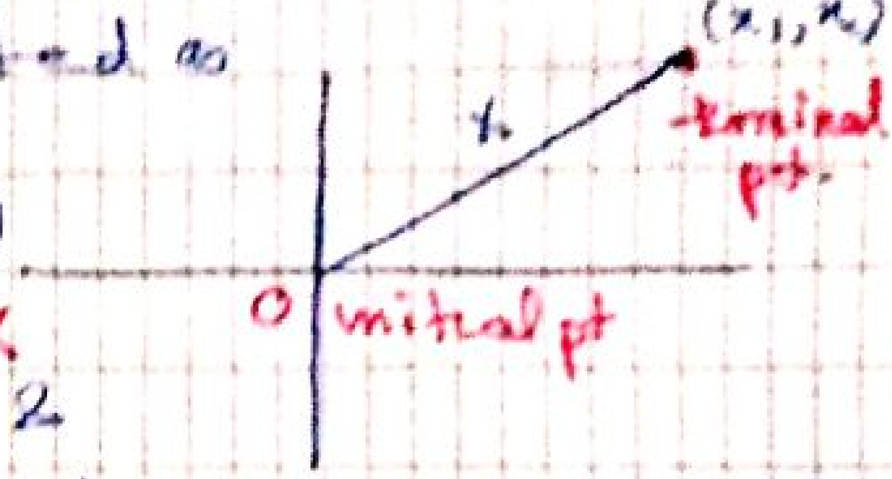
بسم الله الرحمن الرحيم

4.1 Vectors in $\mathbb{R}^n$

* Vectors in the plane $\mathbb{R}^2$:

is represented by a directed line segment with its initial pt at the origin and its terminal pt at (x_1, x_2) and is denoted as



$X = \underbrace{\langle x_1, x_2 \rangle}_{\text{component of a vector } X}$ or (x_1, x_2)

If two vector $u = \langle u_1, u_2 \rangle$ & $v = \langle v_1, v_2 \rangle$ are equal then $u_1 = v_1$ & $u_2 = v_2$ and $u + v = \langle u_1 + v_1, u_2 + v_2 \rangle$

Ex(2) Find $u + v$

(a) $u = \langle 1, 4 \rangle$, $v = \langle 2, -2 \rangle$

$u + v = \langle 1+2, 4+(-2) \rangle = \langle 3, 2 \rangle$

(b) $u = \langle 3, -2 \rangle$, $v = \langle -3, 2 \rangle$

$u + v = \langle 3+(-3), -2+2 \rangle = \langle 0, 0 \rangle = 0$

(c) $u = \langle 2, 1 \rangle$, $v = \langle 0, 0 \rangle$

$u + v = \langle 2+0, 1+0 \rangle = \langle 2, 1 \rangle$

* Scalar multiplication if $v = \langle v_1, v_2 \rangle$, $c \in \mathbb{R}$ c is scalar then $cv = \langle cv_1, cv_2 \rangle$ & if $c = -1$ then $(-1)v = -v$

$\Rightarrow u - v = u + (-v)$

Ex(3) Let $v = \langle -2, 5 \rangle$, $u = \langle 3, 4 \rangle$

Find

a. $\frac{1}{2} v = \frac{1}{2}\langle -2, 5\rangle = \langle \frac{-2}{2}, \frac{5}{2} \rangle = \langle -1, \frac{5}{2} \rangle$

b. $u - v = \langle 3-(-2), 4-5 \rangle = \langle 5, -1 \rangle$

c. $\frac{1}{2} v + u = \langle -1, \frac{5}{2} \rangle + \langle 3, 4 \rangle = \langle -1+3, \frac{5}{2}+4 \rangle$

$= \langle 2, \frac{13}{2} \rangle$

vectors in $\mathbb{R}^n$:

$\mathbb{R} = \mathbb{R}^1 = 1$ space = set of all real numbers

$\mathbb{R}^2 =$ 2-space the set of all ordered pairs of real numbers

$\mathbb{R}^3 =$ The 3-space: set of all ordered triples of real numbers

⋮

$\mathbb{R}^n =$ n-space = Set of all ordered n-tuples of real numbers

$x \in \mathbb{R}^n \Rightarrow x = (x_1, x_2, \dots, x_n)$

& if $u = (u_1, u_2, u_3, \dots u_n)$, $v = (v_1, v_2, \dots v_n)$

then $\vec{u} + v = (u_1+v_1, u_2+v_2, \dots, u_n+v_n)$

$cu = (cu_1, cu_2, cu_3, \dots, cu_n)$

$-u = (-1)u = (-u_1, -u_2, -u_3, \dots, -u_n)$

$u - v = (u_1-v_1, u_2-v_2, \dots, u_n - v_n)$

Ex(4) If $u = (-1, 0, 1)$ $v = (2, -1, 5)$ in $\mathbb{R}^3$

Find

(a) $u + v = (-1+2,\ 0+(-1),\ 1+5) = (1, -1, 6)$

(b) $2u = (2(-1),\ 2(0),\ 2(1)) = (-2, 0, 2)$

(c) $v - 2u = (2-(-2),\ -1-0,\ 5-2) = (4, -1, 3)$

Properties of vectors $\mathbb{R}^n$

let u, v & w are vecters in $\mathbb{R}^n$ and let $c, d \in \mathbb{R}$

1. $u + v$ are vecters in $\mathbb{R}^n$
2. $u + v = v + u$
3. $(u + v) + w = u + (v + w)$
4. $u + 0 = u$ $\quad 0 = (0, 0, \dots, 0)$
5. $u + (-u) = 0$
6. cu is a vecter in $\mathbb{R}^n$
7. $c(u + v) = cu + cv$
8. $(c + d)u = cu + du$
9. $c(du) = (cd)u$
10. $1(u) = u$

Ex(5) let $u = (2, -1, 5, 0)$, $v = (4, 3, 1, -1)$ and $w = (-6, 2, 0, 3)$ be vectors in $\mathbb{R}^4$. Find

a. $x = 2u - (v + 3w)$, $3w = (-18, 6, 0, 9)$

$= (2(2),\ 2(-1),\ 2(5),\ 2(0)) - \left[(4+(-18),\ 3+6,\ 1+0,\ -1+9)\right]$

$= (4, -2, 10, 0) - (-14, 9, 1, 8)$

$= (4+14,\ -2-9,\ 10-1,\ 0-8) = (18, -11, 9, -8)$

b. $3(x+w) = 2u - v + x$

$\Rightarrow 3x + 3w = 2u - v + x$

$2x = 2u - v - 3w$

$x = \frac{1}{2}(2u - v - 3w)$ from part a.

$\Rightarrow x = \frac{1}{2}(18, -11, 9, -8) = (9, -\frac{11}{2}, \frac{9}{2}, -4)$

$0 = (0, 0, \dots, 0)$ is called zero vector of $\mathbb{R}^n$ & is additive identity in $\mathbb{R}^n$ and the vector $-v$ is additive inverse of v

Properties of additive inverse & identity

1. the additive identity is unique i.e.
   if $v + u = v$ then $u = 0$
2. the additive inverse of v is unique that is if $v + u = 0$ then $u = -v$
3. $0v = 0$
4. $c0 = 0$
5. if $cv = 0$ then either $c = 0$ or $v = 0$
6. $-(-v) = v$

Linear combinations of vectors

if we write one vector x as the sum of scalar multiple of other vectors $v_1, v_2, \dots, v_n$ i.e.

for scalors $c_1, c_2, \dots c_n \in \mathbb{R}$ we have

$$x = c_1 v_1 + c_2 v_2 + \dots + c_n v_n$$

the vecter x is called linear combination of vectors $v_1, v_2, \dots, v_n$

Ex(6) Let $x = (-1,-2,-2)$, $u = (0,1,4)$, $v = (-1,1,2)$ and $w = (3,1,2)$ in $\mathbb{R}^3$ find scalor a, b & c s.t

$$x = au + bv + cw$$

$$\Rightarrow (-1,-2,-2) = a(0,1,4) + b(-1,1,2) + c(3,1,2)$$

$$= (0, a, 4a) + (-b, b, 2b) + (3c, c, 2c)$$

$$= (-b+3c,\ a+b+c,\ 4a+2b+2c)$$

$$\Rightarrow \quad -b+3c = -1 \quad \text{--}(1)$$

$$a+b+c = -2 \quad \text{..}(2)$$

$$4a+2b+2c = -2 \quad \text{..}(3)$$

multiply (2) by -4 & add with (3)

$$(2) \Rightarrow -4a-4b-4c = 8 \quad \text{---}(4)$$

$$(3)+(4) \Rightarrow \quad -2b-2c = 6 \quad \text{--}(5)$$

multiply (1) by -2 & add with 5

$$(1) \Rightarrow \quad 2b-6c = 2 \quad \text{--}(6)$$

$$(5)+(6) \Rightarrow \quad -8c = 8 \Rightarrow \boxed{c=-1} \text{ Put in (1)}$$

$$(1) \Rightarrow -b-3 = -1 \Rightarrow \boxed{b=-2}$$

Put $b=-2, c=-1$ in (2)

$$a-2-1 = -2 \Rightarrow a = +1$$

$$\therefore x = u - 2v - w$$

Exercise (Page 159)

(7) if $u = (1,3)$, $v = (2,-2)$ find $u+v$

$u+v = (1+2, 3-2) = (3,1)$

(13) if $u = (-2,3)$, $w = (-3,-2)$

find $v = u + 2w$

$v = (-2,3) + 2(-3,-2)$

$= (-2,3) + (-6,-4)$

$= (-2-6, 3-4) = (-8,-1)$

$\Rightarrow v = (-8,-1)$

23) if $u = (1,2,3)$ $v = (2,2,-1)$, $w = (4,0,-4)$

find z if $3u - 4z = w$

$-4z = w - 3u \Rightarrow z = -\frac{1}{4}[w - 3u]$

$\Rightarrow z = -\frac{1}{4}[(4,0,-4) - 3(1,2,3)]$

$= -\frac{1}{4}[(4,0,-4) - (3,6,9)] = -\frac{1}{4}(1,-6,-13)$

$\therefore z = (\frac{1}{4}, \frac{3}{2}, \frac{13}{4})$

24) find z where $2u + v - w + 3z = 0$

$\Rightarrow 3z = -2u - v + w$

$\Rightarrow 3z = (-2,4,-6) + (-2,-3,1) + (4,0,-4)$

$\Rightarrow 3z = (0,2,-9) \Rightarrow z = (0, \frac{2}{3}, -3)$