

# Verifiable outsourced ciphertext-policy attribute-based encryption in cloud computing

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**Abstract** In the attribute-based encryption (ABE) systems, users can encrypt and decrypt messages based on their attributes. Because of the flexibility of ABE, it is more and more widely used in various network environments. However, complex functionality of ABE may cause an enormous computational cost. This reason greatly restricts the application of ABE in practice. In order to minimize the local computation of ABE, we introduce the concept of verifiable outsourced ABE system, in which key generation center, encryptor and decryptor, are able to outsource their computing tasks to the corresponding service providers, respectively, to reduce the local load. In addition, they are also able to

verify the correctness of outsourcing calculation efficiently by using the outsourcing verification services. This is useful to save local computational resources, especially for mobile devices. Then, we propose a specific verifiable outsourced ABE scheme and prove its adaptive security in the standard model using the dual-system encryption method. Finally, we introduce how to deploy our outsourced CP-ABE scheme in cloud computing environment.

**Keywords** CP-ABE · Outsourcing · Verifiable · Cloud computing · Adaptively secure

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## 1 Introduction

In 2005, [Sahai and Waters \(2005\)](#) first introduced the concept of attribute-based encryption (ABE). It is an extension of identity-based encryption in which the “identity” of users are dependent upon their attributes (e.g., Shandong Normal University, male, teacher). In such a system, the decryption is correct only if the set of attributes of the user key matches the access policy of the ciphertext. Then, [Goyal et al. \(2006\)](#) give the formal definition of ABE system, and divide this concept into two types: ciphertext-policy ABE (CP-ABE) and key-policy ABE (KP-ABE). In CP-ABE, ciphertexts are associated with access policies and keys are associated with sets of attributes, while in KP-ABE, keys are associated with access policies and ciphertexts are associated with sets of attributes.

In 2007, the first CP-ABE scheme was proposed in the random oracle model by [Bethencourt et al. \(2007\)](#). Then [Cheung and Newport \(2007\)](#) proposed an “AND-gate” CP-ABE scheme without random oracle (the access policies only support “AND-gate”). In 2008, [Waters \(2008\)](#) proposed the first expressive CP-ABE using linear secret-sharing schemes

(LSSS) as access policies and proved security in the selective model. Until 2010, Lewko proposed the first fully secure ABE schemes in [Lewko et al. \(2010\)](#), using the dual-system encryption method. In the current ABE systems, access policies are typically expressed by LSSS, which makes keys and ciphertexts have rich structure.

### 1.1 Motivation and contribution

The main challenge in employing ABE systems in the real network environment is that the complex functionality of ABE may cause an enormous computational cost. This large expense may impact several networking applications, especially for mobile networks.

In this paper, we aim to minimize the local computation for CP-ABE and introduce a new method to construct the outsourced CP-ABE. Our outsourced CP-ABE includes four types of service providers for outsourcing tasks, i.e., outsourcing key generation server provider (OKGSP), outsourcing encryption server provider (OESP), outsourcing decryption server provider (ODSP) and outsourcing verification server provider (OVSP). To improve the computational efficiency, KGC can outsource part of the calculation tasks of key generation to OKGSP, encryptor can outsource part of the calculation tasks of encryption to OESP and decryptor can outsource part of the calculation tasks of decryption to ODSP. In addition, all entities can verify the correctness of the outsourcing calculation by calling the outsourcing verification server provider. It is useful for mobile devices to save local resources. Then, we proposed a specific outsourced CP-ABE scheme and proved its security in the standard model. In our new outsourced CP-ABE scheme, the local computation of each stage is constant, and users can verify the correctness of the outsourcing results efficiently. Finally, we introduced how to deploy our outsourced CP-ABE scheme in cloud computing environment and introduced a specific application in secure cloud storage system.

### 1.2 Related work

To reduce the local computation cost, the natural idea is to deliver most of computational tasks outside. In actual fact, theoretical computer science community has paid attention to the problem that how to outsource different kinds of expensive computations securely ([Atallah et al. 2001](#); [Atallah and Li 2005](#); [Benjamin and Atallah 2008](#); [Chen et al. 2012](#)). Recently, Chen et al. presented efficient algorithms for secure outsourcing of modular exponentiations ([Chen et al. 2014](#)), linear equations ([Chen et al. 2015a](#)) and bilinear pairings ([Chen et al. 2015b](#)). However, we note that though kinds of expensive computations can be securely outsourced, they are difficult to be used directly to construct the provably secure outsourced CP-ABE.

To reduce the local load of ABE, [Green et al. \(2011\)](#) provided a novel method for outsourcing the decryption of ABE, while [Li et al. \(2012\)](#) presented a novel ABE scheme, which could outsource both encrypting computation and decrypting computation to MapReduce cloud securely. Then, [Lai et al. \(2013\)](#) proposed a verifiable outsourced ABE scheme, which allows to verify the correctness of decryption. Their scheme appends additional information with the ciphertexts and uses this information for verification. In 2014, [Li et al. \(2014\)](#) presented a new outsourced ABE scheme, which allows to outsource key generating computation and enables checkability on returned results from cloud. Recently, [Wang et al. \(2015\)](#) proposed an adaptively secure outsourced CP-ABE scheme in the standard model; however, they only considered the outsourcing needs of the decryption and did not verify the outsourcing results.

Furthermore, because of the flexibility of ABE, it is more and more widely used in various network environments, such as cloud computing ([Zhangjie et al. 2015b](#); [Ren et al. 2015](#); [Zhangjie et al. 2015a](#)), social networking ([Shen et al. 2015a](#)), web of things ([Shen et al. 2015b](#); [He et al. 2015](#); [He et al. 2016b](#)), access control system ([He et al. 2016a](#)), authentication system ([Huang et al. 2014](#); [Huang et al. 2015](#)), and so on.

### 1.3 Organization

We introduce the preliminaries in Sect. 2 and provide the relevant definitions for outsourced ABE in Sect. 3. In Sect. 4 and Sect. 5, we construct a specific outsourced CP-ABE scheme and give the security proof, respectively. Then, we introduce how to deploy our outsourced CP-ABE scheme in cloud computing environment in Sect. 6. Finally, we give the conclusion in Sect. 7.

## 2 Preliminaries

### 2.1 Composite order bilinear group

The concept of composite order bilinear group was first introduced by [Boneh et al. \(2005\)](#).

$\mathcal{G}(\lambda) \rightarrow (N, G, G_T, e)$ :  $\mathcal{G}$  is a group generation algorithm. It takes a security parameter  $\lambda$  as input and outputs a description of bilinear cyclic group with order  $N$ , where  $N = \prod_{i=1}^n p_i$  is a product of  $n$  distinct primes, and  $e : G^2 \rightarrow G_T$  is map such that:

- (Bilinear)  $\forall g, h \in G, a, b \in \mathbb{Z}_N, e(g^a, h^b) = e(g, h)^{ab}$ .
- (Non-degenerate)  $\exists g \in G$  such that  $e(g, g)$  has order  $N$  in  $G_T$ .
- (Computable) The group operations in  $G$  and  $G_T$  and the map  $e$  are computable in polynomial time with respect to  $\lambda$ .

- (Orthogonal) If  $f_i \in G_{p_i}$  and  $f_j \in G_{p_j}$  for  $i \neq j$ , then  $e(f_i, f_j)$  is the identity element in  $G_T$ .

In this paper, we set  $n = 3$ , i.e.,  $N = p_1 p_2 p_3$ . For any non-empty set  $Z \subseteq \{1, 2, 3\}$ , there is a corresponding subgroup of  $G$  of order  $\prod_{i \in Z} p_i$ . Let  $G_Z$  denote this subgroup.

## 2.2 Complexity assumption

**Assumption 1** Lewko and Waters (2012) Let  $Z_0, Z_1, Z_2, \dots, Z_k$  denote a collection of non-empty subsets of  $\{1, 2, 3\}$  where each  $Z_i$  for  $i \geq 2$  satisfies either  $Z_0 \cap Z_i \neq \emptyset \neq Z_1 \cap Z_i$  or  $Z_0 \cap Z_i = \emptyset = Z_1 \cap Z_i$ . We define the following distribution:

$$\begin{aligned} \mathbf{G} &:= (N = p_1 p_2 p_3, G, G_T, e) \xleftarrow{R} \mathcal{G}, \\ g_{Z_2} &\xleftarrow{R} G_{Z_2}, \dots, g_{Z_k} \xleftarrow{R} G_{Z_k}, \\ D &:= (\mathbf{G}, g_{Z_2}, \dots, g_{Z_k}), \\ T_0 &\xleftarrow{R} G_{Z_0}, T_1 \xleftarrow{R} G_{Z_1} \end{aligned}$$

Fixing the collection of sets  $Z_0, \dots, Z_k$ , the advantage of an algorithm  $\mathcal{A}$  in breaking this assumption is defined to be:

$$\text{Adv}_{\mathcal{G}, \mathcal{A}}^1(\lambda) := |\Pr[\mathcal{A}(D, T_0) = 1] - \Pr[\mathcal{A}(D, T_1) = 1]|.$$

If  $\text{Adv}_{\mathcal{G}, \mathcal{A}}^1(\lambda)$  is a negligible function, we say that  $\mathcal{G}$  satisfies Assumption 1.

**Assumption 2** Lewko and Waters (2010), Lewko et al. (2010), Lewko and Waters (2012) We define the following distribution:

$$\begin{aligned} \mathbf{G} &:= (N = p_1 p_2 p_3, G, G_T, e) \xleftarrow{R} \mathcal{G}, \\ g_1 &\xleftarrow{R} G_{p_1}, g_2, X_2, Y_2 \xleftarrow{R} G_{p_2}, g_3 \xleftarrow{R} G_{p_3}, \\ \alpha, s &\xleftarrow{R} \mathbb{Z}_N \\ D &:= (\mathbf{G}, g_1, g_2, g_3, g_1^\alpha X_2, g_1^s Y_2), \\ T_0 &= e(g_1, g_1)^{\alpha s}, T_1 \xleftarrow{R} G_T \end{aligned}$$

The advantage of an algorithm  $\mathcal{A}$  in breaking this assumption is defined to be:

$$\text{Adv}_{\mathcal{G}, \mathcal{A}}^2(\lambda) := |\Pr[\mathcal{A}(D, T_0) = 1] - \Pr[\mathcal{A}(D, T_1) = 1]|.$$

If  $\text{Adv}_{\mathcal{G}, \mathcal{A}}^2(\lambda)$  is a negligible function, we say that  $\mathcal{G}$  satisfies Assumption 2.

## 3 Definition of outsourced ABE

### 3.1 Algorithms

In an outsourced ABE system, there are seven types of entities, namely key generation center (KGC), encryptor, decryptor, outsourcing key generation server provider (OKGSP), outsourcing encryption service provider (OESP), outsourcing decryption service provider (ODSP), and outsourcing verification service provider (OVSP). They interact as in Fig. 1 and run the following seven algorithms:

- **Setup**( $\lambda, \mathcal{U}$ )  $\rightarrow PP, MSK$  The setup algorithm takes security parameter  $\lambda$  and attribute universe  $\mathcal{U}$  as input and outputs public parameters  $PP$  and master secret key  $MSK$ .  $PP$  may be widely distributed, while  $MSK$  is known only to key generation center (KGC).
- **Outsourcing-KeyGen-PreProc**( $PP, MSK$ )  $\rightarrow MSK_O, MSK_L$  The outsourcing key generation preprocessing algorithm is run by key generation center (KGC); it takes the public parameters  $PP$  and the master secret key  $MSK$  as input. It outputs an outsourcing master secret key  $MSK_O$  and a local master secret key  $MSK_L$ . Then, it delegates the  $MSK_O$  to its outsourcing key generation server provider (OKGSP) and keeps  $MSK_L$  locally.
- **Outsourcing-KeyGen**( $PP, MSK_O, I_{key}$ )  $\rightarrow IK_{I_{key}}$  The outsourcing key generation algorithm is run by outsourcing key generation service provider (OKGSP); it takes the  $PP, MSK_O$ , and an attribute set (resp. access structure)  $I_{key}$  as input. It outputs an intermediate key  $IK_{I_{key}}$  and sends  $IK_{I_{key}}$  to KGC.
- **KeyGen**( $PP, MSK_L, IK_{I_{key}}$ )  $\rightarrow SK_{I_{key}}$  The key generation algorithm is run by key generation center (KGC); it takes the public parameters  $PP$ , the local master secret key  $MSK_L$ , the intermediate key  $IK_{I_{key}}$  as input, and outputs a user secret key  $SK_{I_{key}}$  for attribute set (resp. access structure)  $I_{key}$ .

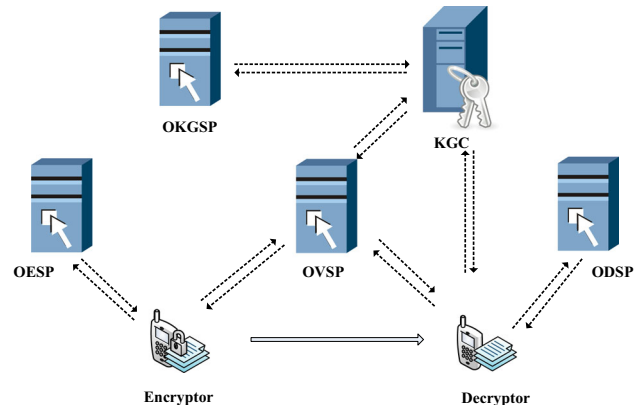


Fig. 1 Outsourced ABE System

- **Outsourcing-Encryption-PreProc**( $PP$ )  $\rightarrow EP_O, EP_L$   
The outsourcing encryption preprocessing algorithm is run by encryptor; it takes as input the public parameters  $PP$  and outputs an outsourcing encryption parameters  $EP_O$  and a local encryption parameters  $EP_L$ . The encryptor keeps  $EP_L$  locally and delegates the  $EP_O$  to its outsourcing encryption service provider (OESP).
- **Outsourcing-Encryption**( $PP, EP_O, I_{enc}$ )  $\rightarrow IC_{I_{enc}}$  The outsourcing encryption algorithm is run by OESP; it takes as input the public parameters  $PP$ , the outsourcing encryption parameters  $EP_O$ , and an access structure (resp. attribute set)  $I_{enc}$ . It outputs an intermediate ciphertext  $IC_{I_{enc}}$ .
- **Encryption**( $PP, EP_L, IC_{I_{enc}}, m$ )  $\rightarrow CT_{I_{enc}}$  The encryption algorithm is run by encryptor; it takes as input the public parameters  $PP$ , the local encryption parameters  $EP_L$ , the intermediate ciphertext  $IC_{I_{enc}}$  and a message  $m$ . It outputs the ciphertext  $CT_{I_{enc}}$ .
- **Outsourcing-Decryption-PreProc**( $SK$ )  $\rightarrow (SK_O, SK_L)$   
The outsourcing decryption preprocessing algorithm is run by decryptor; it takes its secret key  $SK$  as input and outputs an outsourcing secret key  $SK_O$  and a local secret key  $SK_L$ . The decryption user keeps  $SK_L$  locally and delegates the  $SK_O$  to its outsourcing decryption service provider (ODSP).
- **Outsourcing-Decryption**( $SK_O, CT$ )  $\rightarrow CT'$  The outsourcing decryption algorithm is run by ODSP; it takes as input an outsourcing secret key  $SK_O$  for attribute set (resp. access structure)  $I_{key}$  and a ciphertext  $CT$  that was encrypted under access structure (resp. attribute set)  $I_{enc}$ . It outputs the partially decrypted ciphertext  $CT'$  if  $f(I_{enc}, I_{key}) = 1$  and the error symbol  $\perp$  otherwise.
- **Decryption**( $CT', SK_L$ )  $\rightarrow m$  The decryption algorithm is run by decryptor; it takes  $SK_L$  for  $I_{key}$  and  $CT'$  for  $I_{enc}$  as input. It outputs the original message  $m$  if  $f(I_{enc}, I_{key}) = 1$  and the error symbol  $\perp$  otherwise.
- **Outsourcing-Verification** The outsourcing verification algorithm is run by outsourcing verification service provider (OVSP) and provides verification services to confirm whether the outsourcing is correct. It contains two sub-algorithms:
  - **Outsourcing-KeyGen-Verification**( $PP, MSK_O, IK_{I_{key}}$ )  $\rightarrow b \in \{0, 1\}$  It takes the public parameters  $PP$ , an outsourcing master secret key  $MSK_O$  and an intermediate key  $IK_{I_{key}}$  as input. It outputs 1 if  $IK_{I_{key}}$  is correctly calculated by OKGSP using  $MSK_O$  and 0 otherwise.
  - **Outsourcing-Encryption-Verification**( $PP, EP_O, IC_{I_{enc}}$ )  $\rightarrow b \in \{0, 1\}$  It takes the public parameters  $PP$ , an outsourcing encryption parameters  $EP_O$  and an intermediate ciphertext  $IC_{I_{enc}}$  as input. It outputs 1

if  $IC_{I_{enc}}$  is correctly calculated by OESP using  $EP_O$  and 0 otherwise.

Note, in decryption phase, the outsourcing verification service is not used, because the decryptor can verify the correctness of the outsourcing calculation by the plaintext  $m$  easily.

### 3.2 Security model

The security experiment  $\text{OABE-Exp}_{\mathcal{A}, \Pi}(\lambda, \mathcal{U})$  for outsourced ABE scheme  $\Pi$ , adversary  $\mathcal{A}$ , parameter  $\lambda$  and attribute universe  $\mathcal{U}$  is defined as follow:

**Initial** The challenger runs the setup algorithm to obtain  $PP$  and  $MSK$ . Then, it runs the Outsourcing-KeyGen-PreProc algorithm on  $(PP, MSK)$  to obtain  $MSK_O$  and  $MSK_L$ . It gives  $PP$  and  $MSK_O$  to the adversary.

Note: In real-world situations, we allow the adversary to corrupt OKGSP, so that the adversary can obtain the  $MSK_O$  in this experiment.

**Phase 1** The challenger sets two integer counters  $\zeta, \xi = 0$ , two empty tables  $\mathcal{T}_1, \mathcal{T}_2$ , and an empty set  $\mathcal{D}$ . The adversary can make the following queries repeatedly:

- **Create**( $I_{key}$ ) The challenger sets  $\zeta := \zeta + 1$ . It runs Outsourcing-KeyGen algorithm on  $I_{key}$  to obtain  $IK_{I_{key}}$ , runs KeyGen algorithm on  $IK_{I_{key}}$  to obtain  $SK_{I_{key}}$  and runs Outsourcing-Decryption-PreProc algorithm on  $SK_{I_{key}}$  to obtain  $SK_{I_{key}, O}$  and  $SK_{I_{key}, L}$ . Then, it stores in table  $\mathcal{T}_1$  the entry  $(\zeta, I_{key}, IK_{I_{key}}, SK_{I_{key}}, SK_{I_{key}, O}, SK_{I_{key}, L})$ , and gives  $(\zeta, I_{key}, IK_{I_{key}}, SK_{I_{key}, O})$  to adversary.
- Note: In real-world situations, we allow the adversary to corrupt OKGSP and ODSP, so that the adversary can obtain the  $IK_{I_{key}}$  and  $SK_{I_{key}, O}$  in this experiment.
- **Corrupt**( $i$ ) If the  $i$ th entry exists in table  $\mathcal{T}_1$ , the challenger gets the entry  $(i, I_{key}, IK_{I_{key}}, SK_{I_{key}}, SK_{I_{key}, O}, SK_{I_{key}, L})$ , sets  $\mathcal{D} := \mathcal{D} \cup \{I_{key}\}$  and returns the key set  $(i, SK_{I_{key}}, SK_{I_{key}, L})$  to the adversary, else it returns  $\perp$ .
- **Outsourcing-Encryption**( $I_{enc}$ ) The challenger sets  $\xi := \xi + 1$ . It runs Outsourcing-Encryption-PreProc algorithm to obtain  $EP_O$  and  $EP_L$  and runs Outsourcing-Encryption algorithm on  $(EP_O, I_{enc})$  to obtain  $IC_{I_{enc}}$ . Then, it stores in table  $\mathcal{T}_2$  the entry  $(\xi, I_{enc}, EP_O, EP_L, IC_{I_{enc}})$  and gives  $(\xi, I_{enc}, EP_O, IC_{I_{enc}})$  to adversary.

Note: In real-world situations, we allow the adversary to corrupt OESP, so that the adversary can obtain the  $EP_O$  and  $IC$  in this experiment.

**Challenge** The adversary provides two messages  $m_0^*$  and  $m_1^*$  with equal length, an index  $j$  of  $\mathcal{T}_2$ . If there exists a  $j^{th}$  entry  $(j, I_{enc}^*, EP_O, EP_L, IC_{I_{enc}^*})$  in table  $\mathcal{T}_2$ , and for all  $I_{key} \in \mathcal{D}$ ,  $f(I_{enc}^*, I_{key}) \neq 1$ , the challenger chooses a random bit  $b$  and encrypts the message  $m_b^*$  under  $(I_{enc}^*, EP_L, IC_{I_{enc}^*})$ . The



ciphertext  $CT^*$  is returned to the adversary. Otherwise, it returns  $\perp$ .

**Phase 2** Just like Phase 1, except that the adversary cannot obtain the suitable keys for the challenge ciphertext. That is, the adversary cannot issue any *Corrupt* queries that would added  $I_{key}$  which satisfies  $f(I_{enc}^*, I_{key}) = 1$  to  $\mathcal{D}$ .

**Guess** The adversary outputs a guess  $b'$  of  $b$ . The output of the experiment is 1 if and only if  $b = b'$ .

**Definition 1** An outsourced ABE scheme is secure if for all probabilistic polynomial-time adversaries  $\mathcal{A}$ , there exists a negligible function  $negl$  such that:

$$\Pr[\text{OABE-Exp}_{\mathcal{A}, \Pi}(\lambda, \mathcal{U}) = 1] \leq \frac{1}{2} + negl(\lambda).$$

## 4 Outsourced CP-ABE scheme

### 4.1 Construction

In this section, we proposed a specific outsourced CP-ABE scheme. In our construction, we will use linear secret-sharing schemes (LSSS) to express the access structure. This technology is proposed by [Beimel \(1996\)](#) and widely used in ABE scheme, such as [Waters \(2011\)](#), [Lewko and Waters \(2012\)](#) etc. The proposed scheme consists of the following 12 algorithms:

**Setup** $(\lambda, \mathcal{U}) \rightarrow PP, MSK$  The setup algorithm chooses a bilinear group  $G$  of order  $N = p_1 p_2 p_3$  (3 distinct primes). We let  $G_{p_i}$  denote the subgroup of order  $p_i$  in  $G$ . It then chooses random exponents  $\alpha, a, \kappa \in \mathbb{Z}_N$ , a random group element  $g_1 \in G_{p_1}$  and a random group element  $g_3 \in G_{p_1}$ . For each attribute  $i \in \mathcal{U}$ , it chooses a random value  $h_i \in \mathbb{Z}_N$ . In addition, we assume that the maximum number of rows of the matrix in any LSSS access structure is bounded by  $L$ . The public parameters  $PP$  contain  $\{N, g = g_1, g^a, g^\kappa, e(g, g)^a, \{H_i = g^{h_i}\}_{i \in \mathcal{U}}, g_3, L\}$ . The master secret key  $MSK$  additionally contains  $\alpha$ .

**Outsourcing-KeyGen-PreProc** $(PP, MSK) \rightarrow MSK_O, MSK_L$  The outsourcing key generation preprocessing algorithm is run by KGC. It chooses  $\alpha_0 \in \mathbb{Z}_N$  randomly, and outputs

$$MSK_O = g^{\alpha_0}, MSK_L = g^{\alpha - \alpha_0}.$$

**Outsourcing-KeyGen** $(PP, MSK_O, S \subseteq \mathcal{U}) \rightarrow IK_S$  Here  $S$  is an attribute set. The outsourcing key generation algorithm is run by OKGSP. It chooses random exponents  $t \in \mathbb{Z}_N$ , random elements  $R, R', R'', \{R_i\}_{i \in S} \in G_{p_3}$ , and calculates

$$K_O = g^{\alpha_0} g^{at} R, K'_O = g^t R', \\ \{K'_{O,i} = H_i^t R_i\}_{i \in S}.$$

Then, it outputs  $IK_S = (S, K_O, K'_O, \{K_{O,i}\}_{i \in S})$ .

**Outsourcing-KeyGen-Verification** $(PP, MSK_O, IK_S) \rightarrow b \in \{0, 1\}$  The outsourcing key generation verification algorithm is run by OVSP, when OVSP received a verification request from the KGC. It verifies the following equation:

$$e(K_O, g) \stackrel{?}{=} e(MSK_O, g) e(K'_O, g^a); \\ e(K'_O, H_i) \stackrel{?}{=} e(K'_{O,i}, g) \quad \forall i \in S.$$

It outputs  $b = 1$  if all of the above equations are satisfied, and  $b = 0$  otherwise.

**KeyGen** $(PP, MSK_L, IK_S) \rightarrow SK_S$  The key generation algorithm is run by KGC. It chooses  $u \in \mathbb{Z}_N$  and  $\tilde{R}, \tilde{R}', \tilde{R}'', \{\tilde{R}_i\}_{i \in S} \in G_{p_3}$  randomly, and calculates

$$K = g^{\alpha - \alpha_0} K_O g^{\kappa \cdot u} \tilde{R}, K' = K'_O \tilde{R}', K'' = g^u \tilde{R}'', \\ \{K_i = K_{O,i} \tilde{R}_i\}_{i \in S}.$$

Then, it outputs  $SK_S = (S, K, K', K'', \{K_i\}_{i \in S})$ .

**Outsourcing-Encryption-PreProc** $(PP) \rightarrow EP_O, EP_L$  The outsourcing encryption preprocessing algorithm is run by encryptor. For  $\forall j \in [1, L]$ , it chooses random exponent  $\lambda'_j \in \mathbb{Z}_N$ , and calculates  $ep_j = (g^a)^{\lambda'_j}$ .

Then, it outputs  $EP_O = \{ep_j\}_{j \in [1, L]}, EP_L = \{\lambda'_j\}_{j \in [1, L]}$ .

**Outsourcing-Encryption** $(PP, EP_O, \mathbb{A} = (\mathbf{M}, \rho)) \rightarrow IC$  Here  $(\mathbf{M}, \rho)$  is an access structure, where  $\mathbf{M}$  is an  $l \times n$  matrix, and  $\rho$  is a function, associates rows of  $\mathbf{M}$  to attributes. The outsourcing encryption algorithm is run by OESP. For  $\forall j \in [1, l]$ , it chooses random exponent  $r_j \in \mathbb{Z}_N$ , and calculates

$$IC_{j,1} = (g^a)^{\lambda'_j} H_{\rho(j)}^{-r_j}, IC_{j,2} = g^{r_j}.$$

Then, it outputs  $IC = ((\mathbf{M}, \rho), \{IC_{j,1}, IC_{j,2}\}_{j \in [1, l]})$ .

**Outsourcing-Encryption-Verification** $(PP, EP_O, IC) \rightarrow b' \in \{0, 1\}$  The outsourcing encryption verification algorithm is run by OVSP, when OVSP received a verification request from the encryptor. It verifies the following equation:

$$e(IC_{j,1}, g) e(IC_{j,2}, H_{\rho(j)}) \stackrel{?}{=} e(ep_j, g) \quad \forall j \in [1, l].$$

It outputs  $b' = 1$  if all of the above equations are satisfied, and  $b' = 0$  otherwise.

**Encryption** $(PP, m, EP_L, IC) \rightarrow CT$  The encryption algorithm chooses random exponents  $x, s, v_2, \dots, v_n \in \mathbb{Z}_N$ , and sets  $\mathbf{v} = (s, v_2, \dots, v_n)$ , then calculates

$$(\lambda_1, \dots, \lambda_l) = \mathbf{M} \cdot \mathbf{v}, \\ C_0 = m \cdot e(g, g)^{\alpha \cdot s}, C_1 = g^s, C_2 = (g^\kappa)^s, \\ \{C_{j,1} = (g^a)^x IC_{j,1}, C_{j,2} = IC_{j,2}, \\ C_{j,3} = \lambda_j - \lambda'_j - x\}_{j \in [1, l]}.$$

It outputs

$$CT = ((\mathbf{M}, \rho), C_0, C_1, C_2, \{C_{j,1}, C_{j,2}, C_{j,3}\}_{j \in [1,l]}).$$

**Outsourcing-Decryption-PreProc**( $SK$ )  $\rightarrow (SK_O, SK_L)$

The outsourcing decryption preprocessing algorithm chooses random exponent  $\alpha_U \in \mathbb{Z}_N$  and calculates  $\bar{K} = K \cdot g^{\alpha_U}$ . Then, it outputs

$$SK_L = g^{\alpha_U}, SK_O = (S, \bar{K}, K', K'', \{K_i\}_{i \in S}).$$

**Outsourcing-Decryption**( $SK_O, CT$ )  $\rightarrow CT'$  If  $S$  does not satisfy the access structure  $(\mathbf{M}, \rho)$ , then the algorithm issues an error message. Otherwise, it computes constants  $\omega_j \in \mathbb{Z}_N$  such that  $\sum_{\rho(j) \in S} \omega_j \cdot \mathbf{M}_j = (1, 0, \dots, 0)$ , where  $\mathbf{M}_j$  is the  $j$ th row of  $\mathbf{M}$ . Then it calculates and outputs

$$CT' = \frac{e(C_1, \bar{K})e(C_2, K'')^{-1}}{\prod_{\rho(j) \in S} (e(C_{j,1}, K')e(g^{a \cdot C_{j,3}}, K')e(C_{j,2}, K_{\rho(j)}))^{\omega_j}} \\ = e(g, g)^{(\alpha + \alpha_U) \cdot s}$$

**Decryption**( $CT', SK_L$ )  $\rightarrow m$  The decryption algorithm calculates and outputs

$$m = \frac{C_0 \cdot e(C_1, SK_L)}{CT'}.$$

## 4.2 Comparison

In Table 1, we give the comparison of local computation, where  $E$  expresses exponentiation and  $P$  expresses pairing. The local computation of our scheme is constant at every stage.

## 5 Security

In this section, we use the dual-system encryption technique (Waters 2009; Lewko et al. 2010; Lewko and Waters 2012) to prove the adaptive security of our scheme.

At the beginning, we will introduce the definitions of semi-functional keys and semi-functional ciphertexts:

**Semi-functional keys** Based on the normal key  $(K, K', K'', \{K_i\}_{i \in S})$ , we choose random elements  $W, W'' \in G_{p_2}$ , the semi-functional key is defined as:

$$SK^{semi} = (S, KW, K', K''W'', \{K_i\}_{i \in S}).$$

The outsourcing secret key  $SK_O^{semi}$  is semi-functional, if it is created using a semi-functional key  $SK^{semi}$ .

**Semi-functional ciphertexts** Based on the normal ciphertext  $((\mathbf{M}, \rho), C_0, C, C', \{C_{j,1}, C_{j,2}, C_{j,3}\}_{j \in [1,l]})$ , we randomly choose  $a', \kappa', s' \in \mathbb{Z}_N$ ,  $\mathbf{w} \in \mathbb{Z}_N^n$ ,  $\eta'_{\rho(j)}, \gamma'_j \in \mathbb{Z}_N$  for each  $j \in [l]$ , where  $s'$  is the first entry of vector  $\mathbf{w}$ . The semi-functional ciphertext  $CT^{semi}$  is defined as:

$$((\mathbf{M}, \rho), C_0, Cg_2^{s'}, C'g_2^{s'\kappa'}, \{C_{j,1}g_2^{a'\mathbf{M}_j \cdot \mathbf{w}}, C_{j,2}, C_{j,3}\}_{j \in [1,l]}).$$

Then, we will employ the hybrid argument method. The security game  $Game_{real}$  is defined in Sect. 3.2. We suppose that the attacker would make  $Q$  create key queries. For  $k \in [0, Q]$ , the  $k$ th game  $Game_k$  is defined as follow.

$Game_k$  In this game, the intermediate ciphertext and ciphertext are semi-functional, as are the first  $k$  keys in  $\mathcal{T}_1$ , and the other keys are normal.

In this hybrid argument,  $Game_{real}$  will be translated to  $Game_0$ , then to  $Game_1$ , next to  $Game_2, \dots$ , finally to  $Game_Q$ . Then,  $Game_Q$  will be translated to  $Game_{final}$ , in which the ciphertext is a semi-functional encryption of a random message. That is to say, any attacker has no advantage in this game.

**Lemma 1** Under Assumption 1, no PPT attacker has non-negligible advantage in distinguishing  $Game_{real}$  and  $Game_0$ .

*Proof* Suppose there is a PPT attacker  $\mathcal{A}$ , who has non-negligible advantage in distinguishing  $Game_{real}$  and  $Game_0$ . We can construct a PPT algorithm  $\mathcal{B}$  to break Assumption 1. We set  $Z_0 := \{1\}$ ,  $Z_1 := \{1, 2\}$ ,  $Z_2 := \{1\}$ ,  $Z_3 := \{3\}$ , and give  $g_1, g_3, T$  to  $\mathcal{B}$ , where  $g_1 \in G_{p_1}$ ,  $g_3 \in G_{p_3}$ ,  $T$  is either a random element of  $G_{p_1}$  or a random element of  $G_{p_1 p_2}$ .

**Initial** We assume that the maximum number of rows of the matrix in any LSSS access structure is bounded by  $L$ .  $\mathcal{B}$  chooses  $\alpha, \alpha_0, a, \kappa, \{h_i\}_{i \in \mathcal{U}} \in \mathbb{Z}_N$  randomly, and sets  $PP = (N, g = g_1, g^a = g_1^a, g^\kappa = g_1^\kappa, e(g, g)^\alpha = e(g_1, g_1)^\alpha, \{H_i = g_1^{h_i}\}_{i \in \mathcal{U}}, g_3, L)$ ,  $MSK_O = g^{\alpha_0}$ .  $PP$  and  $MSK_O$  are given to  $\mathcal{A}$ . Note,  $\mathcal{B}$  has the  $MSK = \alpha$  and  $MSK_L = g^{\alpha - \alpha_0}$ .

**Phase 1&2**  $\mathcal{B}$  sets two integer counters  $\zeta, \xi = 0$ , two empty tables  $\mathcal{T}_1, \mathcal{T}_2$ , and an empty set  $\mathcal{D}$ .  $\mathcal{A}$  can make the following queries to  $\mathcal{B}$  repeatedly:

**Table 1** Comparison of local computation

Schemes	KeyGen	Encryption	Decryption	Group order	Security
Waters (2011)	$( S  + 3)E$	$(3l + 2)E$	$(2l + 1)P + lE$	Prime	Selective, wRO
Lewko and Waters (2012)	$( S  + 5)E$	$(3l + 3)E$	$(2l + 2)P + lE$	Composite	Adaptive, wRO
Rouselakis and Waters (2015)	$( S  + 3)E$	$(6l + 1)E$	$3lP + lE$	Prime	Adaptive, RO
Our scheme	$2E$	$4E$	$1P$	Composite	Adaptive, wRO

- *Create (S)*  $\mathcal{B}$  can response  $\mathcal{A}$ 's query on attribute set  $S$  by running Outsourcing-KeyGen, KeyGen and Outsourcing-Decryption-PreProc algorithms, because it has  $MSK, MSK_O$  and  $MSK_L$ .
- *Corrupt (k)* When  $\mathcal{A}$  make a corrupt query on index  $k$ ,  $\mathcal{B}$  sets  $\mathcal{D} = \mathcal{D} \cup \{S_k\}$ , and gives  $(SK_{S_k,L}, SK_{S_k})$  to  $\mathcal{A}$ , if exists a  $k$ th entry  $(k, S_k, IK_{S_k}, SK_{S_k}, SK_{S_k,O}, SK_{S_k,L})$  in table  $\mathcal{T}_1$ , and outputs  $\perp$  otherwise.
- *Outsourcing-Encryption (A)*  $\mathcal{B}$  can response this query by running Outsourcing-Encryption-PreProc and Outsourcing-Encryption algorithm.

**Challenge**  $\mathcal{A}$  submits two messages  $m_0^*, m_1^*$  with equal length, and an index  $j$  of  $\mathcal{T}_2$ . If there exists a  $j^{th}$  entry  $(j, \mathbb{A}^* = (\mathbf{M}^*, \rho^*), EP_O = \{(g^a)^{\lambda'_j}\}_{j \in [1,L]}, EP_L = \{\lambda'_j\}_{j \in [1,L]}, \{(g^a)^{\lambda'_j} H_{\rho^*(j)}^{-r_j}, g^{r_j}\}_{j \in [1,L]})$  in table  $\mathcal{T}_2$ , and for all  $S \in \mathcal{D}, S \notin \mathbb{A}^*, \mathcal{B}$  chooses  $b \in \{0, 1\}$  randomly, and encrypts  $m_b$  as follows. It chooses vector  $\tilde{\mathbf{v}} \in \mathbb{Z}_N^n$  randomly, except the first entry is 1, and computes the ciphertext as:

$$C_0 = m_b e(g_1, T)^\alpha, \quad C = T, C' = T^\kappa, \\ C_{j,1} = T^{a\mathbf{M}_j^* \cdot \tilde{\mathbf{v}}} (g^a)^{\lambda'_j} H_{\rho^*(j)}^{-r_j}, C_{j,2} = g^{r_j}, C_{j,3} = -\lambda'_j \quad \forall j.$$

Note,  $\mathcal{B}$  implicitly sets  $g^s$  equal to the  $G_{p_1}$  part of  $T$ , and  $\mathbf{v} = s\tilde{\mathbf{v}}$ . If  $T \in G_{p_1}$ , the ciphertext is normal, and  $\mathcal{B}$  simulates  $Game_{real}$  with  $\mathcal{A}$  properly. If  $T \in G_{p_1 p_2}$ , the ciphertext is semi-functional,  $\mathcal{B}$  simulates  $Game_0$  with  $\mathcal{A}$  properly. Therefore, if  $\mathcal{A}$  has non-negligible advantage in distinguishing  $Game_{real}$  and  $Game_0$ ,  $\mathcal{B}$  can use this advantage to break Assumption 1.  $\square$

**Lemma 2** Under Assumption 1, no PPT attacker has non-negligible advantage in distinguishing  $Game_{k-1}$  and  $Game_k$ .

*Proof* Suppose there is a PPT attacker  $\mathcal{A}$ , who has non-negligible advantage in distinguishing  $Game_{k-1}$  and  $Game_k$  for some  $k \in [1, Q]$ . We can construct a PPT algorithm  $\mathcal{B}$  to break Assumption 1. We set  $Z_0 := \{1, 3\}, Z_1 := \{1, 2, 3\}, Z_2 := \{1\}g, Z_3 := \{3\}, Z_4 := \{1, 2\}, Z_5 := \{2, 3\}$ , and give  $g_1, g_3, X_1 X_2, Y_2 Y_3, T$  to  $\mathcal{B}$  as input, where  $g_1, X_1 \in G_{p_1}, X_2, Y_2 \in G_{p_2}, g_3, Y_3 \in G_{p_3}$ , and  $T$  is either a random element of  $G_{p_1 p_2}$  or a random element of  $G_{p_1 p_2 p_3}$ .

**Initial** We assume that the maximum number of rows of the matrix in any LSSS access structure is bounded by  $L$ .  $\mathcal{B}$  chooses  $\alpha, \alpha_0, a, \kappa, \{h_i\}_{i \in \mathcal{U}} \in \mathbb{Z}_N$  randomly, and sets  $PP = (N, g = g_1, g^a = g_1^a, g^\kappa = g_1^\kappa, e(g, g)^\alpha = e(g_1, g_1)^\alpha, \{H_i = g_1^{h_i}\}_{i \in \mathcal{U}}, g_3, L), MSK_O = g^{\alpha_0}$ .  $PP$  and  $MSK_O$  are given to  $\mathcal{A}$ . Note,  $\mathcal{B}$  has the  $MSK = \alpha$  and  $MSK_L = g^{\alpha - \alpha_0}$ .

**Phase 1&2**  $\mathcal{B}$  sets two integer counters  $\zeta, \xi = 0$ , two empty tables  $\mathcal{T}_1, \mathcal{T}_2$ , and an empty set  $\mathcal{D}$ .  $\mathcal{A}$  can make the following queries to  $\mathcal{B}$  repeatedly:

- *Create (S)* For the first  $k - 1$  queries,  $\mathcal{B}$  uses the Outsourcing-KeyGen and KeyGen algorithms to generate a normal key  $SK_S = \{K, K', K'', \{K_i\}_{i \in S}\}$ , and chooses  $\tau, \tau' \in \mathbb{Z}_N$  randomly. The semi-functional is formed as:

$$SK_S^{semi} = (K(Y_2 Y_3)^\tau, K', K''(Y_2 Y_3)^{\tau'}, \{K_i\}_{i \in S})$$

Then, it uses the Outsourcing-Decryption-PreProc algorithm on  $SK_S^{semi}$  to obtain corresponding outsourcing secret key.

Note,  $SK_S^{semi}$  is distributed properly, as  $Y_2^\tau$  and  $Y_2^{\tau'}$  are distributed as uniformly random elements of  $G_{p_2}$ .

- *Corrupt (k)* When  $\mathcal{A}$  make a corrupt query on index  $k$ ,  $\mathcal{B}$  sets  $\mathcal{D} = \mathcal{D} \cup \{S_k\}$ , and gives  $(SK_{S_k,L}, SK_{S_k})$  to  $\mathcal{A}$ , if exists a  $k$ th entry  $(k, S_k, IK_{S_k}, SK_{S_k}, SK_{S_k,O}, SK_{S_k,L})$  in table  $\mathcal{T}_1$ , and outputs  $\perp$  otherwise.
- *Outsourcing-Encryption (A)*  $\mathcal{B}$  can response this query by running Outsourcing-Encryption-PreProc and Outsourcing-Encryption algorithm.

**Challenge**  $\mathcal{A}$  submits two messages  $m_0^*, m_1^*$  with equal length, an index  $j$  of  $\mathcal{T}_2$ . If there exists a  $j$ th entry  $(j, \mathbb{A}^* = (\mathbf{M}^*, \rho^*), EP_O = \{(g^a)^{\lambda'_j}\}_{j \in [1,L]}, EP_L = \{\lambda'_j\}_{j \in [1,L]}, \{(g^a)^{\lambda'_j} H_{\rho^*(j)}^{-r_j}, g^{r_j}\}_{j \in [1,L]})$  in table  $\mathcal{T}_2$ , and for all  $S \in \mathcal{D}, S \notin \mathbb{A}^*, \mathcal{B}$  chooses  $b \in \{0, 1\}$  randomly, and encrypts  $m_b$  as follows. It chooses a random vector  $\tilde{\mathbf{v}} \in \mathbb{Z}_N^n$ , in which the first entry is 1. This means that  $g^s = X_1$  and  $\mathbf{v} = s\tilde{\mathbf{v}}$ . It computes the ciphertext as:

$$C_0 = m_b e(g_1, X_1 X_2)^\alpha, \quad C = X_1 X_2, \quad C' = (X_1 X_2)^\kappa, \\ C_{j,1} = (X_1 X_2)^{a\mathbf{M}_j^* \cdot \tilde{\mathbf{v}}} (g^a)^{\lambda'_j} H_{\rho^*(j)}^{-r_j}, \\ C_{j,2} = g^{r_j}, C_{j,3} = -\lambda'_j \quad j \in [1, l].$$

Note, this means that  $g_2^{s'} = X_2, \kappa' = \kappa$  modulo  $p_2$ , and  $a' = a$  modulo  $p_2$ . The semi-functional ciphertexts are properly distributed.

For the  $k$ th create key query on  $S$ ,  $\mathcal{B}$  chooses  $t \in \mathbb{Z}_N$ , and  $R, R', R'', \{R_i\} \in G_{p_3}$  randomly. It sets:

$$K = (g_1^{a+at} T^\kappa) R, \\ K' = g^t R', K'' = T R'', \{K_i = H_i^t R_i\}_{i \in S}.$$

If  $T \in G_{p_1 p_3}$ ,  $K$  is a normal key. If  $T \in G_{p_1 p_2 p_3}$ ,  $K$  is a semi-functional key. Thus, when  $T \in G_{p_1 p_3}$ ,  $\mathcal{B}$  simulates  $Game_{k-1}$ , and when  $T \in G_{p_1 p_2 p_3}$ ,  $\mathcal{B}$  simulates  $Game_k$ . Therefore, if  $\mathcal{A}$  has non-negligible advantage in distinguishing  $Game_{k-1}$  and  $Game_k$ ,  $\mathcal{B}$  can use this advantage to break Assumption 1.  $\square$

**Lemma 3** Under Assumption 2, no PPT attacker has non-negligible advantage in distinguishing  $Game_Q$  and  $Game_{final}$ .

*Proof* Suppose there is a PPT attacker  $\mathcal{A}$ , who has non-negligible advantage in distinguishing  $Game_Q$  and  $Game_{final}$ . We can construct a PPT algorithm  $\mathcal{B}$  to break Assumption 2. We give  $g_1, g_2, g_3, g_1^\alpha X_2, g_1^s Y_2, T$  to  $\mathcal{B}$  as input, where  $T$  is either  $e(g_1, g_1)^{\alpha s}$  or a random element of  $G_T$ .

**Initial** We assume that the maximum number of rows of the matrix in any LSSS access structure is bounded by  $L$ .  $\mathcal{B}$  chooses  $\alpha, \alpha_0, a, \kappa, \{h_i\} \in \mathbb{Z}_N$  randomly, sets  $PP = (N, g = g_1, g^a = g_1^a, g^\kappa = g_1^\kappa, e(g, g)^\alpha = e(g_1, g_1)^\alpha, \{H_i = g_1^{h_i}\}_{i \in \mathcal{U}}, L)$ , and sets  $MSK_O = g^{\alpha_0}$ . It gives  $PP$  and  $MSK_O$  to  $\mathcal{A}$ . Note,  $\mathcal{B}$  knows  $MSK = \alpha$  and  $MSK_L = g^{\alpha - \alpha_0}$ .

**Phase 1&2**  $\mathcal{B}$  sets two integer counters  $\zeta, \xi = 0$ , two empty tables  $\mathcal{T}_1, \mathcal{T}_2$ , and an empty set  $\mathcal{D}$ .  $\mathcal{A}$  can make the following queries to  $\mathcal{B}$  repeatedly:

- *Create* ( $S$ )  $\mathcal{B}$  chooses  $t, u, \gamma, \gamma' \in \mathbb{Z}_N$ , and  $R, R', R''$ ,  $\{R_i\}_{i \in S} \in G_{p_3}$  randomly, and formed  $SK_S^{semi}$  as:

$$K = (g_1^\alpha X_2) g_1^{(at + \kappa u)} R g_2^{\gamma'}, K' = g_1^t R', K'' = g_1^u R'' g_2^{\gamma'}, \\ \{K_i = H_i^t R_i\}_{i \in S}$$

Then, it uses the Outsourcing-Decryption-PreProc algorithm on  $SK_S^{semi}$  to obtain corresponding outsourcing secret key.

- *Corrupt* ( $k$ ) When  $\mathcal{A}$  make a corrupt query on index  $k$ ,  $\mathcal{B}$  sets  $\mathcal{D} = \mathcal{D} \cup \{S_k\}$ , and gives  $(SK_{S_k, L}, SK_{S_k})$  to  $\mathcal{A}$ , if exists a  $k$ th entry  $(k, S_k, IK_{S_k}, SK_{S_k}, SK_{S_k, O}, SK_{S_k, L})$  in table  $\mathcal{T}_1$ , and outputs  $\perp$  otherwise.
- *Outsourcing-Encryption* ( $\mathbb{A}$ )  $\mathcal{B}$  can response this query by running Outsourcing-Encryption-PreProc and Outsourcing-Encryption algorithm.

**Challenge**  $\mathcal{A}$  submits two messages  $m_0^*, m_1^*$  with equal length, an index  $j$  of  $\mathcal{T}_2$ . If there exists a  $j$ th entry  $(j, \mathbb{A}^* = (\mathbf{M}^*, \rho^*), EP_O = \{(g^a)^{\lambda'_j}\}_{j \in [1, L]}, EP_L = \{\lambda'_j\}_{j \in [1, L]}, \{(g^a)^{\lambda'_j} H_{\rho^*(j)}^{-r_j}, g^{r_j}\}_{j \in [1, l]})$  in table  $\mathcal{T}_2$ , and for all  $S \in \mathcal{D}$ ,  $S \notin \mathbb{A}^*$ ,  $\mathcal{B}$  chooses  $b \in \{0, 1\}$  randomly, and encrypts  $m_b$  as follows. It chooses a random vector  $\tilde{v} \in \mathbb{Z}_N^n$ , in which the first entry is 1. This means that  $v = s\tilde{v}$ . It produces the ciphertext as:

$$C_0 = m_b T, \quad C = g_1^s Y_2, \quad C' = (g_1^s Y_2)^\kappa, \\ C_{j,1} = (g_1^s Y_2)^{a \mathbf{M}_{j,1}^* \cdot \tilde{v}} (g^a)^{\lambda'_j} H_{\rho^*(j)}^{-r_j}, \\ C_{j,2} = g^{r_j}, C_{j,3} = -\lambda'_j \quad \forall j.$$

If  $T = e(g_1, g_1)^{\alpha s}$ , this is a semi-functional ciphertext of  $m_b$ , and  $\mathcal{B}$  simulates  $Game_Q$ . If  $T$  is a random element of  $G_T$ ,

this is a semi-functional ciphertext of a random message, and  $\mathcal{B}$  simulates  $Game_{final}$ . Therefore, if  $\mathcal{A}$  has non-negligible advantage in distinguishing  $Game_Q$  and  $Game_{final}$ ,  $\mathcal{B}$  can use this advantage to break Assumption 2.  $\square$

**Theorem 1** Under Assumptions 1 and 2, the outsourced CP-ABE scheme is secure.

If Assumptions 1 and 2 hold, Lemmas 1–3 show that  $Game_{real}$  is indistinguishable from  $Game_{final}$ , in which  $b$  is hidden from  $\mathcal{A}$ . Therefore,  $\mathcal{A}$  have no advantage to break  $Game_{real}$ .

## 6 Outsourced CP-ABE in cloud computing environment

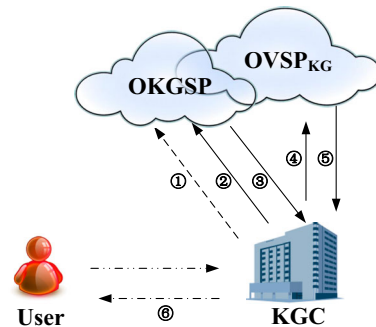
In this section, we introduce how to deploy our outsourced CP-ABE scheme in cloud computing environment and introduce a specific application for secure cloud storage system.

### 6.1 Deployment

There are three phases in the running of the outsourced CP-ABE scheme, i.e., key generation phase, encryption phase and decryption phase.

- **Key generation phase** In this phase, KGC issues private keys to users. To calculate the private key effectively, KGC calls the outsourcing key generation service, which is provided by OKGSP-cloud, to share calculation tasks. After receiving the outsourcing results, KGC calls the outsourcing verification service, which is provided by OVSP<sub>KG</sub>-cloud, to verify the correctness of the calculation. The details are as follows (Fig. 2):

1. KGC runs the **Outsourcing-KeyGen-PreProc** algorithm to obtain  $(MSK_O, MSK_L)$  and delegates the  $MSK_O$  to OKGSP-cloud, which provide the outsourcing key generation service.



**Fig. 2** Key generation phase



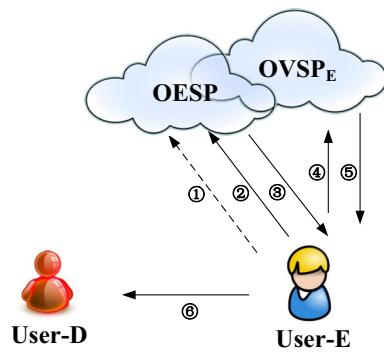


Fig. 3 Encryption phase

2. After KGC receives a key generation request from some user on attributes set  $S$ , it submits this request to OKGSP-cloud.
3. According to the request on attribute set  $S$  from KGC, OKGSP-cloud runs **Outsourcing-KeyGen** to obtain an intermediate key  $IK_S$ , and returns the  $IK_S$  to KGC.
4. After receiving  $IK_S$  from OKGSP-cloud, KGC submits  $(MSK_O, IK_S)$  to  $OVSP_{KG}$ -cloud to verify the correctness of outsourcing calculation.
5. The  $OVSP_{KG}$ -cloud runs **Outsourcing-KeyGen-Verification** algorithm on  $(MSK_O, IK_S)$  and returns a boolean value to KGC.
6. If  $IK_S$  is correct, KGC runs **KeyGen** algorithm on  $(MSK_L, IK_S)$  and returns the secret key  $SK_S$  to user.

– **Encryption phase** In encryption phase, the encryptor calls the outsourcing encryption service, which is provided by OESP-cloud, to share calculation tasks. After receiving the outsourcing results, encryptor calls the outsourcing verification service, which is provided by  $OVSP_E$ -cloud, to verify the correctness of the calculation. The details are as follows (Fig. 3):

1. The encryptor runs **Outsourcing-Encryption-PreProc** algorithm to obtain  $(EP_O, EP_L)$  and delegates the  $EP_O$  to OESP-cloud, which provide the outsourcing encryption service.
2. When encryptor needs to encrypt a message on some access structure  $\mathbb{A}$ , it submits  $\mathbb{A}$  to OESP-cloud.
3. OESP-cloud runs **Outsourcing-Encryption** to obtain an intermediate ciphertext  $IC$  and returns the  $IC$  to the encryptor.
4. After receiving  $IC$  from OESP-cloud, the encryptor submits  $(EP_O, IC)$  to  $OVSP_E$ -cloud to verify the correctness of outsourcing calculation.
5. The  $OVSP_E$ -cloud runs **Outsourcing-Encryption-Verification** algorithm on  $(EP_O, IC)$  and returns a boolean value to the encryptor.

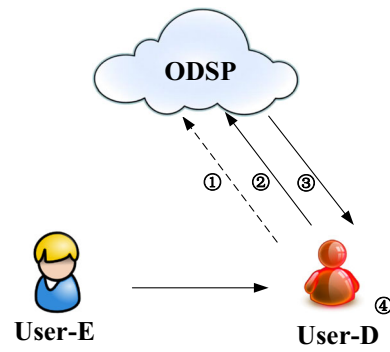


Fig. 4 Decryption phase

6. If  $IC$  is correct, the encryptor runs **Encryption** algorithm on  $(EP_L, IC)$  and sends the ciphertexts  $CT$  to the decryptor.

– **Decryption phase** In decryption phase, the decryptor calls the outsourcing decryption service, which is provided by ODSP-cloud, to share calculation tasks. After receiving the outsourcing results, decryptor calls the outsourcing verification service, which is provided by  $OVSP_D$ -cloud, to verify the correctness of the calculation. The details are as follows (Fig. 4):

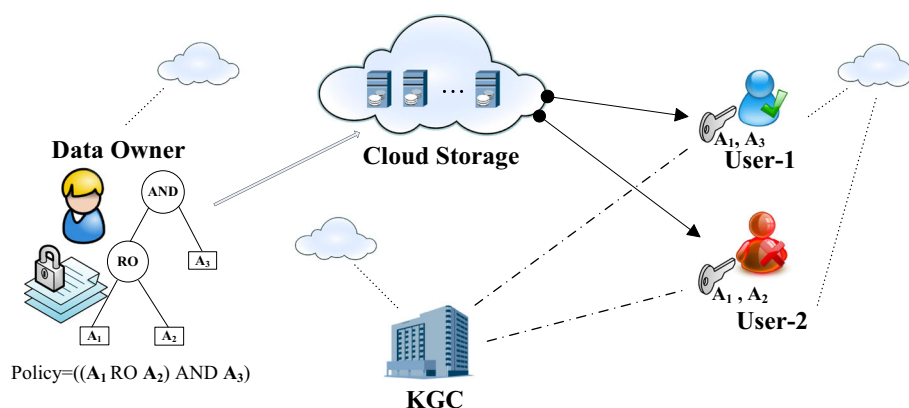
1. The decryptor runs **Outsourcing-Decryption-PreProc** algorithm to obtain  $(SK_O, SK_L)$  and delegates the  $SK_O$  to ODSP-cloud, which provide the outsourcing encryption service.
2. When decryptor needs to decrypt a ciphertext  $CT$ , it submits  $CT$  to ODSP-cloud.
3. OESP-cloud runs **Outsourcing-Decryption** to obtain a partially decrypted ciphertext  $CT'$  and returns the  $CT'$  to the decryptor.
4. The decryptor runs **Decryption** algorithm on  $(SK_L, CT')$  to obtain plaintext  $m$ .

## 6.2 Application in secure cloud storage system

In this section, we constructed a new data access control scheme for cloud storage systems using our outsourced CP-ABE scheme. Figure 5 shows the architecture of secure cloud storage system, which consists of the following system entities:

- **KGC** Its task is to authenticate the user's attributes and to issue corresponding keys for users.
- **Data owner** Data owner can share the data in the cloud storage server for specific users.
- **Cloud storage server** It is a semi-trusted server. It can provide storage space for data owners and allow legitimate users to access data.

**Fig. 5** Secure cloud storage system



- **User** In this system, each user has a set of attribute keys corresponding to their attributes and can access the cloud storage services.

Running of the data access control scheme for cloud storage contains three phases, i.e., key generation phase, encryption phase and decryption phase. They are as described in Sect. 6.1, except that, at the end of encryption phase, data owner stores the ciphertext on the storage cloud.

## 7 Conclusion

In this paper, we introduced the verifiable outsourced CP-ABE system. In this system, KGC, encryptor and decryptor, are able to outsource their computing tasks to the corresponding service providers, respectively, to improve the computational efficiency, and they are also able to verify the correctness of outsourcing calculation efficiently by using the corresponding outsourcing verification services. Then, we proposed a specific outsourced CP-ABE scheme and prove its adaptive security in the standard model. Finally, we introduced how to deploy our outsourced CP-ABE scheme in cloud computing environment and introduced a specific application for secure cloud storage system.

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### Compliance with ethical standards

**Conflict of interest** The authors declare that they have no conflict of interest.

**Ethical approval** This article does not contain any studies with human participants or animals performed by any of the authors.

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