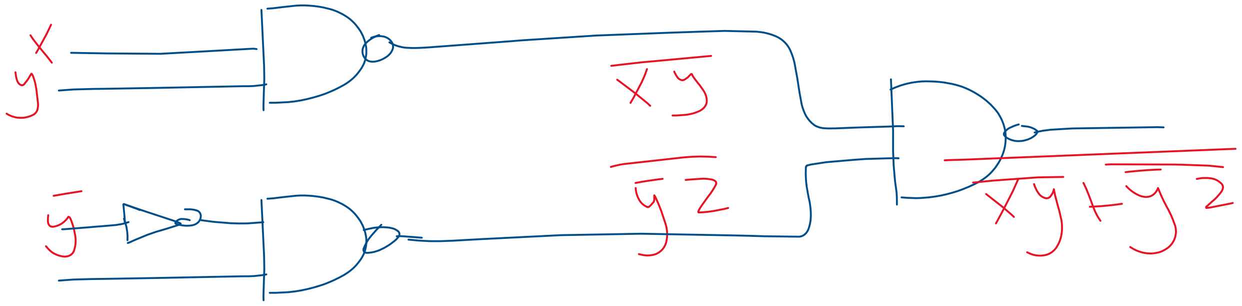
(c) With AND and inverter gates

$$= \overline{\overline{(xy)} \,.\, \overline{(\bar{y}z)}}$$



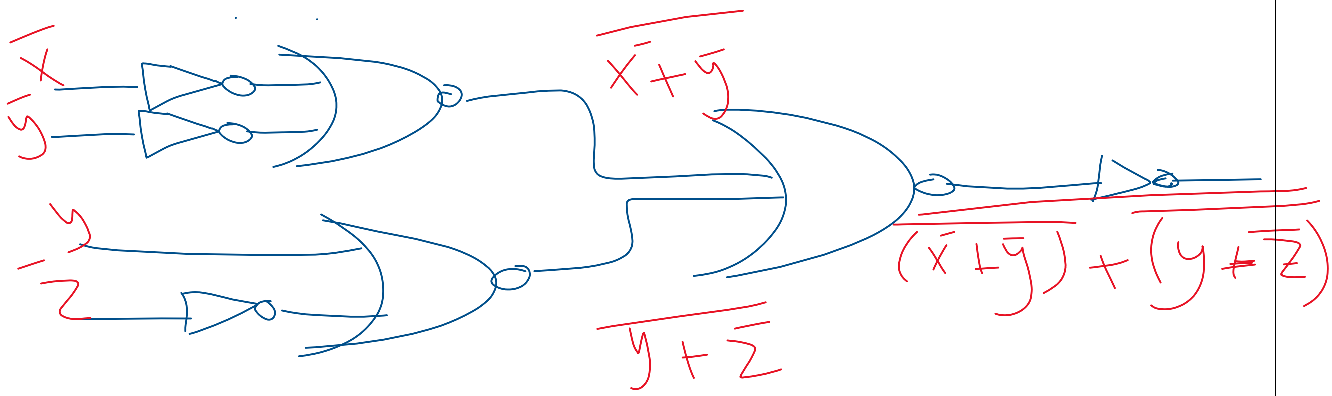
(d) With NAND and inverter gates

$$= \overline{\overline{(xy)} \,.\, \overline{(\bar{y}z)}}$$



(e) With NOR and inverter gates

$= \overline{(\bar{x} + \bar{y})} + \overline{(y + \bar{z})} = \overline{\overline{(\bar{x} + \bar{y})} + \overline{(y + \bar{z})}}$



[8] Express the following function as a sum of minterms and as a product of

$F(A,B,C,D) = \bar{B}\ D + \bar{A}D + BD$

$=D(\bar{B}\ + \bar{A} + B)= D\bar{A}$

A	$\bar{A}$	B	C	D	$F=D\bar{A}$
0	1	0	0	0	0
0	1	0	0	1	1
0	1	0	1	0	0
0	1	0	1	1	1
0	1	1	0	0	0
0	1	1	0	1	1
0	1	1	1	0	0
0	1	1	1	1	1
1	0	0	0	0	0
1	0	0	0	1	0
1	0	0	1	0	0
1	0	0	1	1	0
1	0	1	0	0	0
1	0	1	0	1	0
1	0	1	1	0	0
1	0	1	1	1	0

F(A, B, C, D) = Σ(1,3,5,7)

F(A, B, C, D) = Π(0,2,4,6,8,9,10,11,12,13,14,15)

[9] Express the complement of the following functions in sum-of-minterms form :

(a) $F(A,B\ ,C,D) = \sum(2,4,7,10,12,14)$

$\bar{F}$ (A,B,C,D) = ∑ (0,1,3,4,5,6,8,9,11,13,15)

(b) $F(\ x,y,z) = \prod(3,5,7)$

$\bar{F}\ (\ x,y,z) = \sum(3,5,7)$

[10] Convert each of the following to the other canonical form :

(a) $F(x,y,z) = \sum(1,3,5)$

F(x,y,z) = $\prod(0,2,4,6,7)$

(b) $F(A,B,C,D) = \prod(3, 5, 8, 11)$

F(A,B,C,D) = $\sum(0,1,2,4,6,7,9,10,12,13,14,15)$

[11] Convert each of the following expressions into sum of products and product of sums :

(a) $F = (AB + C)(B + \bar{C}D)$

$= ABB + AB\bar{C}D + CB + C\bar{C}D$

$= AB + AB\bar{C}D + CB + 0$

$= AB(1 + \bar{C}D) + CB$

$= AB + CB$ (SOP)

=B(A+C) (POS)

(b) $F = \overline{x} + x(x + \bar{y})(y + \bar{z})$

$= \overline{x} + \overline{z}x + xy + zx\overline{y} + xy\overline{y}$

$= \bar{x} + \bar{z}x(1 + \bar{y}) + xy$

$= \bar{x} + \bar{z}x + xy$

$= \bar{x} + \bar{z}x + \bar{x} + xy$

$= \bar{x} + \bar{z} + \bar{x} + y$

$= \bar{x} + \bar{y} + z$ (SOP) & (POS)

Where we used

$\bar{x}$ +xy

= $\bar{x}$ (y+ $\bar{y}$)+xy

= x (y+y+ $\bar{y}$)+xy

= $\bar{x}$ (y+ $\bar{y}$)+ $\bar{x}$ y+xy

= $\bar{x}$ +($\bar{x}$ +x)y

= $\bar{x}$ +y

[12] Express the following functions as CSOP and CPOS expressions:

(a) F(x,y,z) = (x + y' + z)(x + z') + yz in terms of x, y, and z

=xx+x$\bar{z}$ + $\bar{y}$ x+$\bar{y}$ $\bar{z}$ +zx +z$\bar{z}$ + yz

=x(1+$\bar{z}$ + $\bar{y}$)+$\bar{y}$ z+zx+yz

=x+$\bar{y}$ z+zx+yz

=x+z($\bar{y}$ +y+x)

=x+z

(b) F(w, x, y, z)=$\overline{xy}$ (wz+$\overline{w}\ \overline{z}$)(w+$\overline{xz}$)   in terms of w, x, y, and z

=$\overline{xy}$ (1)(w+$\overline{xz}$)

=$\overline{xy}$ (w+$\overline{xz}$)

=$\overline{xy}$ w + $\overline{xyz}$

[13] Given the following pulse trains for A, B and C, draw the pulse trains for F1 and F2.