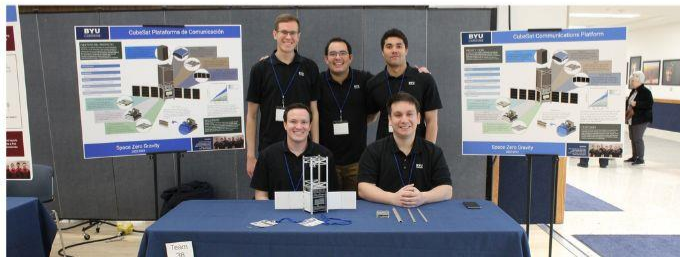


# CAPSTONE DESIGN FAIR

THURSDAY, APRIL 4  
WSC BALLROOM  
11AM - 1 PM



**Stop by and  
check out  
over 50  
unique  
student  
projects!**

Free treats while you  
explore the projects



# Spiritual Thought – Enduring Well

**Mosiah 14:5** But he was wounded for our transgressions, he was bruised for our iniquities; the chastisement of our peace was upon him; and with his stripes we are healed.

**7** He was oppressed, and he was afflicted, yet he opened not his mouth; he is brought as a lamb to the slaughter, and as a sheep before her shearers is dumb so he opened not his mouth.

**Moroni 7:45** And charity suffereth long, and is kind, and envieth not, and is not puffed up, ... beareth all things, ..., endureth all things.

As in all things, the Savior provides a perfect example for us

What does suffering long, and being kind, and envying not, and not getting puffed up really mean?

Why is “suffering long”, “bearing all things”, and “enduring all things” (critical to developing charity)?

# **21 – Propagation of Uncertainty (Sensitivity Analysis)**

# Announcements

- 3<sup>rd</sup> Milestone due March 29 (Design Stage Uncertainty Analysis)
- 4<sup>th</sup> Milestone due April 2 (Project Presentation)
  - Student grades for each presentation due April 4
- 5<sup>th</sup> Milestone due April 9 (Project Report Draft due)
- Peer Review of Draft Project Report (in class) April 11
- Project Final Report due April 17
- In LS, go to Drive Access in the Content folder, then click on Final Project

# Presentation Schedule

	<u>Section</u>	<u>Member #1</u>	<u>Member #2</u>	<u>Member #3</u>	<u>2-Apr</u>	<u>4-Apr</u>
Group1.1	1	Denver Toner	Jacob Boyer	Christian Devey	X	
Group1.2	1	Brian Stewart	Caleb Becker	Chase Christopherson	X	
Group1.3	1	Davis Wing	Natalie Jones	Matt Bozer	X	
Group2.1	2	Lexie Rhodes	Rachel Day	Bentley Cook	X	
Group2.2	2			Erik Villa	X	
Group2.3	2	Kirsten Steele	Ashley Quinn	Kaj Call	X	
Group2.4	2	Chase Williams	Noa Leitualla	Hans Klomp	X	
Group3.1	3	Seth Nelson	Simon Calabuig	Jake Limburg		X
Group3.2	3	Ayden Bennett	Spencer Peterson	Callan Bradford		X
Group3.3	3	Corinne Jackson	Spencer Shirley	Jackson Jones		X
Group4.1	4	Parker Breit	Mark Griffiths	Connor Crandall		X
Group4.2	4	Michelle Arias	Rylee McLaughlin			X
Group2.X	2	Jacob Cox	Vincent Carter			X

# Presentation Grade Sheet

NAME:

from 1 to 10

from 1 to 10

Date	Order	Team Members		
4/2 and 4/4 2024	1			
	2			
	3			
	4			
	5			
	6			
	7			
	8			
	9			
	10			
	11			
	12			
	13			
	14			
	15			
	16			

Slide Quality	Presentation Quality

[https://docs.google.com/spreadsheets/d/1g9\\_LBir5QW5tF9nTUx66\\_AEpSB0vbg9X/edit?usp=drive\\_link&oid=109260467525861689247&rtpof=true&sd=true](https://docs.google.com/spreadsheets/d/1g9_LBir5QW5tF9nTUx66_AEpSB0vbg9X/edit?usp=drive_link&oid=109260467525861689247&rtpof=true&sd=true)

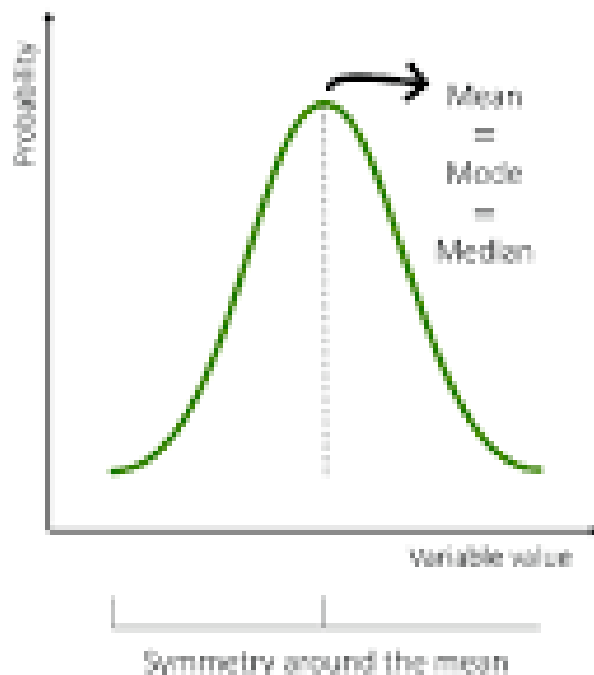
Part of your grade for the presentation will consist of how rigorously you graded your peers.

# Abnormal Distributions

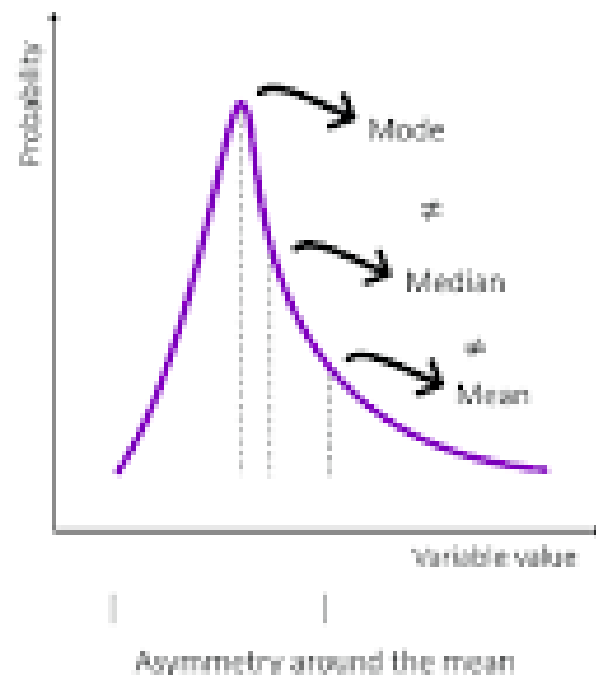
- Abnormal or non-normal distributions may lack symmetry, may have extreme values, or may have a flatter or steeper “dome” than a typical bell. There is nothing inherently wrong with non-normal data; some traits simply do not follow a bell curve. For example, data about coffee and alcohol consumption are rarely bell shaped.

- Plotting sample means will result in a much more normal or gaussian distribution.

NORMAL DISTRIBUTION

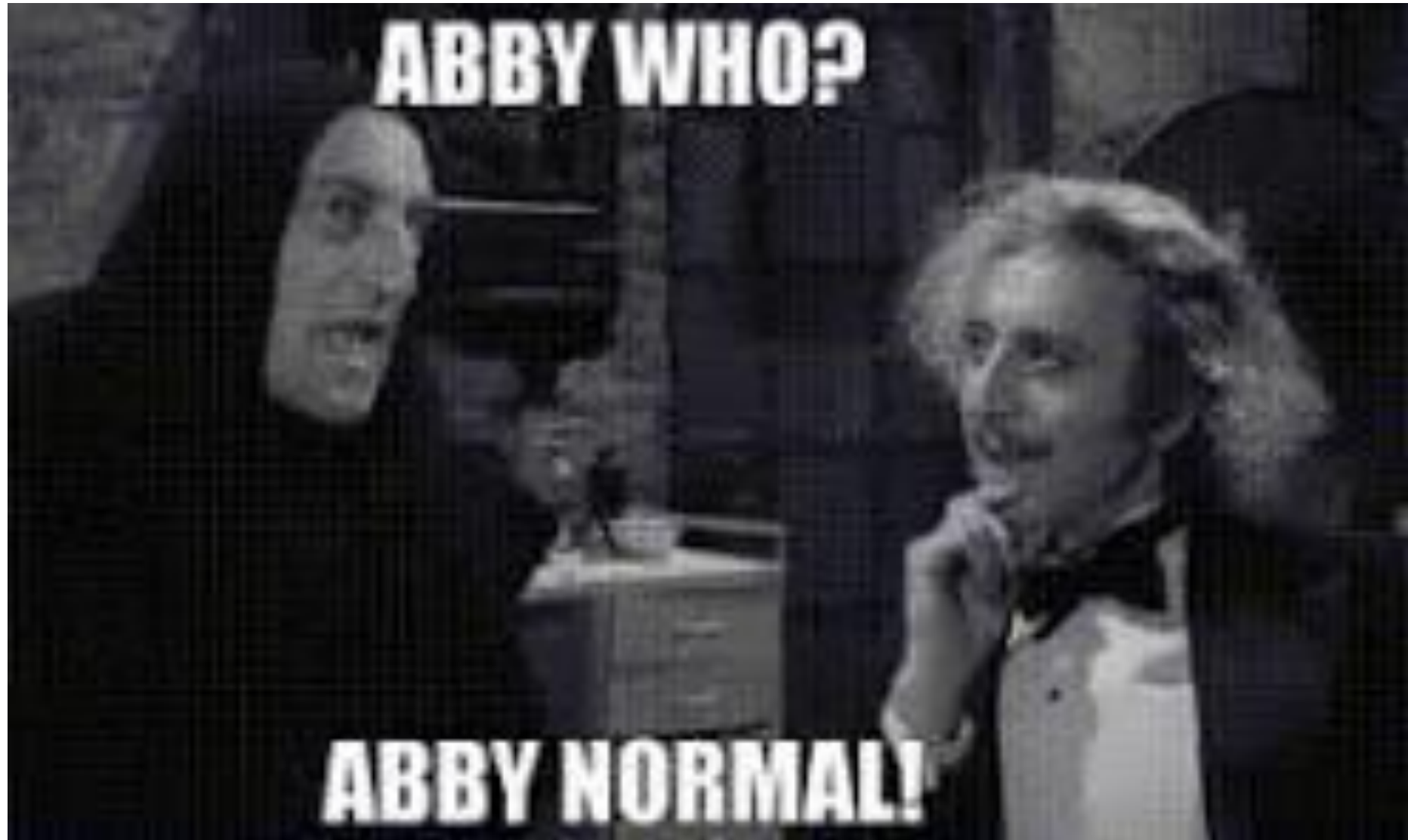


ABNORMAL DISTRIBUTION





# Young Frankenstein Quote





# Big Picture View

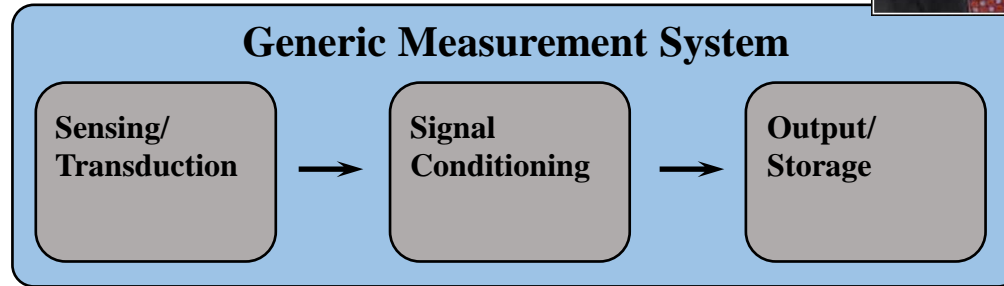


When we deal in generalities, we shall never succeed. When we deal in specifics, we shall rarely have a failure. When performance is measured, performance improves. When performance is measured and reported, the rate of performance accelerates.

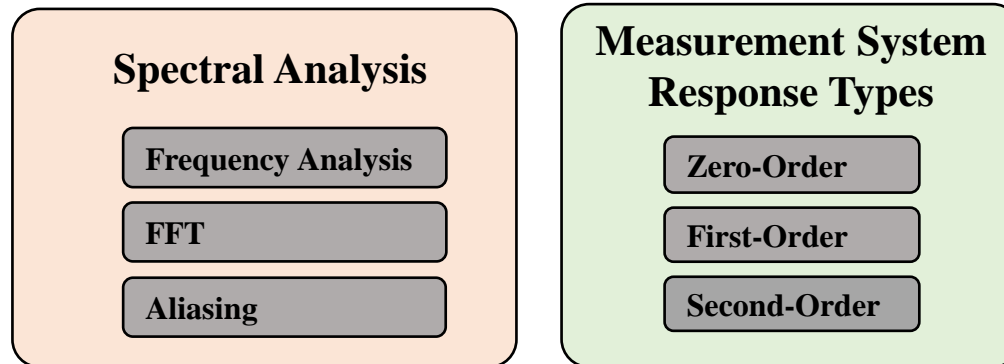
— Thomas S. Monson —

AZ QUOTES

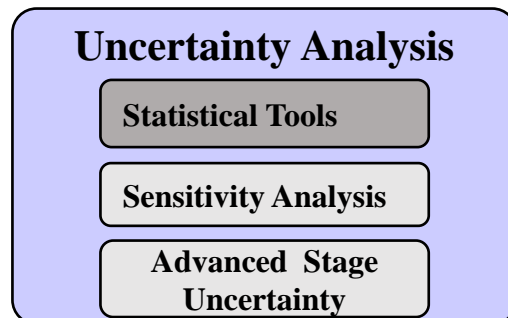
## PART I



## PART II



## PART III



## Labs

1. Thermocouple
2. Strain Gage
3. Pressure/Temp
4. Accel Integration
5. Density Uncertainty
6. Frequency Analysis
7. 3D Imaging

Memo report  
Memo report revise

**Final Exam**

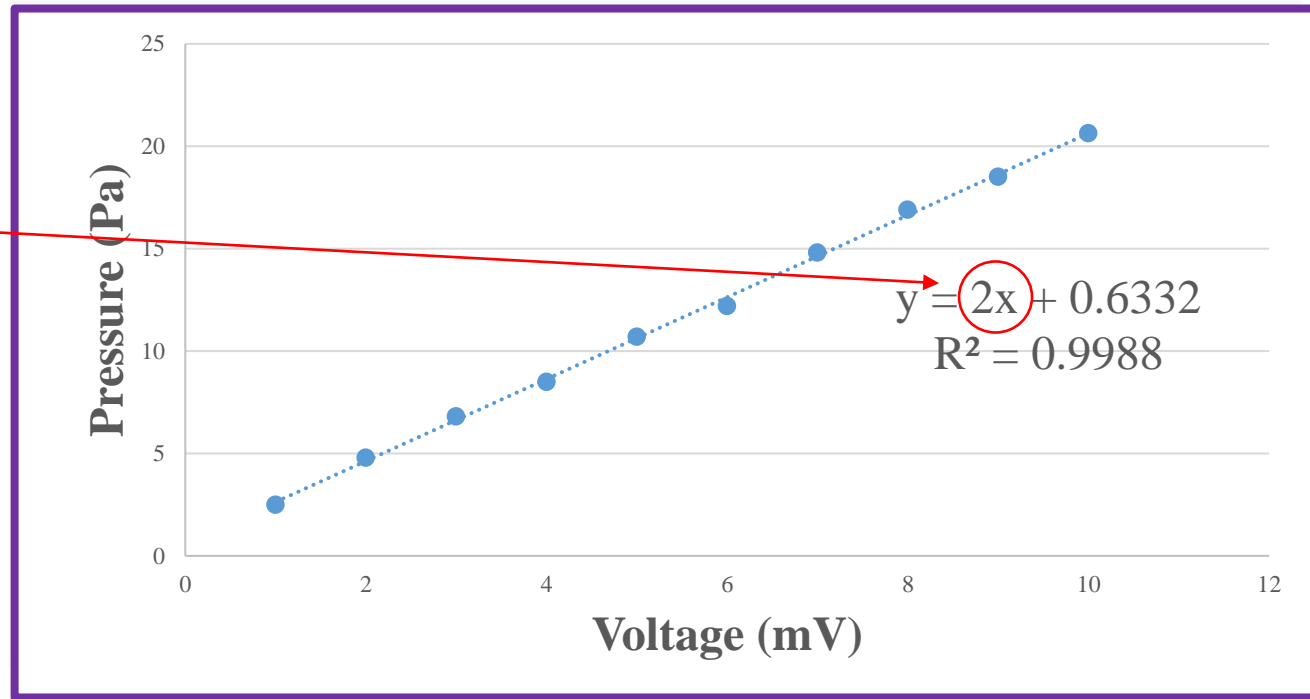
**Project**  
**Final Project/Report**

# Design Stage Uncertainty Review

## Pressure Sensor

- Resolution = 2 mV
- Sensitivity = 2 Pa/mV

Where did this come from?



$$u_{lin} = \pm 3 \text{ Pa}$$

$$u_{hys} = \pm 4 \text{ Pa}$$

$$u_D = ?$$

$$u_D = \pm \sqrt{u_I^2 + u_o^2}$$

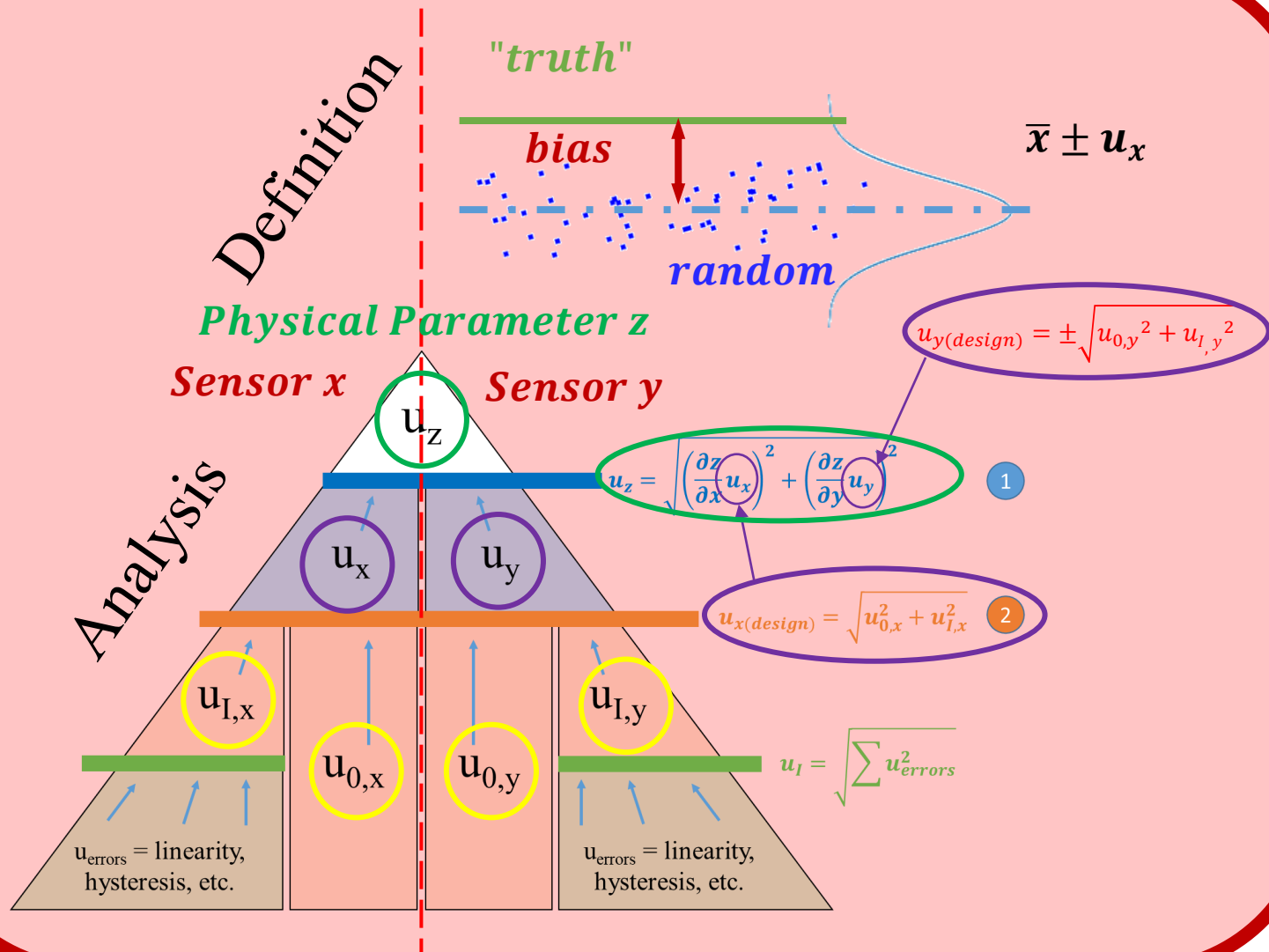
$$u_0 = ?$$

$$u_0 = \pm 2 \frac{\text{Pa}}{\text{mV}} 2 \text{ mV} = \pm 4 \text{ Pa}$$

$$u_I = \pm \sqrt{u_{lin}^2 + u_{hys}^2} = \pm \sqrt{(3 \text{ Pa})^2 + (4 \text{ Pa})^2} = \pm 5 \text{ Pa}$$

$$u_D = \pm \sqrt{(5 \text{ Pa})^2 + (4 \text{ Pa})^2} = \pm 6.4 \text{ Pa}$$

# Uncertainty Overview



# Propagation of uncertainty



# Propagation of uncertainty

$$KE = \frac{1}{2}mv^2$$

$u_{KE} = ?$

Arrows indicate uncertainty contributions:  $u_m$  from the mass  $m$  and  $u_v$  from the velocity  $v$ .



Is this how we combine uncertainties?

$$u_{KE} = \pm \sqrt{u_m^2 + u_v^2} ?$$

$$m = 500 \pm 1 \text{ kg}$$

How is mass measured?

$$v = 116.4 \pm 1.2 \frac{\text{m}}{\text{s}}$$

How is velocity measured?

We combine uncertainties using a Taylor series expansion approximation.

$$u_{KE} = \pm \sqrt{\left(\frac{\partial E}{\partial m} u_{D,m}\right)^2 + \left(\frac{\partial E}{\partial v} u_{D,v}\right)^2}$$

$$u_{KE} = \pm \sqrt{\left(\frac{1}{2}v^2 u_m\right)^2 + (mv u_v)^2} = \pm 70.2 \text{ kJ}$$

$$KE = \frac{1}{2} 500 * 116.4^2 = 3,387.2 \text{ kJ}$$

$$\frac{u_{KE}}{KE} = \pm 2.1\%$$

# Propagation of uncertainty

$$R = f(x_1, x_2, \dots, x_n)$$

$$u_R = \pm \sqrt{\underbrace{\left(\frac{\partial R}{\partial x_1} u_{x_1}\right)^2}_{\text{Term 1}} + \underbrace{\left(\frac{\partial R}{\partial x_2} u_{x_2}\right)^2}_{\text{Term 2}} + \dots + \underbrace{\left(\frac{\partial R}{\partial x_n} u_{x_n}\right)^2}_{\text{Term n}}}$$

1. Have to have same confidence interval (e.g. 95%) for each uncertainty
2. All terms have same units
3. Derivative and u term will tell you which variables contributes most uncertainty
4. Valid only for the measured values, particularly for nonlinear functions

Propagation of uncertainty tends to overestimate uncertainty!

Sensitivity analysis used in other applications like optimization, Taguchi, Kalman filters, etc.  
Like: how would my house payment change due to changes in rate, loan life, etc.?





# Propagation of uncertainty

$$V = 100 \pm 2 \text{ Volts}$$

$$I = 10 \pm 0.5 \text{ Amps}$$

$$u_R = \pm \sqrt{\left(\frac{\partial R}{\partial x_1} u_{x_1}\right)^2 + \left(\frac{\partial R}{\partial x_2} u_{x_2}\right)^2 + \cdots + \left(\frac{\partial R}{\partial x_n} u_{x_n}\right)^2}$$

Find uncertainty of power

$$u_P = ?$$

$$P = IV$$

Which causes more uncertainty, V, or I?

$$\frac{\partial P}{\partial I} = V \quad \frac{\partial P}{\partial V} = I$$

$$u_P = \pm \sqrt{(Vu_I)^2 + (Iu_V)^2}$$

$$u_P = \pm \sqrt{(100V * 0.5A)^2 + (10A * 2V)^2}$$

$$u_P = \pm \sqrt{(50W)^2 + (20W)^2}$$

$$u_P = \pm 53.8W$$

$$P = 1000 \pm 53.8W$$

# Static vs Regression line

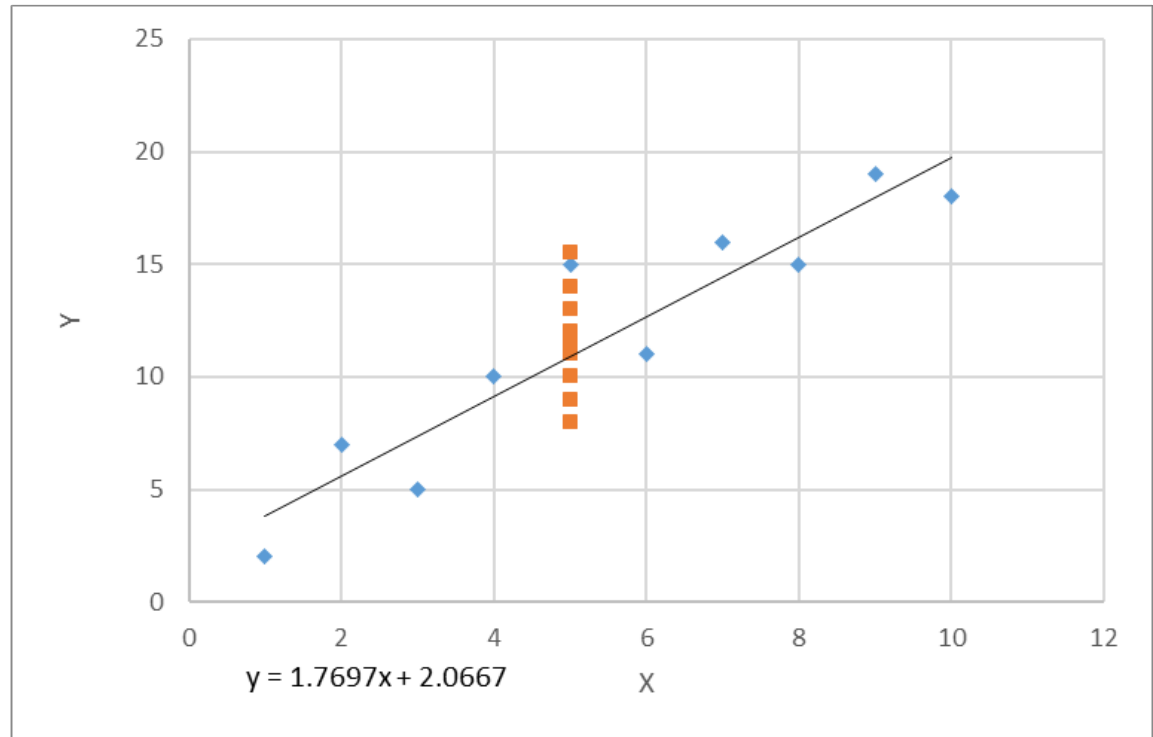
- Confidence interval

$$\pm t_{95,v} s_x$$

$$s_x = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2}$$

For N=10,  $t_{95}= 2.262$

$$\text{CI} = 10.0 \pm 0.936$$



How do we characterize uncertainty, variation, some kind of confidence interval with this kind of data?

# Confidence around a regression line

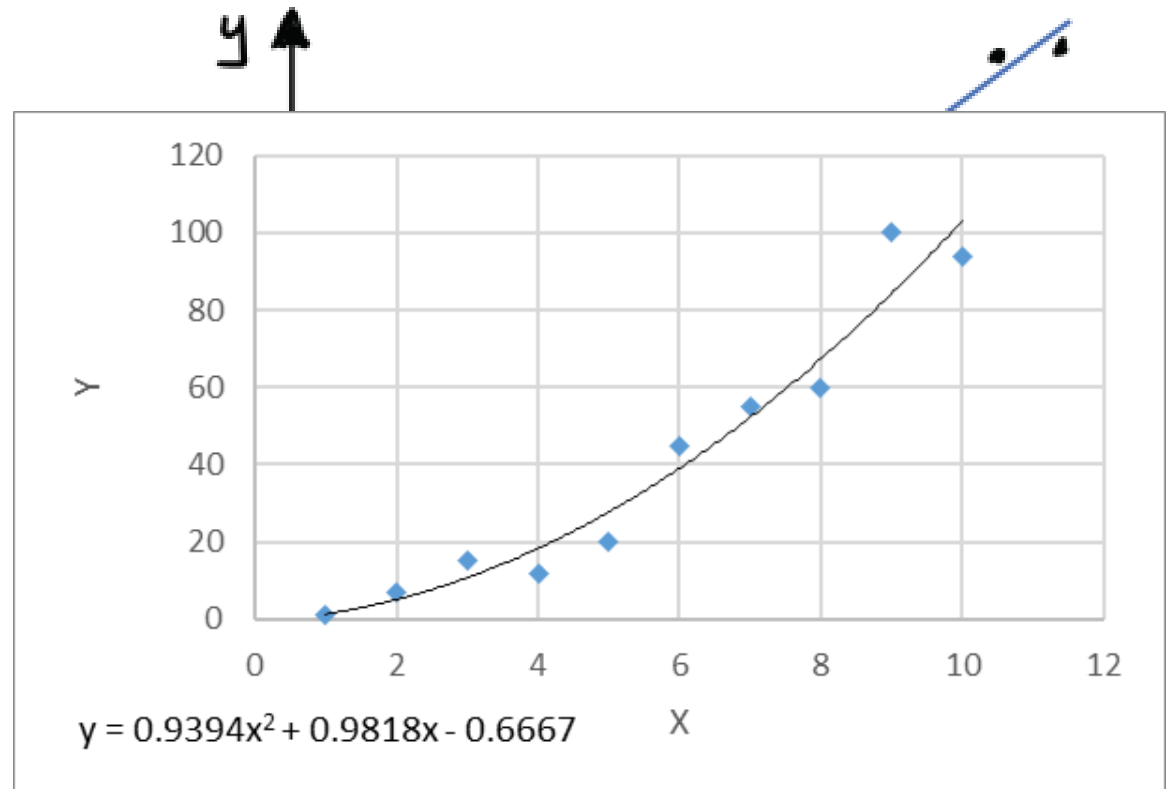
- Regression line

$$\pm t_{95,v} s_{yx}$$

$$s_{yx} = \sqrt{\frac{1}{v} \sum_{i=1}^N (y_i - y_{ci})^2}$$

$$v = N - (m + 1)$$

order of the fit



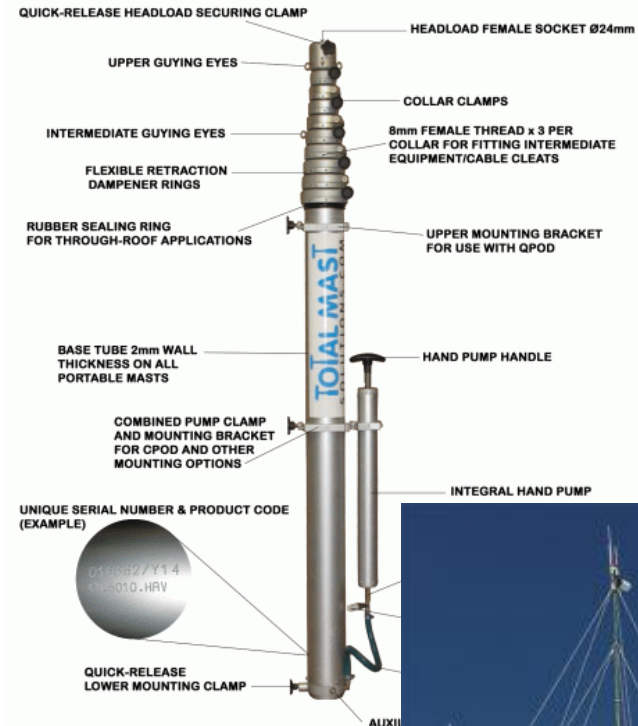
# Review

Term	Definition (in plain English)	Formulas
Confidence interval	A estimation of the interval in which future measurements will occur.	
Confidence interval of the population	An estimation of the interval in which the full population will occur.	$\bar{x} \pm t_{v,95} S_x$ $s_x = \sqrt{\frac{1}{v} \sum_{i=1}^N (x_i - \bar{x})^2}$ $v = N - 1$
Confidence interval of the mean	An estimation of the interval in which future average values will occur.	$\bar{x} \pm t_{v,95} S_{\bar{x}}$ $S_{\bar{x}} = \frac{S_x}{\sqrt{N}}$
Confidence interval of data about a regression line	An estimation of the interval in which future data points will occur around a regression line	$y_{pred} \pm t_{95,v} S_{yx}$ $s_{yx} = \sqrt{\frac{1}{v} \sum_{i=1}^N (y_i - y_{ci})^2}$ $v = N - (m + 1)$

# Story – Overestimating Uncertainty

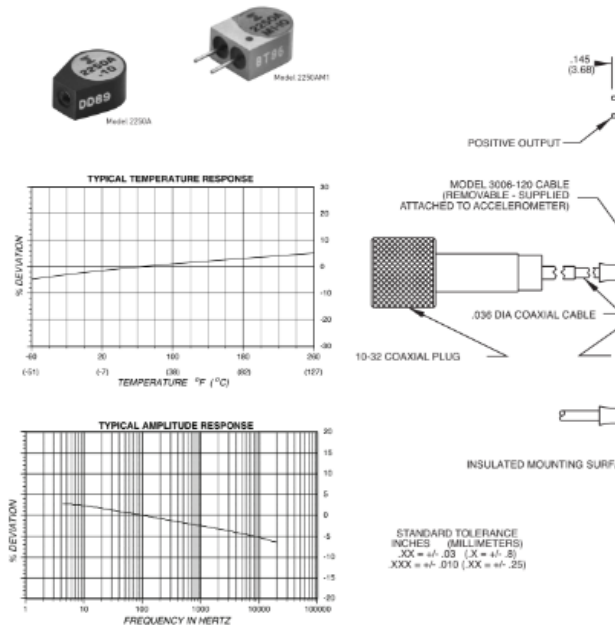
- Story of Radar Target Mast design
  - I asked a colleague what FS I should use, and he matter-of-factly said, since someone could die, use an FS of 10
  - Used 2" square tubing (0.25" thickness)
  - Designed a massive steel structure that was too heavy for a truck bed
  - Then had to add a trailer
  - Then had to add outriggers to the trailer
- Could have been a simple bolt-in-the-back-of-a-truck-bed if had used a more reasonable FS

ALL MASTS ARE FULLY ANODISED AND ALL FIXINGS SECURED WITH ADHESIVE DURING MANUFACTURE



# Isotron® accelerometer

## Model 2250A / AM1-10



The Endevco® model's 2250A/AM1 are extremely small, adhesive mounting pie accelerometers with integral electronics, designed specifically for measuring structures and small objects. These accelerometers offer high resonance frequency bandwidth, their lightweight (0.4 gm) effectively eliminates mass loading effect miniature cable is supplied with the 2250A-10, and small gage, lightweight hook with the 2250AM1-10.

Models 2250A/AM1 feature Endevco's Piezite® type P-8 crystal element, operate in a mode, which exhibits excellent output sensitivity stability over time. These accelerometers feature an internal hybrid signal conditioner in a two-wire system, which transmits its output through the same cable that supplies the constant current power. Signify the mounting surface by a ceramic mounting base. A tool is included in the package for removal of the accelerometer from its mounting surface.

Endevco signal conditioner Models 4416B, 133, 2792B, 2793, 2775B or Oasis 2000 computer-controlled system are recommended for use with these accelerometers.

# Isotron® accelerometer

## Model 2250A / AM1-10

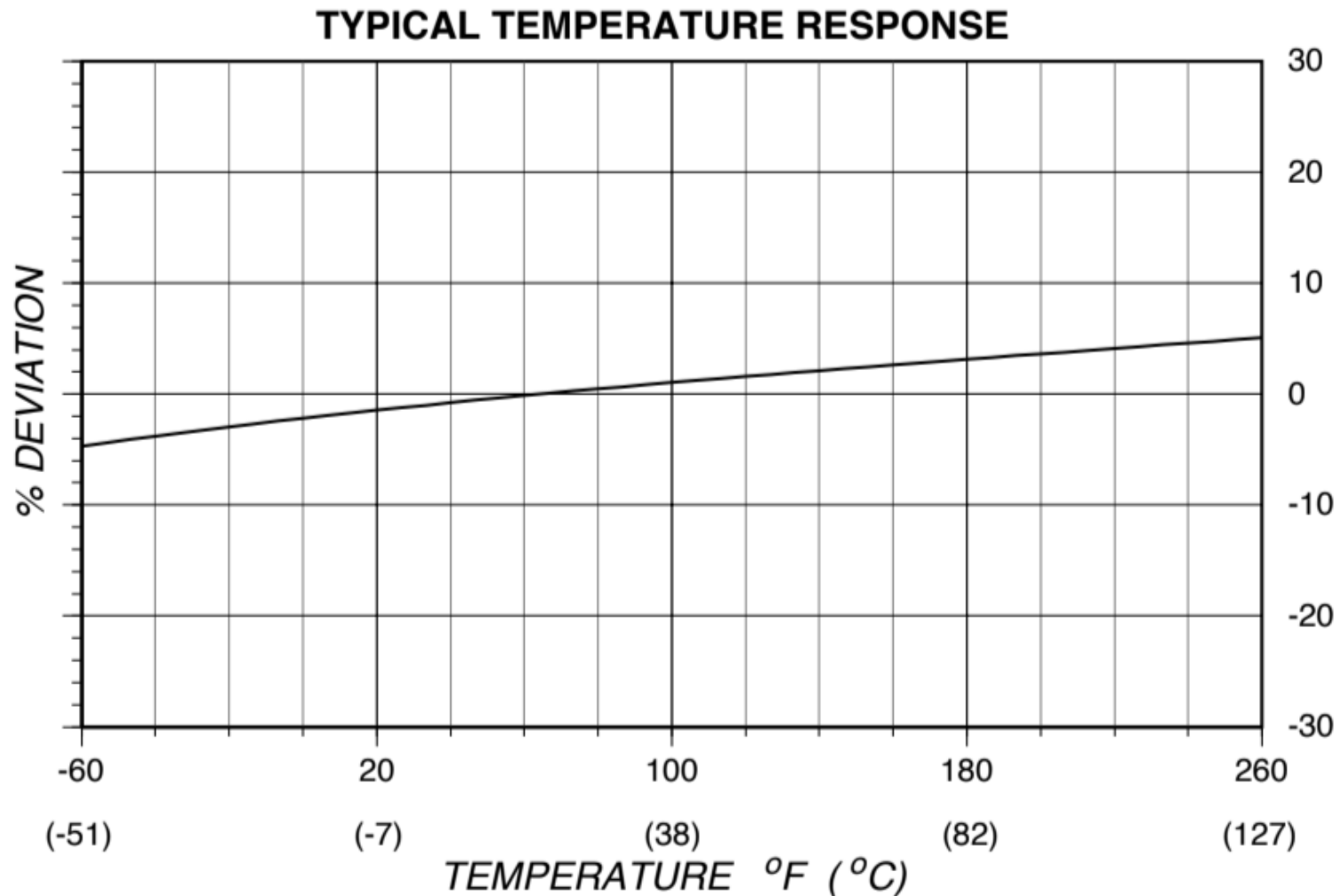
### Specifications

The following performance specifications conform to ISA-RP-37.2 (1964) and are typical values, referenced at +75°F (+24°C) and 100 Hz, unless otherwise noted. Calibration data, traceable to National Institute of Standards and Technology (NIST), is supplied.

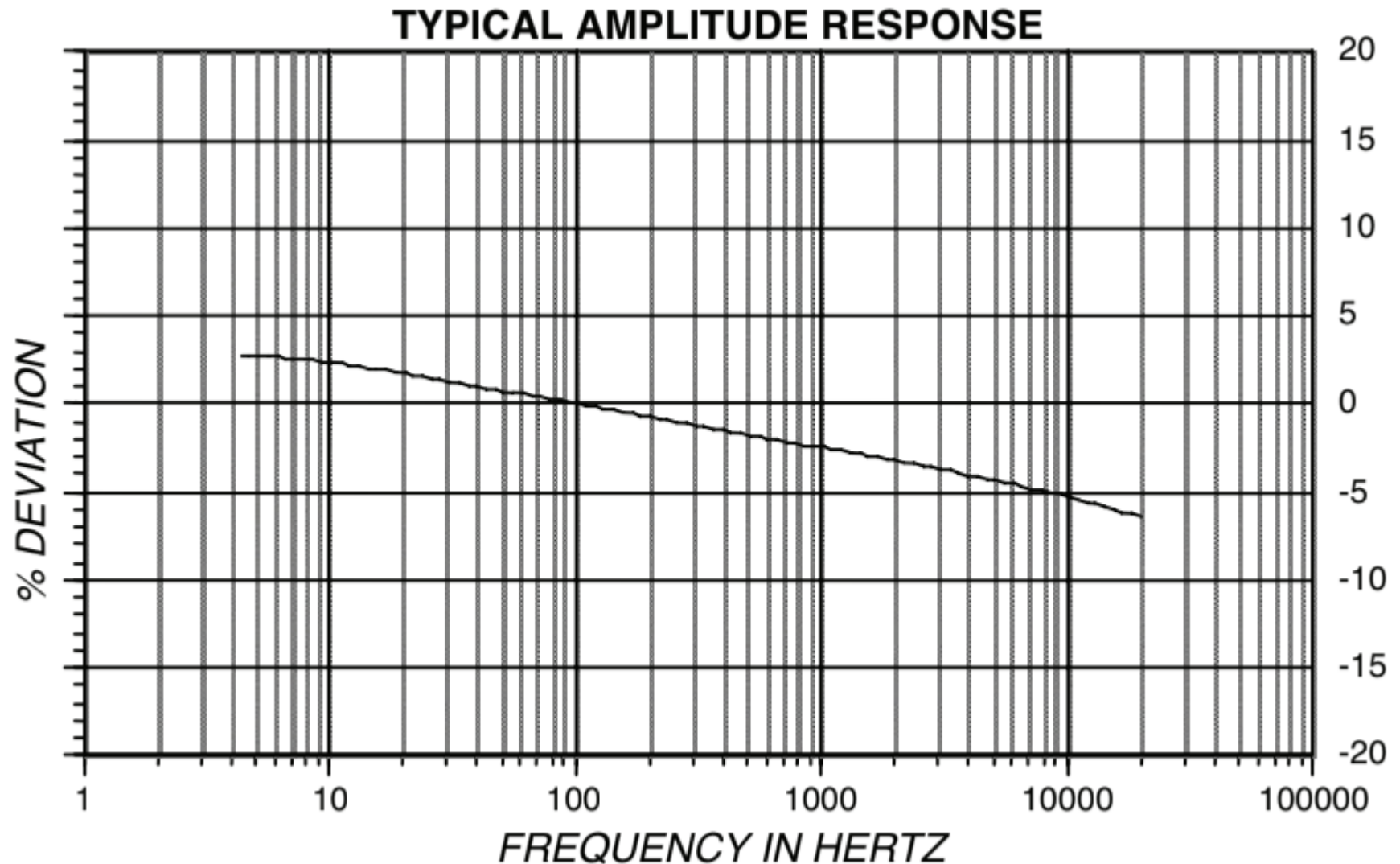
	Units	
<b>Dynamic Characteristics</b>		
Range	g	±500
Voltage sensitivity	mV/g	10
±5%		
Frequency response		See typical amplitude response
Resonance frequency	KHz	80
Amplitude response		
±1dB	Hz	2 to 15,000
Temperature response		See typical curve
Transverse sensitivity	%	≤ 5
Amplitude linearity [4]	%	1 to 500 g
<b>Output characteristics</b>		
Output polarity		Acceleration directed into the base of unit produces positive output
Compliance voltage	Vdc	18 to 30
Supply current	mA	2 to 20
DC output bias voltage	Vdc	6.5 to 12.5
Output impedance	Ω	≤ 100
Residual noise	equiv. g rms	0.0015
2 Hz to 25 kHz, broadband		
Grounding		Signal ground connected to case but isolated from mounting surface
<b>Environmental characteristics</b>		
Temperature range		-67°F to +257°F [-55°C to +125°C]
Humidity		Epoxy sealed, non-hermetic
Sinusoidal vibration limit	g pk	1000
Shock limit	g pk	2000
Base strain sensitivity	equiv. g pk/μ strain	0.0004
Thermal transient sensitivity	equiv. g pk/F* [/*°C]	0.1 (0.18)
Electromagnetic sensitivity	equiv. g rms/gauss	0.0001
<b>Physical characteristics</b>		
Dimensions		See outline drawing
Weight	gm (oz)	0.4 (0.01)
Case material		Anodized aluminum alloy case, beryllium copper lid, alumina mounting surface
Connector	2250A-10;	1.2 UNM threads. Recommended connector torque, 0.8 (lb-in [0.09 Nm]) or finger tight using wrench.
	2250AM1-10;	Solder terminal, "+" denoted by red dot.
		Flat surface provided for adhesive mounting
<b>Mounting [1]</b>		
<b>Calibration</b>		
Supplied:		
Sensitivity	mV/g	
Maximum transverse sensitivity	%	
Frequency response	%	
	dB	



# Temperature Uncertainty



# Amplitude Uncertainty



## OPERATING INSTRUCTIONS AND SPECIFICATIONS NI 9233

4-Channel,  $\pm 5$  V, 24-Bit IEPE Analog Input Module

Français Deutsch 日本語 한국어 简体中文

[ni.com/manuals](http://ni.com/manuals)

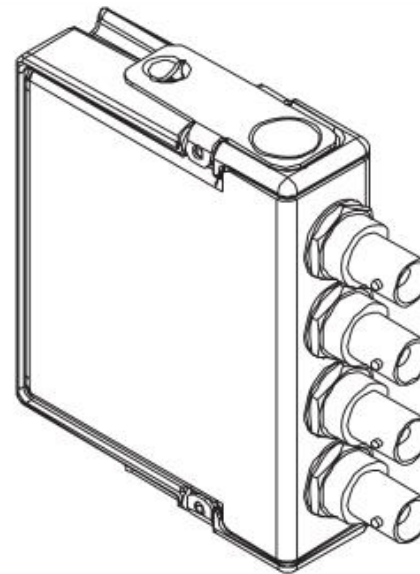
LSB = ?

$$\text{LSB} = 1/2^{24} = 6 \times 10^{-8}$$

$$\text{Resolution} = 10\text{V} \times 6 \times 10^{-8}$$

$$\text{Resolution} = 6 \times 10^{-7} \text{ V}$$

$$\text{Resolution} = \pm 0.06 \mu\text{V}$$



Essentially a  
multi-channel  
ADC

**Questions?**

# **22 – Advanced Stage Uncertainty**

# Advanced Stage

Why do we care about uncertainty  
(noise, instrument, resolution, etc.)?

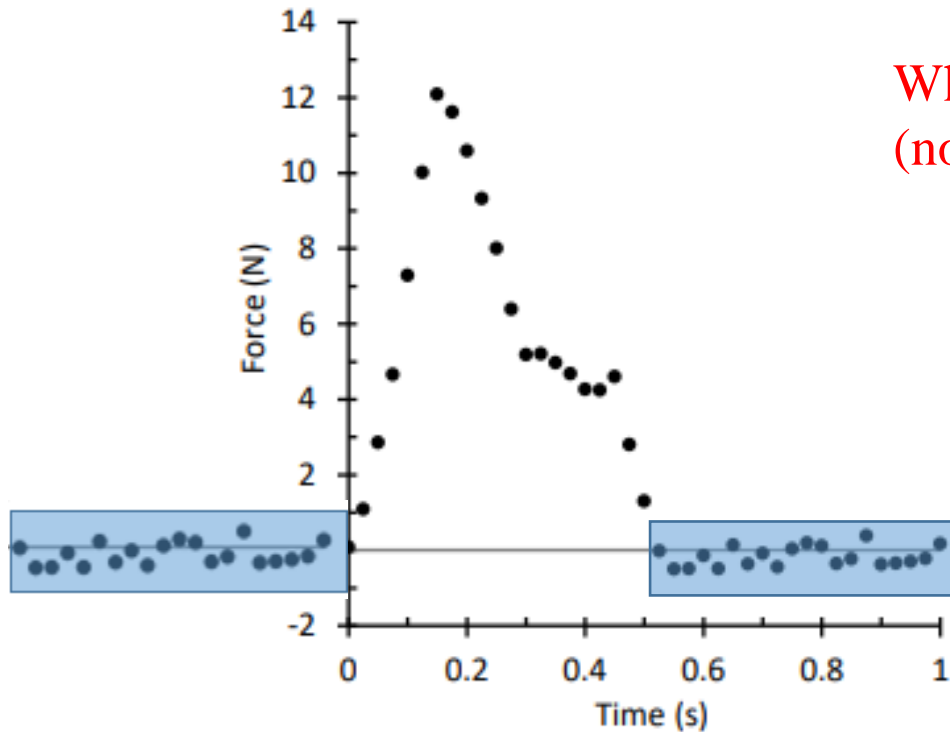


Fig. 3: Measured force vs. time data for the model rocket engine.

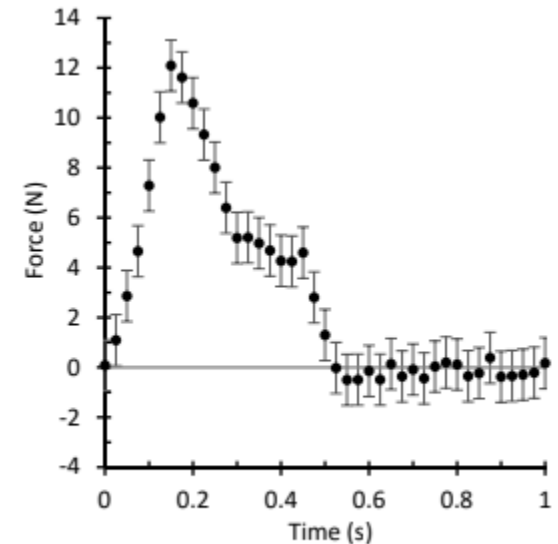


Fig. 7: Force vs. time data with error bars denoting  $\pm u_F$ .



# Advanced Stage

$$S_x$$

- Uses  $\bar{x}$  to calculate  $s_x$  from data
  - thus  $v = N-1$

$$s_x = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2}$$

$$S_{yx}$$

- Regression, use data once to calculate average
- Use average to minimize error of regression **line**
  - Since  $v = N - (m + 1)$
  - $v = N-2$

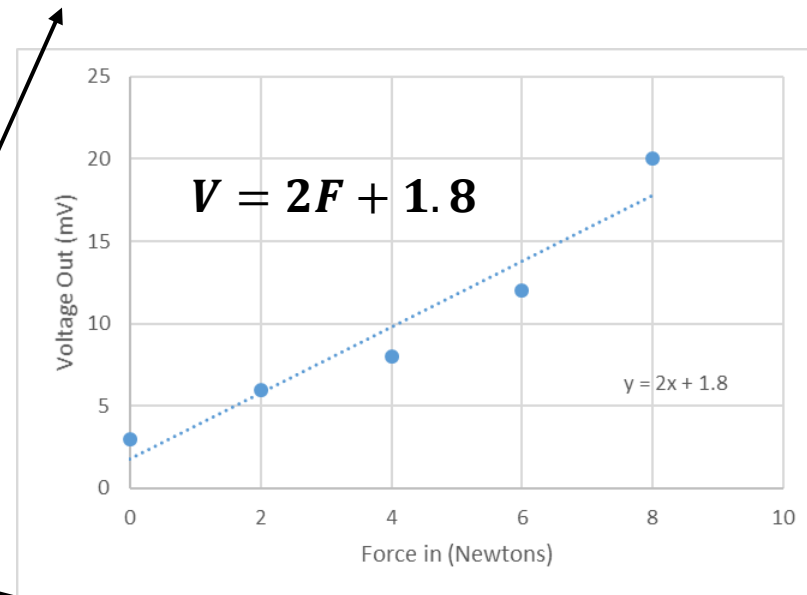
$$s_{yx} = \sqrt{\frac{1}{v} \sum_{i=1}^N (y_i - y_{ci})^2}$$

# Example #1

- Have cantilever beam with strain gages. We add weight to the end of the bar and measure voltage out, using a digital system with a resolution of 1 mV.

Force (N)	Voltage (mV)
0	3
2	6
4	8
6	12
8	20

Force (N)	Voltage (mV)
0	1
0	3
0	0
0	2
0	1



$$u_V = \pm \sqrt{(u_I)^2 + (u_0)^2 + \dots + (u_{noise})^2}$$

- Find  $u_V$

$$u_I = \pm t_{95,v} s_{yx} \quad s_{yx} = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (y_i - \bar{y})^2}$$

$$u_0 = \pm (\text{resolution})$$

$$u_{Noise} = \pm t_{95,v} s_x$$

$$s_x = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2}$$

# Example #1

**Table 4.4** Student's *t* Distribution

<i>v</i>	<i>t</i> <sub>50</sub>	<i>t</i> <sub>90</sub>	<i>t</i> <sub>95</sub>	<i>t</i> <sub>99</sub>
1	1.000	6.314	12.706	63.657
2	0.816	2.920	4.303	9.925
3	0.765	2.353	3.182	5.841
4	0.741	2.132	2.770	4.604
5	0.727	2.015	2.571	4.032
6	0.718	1.943	2.447	3.707
7	0.711	1.895	2.365	3.499
8	0.706	1.860	2.306	3.355
9	0.703	1.833	2.262	3.250
10	0.700	1.812	2.228	3.169
11	0.697	1.796	2.201	3.106
12	0.695	1.782	2.179	3.055
13	0.694	1.771	2.160	3.012
14	0.692	1.761	2.145	2.977
15	0.691	1.753	2.131	2.947
16	0.690	1.746	2.120	2.921
17	0.689	1.740	2.110	2.898
18	0.688	1.734	2.101	2.878
19	0.688	1.729	2.093	2.861
20	0.687	1.725	2.086	2.845
21	0.686	1.721	2.080	2.831
30	0.683	1.697	2.042	2.750
40	0.681	1.684	2.021	2.704
50	0.680	1.679	2.010	2.679
60	0.679	1.671	2.000	2.660
∞	0.674	1.645	1.960	2.576

Force (N)	Voltage (mV)	Force (N)	Voltage (mV)
0	3	0	1
2	6	0	3
4	8	0	0
6	12	0	2
8	20	0	1

$$V = 2F + 1.8 \quad \text{Resolution} = 1 \text{ mV}$$

$$u_I = \pm t_{95,v} s_{yx}$$

$$s_{yx} = \sqrt{\frac{1}{v} \sum_{i=1}^N (y_i - y_{ci})^2}$$

$$u_{\text{Noise}} = \pm t_{95,v} s_x$$

$$s_x = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2}$$

# Example #1

$$u_V = \pm \sqrt{(u_I)^2 + (u_0)^2 + \dots + (u_{noise})^2}$$

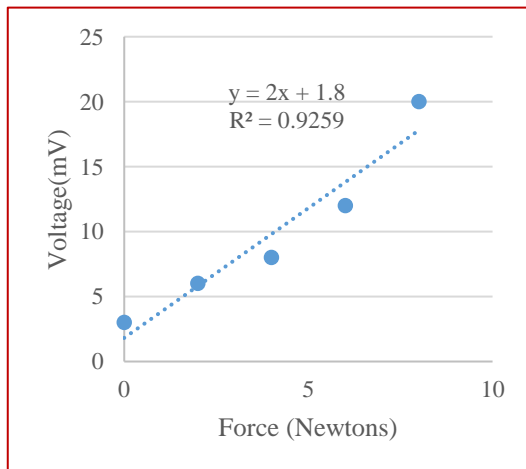
$$u_I = \pm t_{95,v} s_{yx} = \pm 6.57 \text{ mV} \quad (s_{yx} = 2.066, t_{95,3} = 3.182)$$

$$u_0 = \pm \text{resolution} = \pm 1 \text{ mV}$$

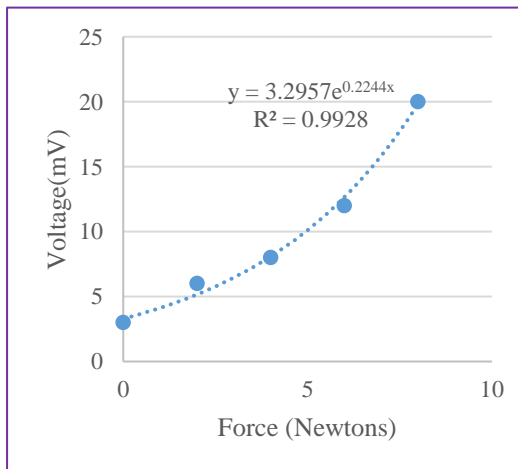
$$u_{Noise} = \pm t_{95,v} s_x = \pm 3.16 \text{ mV} \quad (s_x = 1.14, t_{95,4} = 2.770)$$

$$u_V = \pm 7.36 \text{ mV}$$

A



B



Difference between A vs B?

Which is better?

Why?

# Example #2 Mini Baja



## LPPS-22 Series Linear Potentiometer Position Sensor with Rod Ends

$$u_V = \pm \sqrt{(u_I)^2 + (u_0)^2 + \dots + (u_{noise})^2}$$

What units do we want our uncertainty in?

$$u_0 = \pm resolution = 1mV$$

$$u_I = .01 * 10V = 100 mV$$



### Specifications

$V_{in} = 10VDC$

**Output:**

0 to 100% of Input Voltage (potentiometer circuit)

**Non-Linearity, Full Stroke:  
Best Fit Straight Line (BFSL)**

$\pm 0.50\%$  (typical),  $\pm 1.0\%$  (max)

**Operating Temperature:  
Temperature Coefficient:**

$-40$  to  $+95^{\circ}C$  ( $-40$  to  $+203^{\circ}F$ )  
 $\leq \pm 0.03\%$  of FS /  $^{\circ}C$

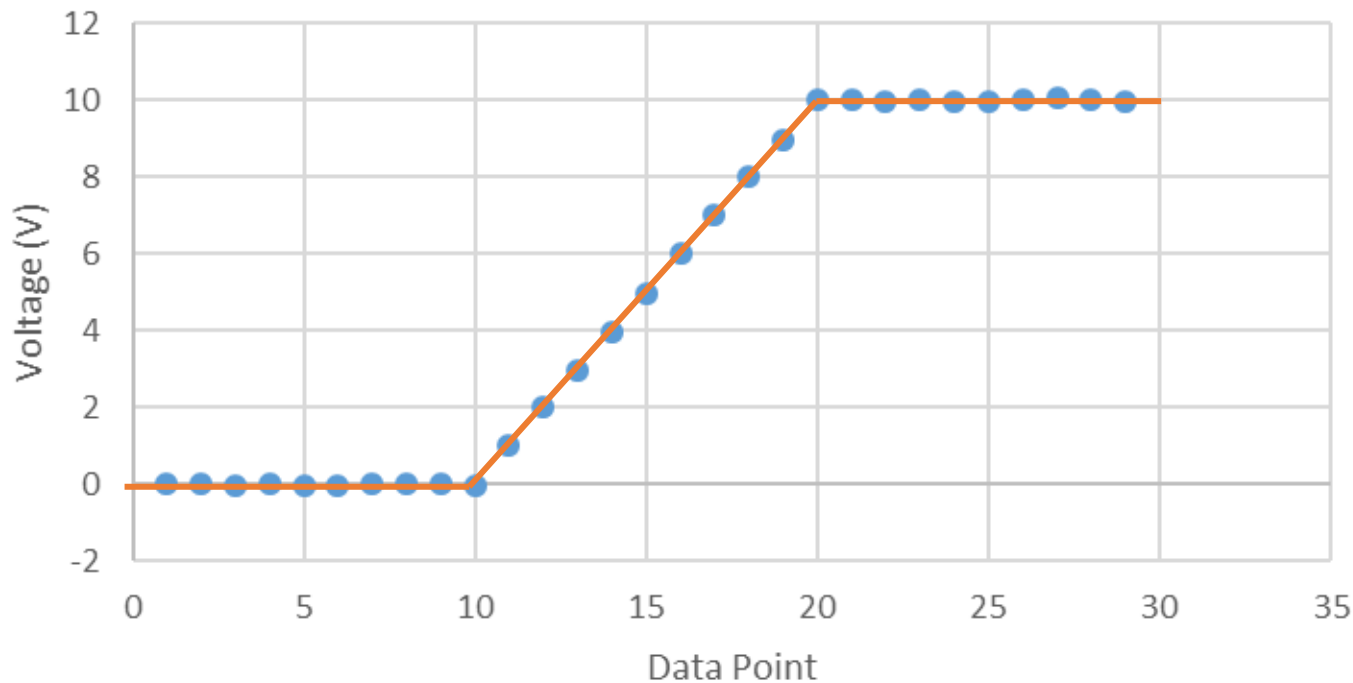
# Example #2 Mini Baja



## LPPS-22 Series Linear Potentiometer Position Sensor with Rod Ends



Potentiometer Plot





# Example #2 Mini Baja

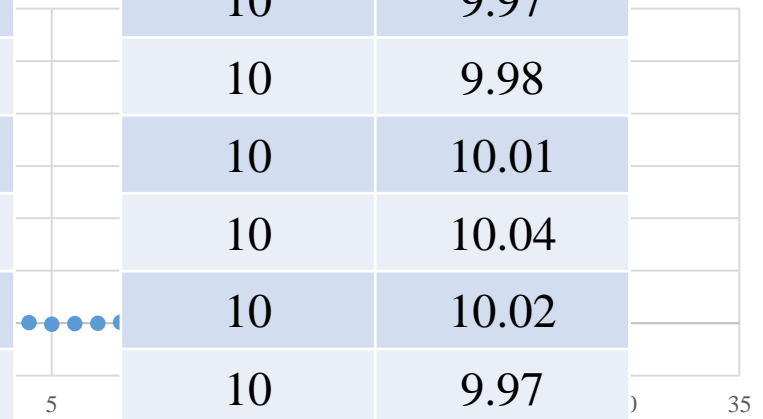


## LPPS-22 Series Linear Potentiometer Position Sensor with Rod Ends

Displacement (inches)	Voltage (Volts)
0	0.01
0	0.03
0	-0.02
0	0.02
0	-0.04
0	-0.01
0	0
0	0.03
0	0.01
0	-0.02

Displacement (inches)	Voltage (Volts)
0	-0.02
1	1.01
2	2.03
3	2.97
4	3.99
5	4.98
6	6.01
7	7
8	8.03
9	8.98
10	10

Displacement (inches)	Voltage (Volts)
10	10
10	10.03
10	9.07
10	10.03
10	9.97
10	9.98
10	10.01
10	10.04
10	10.02
10	9.97



Data Point



# Example #2 Mini Baja



## LPPS-22 Series Linear Potentiometer Position Sensor with Rod Ends

Displacement (inches)	Voltage (Volts)
0	0.01
0	0.03
0	-0.02
0	0.02
0	-0.04
0	-0.01
0	0
0	0.03
0	0.01
0	-0.02

$$s_{x,0} = 0.023V$$

$$s_x = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2}$$

$$N = 10$$

$$s_{pool} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$s_{pool} = \sqrt{\frac{(9)s_1^2 + (9)s_2^2}{18}} = 0.0257$$

$$t_{95,18} = 2.101$$

$$u_{Noise} = \pm t_{95,v} s_x = \pm 0.054 V$$

Displacement (inches)	Voltage (Volts)
10	10
10	10.03
10	9.97
10	10.03
10	9.97
10	9.98
10	10.01
10	10.04
10	10.02
10	9.97

$$s_{x,10} = 0.027V$$

# Example #2 Mini Baja

## LPPS-22 Series Linear Potentiometer Position Sensor with Rod Ends

Displacement (inches)	Voltage (Volts)
0	-0.02
1	1.01
2	2.03
3	2.97
4	3.99
5	4.98
6	6.01
7	7
8	8.03
9	8.98
10	10

$$s_{yx} = \sqrt{\frac{1}{v} \sum_{i=1}^N (y_i - y_{ci})^2}$$

$$v = 9$$

$$s_{yx} = 0.021V$$

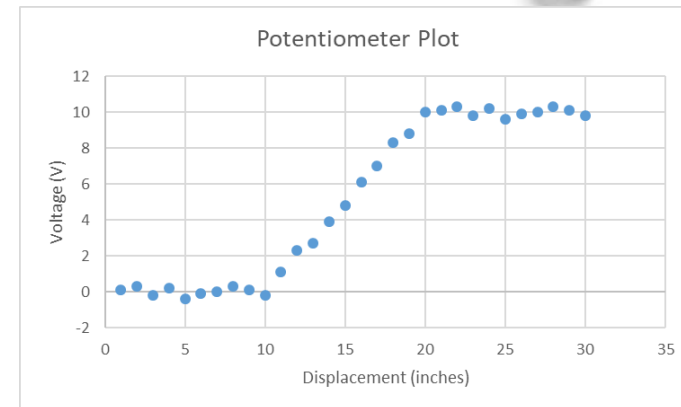
$$t_{95,9} = 2.262$$

$$u_I = \pm t_{95,v} s_{yx} = \pm 0.048V$$

$$u_{Noise} = \pm 0.054 V$$

$$u_V = \pm \sqrt{(u_I)^2 + (u_0)^2 + \dots + (u_{noise})^2}$$

$$u_V = \pm 72mV$$



**Questions?**