ANALYZING EFFECT OF UNCERTAINTY IN OPTIMIZATION PROBLEMS SUCH AS SCHEDULING AND PRODUCTION PLANNINGPROBLEM AND WAYS TO HANDLE THEM EFFECTIVELY

1.0 ABSTRACT

This project delves into the critical realm of optimization problems under uncertainty, with a specific focus on the challenges presented in scheduling and production planning. The study addresses the inherent complexity of decision-making when faced with incomplete knowledge offuture outcomes, emphasizing the ubiquity of uncertainty in diverse sectors such as electrical power generation, reservoir operation, inventory management, portfolio selection, and more. By exploring various modeling philosophies, including stochastic programming and fuzzy programming, the research aims to contribute to the evolving methodologies for robust decision- making in the presence of uncertainty. The mathematical formulation of a Production Planning Problem (PPP) serves as the foundation for two distinct analytical approaches: a naive method involving multiple runs and a robust optimization approach. The findings provide insights into the impact of uncertainty on profits and the trade-off between robustness and optimality.

2.0 INTRODUCTION

The most prominent example of optimization under uncertainty difficulties are fluctuations in the commodities market and general stock buying and selling issues. These problems involve makingdecisions without fully understanding the consequences of those decisions. These kinds of issues arise across a wide range of application domains and pose intriguing conceptual and computationalchallenges in the quest for dependable solutions. Most of the optimization literature makes the assumption that all data have known, constant values. However, in practice, uncertainty is common in many production, scheduling, and other problems because of the lack of precise process modelsand the variability of environmental and process data. As a result, the meaning of a decisive optimal solution is lost, and instead, the focus shifts to identifying the most likely optimal solution or making a confident prediction about it. In order to solve this issue, produce dependable/probablefindings that are still achievable in the face of parameter uncertainty, and more realistically simulate real-world scenarios, a new field of study has already begun to take shape. Different approaches, such as stochastic, probabilistic, and fuzzy programming methods, have been used tosolve the scheduling under uncertainty problem. We will discuss these approaches as we go alongand outline our plan to address the problem analytically while also making adjustments to the algorithm, which we researched while drawing ideas from previously suggested strategies.

During our modelling of uncertainty and discussion we'll make certain assumptions justifying theirapplicability in certain use cases validating our uncertainty model. Note the purpose of this project is to depict how introducing uncertainty causes deviation from classic mathematicaltechniques and discuss ways to mitigate or incorporate uncertain parameters within our model making naïve attempts as of now to produce results which remain feasible even under variability of defining parameters with certain level of confidence.

Before Beginning our discussion on we'll like to highlight some areas where need to optimize under uncertainty arises greatly further highlighting importance and need of this project.

2.1 Uncertainty:

Making decisions frequently requires facing uncertainty. Decisions made now will have repercussions that won't become completely apparent for some time. However, as more information becomes available, there might be opportunities for subsequent corrective action or even multiple opportunities for recourse.

Numerous modeling philosophies, such as expectation minimization, minimization of goal deviations, minimization of maximum costs, and optimization over soft constraints, have been applied to optimization under uncertainty. The primary methods for optimizing under uncertainty are as follows: stochastic dynamic programming, fuzzy programming (possibilistic and flexible programming), and stochastic programming (recourse models, robust stochastic programming, andprobabilistic models). We can also discuss applications, the state-of-the-art in computations, and significant algorithmic advancements made by the process systems engineering community.

Since our solution is based on modeling parameter uncertainty using the concept of random variables, we primarily focus on stochastic modeling.

2.2 Modelling of uncertainty:

A problem's uncertainties must be represented so that their implications on current decision- making can be appropriately considered. This is an intriguing and difficult topic, but our understanding of probability can help.

Stochastic modeling: The theory of probability can be used to model the uncertain components of a problem as random variables. Such elements need to have a "known" probability distribution inorder to serve this purpose.

Degrees of knowledge: As with weather variables, such a distribution may be derived from statistical data, or it may simply be "subjective" probabilities based on educated guesswork, as in the case of interest rates or election prospects. The mathematical approach is the same in either case.

Stochastic formulations incorporate uncertainty by taking advantage of the fact that probability distributions governing the data are known or can be estimated. The goal then is to find an optimal solution that is feasible for all, or almost all, of the instances of the uncertain parameters while maximizing the expectation of some function of the problem variables and the random variables.

3. MATHEMATICAL FORMULATION / SOLUTION STRATEGY:

We begin with the use of existing Mathematical formulation of PPP as discussed in class and trytwo approaches and visualize their result.

3.1 MATHEMETICAL MODELING OF PPP:

$$PROFIT = \sum_j SP(j) * X(j) - \sum_j (PC_l(j) * L(j) + PC_m(j) * M(j) + PC_h(j) * H(j))$$

$$X(j) = cl(j) * L(j) + cm(j) * M(j) + ch(j) * H(j)$$

$$H(j) = 1 - Y(j)$$

$$L(j) \leq Y(j)$$

$$L(j) + M(j) + H(j) = Z(j)$$

$$X(j) \leq U * z(j)$$

$$\sum_j rm(k, j) * X(j) \leq R(k)$$

$$\sum_j (IC_l(j) * L(j) + IC_m(j) * M(j) + IC_h(j) * H(j)) \leq B$$

3.1.1 Naïve approach to solve uncertainty with multiple runs:

In this approach we,

- Execute the optimization model with different scenarios of raw material availability.
- Observe variations in profits to understand the impact of uncertainty.
- Analyze the results to identify patterns or trends in the relationship between raw materialavailability and profit.

We model the random perturbation as:

$$\tilde{a}_{lm} = (1 + \epsilon\xi_{lm})a_{lm}$$
$$\tilde{b}_{lk} = (1 + \epsilon\xi_{lk})b_{lk}$$
$$\tilde{p}_l = (1 + \epsilon\xi_l)p_l$$

where ξlm, ξ lk and ξ l are independent random variables and $\epsilon > 0$ is a given (relative) uncertainty level.

We see the variation of profit w.r.t. to variation in availability of raw materials, trying to identifywhether such variation will be linear or not does it actually even impact the problem and if it does than to under what extent. Irrespective of type, it will provide sufficient analytic data to visualize the impact of uncertainty in raw materials giving analytical intuition to choose the result from the most feasible region.

$$\tilde{R}_{lk} = (1 + \epsilon \cdot \xi) \cdot R_{lk}$$

To achieve Normal distribution in Gams we used,

$$R(r1) = ROrignal(r1) * (1 + 0.5 \cdot Normal(0,0.2))$$

To achieve Uniform distribution in Gams we used,

$$R(r1) = ROrignal(r1) * (1 + 0.5 \cdot Uniform(-1,1))$$

3.1.2 <u>Robust optimization approach and formulation:</u>

Robust optimization is a field of mathematical optimization theory that deals with optimizationproblems in which a certain measure of robustness is sought against uncertainty that can be represented as deterministic variability in the value of the parameters of the problem itself and/orits solution.

The underlying mathematical framework is based on a robust optimization methodology first introduced for linear programming (LP) problems by and extended in Lin et al. (2004) and this work for mixed-integer linear programming problems. The approach produces "robust" solutionsfor uncertainties in both the coefficients and right-hand-side parameters of the linear inequality constraints and can be applied to address the problem of production scheduling with uncertain parameters.

Consider the following generic mixed-integer linear programming problem

$$\begin{aligned} \underset{x,y}{\text{Min/Max}} \quad & c^{\mathrm{T}}x + d^{\mathrm{T}}y \\ \text{s.t.} \quad & Ex + Fy = e \\ & Ax + By \le p \\ & \underline{x} \le x \le \bar{x} \\ & y = 0, 1. \end{aligned}$$

Assume that the uncertainty arises from both the coefficients and the right-hand-side parametersof the inequality constraints, namely, a_{lm}, b_{lm} *and* p_l where l is the index of the uncertain inequality, m is the index of the continuous terms, and k is the index of the binary terms. Thus, we are concerned about the feasibility of the following inequality

$$\sum_m \tilde{a}_{lm} x_m + \sum_k \tilde{b}_{lk} y_k \leq \tilde{p}_l$$

Where a_{lm}, b_{lk} *and* p_l are the nominal values of the uncertain parameters and $\tilde{}a_{lm}$, $\tilde{}b_{lk}$, and $\tilde{}p_l$are the “true” values of the uncertain parameters.

Assume that for inequality constraint l, the true values of the uncertain parameters are obtainedfrom their nominal values by random perturbations

$$\tilde{a}_{lm} = (1 + \epsilon\xi_{lm})a_{lm}$$
$$\tilde{b}_{lk} = (1 + \epsilon\xi_{lk})b_{lk}$$
$$\tilde{p}_l = (1 + \epsilon\xi_l)p_l$$

where ξlm, ξ lk and ξ l are independent random variables and $\epsilon > 0$ is a given (relative) uncertainty level.

In this situation, we call a solution (x, y) robust if it satisfies the following

(i) (x, y) is feasible for the nominal problem, and

(ii) for every inequality l, the probability of violation of the uncertain inequality in Eq. (2) (i.e.,the left-hand-side exceeds the right-hand-side) is at most κ,

The formulation allows for control over the degree of conservatism of the solution in terms of theprobabilistic bounds on constraint violation producing results which remain feasible even under uncertainty.

Based on the above we incorporate the formulation by relaxing inequality of availability of rawmaterials:

$$R_{lk} = (1 + \epsilon \cdot \xi) \cdot R_{lk}$$

$$\sum_j rm(k, j) * X(j) \leq R(k)$$

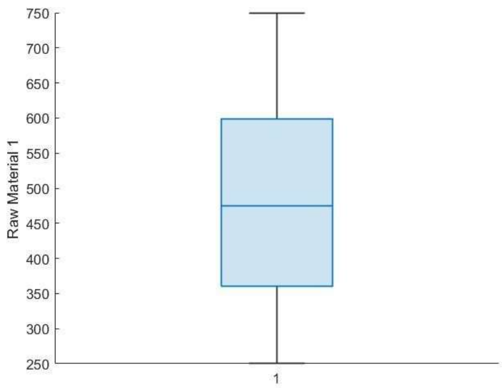
$$\sum_j rm(k, j) * X(j) \leq R(k) - \varepsilon * (1 - 2 * k) * R(k) + \delta * \max(1, R(k))$$

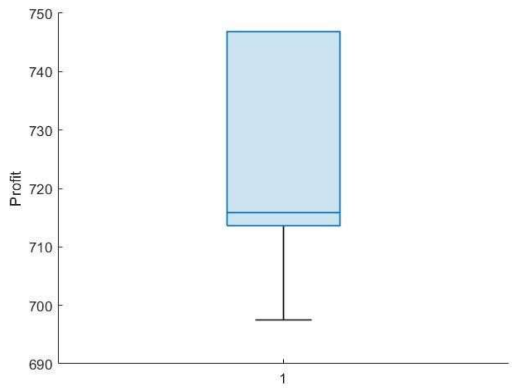
This ensures that the results produced remain feasible under given uncertainties with a certainlevel of tolerance (δ) and reliability (k) resulting in effective and reliable decision making.
While also ensuring less computational load compared to naïve approach where each uncertaintywe would have to reiterate resulting in greater computational loss.

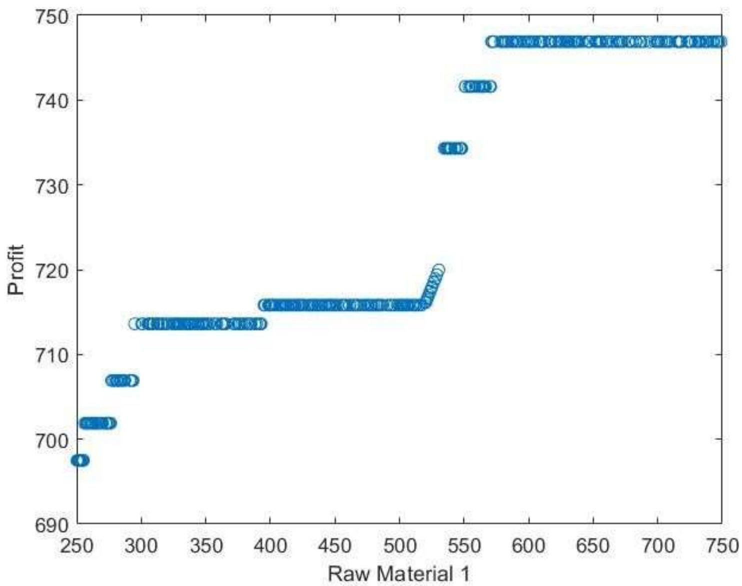
4.0 Results & Discussions:

4.1 Naïve approach

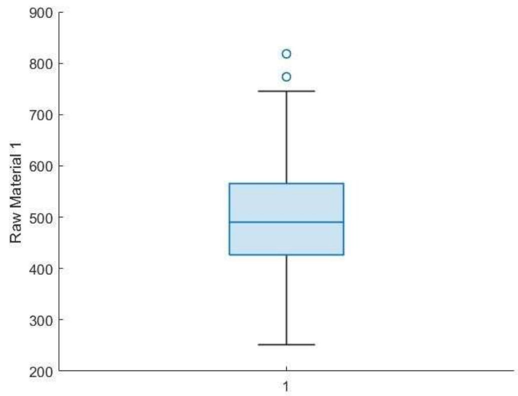
4.1.1. UNDER UNIFORM DISTRIBUTION (-0.5 to 0.5) OF UNCERTAINITY IN RAW MATERIAL

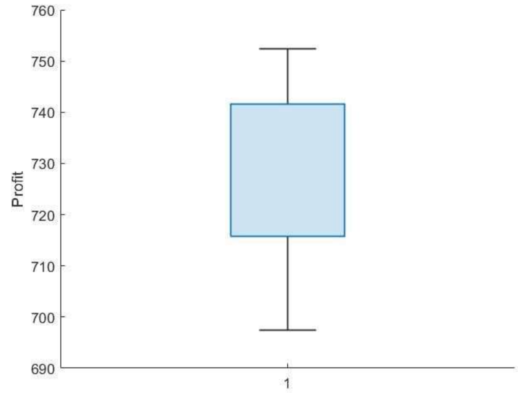


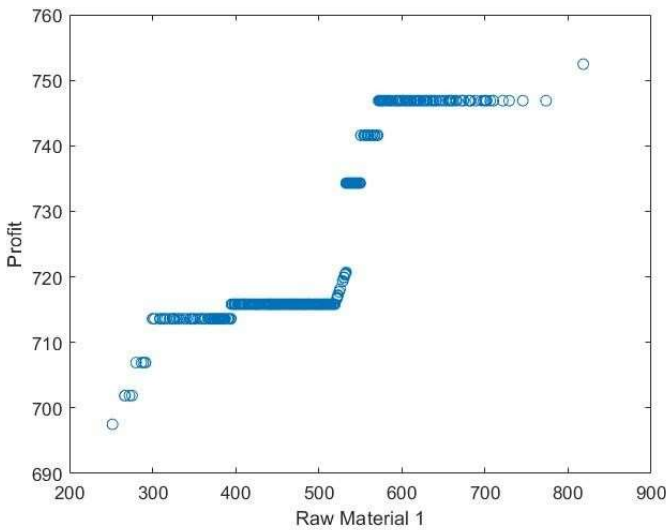




4.1.2. <u>UNDER NORMAL DISTRIBUTION (MEAN 0, STD 0.2) OF UNCERTAINITY</u>







Intuitively, we expect that as our Model has Linear dependency on Raw materials, we expect them to be a linearly increasing graph. But we find it to be constant within certainbands. So, if we find our raw martial supply to vary within one of the bands, we can rest assure that our profit will not change

4.2. <u>Robust optimization</u>

4.2.1 Uncertainty Modeling:

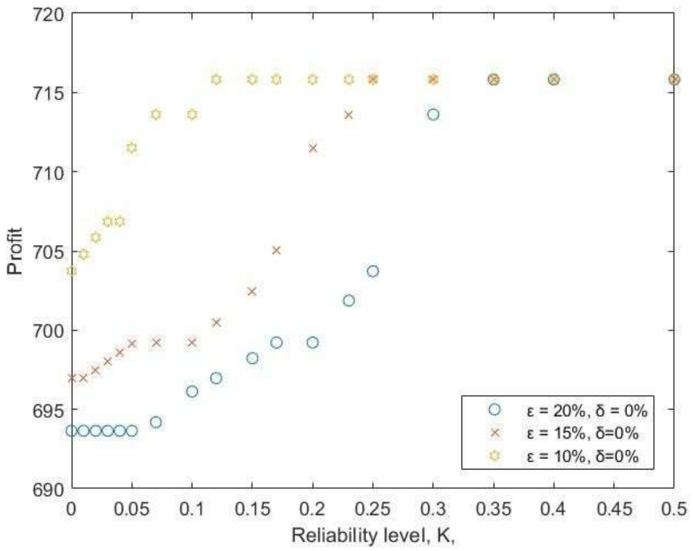
- We define the random perturbations in uncertain parameters using independentrandom variables and assuming the distribution of such uncertainty.
- Specify the uncertainty level (ϵ) to control the degree of conservatism.

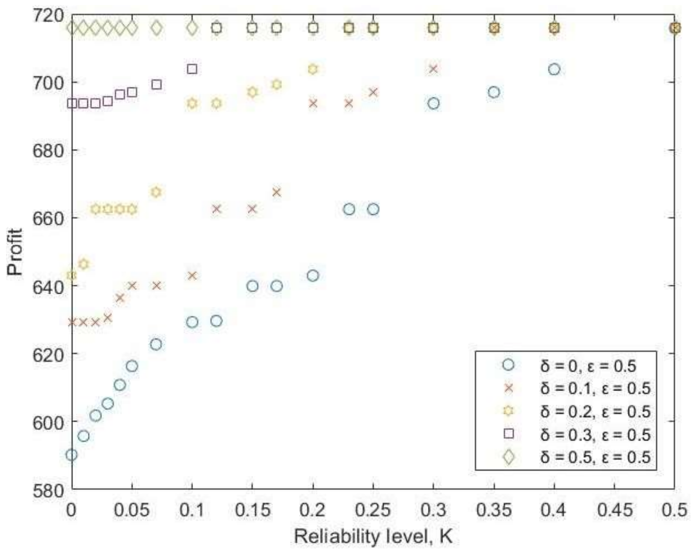
4.2.2 Degree of Conservatism:

The term "conservatism" refers to the cautious approach taken to ensure that solutions remain feasible even in the face of uncertain parameters. It represents a deliberate trade-off between pursuing optimal outcomes and safeguarding against potential risks introduced by the variability of parameters.

- We discuss how adjusting the uncertainty level (ϵ) allows for tuning theconservatism of the solution.
- Highlight the trade-off between robustness and optimality in the face ofuncertainty.

Intuitively, we can expect that as we increase uncertainty in the raw material the expected profitwe can obtain should decrease as provide safe side result which remain feasible with greater probability should be a lower value more over as we allow the deviation from given bounds of certain inequality the resulting value should increase at the expense of violation in certain case.





The above graph does agree with our intuitive understanding, moreover if we look at givenreliability level K, we find that the band variation in profit still holds true. Though Important point to note here is that while naïvely we need to do multiple runs to arrive at aconclusive result and analytical analysis, this approach does produce satisfactory results byincorporating uncertainty bounds inside deterministic MILP formulation saving us from expensive re-computations and analytical procedure giving us satisfactory results in single run. The result obtained can be considered as a worst-case result under given ε, δ *and* k **.**

5.0 CONCLUSION AND FUTURE WORKS:

This project contributes to the field of optimization under uncertainty, specifically addressing challenges in production planning production, taking inspiration from available literature on scheduling problems. The study explores two distinct analytical approaches: a naive method involving multiple runs and a robust optimization approach. The naive approach provides valuable insights into the impact of uncertainty on profits, showcasing variations in different scenarios. On the other hand, the robust optimization approach introduces a systematic and computationally efficient methodology that ensures feasibility under specified uncertainty levels.

Through the mathematical formulation of a Production Planning Problem (PPP), this research illuminates the complexities of decision-making in the presence of incomplete knowledge. Thenaive approach, relying on multiple runs with varied scenarios, sheds light on the variability ofprofits under different conditions. However, it lacks the systematic control over robustness thatthe robust optimization approach offers.

The robust optimization approach, with its foundation in mathematical rigor, introduces a trade-off between optimality and robustness. Decision-makers can fine-tune the degree of conservatism, providing solutions that remain feasible even under uncertain conditions. This approach is computationally efficient, reducing the need for repetitive computations and offeringa reliable framework for decision-making. Though modelling under this approach and probabilistic bounding is quite rigorous and time consuming.

5.1 Naive Approach vs. Robust Optimization Approach: A ComparativeAnalysis

5.1.1 Basic Principle:

- *Naive Approach:* The naive approach involves executing the optimization model with different scenarios of raw material availability without fundamentally altering the originalmodel. It relies on multiple runs to observe variations in profits and analyze the impact ofuncertainty.
- *Robust Optimization Approach:* The robust optimization approach introduces a systematic methodology to address uncertainty. It modifies the optimization model to produce solutions that remain feasible under uncertainty, introducing probabilistic boundson constraint violation.

5.1.2 Handling Uncertainty:

- *Naive Approach:* Handles uncertainty by running the optimization model multiple timeswith different scenarios of raw material availability. Provides insights into the variabilityof profits but may lack a systematic approach to ensure feasibility under uncertainty.
- *Robust Optimization Approach:* Addresses uncertainty through a rigorous mathematical formulation that considers both coefficients and right-hand-side parameters as uncertain.Introduces a level of conservatism to ensure feasibility under a specified level of uncertainty.

5.1.3 Computational Efficiency:

- *Naive Approach:* Involves running the optimization model multiple times, which can becomputationally expensive, especially for complex models with numerous parameters.
- *Robust Optimization Approach:* Typically results in a more computationally efficient solution as it incorporates uncertainty bounds within a deterministic framework. Reducesthe need for repetitive computations.

5.1.4 Trade-off Between Optimality and Robustness:

- *Naive Approach:* May obtain more optimal results for specific scenarios but lacks a systematic control over robustness. The obtained solutions may not be feasible undercertain conditions of uncertainty.
- *Robust Optimization Approach:* Provides a trade-off between optimality and robustness.Decision-makers can control the degree of conservatism by adjusting parameters, allowing them to choose solutions that are both optimal and resilient to uncertainty.

5.1.5 Application to Decision-Making:

- *Naive Approach:* Offers insights into the impact of uncertainty on profits but may notprovide actionable solutions that guarantee feasibility under all conditions.
- *Robust Optimization Approach:* Results in solutions that are guaranteed to be feasible under specified uncertainty levels, offering decision-makers more reliable strategies forproduction planning and scheduling.

5.1.6 Flexibility in Handling Uncertainty Models:

- *Naive Approach:* Relies on simplistic models of uncertainty, such as normal or uniformdistributions, and may not capture the full complexity of real-world uncertainty.
- *Robust Optimization Approach:* Provides a more flexible framework to model uncertainty, allowing for the incorporation of various distribution types and offering amore nuanced understanding of uncertainty patterns.

5.2 Future Works:

- **Refinement of Robust Optimization Model:** Further refinement and fine-tuning of therobust optimization model can be explored. Adjustments in the formulation parameters and exploration of additional constraints may enhance the model's performance in specific scenarios.
- **Integration of Real-Time Data:** Incorporating real-time data and adaptive strategies toupdate the optimization model dynamically based on the latest information can be explored. This would make the decision-making process more responsive to changing conditions.
- **Comparative Analysis:** A comprehensive comparative analysis between different uncertainty modeling approaches, such as stochastic programming and fuzzy programming, can be conducted. Understanding the strengths and limitations of eachapproach in various contexts would contribute to the development of a more versatiledecision-making framework.
- **Application to Specific Industries:** The research can be extended to focus on specific industries, such as electrical power generation, reservoir operation, or inventory management. Tailoring the optimization models to the unique challenges of each industrywould provide more targeted and actionable insights.
- **Incorporation of Multi-Objective Optimization:** Exploring the integration of multi- objective optimization techniques to balance conflicting goals in the face of uncertainty.This would allow decision-makers to consider not only profit but also other critical factors like risk mitigation and resource utilization.
- **Machine Learning Integration:** Investigating the integration of machine learning techniques to predict and adapt to uncertainty patterns over time. This adaptive approachcould enhance the robustness of the optimization model in dynamic and evolving environments