OPTIMIZATION UNDER UNCERTAINTY

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CONTENT

PROBLEM STATEMENT

- To depict how introducing uncertainty causes deviation from classical mathematical techniques and discuss ways to mitigate or incorporate uncertain parameters within our model using Production Planning Problem.

INTRODUCTION

1. UNCERTAINTY

Decisions frequently need to be made amidst uncertainty. The actions chosen today will have outcomes that may not be fully understood until later. However, there may be chances for adjustments down the line or even several opportunities to take corrective steps as more information comes to light.

2. MODELLING OF UNCERTAINTY

Stochastic modeling involves representing the uncertain aspects of a problem as random variables, allowing the application of probability theory. For this approach to be effective, these elements must have a "known" probability distribution.

USE OF UNIFORM DISTRIBUTION

Uniform distributions are particularly effective for modeling uncertainties where every value within a specified range is equally likely. For example:

•**Process Variability**: Inherent variations in manufacturing or production processes can be modeled with a uniform distribution. For instance, the time required to complete a task or the amount of resources consumed may follow a uniform distribution within a given range.

•**Demand Variability**: When exact demand for a product or service is uncertain, a uniform distribution can be applied. The range would reflect the possible variations in customer demand.

USE OF NORMAL DISTRIBUTION

The normal distribution is widely used in various fields due to the central limit theorem and its applicability to many natural phenomena. For example :

Sampling Distributions: According to the central limit theorem, the distribution of sample means (or sums) from a sufficiently large number of samples tends to be normally distributed, even if the underlying population distribution is not normal.

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Mathematical Formulation

MILP FORMULATION OF PPP

$$PROFIT = \sum_j SP(j) * X(j) - \sum_j (PC_l(j) * L(j) + PC_m(j) * M(j) + PC_h(j) * H(j))$$

$$X(j) = cl(j) * L(j) + cm(j) * M(j) + ch(j) * H(j)$$

$$H(j) = 1 - Y(j)$$

$$L(j) \leq Y(j)$$

$$L(j) + M(j) + H(j) = Z(j)$$

$$X(j) \leq U * z(j)$$

$$\sum_j rm(k, j) * X(j) \leq R(k)$$

$$\sum_j (IC_l(j) * L(j) + IC_m(j) * M(j) + IC_h(j) * H(j)) \leq B$$

NAIVE APPROACH

We make following additions in the existing MILP formulation:

$R_{lk} = (1 + \epsilon \cdot \xi) \cdot R_{lk}$

To achieve Normal distribution in Gams we used,

$$R(r1) = ROrignal(r1) * (1 + 0.5 \cdot Normal(0, 0.2))$$

To achieve Uniform distribution in Gams we used,

$$R(r1) = ROrignal(r1) * (1 + 0.5 \cdot Uniform(-1, 1))$$

In this approach we,

- **Execute the optimization model with different scenarios of raw material availability.**
- **Observe variations in profits to understand the impact of uncertainty.**
- **Analyze the results to identify patterns or trends in the relationship between raw material availability and profit.**

ROBUST APPROACH

Based on the proposed formulation of robust optimization we incorporate the formulation by relaxing inequality of availability of raw materials in existing formulation under K(Reliability level),δ(feasibility tolerance) and ε(max Error):

$$\tilde{R}_{lk} = (1 + \epsilon \cdot \xi) \cdot R_{lk}$$

$$\sum_j rm(k, j) * X(j) \leq R(\widetilde{k})$$

$$\sum_j rm(k, j) * X(j) \leq R(k) - \varepsilon * (1 - 2 * k) * R(k) + \delta * \max(1, R(k))$$

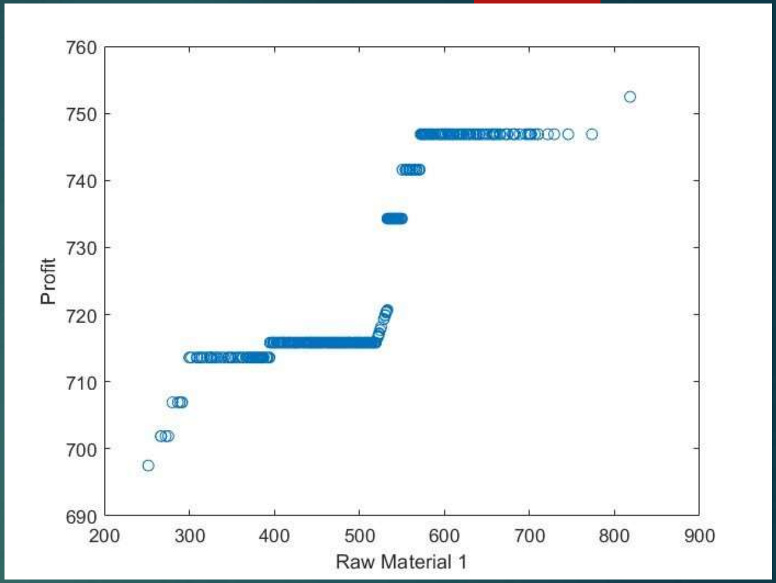
This ensures that the results produced remain feasible under given uncertainties with a certain level of tolerance and reliability resulting in effective and reliable decision making. While also ensuring less computational load compared to naïve approach where each uncertainty we would have to reiterate resulting in greater computational loss. here:

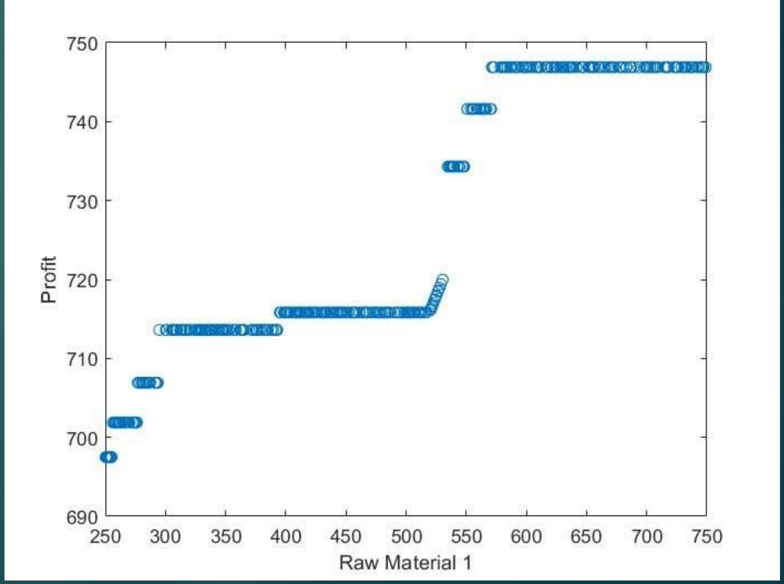
K(Reliability level)
δ(feasibility tolerance)
ε(max Error)

Observation

Naive Method

- Initially we were trying to compare and analyze the distribution of profit with different distribution of Raw material (R.V. - Normal/Uniform)
- We expected somewhat linear dependency of raw material with profits
- We observed bands of const. profit for wide range of Raw material
- Profit distribution were similar to the RV used i.e. in Normally distributed Raw matrial we saw that Profit was concentrated in the middle distributed





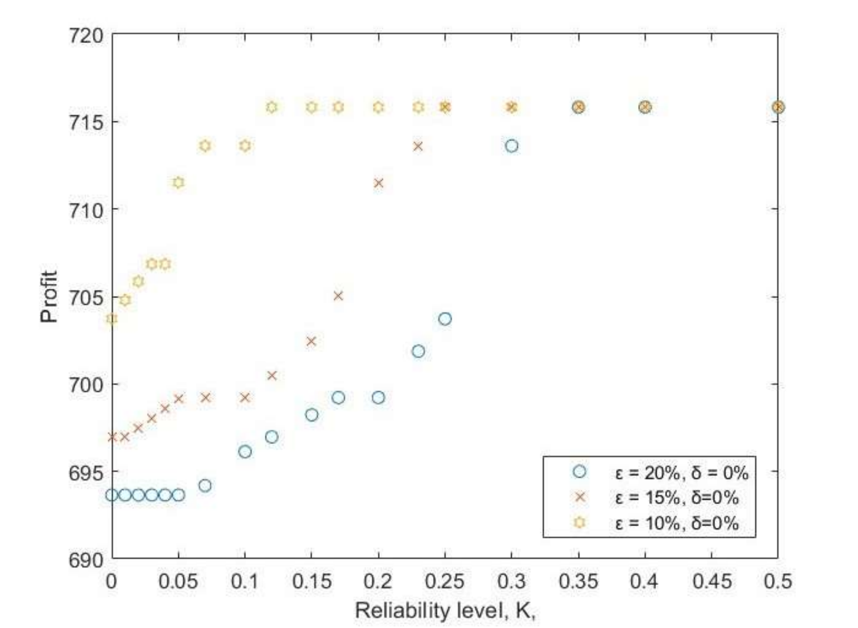
Observation

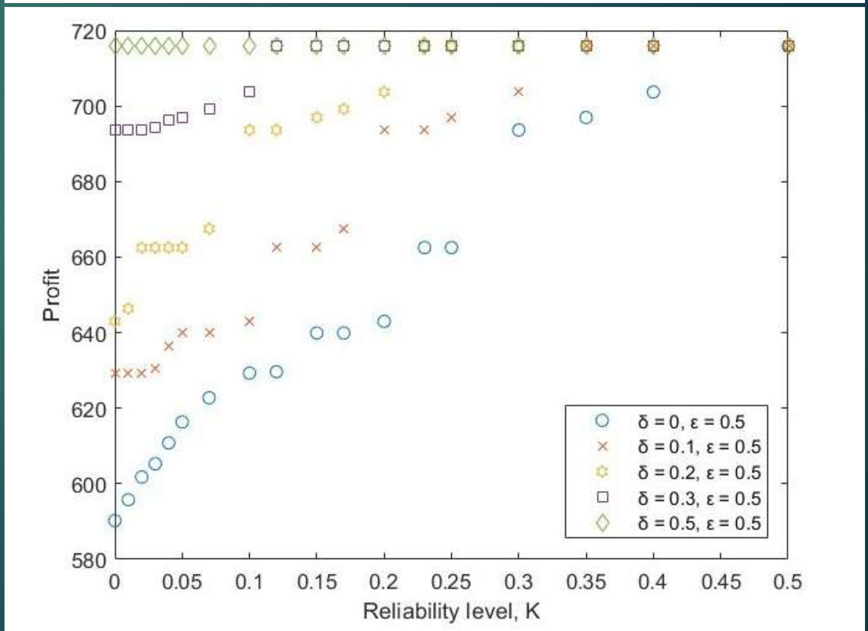
Robust Method

- A new constrain was added (prev. slide) where, K(Reliability level)
  δ(feasibility tolerance)
  ϵ(max Error)
- At the time of planning we don't have the actual value of the amount raw material available at time of production. So Robust Method says the probability of a plan to violate the Raw material constrain is k.

$$\mathbf{P}\left(\sum_j \mathbf{rm}(k,j) * \mathbf{X}(j) > \mathbf{R}(k) \right) = \mathbf{K}$$

- So we can make estimated decisions like what plan should we use with 95%chances of working withing bound i.e. (1-K=0.95)





Code analysis

Naive Method

- PPP code was modified
- Loop was added
- Value of Raw material 1 was changed using Uniform/Normal inbuilt functions
- Max profit was calculated
- File I/O was used in gams to save the data into .txt file
- Data was taken into exel and then to Matlab for plots and analysis

```
Parameter Solution1;

File outputTextFile / 'output.txt' /;

put outputTextFile;

put "Profit      Raw Matrial 1"   /;

outputTextFile.ap=1;

loop(i,

*       R('r1') = R_orignal('r1')*(1 + Normal(0,0.2));
        R('r1') = R_orignal('r1')*(1 + 0.5*uniform(-1,1));

    SOLVE petrochemical USING MIP MAXIMIZING OBJ;

    Solution1  = OBJ.l;

    put Solution1 , ';', R('r1')  /;

)
```

Code analysis

Robust Method

- PPP code was modified
- New Constrain was added
- value of k i.e. reliability level (d in the code) was iterated in loop
- values for epsilon and delta was manually set
- Max profit was calculated
- File I/O was used in gams to save the data into .txt file
- Data was taken into exel and then to Matlab for plots and analysis

```
EqnRobust(k)..
    sum(j, rm(k, j) * X(j)) =l=
    R(k) - epsilon*(1 - 2*d)*R(k) + delta*max(1,R(k));

epsilon = 0.5;
delta = 0.3;

MODEL petrochemical /all/;

File outputTextFile / 'output.txt' /;

put outputTextFile;

put "Profit     K"  /;

outputTextFile.ap=1;

loop(o,
    d=rl(o);
    SOLVE petrochemical USING MIP MAXIMIZING OBJ;
    Solution1  = OBJ.l;
    put Solution1 , ';', d  /;
)
```

Conclusion

This project contributes to the field of optimization under uncertainty, focusing on challenges in production planning by drawing from existing literature on scheduling problems. The study explores two analytical approaches: a naive method involving multiple iterations and a robust optimization approach. The naive method offers valuable insights into how uncertainty affects profits, demonstrating variations across different scenarios. In contrast, the robust optimization approach introduces a systematic and computationally efficient method that ensures feasibility under specific levels of uncertainty.

Through the mathematical formulation of a Production Planning Problem (PPP), this research highlights the complexities of decision-making when dealing with incomplete information. While the naive method, based on multiple scenario runs, reveals the variability of profits under different conditions, it lacks the systematic control over robustness that the robust optimization approach provides.

The robust optimization approach, grounded in mathematical rigor, presents a trade-off between optimality and robustness. Decision-makers can adjust the level of conservatism to obtain solutions that remain feasible even under uncertain conditions. This approach is computationally efficient, minimizing the need for repetitive calculations and offering a reliable decision-making framework. However, modeling under this approach and probabilistic bounding is notably rigorous and time-consuming.