01_Probabilistic_AVO_analysis

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1 Probabilistic AVO analysis

1.1 Introduction

This notebook presents an example and methodology of probabilistic amplitude variation with offset (AVO) analysis.

Avseth, et.al. state that "*we show how we can do probabilistic AVO analysis taking into account the natural variability and uncertainties in rock properties*".

In this example, well-log analysis (provided by Avseth, et.al.), in the key well (well 2) describe 10 different lithofacies. These are are *shale* (`shale`), *silty shale* (`sltShale`), *clean sand* (`clnSand`), *silty sand 1* (`sltSand1`), *silty sand 2* (`sltSand2`), and *cemented sand* (`cemSand`) along with two fluid scenarios (*brine* and *oil*).

Non-parametric probability density functions (pdfs) of AVO input parameters (P-wave, S-wave and density) for the different lithofacies combinations are created and used to "*assess uncertainties in seismic signatures related to the natural variability within each facies.*"

Extract taken from pages 225-226 (Chapter 4, Common techniques for quantitative seismic interpretation.), Quantifying AVO uncertainties related to variability in rock properties

1.2 Methodology

Non-parametric pdfs are created by calculating kernel density estimates (kde) of the 10 lithofacies histograms for P-wave (Vp), S-wave (Vs) and density (rho). Following this, a reflection coefficient, R, and a gradient, G, over reflection angles 0-40 degrees are calculated. The calculation of R and G is through an AVO approximation equation (see Reference) that takes in as input Vp, Vs and rho for two adjacent rocks (cap-rock over reservoir).

Random sampling is then performed from the multi-variate (Vp-Vs-rho) kde for each lithofacies group.

Probabilistic AVO analysis is part of a greater topic, Statistical rock physics which is covered in Chapter 3.

1.3 Quantifying uncertainty

The repeated random sampling and calculation of the AVO quantifies the uncertainty associated with each input log and thus the AVO response for each rock pair.

1.3.1 Import libraries

The `packages` folder contains a `function.py` file that contains some helper functions.

[1]:
```python
from packages.functions import *  # vshale_from_gr, vrh
import warnings
warnings.filterwarnings("ignore")
```

```
['./data/well_2.las']
```

1.3.2 Load well data

[2]:
```python
df = load()[0]
```

1.3.3 Add feature columns

Add VP, VS, VSH (IGR, Iarionov, Steiber,), IP, IS, VP/VS, sand-shale indicator, facies code, reservoir code,

[3]:
```python
well2 = well_add_features(df)
```

1.3.4 Well header

[4]:
```python
well2.head()
```

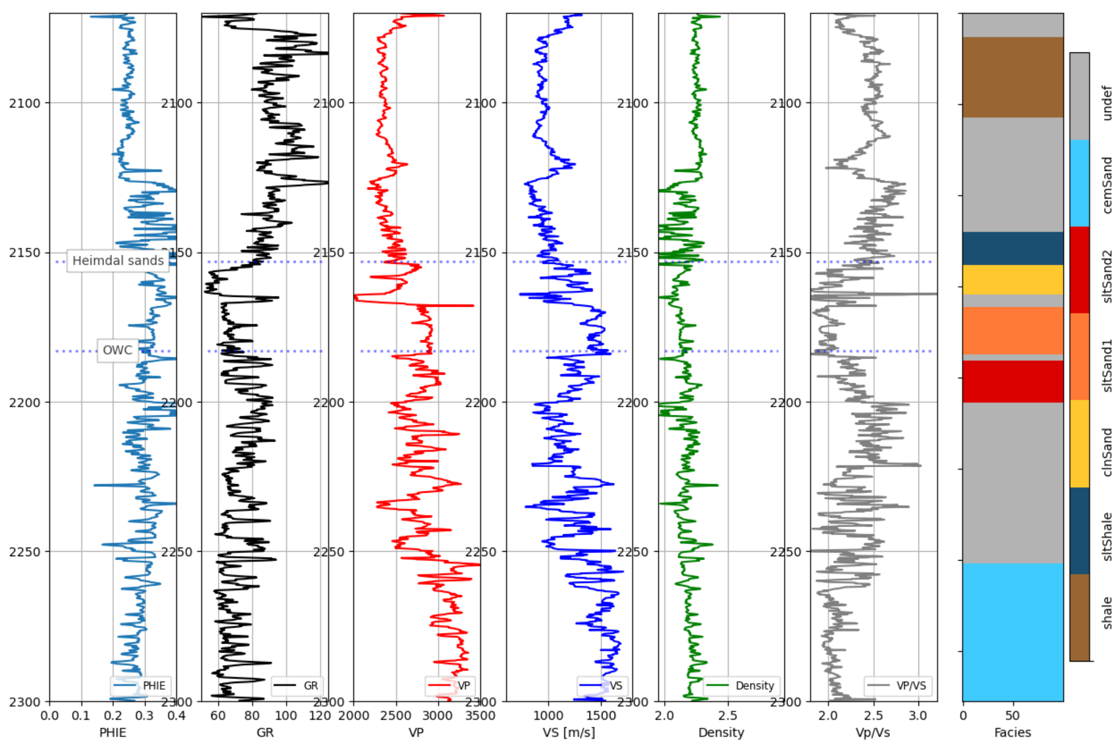
[4]:
```
       DEPTH      VP      VS    RHOB       GR    NPHI       IGR  VSH_clavier  \
0  2013.2528  2.2947  0.8769  1.9972  91.8785  0.4908  0.493621     0.301691
1  2013.4052  2.2967  0.9430  2.0455  86.8004  0.4833  0.436010     0.254496
2  2013.5576  2.2904  0.9125  2.1122  86.0021  0.4474  0.426953     0.247424
3  2013.7100  2.2775  0.8916  2.1960  87.3570  0.4140  0.442325     0.259481
4  2013.8624  2.2620  0.8905  2.2020  90.4024  0.4293  0.476875     0.287567

   VSH_larionovO  VSH_steiber  …      PHIE          IP          IS  \
0       0.324189     0.245246  …  0.408000  4582.97484  1751.34468
1       0.273974     0.204894  …  0.377812  4697.89985  1928.90650
2       0.266438     0.198944  …  0.336125  4837.78288  1927.38250
3       0.279284     0.209102  …  0.283750  5001.39000  1957.95360
4       0.309177     0.233048  …  0.280000  4980.92400  1960.88100

       VPVS  sandy-shaly         KO  FACIES  RESERVOIR  LABELS  FCODES
0  2.616832        sandy  22.676699       0          0       0       6
1  2.435525        sandy  24.820085       0          0       0       6
2  2.510027        sandy  25.646882       0          0       0       6
3  2.554397        sandy  25.894306       0          0       0       6
4  2.540146        sandy  25.048897       0          0       0       6

[5 rows x 22 columns]
```

1.3.5 Well plot

Reference: Agile Geoscience

[5]:
```
import matplotlib.colors as colors
ccc = ['#996633', '#1B4F72', '#FFC82E','#FF7A36', '#DB0000', '#40CBFF',␣
 ↪'#B3B3B3']
cmap_facies = colors.ListedColormap(ccc[0:len(ccc)], 'indexed')

ztop=2070; zbot=2300  #ztop=2140; zbot=2200
ll=well2[(well2.DEPTH>=ztop) & (well2.DEPTH<=zbot)]

cluster=np.repeat(np.expand_dims(ll['FCODES'].values,1), 100, 1)

f, ax = plt.subplots(nrows=1, ncols=7, figsize=(15, 10))

ax[0].plot(ll.PHIE,    ll.DEPTH,         label='PHIE')
ax[1].plot(ll.GR,      ll.DEPTH, '-k',   label='GR')
ax[2].plot(ll.VP*1000, ll.DEPTH, '-r',   label='VP')
ax[3].plot(ll.VS*1000, ll.DEPTH, 'blue', label='VS')
ax[4].plot(ll.RHOB,    ll.DEPTH, '-g',   label='Density')
ax[5].plot(ll.VPVS,    ll.DEPTH, '-', color='0.5', label='VP/VS')
im=ax[6].imshow(cluster, interpolation='none',␣
 ↪aspect='auto',cmap=cmap_facies,vmin=0,vmax=len(ax)-1)#4)

cbar=plt.colorbar(im, ax=ax[6], aspect=32)
cbar.set_label((9*' ').join(['shale', 'sltShale', 'clnSand', 'sltSand1',␣
 ↪'sltSand2', 'cemSand', 'undef']))
cbar.set_ticks(range(0,1)); cbar.set_ticklabels('')

for i in range(len(ax)-1):
    ax[i].set_ylim(ztop,zbot)
    ax[i].invert_yaxis()
    ax[i].grid()
    ax[i].locator_params(axis='x', nbins=4)
    ax[i].legend(fontsize='small', loc='lower right')

ax[0].set_xlabel("PHIE"),          ax[0].set_xlim(0,0.4)
ax[1].set_xlabel("GR"),            ax[1].set_xlim(50, 125),
ax[2].set_xlabel("VP"),            ax[2].set_xlim(2000, 3500),
ax[4].set_xlabel("Density"),       ax[4].set_xlim(1.95, 2.95),
ax[3].set_xlabel("VS [m/s]"),      ax[3].set_xlim(600, 1800),
ax[5].set_xlabel("Vp/Vs"),         ax[5].set_xlim(1.8,3.2),
ax[6].set_xlabel('Facies'),  ax[6].set_yticklabels([])

tops = {
    "Heimdal sands": 2153,
    "OWC": 2183,
```

```
}

for i in range(len(ax)-1):
    for top in tops.values() :
        f.axes[i].axhline( y = float(top), color = 'b', lw = 2,
                            ls = ':',
                            alpha = 0.5, xmin = 0.05, xmax = 0.95 )

for top, depth in tops.items():
    if (ztop < depth < zbot):
        ax[1].text( x = max(ax[0].xaxis.get_data_interval())*1.0,
                    y = float(depth), s = top,
                             alpha=0.75, color='k',
                             fontsize = '10',
                             horizontalalignment = 'center',
                             verticalalignment = 'center',
                             bbox=dict(facecolor='white', alpha=1.0, lw = 0.25),
                             weight = 'light');
```



1.3.6 Well log section

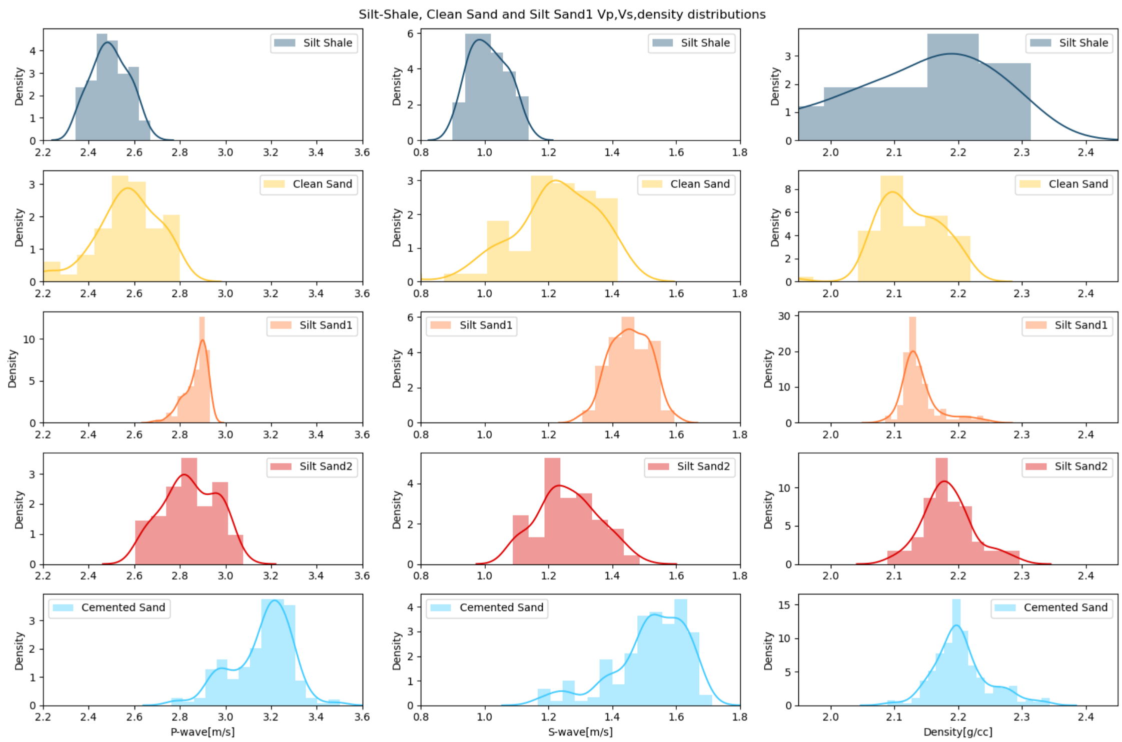
Well 2 around the reservoir section. `sltShale` is cap-rock in this well, with `clnSand`, and `sltSand1` being oil bearing and with `sltSand2` and `cemSand` being below the oil water contact (OWC).

1.3.7 Data histograms and resultant kernel estimates

The plot below shows the P-wave histograms and the calculated kernel estimates.

[6]:
```
fig,
 ↪((ax1,ax2,ax3),(ax4,ax5,ax6),(ax7,ax8,ax9),(ax10,ax11,ax12),(ax13,ax14,ax15))
 ↪= plt.subplots(5, 3, figsize=(15,10))  # sharey=True,
fig.suptitle('Silt-Shale, Clean Sand and Silt Sand1 Vp,Vs,density
 ↪distributions')

sns.distplot(ax=ax1, x=well2[well2.LABELS=='sltShale']['VP'], color=ccc[1],
 ↪label="Silt Shale")
sns.distplot(ax=ax2, x=well2[well2.LABELS=='sltShale']['VS'], color=ccc[1],
 ↪label="Silt Shale")
sns.distplot(ax=ax3, x=well2[well2.LABELS=='sltShale']['RHOB'], color=ccc[1],
 ↪label="Silt Shale")
sns.distplot(ax=ax4, x=well2[well2.LABELS=='clnSand']['VP'], color=ccc[2],
 ↪label="Clean Sand")
sns.distplot(ax=ax5, x=well2[well2.LABELS=='clnSand']['VS'], color=ccc[2],
 ↪label="Clean Sand")
sns.distplot(ax=ax6, x=well2[well2.LABELS=='clnSand']['RHOB'], color=ccc[2],
 ↪label="Clean Sand")
sns.distplot(ax=ax7, x=well2[well2.LABELS=='sltSand1']['VP'], color=ccc[3],
 ↪label="Silt Sand1")
sns.distplot(ax=ax8, x=well2[well2.LABELS=='sltSand1']['VS'], color=ccc[3],
 ↪label="Silt Sand1")
sns.distplot(ax=ax9, x=well2[well2.LABELS=='sltSand1']['RHOB'], color=ccc[3],
 ↪label="Silt Sand1")

sns.distplot(ax=ax10, x=well2[well2.LABELS=='sltSand2']['VP'], color=ccc[4],
 ↪label="Silt Sand2")
sns.distplot(ax=ax11, x=well2[well2.LABELS=='sltSand2']['VS'], color=ccc[4],
 ↪label="Silt Sand2")
sns.distplot(ax=ax12, x=well2[well2.LABELS=='sltSand2']['RHOB'], color=ccc[4],
 ↪label="Silt Sand2")
sns.distplot(ax=ax13, x=well2[well2.LABELS=='cemSand']['VP'], color=ccc[5],
 ↪label="Cemented Sand")
sns.distplot(ax=ax14, x=well2[well2.LABELS=='cemSand']['VS'], color=ccc[5],
 ↪label="Cemented Sand")
sns.distplot(ax=ax15, x=well2[well2.LABELS=='cemSand']['RHOB'], color=ccc[5],
 ↪label="Cemented Sand")

ax1.set_xlim(2.2,3.6), ax2.set_xlim(0.8,1.8), ax3.set_xlim(1.95,2.45)
ax4.set_xlim(2.2,3.6), ax5.set_xlim(0.8,1.8), ax6.set_xlim(1.95,2.45)
ax7.set_xlim(2.2,3.6), ax8.set_xlim(0.8,1.8), ax9.set_xlim(1.95,2.45)
ax10.set_xlim(2.2,3.6), ax11.set_xlim(0.8,1.8), ax12.set_xlim(1.95,2.45)
ax13.set_xlim(2.2,3.6), ax14.set_xlim(0.8,1.8), ax15.set_xlim(1.95,2.45)
```

```
ax13.set_xlabel('P-wave[m/s]'), ax14.set_xlabel('S-wave[m/s]'), ax15.set_xlabel('Density[g/cc]')

plt.tight_layout()
for ax in fig.get_axes():
    ax.legend()
    #ax.label_outer()
```



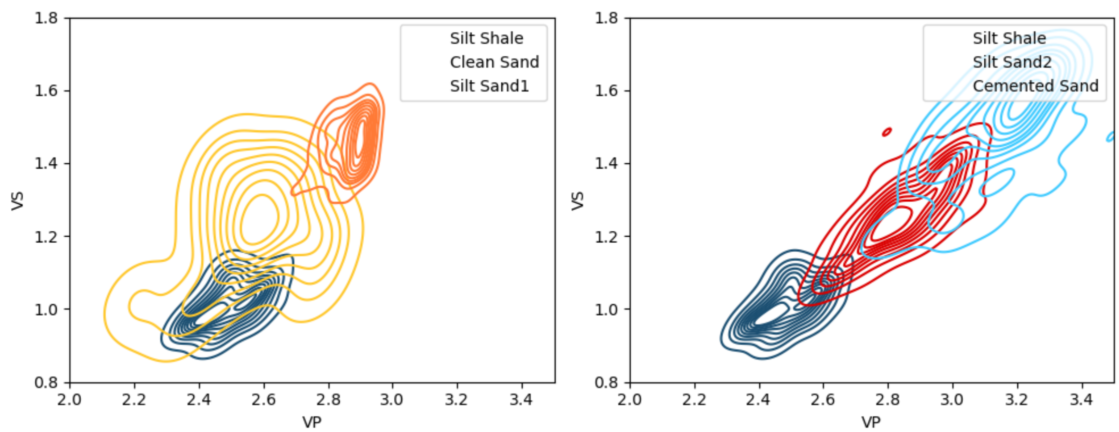
(above) The rows represent the P-wave, S-wave and density of a particular lithofacies. The columns show the comparison of each lithofacies distribution for each log.

1.3.8 Bivariate distribution of P-wave versus S-wave

The plot below demonstrates kernel density estimate of the bivaritate distributions P-wave and S-wave for `sltShale`, `clnSand` and `sltSand1` in plot one and `sltShale`, `sltSand2` and `cemSand` in plot two. This plot shows the two-dimensional distributions of each of the three lithofacies.

[7]:
```
fig, ax = plt.subplots(1,2, figsize=(10,4))
sns.kdeplot(ax=ax[0], data=well2[well2.LABELS=='sltShale'], x='VP', y='VS', color=ccc[1], label="Silt Shale")
sns.kdeplot(ax=ax[0], data=well2[well2.LABELS=='clnSand'], x='VP', y='VS', color=ccc[2], label="Clean Sand")
sns.kdeplot(ax=ax[0], data=well2[well2.LABELS=='sltSand1'], x='VP', y='VS', color=ccc[3], label="Silt Sand1")
```

```
sns.kdeplot(ax=ax[1], data=well2[well2.LABELS=='sltShale'], x='VP', y='VS', color=ccc[1], label="Silt Shale")
sns.kdeplot(ax=ax[1], data=well2[well2.LABELS=='sltSand2'], x='VP', y='VS', color=ccc[4], label="Silt Sand2")
sns.kdeplot(ax=ax[1], data=well2[well2.LABELS=='cemSand'], x='VP', y='VS', color=ccc[5], label="Cemented Sand")

ax[0].set_xlim(2.,3.5); ax[1].set_xlim(2.,3.5)
ax[0].set_ylim(0.8, 1.8); ax[1].set_ylim(0.8,1.8)

plt.tight_layout()
for ax in fig.get_axes():
    ax.legend()
    #ax.label_outer()
```



1.3.9 Monte-Carlo simulation from non-parametric distributions

The function `kde_resample()` creates three-dimensional (3D) kernel density estimates (kde's) for each lithofacies/fluid scenario distribution.

Once the kde has been calculated, the resampling method is called for a specified number of times (here 500). The output of the function is then input to the `r0g()` function below.

[8]:
```
lith_list = ['sltShale', 'clnSand', 'sltSand1', 'sltSand2', 'cemSand']
column = 'LABELS'
logs = ['VP', 'VS', 'RHOB']
num_samples = 500

kde = kde_resample(well2, column, lith_list, logs, num_samples=1000)
```

1.3.10 Calculate R0-G from simulated distributions

The function `r0g()` samples Vp, Vs & rho from the simulated cap-rock kde, `sltShale`, and then the simulated reservoir (`clnSand`, `sltSand1`, `sltSand2` and `cemSand`) kde. The parameters from each lithology are then input to the equation (see References) to obtain a reflection coefficient (R0) and gradient (G) which is then used to determine the reflection coefficient for a particular angle theta for theta between 0-40 degrees.

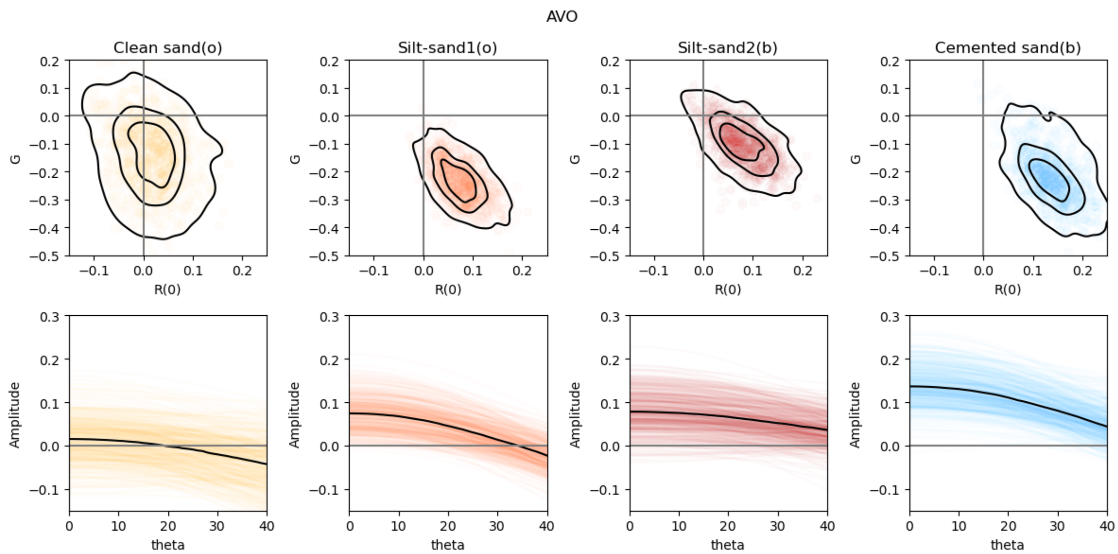
The median of all sampled data for each value of theta is calculated.

[9]:
```
# Assumes that cap rock is in position 0
# ----------------------------- r0g(vp0      , vs0      , rho0     , vp1     ␣
↪, vs1      , rho1)
r01_b, G1_b, Rtheta1_b, med1_b = r0g(kde[0][0], kde[0][1], kde[0][2],␣
↪kde[1][0], kde[1][1], kde[1][2])
r02_b, G2_b, Rtheta2_b, med2_b = r0g(kde[0][0], kde[0][1], kde[0][2],␣
↪kde[2][0], kde[2][1], kde[2][2])
r03_b, G3_b, Rtheta3_b, med3_b = r0g(kde[0][0], kde[0][1], kde[0][2],␣
↪kde[3][0], kde[3][1], kde[3][2])
r04_b, G4_b, Rtheta4_b, med4_b = r0g(kde[0][0], kde[0][1], kde[0][2],␣
↪kde[4][0], kde[4][1], kde[4][2])
#r05_b, G5_b, Rtheta5_b, med5_b = r0g(kde[0][0], kde[0][1], kde[0][2],␣
↪kde[5][0], kde[5][1], kde[5][2])

rc_list = [r01_b, r02_b, r03_b, r04_b]
G_list = [G1_b, G2_b, G3_b, G4_b]
Rtheta_list = [Rtheta1_b, Rtheta2_b, Rtheta3_b, Rtheta4_b]
med_list = [med1_b, med2_b, med3_b, med4_b]
```

1.3.11 Plot R0-G and Amplitude versus angle for each pair

[10]:
```
alpha1=0.02
alpha2=0.02
titles = ['Clean sand(o)', 'Silt-sand1(o)', 'Silt-sand2(b)', 'Cemented sand(b)']

fig, ax = plt.subplots(2, 4, squeeze=False, figsize=(12,6))
fig.suptitle('AVO')

# plot first row:
for i in range(4):
    ax[0,i].set_title(titles[i])
    ax[0,i].scatter(rc_list[i], G_list[i], alpha=alpha1, color=ccc[i+2])
    sns.kdeplot(ax=ax[0,i], x=rc_list[i], y=G_list[i], color='k',␣
↪linestyles="-", levels=4)
    ax[0,i].set_xlabel("R(0)"); ax[0,i].set_ylabel("G")
    ax[0,i].set_xlim(-0.15, 0.25); ax[0,i].set_ylim(-0.5, 0.2)
    ax[0,i].axvline(0, color='gray')
    ax[0,i].axhline(0, color='gray')
```

```
# plot second row
for j in range(4):
    for i in range(num_samples):
        ax[1,j].plot(Rtheta_list[j][i][0], alpha=alpha2, color=ccc[j+2])
    ax[1,j].plot(med_list[j], 'k')
    ax[1,j].set_xlabel("theta"); ax[1,j].set_ylabel("Amplitude")
    ax[1,j].set_xlim(0,40); ax[1,j].set_ylim(-0.15, 0.3)
    ax[1,j].axvline(0, color='gray')
    ax[1,j].axhline(0, color='gray')

plt.tight_layout()
plt.savefig('Probabilistic_AVO_analysis_a.png');
```



Top row: Bivariate distribution of the different seismic lithofacies in the R(0)-G plane, assuming silty shale is the cap rock. The centre of each contour plot represents the most likely set of R(0) and G for each facies. The contours represent iso-probability values, decreasing away from the innermost contour.

Bottom row: AVO pdfs for each lithofacies pair. The cap-rock is represented by a silty shale. The superimposed black lines are the deterministic AVO responses calculated from the median values of the pdfs. The equation $R(\theta) \approx R(0) + G \sin^2 \theta$ is used to calculate these pdfs.

1.4 References

- Avseth, P., Mukerji, T. & Mavko, G. Quantitative Seismic Interpretation. (Cambridge University Press, 2005).

1.4.1 Equations

$$R(\theta_1) \approx \frac{1}{2}(1 - 4p^2V_S^2)\frac{\Delta\rho}{\rho} + \frac{1}{2\cos^2\theta}\frac{\Delta V_P}{V_P} - 4p^2V_S^2\frac{\Delta V_S}{V_S}$$