Final Report

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1 Introduction

The purpose of this report is to give an assessment of my research done over the spring and summer. There are many achievements and projects this summer but only a couple will be outlined in this report.

The first of these projects was the successful creation of a wing optimized to minimize induced drag. The project involved designing a wing that will lift 1.7 Newtons flying at 1 m/s at a pitch angle of 5 degrees and a span of 8.0 meters. The goal was to satisfy all these conditions whilst minimizing induced drag. To achieve the design the vortex lattice method was used to compute lift and drag values whilst a finite difference method generated via IPOPT was used as the optimizer of choice to design the wing.

The second of these projects was to create a computer program that will optimize trim stability on an aircraft given certain lift, twist, static stability, and leading edge constraints. The idea behind this project was that the user could input an initial aircraft design with certain wing densities, air density, etc values as well as certain criteria the aircraft must meet. The program would then create a wing and tail if specified, that is completely stable whilst still satisfying these constraints. This is an ongoing project and will be discussed later on.

2 Methods

2.1 Optimized Induced Drag Methods

The first part of this project involved creating a decently accurate model to get lift and drag coefficients. Luckily there exists a package for Julia that will allow the user to specify a wing which will then be analyzed with the corresponding lift and drag coefficients outputted. Note that these coefficients are defined for a 3d wing and are not airfoil polars.

To allow for a more complex wing shape that allows for less induced drag, the wing was divided into small sections that could each have their own unique chord length. Each chord length was then lined up so that the quarter distance from the leading edge to the the trailing edge of each section was along a straight line.

Once initial chord lengths were specified, the wing was given as an initial condition for the optimizer. Each variable was the individual chord length of a specified section. The optimizer would then take this and optimize an ideal wing that would satisfy the given constraints. The optimizer would then output the chord lengths that each section should have. The wing tended to be rough in shape. To combat this, a second order curve fit was employed to smooth out the wing to create a feasible shape. The smoothed out wing was then tested to find the final lift and drag values. The wing was then plotted and will be discussed later

2.2 Ongoing Method to Increase Accuracy from the Vortex Lattice Method

As part of the project to optimize aircraft stability, another method was derived to find accurate lift and drag coefficients. The reason behind the need is due to inaccurate drag data from the vortex lattice method. This method seeks to combine 2d airfoil theory with 3d wing theory. The idea is that the vortices at the wingtips will create an induced angle of attack throughout the wing. By using the vortex lattice method to find induced angle of attack along each section of the wing, one can then reference the 2d polars with the given angle of attack to get a more accurate 2d lift and drag value. This value is then expanded using geometry to the 3d definition of a lift coefficient. The data referenced from the polars is generated previously by Xfoil.

2.3 Stability Optimization Methods

This project is all based on a computer program created in Julia. Due to this program being in its early stages, the wing complexity is relatively limited. Currently the user can input only two wing chord lengths which means that the wing is trapezoidal in shape. Eventually this program can be used for more complicated shapes. In contrast, the user does have ample freedom to specify changes in each section's leading edge, angle of twist, tail distance, wingspan (tail-span as well), in addition to fluid properties such as density as well as aircraft speeds. The user can also input constraints on leading edge placement, angle of twist, and lift criteria. The last constraint that the aircraft must be statically stable is forced for all optimizations.

Once initial conditions are specified and input into the program, the optimizer Mogensen and Riseth (2018) will then work to create an ideal wing and tail shape and placement that will satisfy all constraints and be trim stable.

Similar to 2.2, this project is ongoing. The aim is to combine the method from 2.2 to provide accurate lift coefficient and drag coefficient data to provide a more accurate measure of trim stability. This provides a quick and accurate way to get data that can be used by an optimizer which has to run the model several times.

3 Results

3.1 Results from the Induced Drag Optimization

The optimization performed led to many different wing designs depending on the initial conditions. The large majority of wings created had very minimal chord lengths at the wingtips with large chord lengths mid-span of the wing.

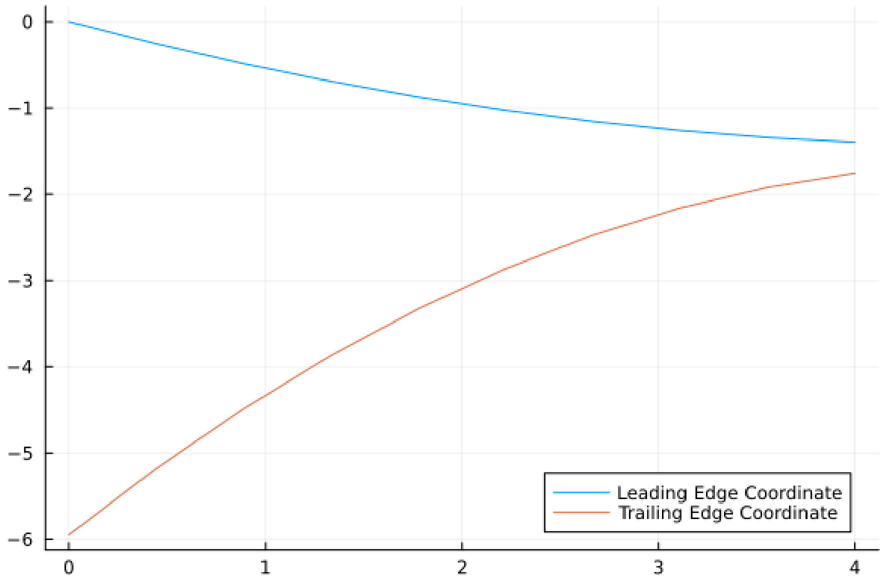
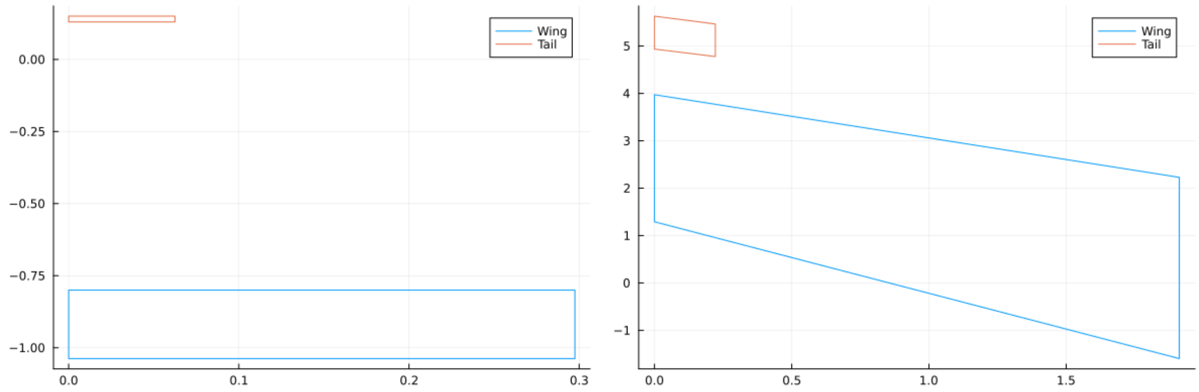


Figure 1: Common Wing Geometry Optimized for Minimal Induced Drag

3.2 Results From Trim Stability Optimization

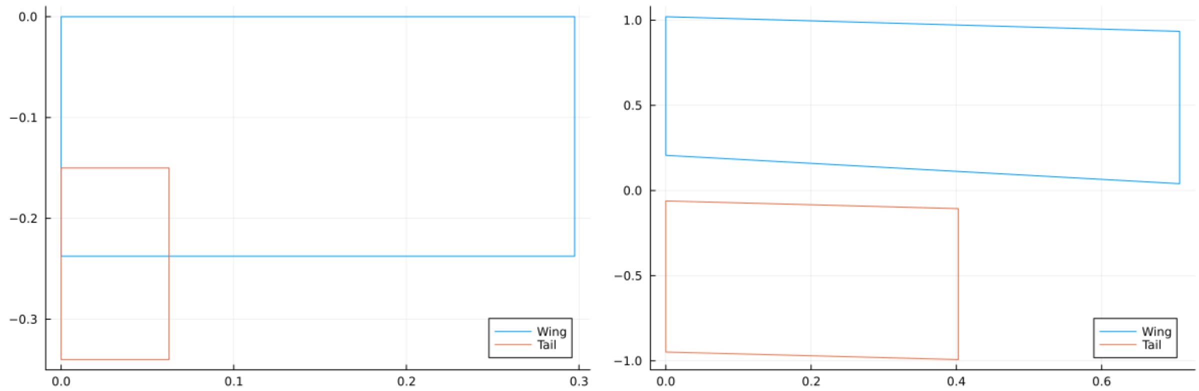
The trim stability optimization led to many results. The first being that there are many different wing configurations that will satisfy the same constraints. The optimizer tended to keep the original configuration(ie tail behind or in front of the wing) whilst still optimizing trim stability. This can be seen in figure 2 where the tail is kept in front after optimizing or in figure 3 where the tail is started behind the wing and ends also behind the wing. It is important to note that this initial condition places the the wing and tail on top of each other which is infeasible but the optimizer did lead to a feasible result as can be seen in figure 3. This is a result of adding an extra constraint that forces the wings to stay separate. Using a general non-linear optimizer allows the user to start in the infeasible region which is a nice perk of this optimizer.

In addition to keeping a similar configuration, the optimizer tended to always increase the size of the tail in each case. The proportion relative to the size of the wing however was not consistent which shows an area that should be researched further. Another result is that the optimizer tended to create a trapezoidal and swept wing. This result will be further expounded in 4.2. Lastly, the optimizer did a complete job and improving trim stability by decreasing the moment coefficient about the center of gravity (Cmg) as shown in figure 3 parts a and b. Ultimately, this optimizer has reduced the Cmg and therefore increased trim stability.



(a) Initial Starting Wing (b) Stabilized Wing

Figure 2: Initial Wing vs Stabilized Wing Example 1



(a) Initial Starting Wing: Cmg = 0.3158 (b) Stabilized Wing: Cmg = -0.000000295

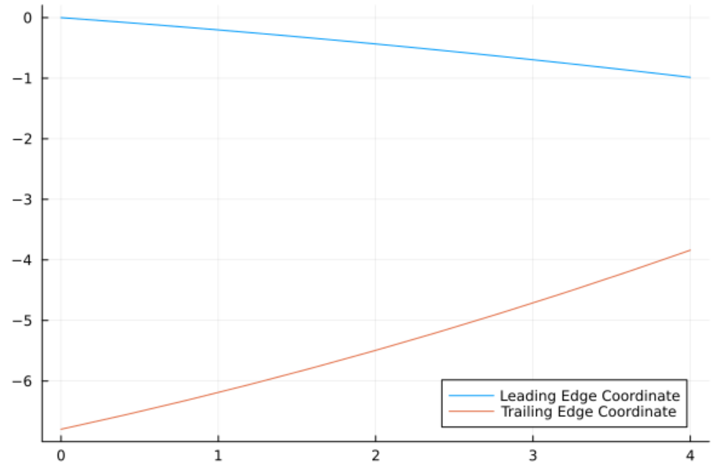
Figure 3: Initial Wing vs Stabilized Wing Example 2

4 Discussion

4.1 Induced Drag Wing Design

The wings designed generally followed the principle of short wingtips and long chords at mid-span. This is a reasonable result due to the fact that less lift is produced at the wingtips then mid-span so therefore the wing should be the largest where there will be the most lift. However, common consensus states that the wing shape with the least amount of induced drag follows an elliptical planform NASA (2023). Some of the optimizations did lead however to an elliptical planform. This can be seen in figure 4 where the wing planform is compared to a Supermarine Spitfire which is a well known aircraft with an elliptical planform Arpingstone (2004) that reduces induced drag.

(a) Supermarine Spitfire Wing Planform



(b) Optimized Wing Planform

Figure 4: Spitfire vs Computer Optimized Wing Comparison

As can be seen from figure 4, many of the optimizations did produce elliptical planform results, but that was wildly dependent on the initial conditions specified. For this particular case the initial conditions had smaller differences in chord lengths compared to many of the other cases that gave results similar to figure 1. The fact that there were so many different wing planforms depending on initial conditions shows that there likely isn't a global solution to this optimization problem. There are many local minimums which will reduce induced drag. By having different minimums, this gives greater flexibility when considering other constraints such as manufacturing feasibility and stability. This project has shown that low fidelity methods can work with optimizers and produce realistic results.

4.2 Trim Stable Wing Design

The results of using an optimizer to design a wing that is trim stable has shown that this is a feasible option. The Cmg was consistently decreased for all experiments be several orders of magnitude. However, there are many limitations to this current model. The wing and tail must have constant density which is usually not the case. Also the mean aerodynamic chord was computed be taking the distance along the chord which would encompass a quarter of the wing area. This is a simplified model to find the mean aerodynamic chord and won't always be the case, but it works to show that this optimization can be performed. There are many other simplifications and assumptions that create inaccuracies in the model, but notwithstanding these limitations, the model did produce wings that could feasibly be created and used in aerospace applications.

In addition to these limitations, the optimizer could be improved by adding additional constraints that were not present in this model. These could be constraints such as sweep, smoothness, monotonicity, and other constraints that could allow the wing to decrease its drag as well as increase manufacturability. For the purpose of this research, these constraints weren't needed in this specific model, but this is an area that needs further research.

Despite these limitations, the wings created were feasible and do reflect patterns seen in creating stable aircraft. One such example of this is the increase in the horizontal stabilizer. This is an important part of stability and often aircraft will have large horizontal stabilizers to balance the pitching created by the wing. Generally the horizontal stabilizer will be placed far behind the wing so as to avoid creating too large of a stabilizer, whereas the optimizer in this study tended to keep the horizontal stabilizer close to the wing which meant it needed a larger planform to compensate for the pitching moment. Despite this, the optimizer approached patterns seen in stabilizing aircraft. Ultimately this shows a successful implementation of optimization algorithms in aerospace applications.

4.3 Ongoing Work on New Wing Analysis Method

One area of further research is the implementation of a more accurate method (method mentioned in 2.2) of finding lift and drag values for an aircraft. Both lift and drag play into pitch stability of an aircraft and an optimizer is only as accurate as the function it uses to optimize. The computer software created to perform this function has created and successfully referenced tabulated airfoil information. It has also successfully computed 3d lift coefficients from 2d lift coefficients and vice versa. The current holdup however, is finding accurate induced angles of attack for each wing section. Once this is accomplished, more accurate lift and drag values will be computed and implemented with the aforementioned optimizer in 4.2 to create stable wings.

5 Appendix

Leading Edge: This is the front edge of the wing where the airflow starts to move over the wing

Polars: These are coefficients of lift and drag values for an airfoil based on its angle of attack

Xfoil: A computational fluid model that predicts polars of an airfoil Drela and Youngren (2013)

Angle of Twist: This is how much the airfoil is rotated about its normal axis (axis coming out of the airfoil).

Static Stability: The ability of an aircraft when angle of attack is displaced, to return to equilibrium position Ning (2018).

Trim Stability (Cmg): This is the natural tendency of an aircraft in equilibrium conditions to want to pitch up or down.This is numerically quantified by the net moment about the center of gravity created by an aircraft in equilibrium position Ning (2018).

Planform: This is the projected wing area into a flat area. It is used for the computing coefficients of drag, lift, and moment.

Induced Angle of Attack: This is the a change in angle of oncoming airflow. It is created by the wingtip vortices from a wing Ning (2022).

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