# A Survey Of Attribute-based Encryption Schemes

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## A. Access Control

As mentioned in introduction, access control and attribute as object of authorization are most essential feature of ABE. Firstly, the least fineness which still has the function of object of authorization in ABE turn into "attribute", instead of "identity" in IBE. We claim that access control is that if and only if attributes satisfy the access policy, the user get the permission to decrypt and will decrypt successfully. In addition, attributes the participant own are usually collected in a set called attribute set. Moreover, those attribute sets that satisfy specific access policy is called authorized attribute set.

Before our survey, the *secret sharing* in access control should be firstly introduced. To put it simply, secret sharing in ABE schemes is the operation that information distributed amongst attributes of each authorized attribute set achieve being reconstructed, while each or many but not all attributes in an authorized set(suppose these attributes can not be a collection of another authorized set) are of no use on their own. We can find that achieving access control is just for the purpose realizing secret share in all concrete ABE schemes, and detailedly obtaining the secret value, usually s, or an implicit expression on secret value. After acquire s in certain form, user can decrypt successful. For the reason that the ABE

syntax has been provided, now the discusses here is just up to gain of secret value s. Additionally, we denote several common notions in our three expressions as shown on Table I.

In order to specifically express the statement "attributes satisfy the policy", intuitionally, we can enumerate all situations, and the access structure A defined as the collection of all authorized attribute sets. Therefore, the statement above can be interpreted as that an authorized attribute set belongs to the set A. we call this expression as enumeration. As far as the easier comprehension of access control and access structure, enumeration performs well, but obviously this expression is not brief at all. Then, Boolean Formula is thought of. In Boolean Formula, the several relations implied in access structure among attributes are pointed out directly. Those relations contain threshold, AND, OR and NOT. Many explorations on how to realize Boolean Formula in specific schemes have been done since the time of birth of ABE, and so far the access tree and circuits have been accepted as the solution. More consideration that whether there is a black-box way to achieve access control is made by researches. In this way, inputs are associated with attributes, while output is the result that policy is satisfied or not. i.e.the way pays little attention to the concrete relationship among attributes. The Boolean Function,  $f: \{1,0\}^n \longrightarrow (0,1)$ , is recalled as theory model of this method. The input of the Boolean Function is a ndimension vector, and every dimension is a Boolean variable with the value of "1" or "0". In ABE context, each value of Boolean variable from input depends on whether user hold corresponding attribute. Then the output is also a boolean variable with the values of "1" or "0". Analogously, the value depends on whether the attributes of user set satisfy the access structure. As for how to realize Boolean Function in specific ABE, [?] utilized the linear secret-sharing scheme(LSSS) to address general monotone access structure.

Monotonicity of access structure and LSSS are introduced in our below. The *access tree* and *circuits* mentioned above are also surveyed in this subsection.

1) Monotonicity: Monotonicity is defined as follows:

**Definition 1** (Monotonicity[?], [?], [?], [?], [?], [?]) access structure  $\mathbb{A}$  is a collection of sets, and  $\mathbb{A}$  is monotone if  $\forall A, B$ , such that A is in  $\mathbb{A}$  and  $A \subseteq B$ , then B is also in  $\mathbb{A}$ . Otherwise  $\mathbb{A}$  is non-monotone.

In Boolean Formula, monotone access structure don't involve *NOT*, therefore the non-monotone access structure should support *NOT*.

In the access control of present ABE schemes, most access structure is monotone, due to that normal methods to express attribute in access structure, such as  $\rho(i)$  in LSSS and leaf node in access tree, only support the situation that each attribute is in a certain authorized set or not in, and don't support

TABLE I SEVERAL COMMON NOTIONS

Notions	Descriptions			
$\overline{p}$	the order of prime group			
$P = \{P_1, P_2 \cdots P_u\}$	$P_j, j \in 1, 2 \cdots u$ , denotes $j - th$ attribute as a unique number defined by system as a selected randomly number			
	P denotes the set of $u$ numbers corresponding all $u$ attributes			
$\gamma$	authorized set that can be used to decrypt successfully			
A	access structure in enumeration			
8	the secret to share			
X	the independent variable of polynomials			

As our discuss above, access control has various forms in expression, so does access structure, we let  $\mathbb{A}$  be the substance of access structure for all expressions because of briefer formulation and easier comprehension of it. Also we will point out the correspondence between  $\mathbb{A}$  and other forms blow.

the one that an attribute is not in any authorized sets, which however is necessary to non-monotone access structure. So, in order to achieve non-monotone access structure, "negative attribute" is supposed ([?]). Main idea of the supposition is that attributes are divided into two values, positive and negative, and preinstall two kinds of SK and ciphertext so that all attributes(with positive and negative values) of users authorized can be expressed in a certain authorized set. Nevertheless, the main problem of this method inclines to inefficient. Two values of each attribute imply the number of attributes in the system will be doubled too ([?]). As for this problem,[?] has found a solution that all attributes have positive values, and negative values of some attributes are added according to specific systems. Additionally, [?] has implicitly proposed a transformation technology to meet the monotone required by LSSS. i.e.this technology can transform non-monotone access structure into monotone one(not real), so that more efficient expressions only supporting monotonic access structure have possibility to address non-monotone access structure using this technology.

2) expressions on access control: In this part, we survey three main expressions from most of present ABE schemes, access tree, circuits and linear secret sharing scheme(LSSS), and unless stated otherwise, by an access structure we mean a monotone access structure. In addition, the comparison among the three methods has been made.

## Access Tree

Let  $\mathcal T$  be a tree as an access structure. Each non-leaf node of the tree represents a threshold gate, described by its children and a threshold value. The  $num_x$  labels the number of children of a node x and  $k_x$  labels its threshold value. Define that  $k_x=1$  if the threshold gate is an OR gate and that  $k_x=num_x$  if it is an AND gate. Each leaf node x of the tree is described by an attribute and a threshold value  $k_x=1$ .

For convenience of description on secret sharing, some extra function are defined here. Define function  $\operatorname{parent}(x)$  as the parent of the node x in the tree, and the function  $\operatorname{att}(x)$  the attribute same as  $P_i$  in Table I associated with the leaf node x only if x is a leaf node. The access tree  $\mathcal T$  also defines an ordering between the children of every node, i.e, the children of a node are numbered from 1 to  $\operatorname{num}$ , and the function  $\operatorname{index}(x)$ , which returns a value associated with the node x, and the index values are uniquely assigned to nodes in the access tree for inputs of secret sharing in an arbitrary manner.

### **Achieving Secret Sharing In Access Tree**

As far as satisfaction between attributes and policy, there

are two parts. One part called set part holds the authorized attribute set, and another called structure part holds the access structure.

In access tree, structure part chooses a polynomial  $q_x$  for each node x by the degree of the polynomial and points with the number of one more than degree. These polynomials are chosen detailedly in a top-down manner as follows. Firstly, beginning at the root node, for each node x, set the degree  $d_x$  of the polynomial  $q_x$  to be one less than the threshold value  $k_x$  of that node, i.e,  $d_x = k_x - 1$ . Then, for the root node x, set  $q_x(0) = s$  and  $d_x$  other points of the polynomial  $q_x$  randomly. After that, for other node x, set  $q_x(0) = q_{parent(x)}(index(x))$  and choose  $d_x$  other points randomly,too. Up to every leaf node in this way, we are finally aware of that the degree of each leaf node is 1, and it only has one point, i.e, each  $q_x$  of leaf node has been determined as a constant by the polynomial of its parent.

We can see the secret value s finally transmits to the constants in leaf nodes associating attributes in authorized set. In fact, process above is the reverse of secret sharing, the distribution for secret value s.

The set part holds the authorized attribute set as mentioned above, and we assume that the part is provided the constants corresponding attributes in the set of it own after the distribution for secret value, and that set part have known the structure of access tree. Label the access tree with root r as  $\mathcal{T}$ , and the subtree of  $\mathcal{T}$  rooted at the node x as  $\mathcal{T}_x$ . In this way,  $\mathcal{T}$  is the same as  $\mathcal{T}_r$ . Every node have two case, being leaf node and non-leaf one. For each leaf node x, specific constant is given if and only if  $att(x) \in \gamma$ , and denote  $\mathcal{T}_x(\gamma) = 1$ , so the  $q_x(X)$  can be calculated by the degree  $d_x = 0$  and the point containing an arbitrary number coupled with constant. After that, set part can find a point in  $q_{parent(x)}(X)$  containing index(x) coupled with  $q_x(0)$ , where X is shown on Table I. For each non-leaf node x, evaluate  $mathcalT_{xt}(\gamma)$  for all children x' of node x, and if and only if at least  $k_x$  children equal to 1, i.e. at least  $k_x$  points of  $q_x(X)$  can be obtained,  $\mathcal{T}_x(\gamma) = 1$ , so the  $q_x(X)$  can be calculated by the degree  $d_x = k_x - 1$  and optional  $k_x$  points gotten from children. After that, set part can again find a point in  $q_{parent(x)}(X)$  containing index(x) coupled with  $q_x(0)$ . Process above continues until acquires the value of  $q_r(0)$ , the secret value s. Additionally, if an extra part without authorized set attempts to pursue the secret value, it can't go on in certain step of obtaining  $q_x(X)$ due to the lack of some points.

Some Analyses And Remarks On Access Tree

Firstly, "attributes satisfy the policy" can be interpret on the expression of access tree as that leaf nodes have the consistency in logic with gates of tree layer by layer, such that finally  $\mathcal{T}_r(\gamma) = 1$ . So, A can be interpret here as the leaf node with the constraints from the structure of the tree. Secondly, general access tree narrated as above is monotone for the reason that if a attribute set  $\gamma$  can achieve secret share, another set, which contains all elements of  $\gamma$  plus an arbitrary attribute having been defined by system, can also achieve that simply by not using the additional attribute. Thirdly, each node of a layer of access tree is just the structure of threshold secret sharing[?] for the reason that  $\mathcal{T}_{x'}(\gamma) = 1$  with the number of at least  $k_x$  children. More specifically, "or" gate implies the threshold value 1, and "and" gate implies the threshold value number of children of the node, i.e, the threshold secret sharing is done by obtaining points with the number of one more that degree of target polynomial. Between two layers, massage is delivered by the point  $(0, q_x(0))$  from nodes of low layer to a node of up layer, and essentially achieving the point  $(0, q_x(0))$ is just the specific form in access tree of outcome that the Boolean Formula of this node x are satisfied by the nodes contacted with x from closest lower layer.

Finally, In many specific schemes [?][?], [?][?], [?], the method to build polynomials by Lagrange's interpolation. So whether there are other methods such us Newton interpolation to build polynomials in ABE schemes is a direction of future work. Moreover, every layer achieves threshold secret sharing by building polynomial, so we can consider the possibility that achieve sharing through other mathematical technologies.

## Circuits

For concision in exposition, we restrict that gates of *circuits* are either AND or OR two inputs. Define the circuit structure as a 5-tuple f=(u,q,A,B,GT), where u shown on Table I is the number of inputs corresponding the set of subscripts of  $P_i$  shown on Table I, and q is the number of gates. Label  $Inputs=\{1,...,u\},\ Wires=\{1,...,u+q\},\$ and  $Gates=\{u+1,...,u+q\}.$  The wire n+q is output wire of the whole circuit. A:GatesWires is a function to identify each gate's first incoming wire, and B:GatesWires is a function to identify each gate's second incoming wire. Finally, GT:GatesAND,OR is a function to identifies a gate as either an AND or OR gate.

Specify that  $\omega > B(\omega) > A(\omega)$ , where  $\omega \in Gates$ , so that the label of a gate  $\omega$  is the same as the label of the outgoing wire from  $\omega$ . For convenience of description, an extra function are defined here. First is layer of gate,  $layer(\omega)$ : the shortest path from gate  $\omega$  to an input belonging to the set Input plus 1, and naturally if  $\omega \in Inputs$ ,  $layer(\omega) = 1$ . we also define the layer of wires, layer'(a) = 1 and specially define the layer of output wire as layer'(u+q),

## **Achieving Secret Sharing In Circuit**

In circuit bounded with layer'(u+q) = l, the structure part firstly produces groups  $G = (G_1, \ldots, G_{l+1})$  of prime order p, with canonical generators  $g_1, \ldots, g_{l+1}$ , and find out a set of bilinear maps  $\{e_{i,j}: \mathbb{G}_i \times \mathbb{G}_j \longrightarrow \mathbb{G}_{i+j} | i,j \geq 1, i+j \leq l+1\}$ , so that the map  $e_{i,j}$  satisfies the following relation:  $e_{i,j}(g_i^a, g_j^b) = g_{ab}^{i+j} \forall a,b \in Z_p$ .

Then, the structure part chooses randomly  $r_1, \ldots, r_{u+q-1}, r_{u+q} \in Z_p$ , where the u+q values have one-to-one correspondence with u+q wires. Then, when those the numbers of subscript k of r are greater than u, where k denote a gate or their output wire, structure part does the calculation as followings.

When k is a AND gate, calculates  $choose1(k): r_k - r_{A(k)} - r_{B(k)}$ , then calculate  $g^c_m hoose1$ ; when k is a OR gate, calculates  $choose2(k): r_k - r_{A(k)}$  and  $choose3(k): r_k - r_{B(k)}$ , then calculate  $g^c_m hoose2$  and  $g^c_m hoose3$ , where m = layer(k). And the secret value is  $g^{r_{u+1}}_{l+1}$  same as s on Table I.

we assume that the set part have known  $e_{i,j}$  defined above,  $g_1$ , and for each attribute in the authorized set of its own, the part is also provided the corresponding  $r_k$ , where  $k \geq u$  is a input. Additionally, each AND gate k have been attached choose1(k); For each OR gate k, attach choose2(k) to k' first incoming wire and choose3(k) to k' second incoming wire.

The set part achieve secret sharing from the bottom up as follows.

For simple narration, denote that function  $C_k(x)=1$ , if and only if the  $input\ k$  is in the authorized set or logic of gate k is true. there are three case for k: input, gate AND and OR. Only when each k let  $C_k(x)=1$  holds, following calculation will be do. When k is a input, calculates  $e_{1,1}(g_1^{r_k},g_1)=g_2^{r_k}$ ; When k is a gate AND, calculates  $e_{m,1}(g_m^{r_A(k)},g_1)\cdot e_{m,1}(g_m^{r_{B(k)}},g_1)\cdot e_{m,1}(g_m^{r_{B(k)}},g_1)$   $e_{m,1}(g_m^{choose1(k)},g_1)=g_{m+1}^{r_k}$ ; When k is a gate OR, there are three cases. Case one is that  $C_t(x)=1$  where A(k) is output wire of gate t and that  $C_t(t)=1$  where  $C_t(t)=1$ 

# Some Analyses And Remarks On Circuit

Firstly, "attributes satisfy policy" can be interpreted on the expression of circuits similar with that of access tree, i.e, inputs have the consistency in logic with gates of circuit layer by layer, such that finally  $C_{u+q}(x) = 1$ . So,  $\mathbb{A}$  can be interpret here as the inputs with the constraints from the structure of the circuit. Secondly, the circuit narrated as above is monotone. As for non-monotone circuit, there is a simple transformation that uses De Morgans rule to transform any general Boolean circuit into an equivalent monotone Boolean circuit with negation gates only allowed at the inputs, and just ignore the layer of the negation gates then non-monotone circuit become our familiar form of the monotone circuit. Thirdly, each gate of a layer in circuit is straightforward Boolean formula that emulate all situations on the incoming wires for that output of the gate's Boolean formula equal to 1, and that do nothing for that output of the gate's Boolean formula equal to 0. More specifically, for the gate in circuit above, according to different situations in incoming wires make the corresponding choices, which is shown on Table II. Between two layers, massage is delivered in the way that logical value as output of lower layer performs

TABLE II
CHOOSING DEPENDS ON DIFFERENT SITUATION IN INCOMING WIRES

Incoming Wire		gate type	output	choosings
The First	The Second	gate type	output	Choosings
1	1	AND	1	choose1
1	0	OR	1	choose2
0	1	OR	1	choose3
1	1	OR	1	choose2 or choose3
*	*	*	0	

meanwhile as input of the gate of higher layer, and accordingly  $g_k$  by pairing operation with a jump element  $g_1$  becomes  $g_{k+1}$ .

Finally, we can see the circuit as well as access tree is designed by fully considering the Boolean Formula, i.e, finding resolutions of that how to achieve the building of specific relationship among attributes. But for circuit, there is apparent drawback that if each gate has more input, then the situations mentioned above becomes more and more. For example, each gate has 3 inputs, so that there are 8 the situations that output of the gate's Boolean formula equals to 1, and 8 choices for all situations despite of 4 distinct choices. In addition, circuit used in ABE schemes bring more cost of communication between two parts due to more prepositive parameter such us those groups and pairing operation.

## **Linear Secret Sharing Scheme**

Access structure in the LSSS is described as follows. Firstly, define a surjection,  $\rho:\{1,\ldots,\ell\}\longrightarrow\{P_1,\ldots,P_u\}$ , where  $P_j$  is shown on Table I. There is a matrix M with  $\ell$  rows and n columns, where  $\ell\geq u\geq n$ . Row index  $i\in\{1,\ldots,\ell\}$  represents the label of ith row of M, and  $M_i$  represents the ith row of M. Let  $\rho(i)=P_j$  so that the row index of M map to attributes. Then define a set  $I=\{i|\rho(i)\in\gamma\}$ , where  $\gamma$  denotes an authorized set shown on Table I, and the  $M_\gamma$ .  $M_\gamma$  is constituted by combining  $M_i$  for all  $i\in I$ .

Now, we reveal how to achieve secret sharing. Structure part generate a column vector  $\mathbf{v} = (s, r_2, \dots, r_n)$ , where s is the secret to be shared, and  $r_2, \dots, r_n \in \mathbb{Z}_p$  are randomly chosen. Then computes  $M_i \cdot \mathbf{v}$  for all  $i \in \{1, \dots, \ell\}$ .

$$\sum_{i\in I}(\omega_i\cdot M_i)=\mathbf{e}$$

For set part,  $M_i \cdot v$  for  $i \in I$  have been known. This part can find out constant  $\omega_i$ , and make equation (III-A2) holds, where e is a n-dimension row vector and  $e = (1, 0, \dots, 0)$ , so that

$$\sum_{i \in I} (\omega_i \cdot (M_i \cdot v)) = (\sum_{i \in I} (\omega_i \cdot M_i)) \cdot v = e \cdot v = s$$

[?] point that each  $\gamma$  can reconstruct the secret in the method by using a linear function of its pieces, attributes in  $\gamma$ . We informally call the secret sharing scheme using linear function is Linear Secret Sharing Scheme. the span program [?] is the known method. Monotone span program (MSP) is associated monotone Boolean Function, and [?] proved that if there is a monotone span program for some Boolean Function then there exists a linear secret sharing scheme for the corresponding access structure, and vice versa [?], so matrix M, and those constants  $\omega$  can be found for certain.

## Some Analyses And Remarks On LSSS

Firstly, "attributes satisfy policy" can be interpreted in LSSS as that the row indexes achieve finding out those  $\omega$  such that equation (III-A2) holds. So,  $\mathbb A$  is interpreted here as row indexes with the constraints from matrix M and mapping  $\rho$ , and actually between these two objects there is a intermediate, the collection of input boolean vectors of Boolean Function.

As above mentioned, general LSSS can only address monotone access structure. When it occur to non-monotone case, use technology of [?].

LSSS achieve access control though the Boolean Function, so that there have no logical judgments or other consideration on relation among attributes. Therefore, LSSS is somewhat more expressive than above two expressions as far as automaticity and mathematics, which implies that not only can general monotone access structure be expressed by Boolean Function but also LSSS can achieve access control based that.

#### **Comparisons**

- B. Design philosophy of classical ABE scheme
  - some introduction goes here.
- 1) KP-ABE: analyze article: Attribute-Based Encryption for Fine-Grained Access Control of Encrypted Data Vipul Goyal Omkant Pandeyy Amit Sahaiz Brent Waters in Proc of Acmccs- 2006
- 2) CP-ABE: analyze article: Ciphertext-Policy Attribute-Based Encryption John Bethencourt Amit Sahai Brent Waters in 2007 IEEE Symposium on Security and Privacy 2007

#### IV. STATE-OF-THE-ART OF THE ABE

#### A. the birth of ABE

Attribute-based encryption is firstly mentioned in[?],this idea originates from Hierarchical identity-based encryption schemes[?] and the schemes of [?] can be achieved due to the inspiration from threshold secret share technology by[?]. After that, researchers pay much attention on general policy, which ties data owner with data user as a series of formalized constraint, compared with traditional point-to-point constraint(corresponding the privilege management infrastructure technology). So the access control technology is recalled, and ABE has perfectly formed just when this technology and the attribute as object of authorization are used in public-key scheme. When access control technology occurs to ABE, there are two types of models realized, i.e KP-ABE and CP-ABE.[?] is the first KP-ABE scheme and [?] is the first CP-ABE, so these two schemes proposed sign the perfect formation of ABE. After then, apart from research on more practical access policies, which are surveyed in section 3, the directions of development of ABE can be summarized those: fuction, efficiency and security.

### B. efficiency

1:Survey on original ABE, and some efficient technology such us constant ciphertext and constant cost in decryption.

- 2:You describe these articles in time sequence.
- 3:And also need a table to compare efficiency among articles mentioned by you.

### C. security

- 1:Survey on original ABE, and some articles, which have promoted in security, based on original ABE.
  - 2: You describe these articles in time sequence.
- 3:Also need a table to compare security among articles mentioned by you.

#### D. function

- 1:Survey on original ABE, and some articles, which have promoted in function, based on original ABE.
  - 2: You describe these articles in time sequence.
- 3:Also need a table to compare security among articles mentioned by you if possible.

## V. EXTENSION

some introduction goes here.

A. Key-abuse Problem

Key-abuse Problem goes here.

B. Outsource

Key-abuse Problem goes here.

C. Proxy Re-encrypton

Proxy Re-encrypton goes here.

D. Multi-authority And distributed ABE

Multi-authority And distributed ABE goes here.

E. Revocation

Revocation goes here.

F. hide-policy ABE

hide-policy ABE goes here.

# VI. RELATED WORK

With the exploration on ABE from aspects of efficiency, security and function, more and more ABE variants has been growing. However, there has been no integrated overview published that contains not only various ABE schemes, but also analysis of design philosophy. [?] does the survey on ABE of access control, but just enumerate CP-ABE, KP-ABE, ABE Scheme with Non-Monotonic Access Structures and so on, so that we cannot obtain the clear relationship among those schemes from the paper, and also Boolean formula of LSSS and circuit do not be mentioned. From the standpoint of difficulties in ABE, [?] describes the access structure, attribute revocation, key-abuse problem and multi-authority scheme, but we cannot find the Non-Monotonic Access Structures and Boolean formula of circuit in the part of access structure from this article. Besides, the ABE variants such as proxy reencryption, outsource ABE are not concluded. [?] points out the direction of development on hot topics of ABE, However the survey on the most essential access policy is not embodied.

VII. FUTURE WORK

Future work goes here.

VIII. CONCLUSION

The conclusion goes here.

#### REFERENCES

[1] J. Donald and M. Martonosi, "Techniques for multicore thermal management: Classification and new exploration," in *Proc. Int. Symp. on Computer Architecture (ISCA)*, June 2006, pp. 78–88.

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